ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2012-2013**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

Math 4307: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. a) Forward elimination changes $Ax = b$ to a row reduced $Rx = d$, and the complete solution is: 3+7

$$x = \begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix} + c \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

   i. Find the 3-by-3 reduced row echelon matrix **R** and the right-hand side **d.**

   ii. If the process of elimination subtracted 3 times row 1 from row 2 and then 5 times row 1 from row 3, what matrix reconnects **R** and **d** to the original **A** and **b**? Use this matrix to find **A** and **b**.

   b) For which values of **c** and **d**, do the following matrices have rank 2? 2+2

$$\text{i. } A = \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix} \qquad \text{ii. } B = \begin{bmatrix} c & d \\ d & c \end{bmatrix}$$

   c) Prove that if **A** has independent columns then $\mathbf{A^T A}$ is invertible. 5

   d) For a linear system $Ax = b$, explain the conditions for solvability. 6

2. a) 15+5

Using Gauss-Jordan Elimination method, calculate the inverse of $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

For your answer, show $AA^{-1} = I$ where matrix-multiplication should be carried out as row-wise operations.

   b) How many operations are required to carry out forward elimination on an augmented matrix $[A|b]$, where **A** is a square matrix of size $m \times m$? 5

3. a) Suppose vector **b** does not lie in the column space of a matrix **A** but its projection **p** does. If **P** is the projection matrix, derive the equation for finding **P** from **A** and **b**. 10

   b) Find the equation of a line that best fits to the points (0, 6), (1, 0), and (2, 0). 15

4. a) Given three independent non-orthogonal vectors **a**, **b**, and **c** as given below, find an orthonormal basis. $a = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}$ 20

   b) Suppose for the square matrix **Q**, $\mathbf{Q^T Q = I}$ is true. What more can you tell about the matrix **Q**? 5