Sc. Engg. HD CSE 3rd Semester

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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION — **WINTER SEMESTER, 2017-2018**

DURATION: 3 Hours — **FULL MARKS: 150**

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8 (eight) questions. Answer any 6 (six) of them.

Figures in the right margin indicate marks.

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1. a) Show that the projection of a vector b on the column space of given matrix A is obtained from $p = A\hat{x}$. Also derive the value of $\hat{x}$. 7

   b) Prove that $A^T A$ is invertible if and only if A has linear independent columns. 5

   c) Suppose a plane is spanned by the linear combination of two vectors $(1, 2, -1)^T$ and $(1,0, 1)^T$. Which vector in this plane is closest to $b = (2, 1, 1)^T$? 8

   d) Suppose A is the 4 by 4 identity matrix with its last column removed: A is 4 by 3. Project $b=(1, 2, 3, 4)^T$ onto the column space of A. What shape is projection matrix P and what is P? 5

2. a) Find the equation $y = mx + c$ of the least-squares line that best fits the data points (2, 1), (5, 2), (7, 3), and (8, 3). What is the magnitude of the error vector? 20

   b) Given an $m \times n$ matrix A with linearly independent columns, let A=QR be a QR factorization of A where Q contains orthonormal columns and R is an upper-triangular matrix. Then, prove that for each b in $\mathbf{R}^m$ space, the equation Ax=b has a unique least-squares solution, given by: $\hat{x} = R^{-1}Q^T b$. 5

3. a) If A=QR then $A^TA=R^TR$. *Gram-Schmidt* method on A corresponds to elimination on A^TA. Show that the pivots for A^TA must be the squares of diagonal entries of R. Find Q and R by Gram-Schmidt for this matrix A: 20

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 2 & 4 \end{bmatrix}$$

   b) Determine whether the following statements are true or false. Give a reason if true or a counter example if false:

      i. The determinant of $I+A$ is $1+\det(A)$. 1

      ii. The determinant of ABC is |A||B||C|. 1

      iii. The determinant of $4A$ is $4\det(A)$. 1

      iv. The determinant of $AB-BA$ is zero. All matrices here are considered square. 2

$$\text{Try with } A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

4. a) The n by n determinant A_n has 1's above and below the main diagonal:

$$A_1 = |0| \qquad A_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \qquad A_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \qquad A_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

      i. What are these determinants A_2, A_3, A_4? 3×5

      ii. By cofactors find the relation between A_n and A_{n-1} and A_{n-2}. Find A_{10}. 5

b) The comers of a triangle are (2, 1) and (3, 4) and (0, 5). Add a corner at (-1, 0) to make a lopsided region (four sides). Find the area of the new region. 5

5. a) Find the eigenvalues for the following matrix A. Calculate the eigenvectors for only nonzero eigenvalues. 20

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

b) Show that if $3I$ is added with a matrix A, then its eigenvalues change but eigenvectors do not. 5

6. a) Factorize the following matrix A in to $S\Lambda S^{-1}$. 7

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

b) The Lucas numbers are like the Fibonacci numbers except they start with $L_1=1$ and $L_2 = 3$. Following the rule $L_{K+2} = L_{K+1} + L_K$ the next Lucas numbers are 4, 7, 11, 18, and so on. Find the Lucas number L_{100}. 18

7. a) List the properties of Markov matrix. Show that the square of a Markov matrix is also a Markov matrix. 2+3

b) Suppose in a city every year 2% of young people become old and 3% of old people become dead. With no births and sudden death of young people, find the steady state for 20

$$\begin{bmatrix} \text{young} \\ \text{old} \\ \text{dead} \end{bmatrix}_{k+1} = [3\times3 \; Markov\ Matrix] \begin{bmatrix} \text{young} \\ \text{old} \\ \text{dead} \end{bmatrix}_k$$

$$\begin{bmatrix} .98 & 0 & \\ .2 & .97 & \\ 0 & .3 & 1 \end{bmatrix}$$

Show all necessary calculations.

8. a) Compute the product of the following two matrices using the concepts of column picture of a linear system. 10

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

b) Find the inverse of matrix A by the Gauss-Jordan method: 10

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

c) For which values of c and d does the following matrix have a rank of 2? Explain your answer. 5

$$A = \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix}$$