B.Sc. Engg. / HD CSE 3rd Semester 13 March 2018 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION WINTER SEMESTER, 2017-2018

DURATION: 1 Hour 30 Minutes FULL MARKS: 75

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 4 (four) questions. Answer any 3 (three) of them.

Figures in the right margin indicate marks.

✓ 1. a) Determine if the following linear system is consistent (has solution) or not. Calculate the determinant of the coefficient matrix A from its row-echelon form. 10+2

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$5x_1 - 8x_2 + 7x_3 = 1$$

b) Prove that – "If a linear system is consistent, then the solution is unique if and only if every column in the coefficient matrix is a pivot column; otherwise there are infinitely many solutions." 3

c) Find the resultant matrix $C=A\times B$ from matrix multiplication, where 10

$$A=\begin{bmatrix} 2 & 5 & 4 \\ -3 & 0 & -3 \\ 7 & -6 & 8 \end{bmatrix} \qquad B=\begin{bmatrix} 1 & 4 & -6 \\ 0 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

All calculations are to be shown from the concepts of column-picture.

✓✓ 2. a) Find the inverse of matrix A using Gauss-Jordan method: 10

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

Show all elimination matrices for performing row eliminations.

b) Factorize the matrix A in Question 2.(a) into its LDU form. 5

c) Find the nullspace of the following matrix A: 10

$$A=\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

Which are the free columns and the pivot columns?

3. a) Find the complete solution to 15

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

b) Find the dimension and basis for the rowspace, columnspace, nullspace and left-nullspace, respectively, for the coefficient matrix A as given in Question 3.(a) 10

1+3+3

4. a) Define a vector subspace. Prove the followings with examples:
   i. The union of two subspaces is not a subspace.
   ii. The intersection of two subspaces is a subspace 9+

b) Check that the solutions to $Ax=0$ are perpendicular to the rows: 1+3

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = E^{-1}R$$

How many independent solutions to $A^T y=0$? Why is the y^T the last row of E?

c) In a $\mathbf{R}^5$ vector-space, suppose two sub-spaces, each being a plane, are orthogonal to each other. Can one of them represent a row-space and the other represents a null-space? Explain your answer. 5