ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
<u>Department of Computer Science and Engineering (CSE)</u>

MID SEMESTER EXAMINATION   **WINTER SEMESTER, 2021-2022**
DURATION: 1.5 Hours   **FULL MARKS: 75**

Math 4341: Linear Algebra
Programmable calculators are not allowed. You need to answer all the 3 questions.
Figures in the right margin indicate marks and CO-PO mapping.

---

1. a) Determine if the matrices A, B and C are invertible. If the matrix is invertible, find its inverse   6+6+6
        by Gauss-Jordan elimination. For matrix C, find the inverse in terms of a.   (CO1)
                                                                                                              (PO1)

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad C = \begin{bmatrix} a & 1 & 1 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}$$

   b) For what values of a is D invertible? How does your formula for D^{-1} breakdown for values   2
       of a for which D is not invertible?   (CO1)
                                                                                                          (PO1)

   c)   5

$$A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$$

(CO1)
(PO1)

   For which three numbers a will elimination fail to give three pivots for this matrix? That is,
   for which values of a is this matrix singular?

2. a) Assume that S and T are lines through (0, 0, 0, 0) in R^4 (4-dimensional space). So, S contains   2+3+3
        all multiples of a nonzero vector v and T contains all multiples of a nonzero vector w.   (CO3)
                                                                                                                  (PO1)

        i.   When is the union $\boldsymbol{S} \cup \boldsymbol{T}$ of the two lines also a subspace?
        ii.   If $\boldsymbol{S} \cup \boldsymbol{T}$ is not a subspace, describe the smallest possible subspace that contains both
              lines S and T.
        iii. If S and T are ANY subspaces of R^4, not necessarily lines, how would you construct
              the smallest subspace that contains both S and T?

   b) Find the complete solution to (with all necessary steps):   10
                                                                                                          (CO2)
$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

(PO1)

   c) Does there exist a matrix B whose column space is spanned by (1, 2, 3) and (1, 0, 1) and   7
       whose nullspace is spanned by (1, 2, 3, 6)? If so, construct B. If not, explain why not.   (CO3)
                                                                                                          (PO3)

3. a) Forward elimination changes $Ax = b$ to a row-reduced form $Rx = d$, and the complete solution for $Ax = b$ is:

10+10
(CO2)
(PO1)

$$x = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

 i.   What is the 3 by 3 reduced row echelon matrix R and what is d?
 ii. If the process of elimination subtracted 3 times row 1 from row 2 and then 5 times row 1 from row 3, what matrix connects R and d to the original A and b? Use this matrix to find A and b. Remember, no upward elimination was involved in the process.

b) Suppose A and B are 3 by 3 matrices.

2+3
(CO3)
(PO1)

 i.   If a vector v is in the column space of AB, why is v also in the column space of A? [Hint: b is in the columnspace of A means there is some x for which $Ax=b$ is true.]
 ii. Give an example of A and B so that $C(AB)$ is SMALLER than $C(A)$.