

Alloy Language & Analysis

Excerpted (mostly), adapted (a bit) from:

SAIL Tutorial at

[http://alloy.mit.edu/alloy/tutorials/day-
course/](http://alloy.mit.edu/alloy/tutorials/day-course/)

by Greg Dennis and Rob Seater

Alloy = logic + language + analysis

- logic
 - first order logic + relational calculus
- language
 - syntax for structuring specifications in the logic
- analysis
 - bounded exhaustive search for counterexample to a claimed property using SAT

alloy language & analysis

- language = syntax for structuring specifications in logic
 - shorthands, puns, sugar
- analysis = tool for finding solutions to logical formulas
 - searches for and visualizes counterexamples



“I'm My Own Grandpa” Song

- popular radio skit originally written in the 1930's
- expanded into hit song by “Lonzo and Oscar” in 1948



“I’m My Own Grandpa” in Alloy

```
module grandpa

abstract sig Person {
  father: lone Man,
  mother: lone Woman
}

sig Man extends Person {
  wife: lone Woman
}

sig Woman extends Person {
  husband: lone Man
}

fact {
  no p: Person |
    p in p.^(mother + father)
  wife = ~husband
  no wife &
    *(mother+father).mother
  no husband &
    *(mother+father).father
}
```

```
assert noSelfFather {
  no m: Man | m = m.father
}

check noSelfFather

fun grandpas [p: Person] : set Person {
  let parent = mother + father +
    father.wife +
    mother.husband |
    p.parent.parent & Man
}

pred ownGrandpa[p: Person] {
  p in grandpas[p]
}

run ownGrandpa for 4 Person
```

language: module header

```
module grandpa
```

- first non-comment of an Alloy model

language: signatures

sig A {}
set of atoms A

sig A {}
sig B {}
disjoint sets A and B (no A & B)

sig A, B {}
same as above

sig B **extends** A {}
set B is a subset of A (B in A)

sig B **extends** A {}
sig C **extends** A {}
B and C are disjoint subsets of A
(B in A && C in A && no B & C)

sig B, C **extends** A {}
same as above

abstract sig A {}
sig B **extends** A {}
sig C **extends** A {}
A is partitioned by disjoint subsets B and C
(no B & C && A = (B + C))

sig B **in** A {}
B is a subset of A – not necessarily
disjoint from any other set

sig C **in** A + B {}
C is a subset of the union of A and B

one sig A {}
lone sig B {}
some sig C {}
A is a singleton set
B is a singleton or empty
C is a non-empty set

grandpa: signatures

```
abstract sig Person {  
    . . .  
}  
  
sig Man extends Person {  
    . . .  
}  
  
sig Woman extends Person {  
    . . .  
}
```

- all men and women are persons
- no person is both a man and a woman
- all persons are either men or women

language: fields

sig A {f: e}

• *f is a binary relation with domain A and range given by expression e*

• *f is constrained to be a function*
(f: A -> one e) or (all a: A | a.f: e)

```
sig A {
  f1: one e1,
  f2: lone e2,
  f3: some e3,
  f4: set e4
}
```

(all a: A | a.fn : m e)

sig A {f, g: m e}

two fields with same constraints

sig A {f: e1 m -> n e2}
(f: A -> (e1 m -> n e2)) or
(all a: A | a.f: e1 m -> n e2)

```
sig Book {
  names: set Name,
  addrs: names -> Addr
}
```

dependent fields

(all b: Book | b.addrs: b.names -> Addr)

grandpa: fields

```
abstract sig Person {  
  father: lone Man,  
  mother: lone Woman  
}  
  
sig Man extends Person {  
  wife: lone Woman  
}  
  
sig Woman extends Person {  
  husband: lone Man  
}
```

- fathers are men and everyone has at most one
- mothers are women and everyone has at most one
- wives are women and every man has at most one
- husbands are men and every woman has at most one

language: facts

```
fact { F }  
fact f { F }  
sig S { ... } { F }
```

*facts introduce constraints that
are assumed to always hold*

```
sig Host {}  
sig Link {from, to: Host}
```

```
fact {all x: Link | x.from != x.to}  
no links from a host to itself
```

```
fact noSelfLinks {all x: Link | x.from != x.to}  
same as above
```

```
sig Link {from, to: Host} {from != to}  
same as above, with implicit 'this.'
```

grandpa: fact

```
fact Biology {  
  no p: Person | p in p.^(mother + father)  
}
```

```
fact Terminology {  wife = ~husband  }
```

```
fact SocialConvention {  
  no wife & *(mother+father).mother  
  no husband & *(mother + father).father  
}
```

- no person is his or her own ancestor
- a man's [woman's] wife [husband] has that man [woman] as a husband [wife]
- a man [woman] does not marry one of his [her] ancestors who is also a mother [father]

language: functions

```
fun f[x1: e1, ..., xn: en] : e { E }
```

- *a function is a named expression with declaration parameters and a declaration expression as a result*
- *invoked by providing an expression for each parameter*

```
sig Name, Addr {}  
sig Book {  
  addr: Name -> Addr  
}
```

```
fun lookup[b: Book, n: Name] : set Addr {  
  b.addr[n]  
}
```

```
fact everyNameMapped {  
  all b: Book, n: Name | some lookup[b, n]  
}
```

language: predicates

pred $p[x_1: e_1, \dots, x_n: e_n] \{ F \}$

named formula with declaration parameters

sig Name, Addr {}

sig Book {

addr: Name -> Addr

}

pred contains[b: Book, n: Name, d: Addr] {

n->d **in** b.addr

}

fact everyNameMapped {

all b: Book, n: Name |

some d: Addr | contains[b, n, a]

}

grandpa: function and predicate

```
fun grandpas[p: Person] : set Person {  
    let parent = mother + father +  
                father.wife + mother.husband |  
    p.parent.parent & Man  
}  
  
pred ownGrandpa[p: Person] {  
    p in grandpas[p]  
}
```

- a one's grandpas are the male parents of one's own parents, where one's parents are one's mother, one's father and one's father's wife and one's mother's husband, if any

language: assertions

assert a { F }

constraint intended to follow from facts of the model



```

sig Node {
  children: set Node
}

one sig Root extends Node {}

fact {
  Node in Root.*children
}

// invalid assertion:
assert someParent {
  all n: Node | some children.n
}

// valid assertion:
assert someParent {
  all n: Node - Root | some children.n
}

```


language: check command

assert a { F }
check a scope

*instructs analyzer to search for
 counterexample to assertion within scope*

*if model has facts M
 finds solution to M && !F*

check a
top-level sigs bound by 3

check a **for** default
top-level sigs bound by default

check a **for** default **but** list
default overridden by bounds in list

check a **for** list
*sigs bound in list,
 invalid if any top level sig unbound*

```
abstract sig Person {}
sig Man extends Person {}
sig Woman extends Person {}
sig Grandpa extends Man {}
```

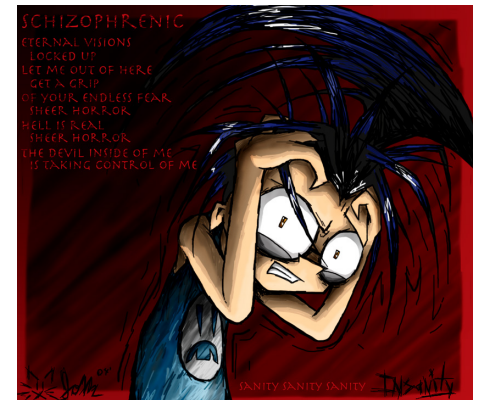
```
check a
check a for 4
check a for 4 but 3 Woman
check a for 4 but 3 Man, 5 Woman
check a for 4 Person
check a for 4 Person, 3 Woman
check a for 3 Man, 4 Woman
check a for 3 Man, 4 Woman, 2 Grandpa
```

```
// invalid:
check a for 3 Man
check a for 5 Woman, 2 Grandpa
```

grandpa: assertion check

```
fact Biology {  
  no p: Person | p in p.^(mother + father)  
}  
  
assert noSelfFather {  
  no m: Man | m = m.father  
}  
  
check noSelfFather
```

- sanity check
- command instructs analyzer to search for counterexample to *noSelfFather* within a scope of at most 3 *Persons*
- *noSelfFather* assertion follows from fact



language: run command

pred $p[x: X, y: Y, \dots] \{ F \}$
run $p \text{ scope}$

*instructs analyzer to search for
instance of predicate within scope*

*if model has facts M , finds solution to
 $M \ \&\& \ (\text{some } x: X, y: Y, \dots \mid F)$*



fun $f[x: X, y: Y, \dots] : R \{ E \}$
run $f \text{ scope}$

*instructs analyzer to search for
instance of function within scope*

*if model has facts M , finds solution to
 $M \ \&\& \ (\text{some } x: X, y: Y, \dots, \text{result}: R \mid \text{result} = E)$*

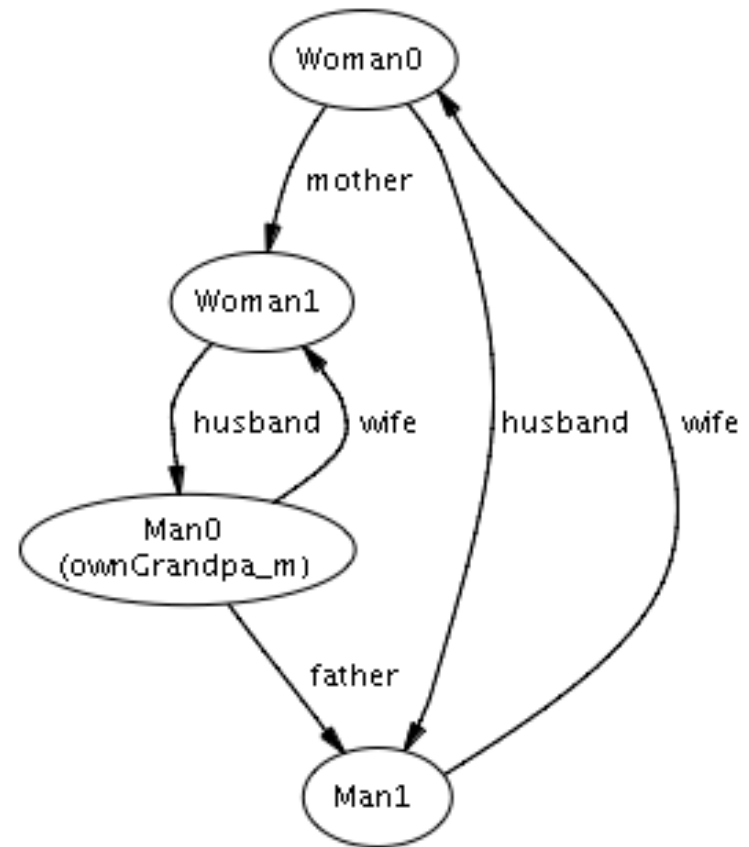
grandpa: predicate simulation

```
fun grandpas [p: Person] : set Person {  
  let parent = mother + father +  
              father.wife +  
              mother.husband |  
  p.parent.parent & Man  
}  
  
pred ownGrandpa[p: Person] {  
  p in grandpas[p]  
}  
  
run ownGrandpa for 4 Person
```

- command instructs analyzer to search for configuration with at most 4 people in which a man is his own grandfather

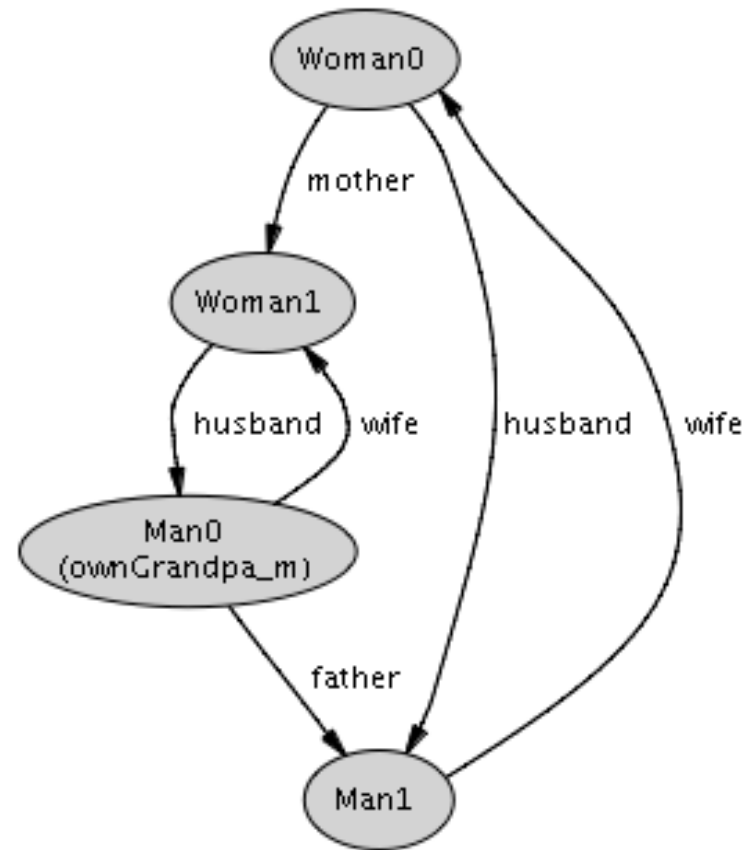
introduction to visualization

- Download *grandpa.als* from the tutorial website
- Click “Execute”
- Click “Show”
- Click “Theme”



superficial

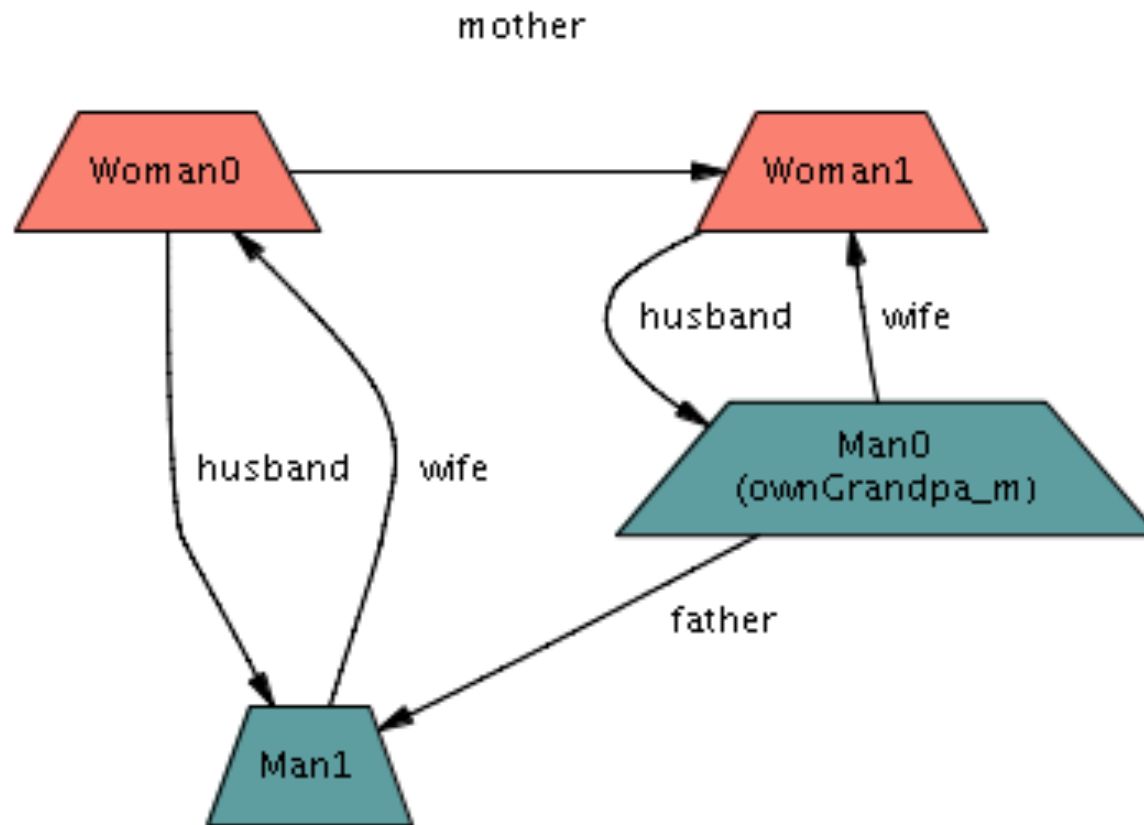
- types and sets
 - default color → gray
 - *Apply*
 - *man* color → blue
 - *woman* color → red
 - *Apply*
- also notice:
 - hide unconnected nodes
 - orientation
 - layout backwards



types & sets

- types: from signatures
 - person shape → trapezoid
 - notice it carries down to man, woman
 - woman: align by type
 - *Apply*

types & sets



types & sets

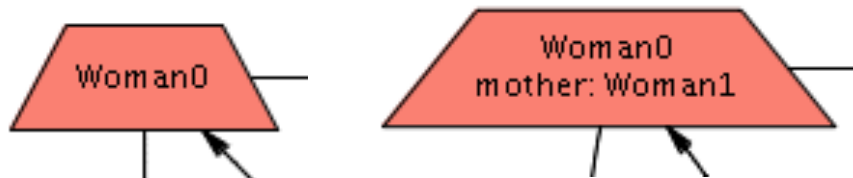
- sets: from existentials, runs, checks
 - somewhat intelligently named
 - \$ownGrandpa_m label → self-grandpa
 - *Apply*



- pitfall: don't show vs. don't show as label
(vs. don't show in customizer...)

relations

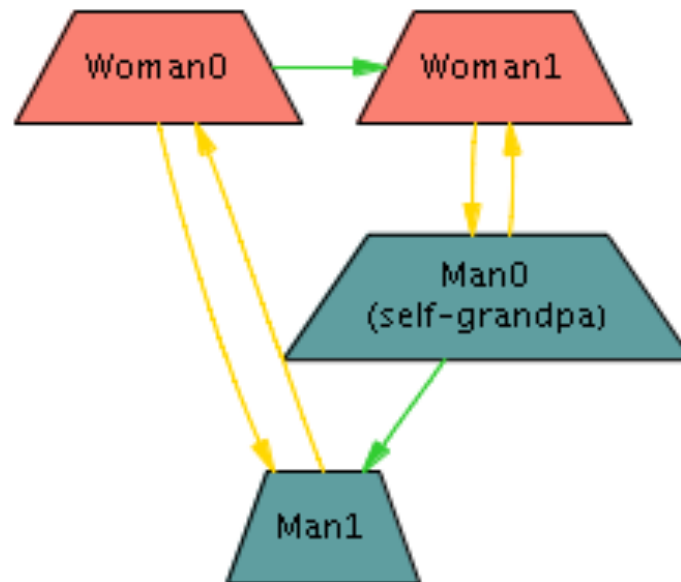
- relations
 - mother: show as attribute → check (still shown as arc)
 - gray = inherited (vs. overridden)
 - *Apply*



relations

- relations
 - mother: show as attribute → uncheck
 - father, mother, husband, wife: label → “ ”
 - father, mother: color → green
 - husband, wife: color → yellow
 - *Apply*

relations



finishing up

- save theme
 - close theme
-
- create your own visualization for the barber exercise!