Lecture 10

Rank one Matrices -

Suggest one rank one matrix -

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix}$$

Dimension and Basis for the columnspace and nullspace?

I can write this matrix as, $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix}$

So, I can write any rank 1 matrix as,

$A = uv^T$, u and v are both column vectors.

Rank 1 matrices are building blocks of all matrices.

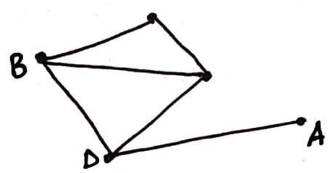
So, if you have a 19 by 12 matrices with rank 5, you need 5 matrices to create that matrix.

Graph:

Graph = {nodes, edges}

Interesting!

Let's think about a graph where a node is a person and an edge between them is a connection between two persons.



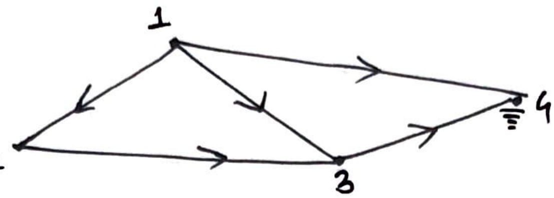
What is the distance between person A and B? → 2. Because A knows D who knows B. D is like mutual friend.

There's a possibility that if you sit beside a completely random person on a flight, there's maximum 6 degrees of separation between you two! Small world.

So the social medias (facebook etc), world wide webs are all saving information as graphs!

Graphs are applications of Linear Algebra.

Let's take a random graph-



$n = 4$

$m = 5$

Let's assume the nodes are potential energy holders and the arrows are electricity flow.

The incidence matrix would be-

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

(-ve) sign for outgoing current.

Loop

This matrix A comes from somewhere, not like our previous examples where we created them out of nowhere!

Are those 4 columns independent?

Let's check!

$Ax = 0$

$$\Rightarrow A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_1 \\ x_4 - x_3 \\ x_4 - x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

~~Give~~ As I said earlier, $X = x_1, x_2, x_3, x_4$ are potentials at the nodes. So what are $x_2 - x_1, x_3 - x_2, \ldots x_4 - x_1$? Potential differences!

Re *

So, what's a solution? If they are all the same.

$$X = c \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

↑ Basis.

$\boxed{\dim N(A) = 1}$.

* So, when all the potentials ~~differences~~ are same, there's no

current flow across the edges.

Now, the rank of the columnspace is 3. If you fix one of the potentials, like node 4, that means you've grounded it. so, the 4th column of A becomes all 0 and all other columns become independent.

Now, what we've worked with? Ohm's Law!

Done.

* Mid Question → 1 mandatory.
* One assignment (coding) after mid.
* Book a class on Tuesday - after mid.