1) a) $(8x^4 + 7x^3 + 2x^2 + 10) + (9x^4 + 6x^2 + x + 7)$

$= (8+9)x^4 + (7+6)x^3 + 2x^2 + x + 17$

$= (17 \bmod 11)x^4 + (13 \bmod 11)x^3 + (2 \bmod 11)x^2$

$+ (1 \bmod 11)x + (17 \bmod 11)$

$= \boxed{6x^4 + 2x^3 + 2x^2 + x + 6}$

b) $(3x^3 + 4x + 3) \times (2x^3 + x^2 + 9x + 7)$

$= 3x^3(2x^3 + x^2 + 9x + 7) + 4x(2x^3 + x^2 + 9x + 7)$

$+ 3(2x^3 + x^2 + 9x + 7)$

$= (6x^6 + 3x^5 + 27x^4 + 21x^3) + (8x^4 + 4x^3 + 36x^2 + 28x)$

$+ (6x^3 + 3x^2 + 27x + 21)$

$= (6x^6 + 3x^5 + 35x^4 + 31x^3 + 39x^2 + 55x + 21) \bmod 11$

$= (6 \bmod 11)x^6 + (3 \bmod 11)x^5 + (35 \bmod 11)x^4$

$+ (31 \bmod 11)x^3 + (39 \bmod 11)x^2 + (55 \bmod 11)x$

$+ (21 \bmod 11)$

$= 6x^6 + 3x^5 + 2x^4 + 9x^3 + 6x^2 + 0x + 10$

$= \boxed{6x^6 + 3x^5 + 2x^4 + 9x^3 + 6x^2 + 10}$

c) $\dfrac{8x^5+6x^4+2x^3+x^2+6x}{2x^3+4x^2+3}$

$$
\begin{array}{r|l}
 & 4x^2+\frac{x}{2} \\
\hline
2x^3+4x^2+3 & 8x^5+6x^4+2x^3+x^2+6x \\
 & 8x^5+5x^4 \qquad\quad +x^2 \\
\hline
 & x^4+2x^3+6x \\
 & x^4+2x^3+\frac{3x}{2} \\
\hline
 & \frac{9x}{2}
\end{array}
$$

$$\frac{8x^5+6x^4+2x^3+x^2+6x}{2x^3+4x^2+3} = 4x^2+\frac{x}{2}+\frac{\frac{9x}{2}}{2x^3+4x^2+3}$$

$$\Rightarrow \left[(2x^3+4x^2+3)\left(4x^2+\frac{x}{2}\right)\right| + \frac{9x}{2}$$

$$\Rightarrow \left(6x^5+16x^4+12x^2+x^4+2x^3+\frac{3x}{2}\right) \bmod 11 + \left(\frac{9}{2} \bmod 11\right)x$$

$$= (8 \bmod 11)x^5 + (16 \bmod 11)x^4 + (12 \bmod 11)x^2 + (1 \bmod 11)x^4$$
$$+ (2 \bmod 11)x^3 + \left(\frac{3}{2} \bmod 11\right)x + \left(\frac{9}{2} \bmod 11\right)x$$

$= 6x^5 + 5x^4 + x^2 + x^4 + 2x^3 + 6x$

$\therefore \left[\left(\frac{3}{2} + \frac{9}{2}\right) \bmod 11\right] x = (6 \bmod 11)\, x$

$= \boxed{6x^5 + 6x^4 + 2x^3 + x^2 + 6x}$

2) a) $(x^2 + x + 1) \times (x + 1)$

$= x^3 + x^2 + x + x^2 + x + 1$

$= x^3 + 2x^2 + 2x + 1$

$= x^3 + 1$ ag $2 \equiv 0 \pmod 2$

$= x^2$ ag $x^3 + x^2 + 1 \equiv 0 \pmod{x^3 + x^2 + 1}$

b) $(x^2 + 1) - (x^2 + x + 1)$

$= -x = x$ ag $-1 \equiv 1 \pmod 2$

$\pmod{x^3 + x^2 + 1}$

c) $(x^2 + 1)\, x = x^3 + x$

$\equiv x^2 + x + 1$ ag $x^3 + x^2 + 1 \equiv 0 \pmod{x^3 + x^2 + 1}$

$= \frac{x^2 + x + 1}{x^2 + 1} = x \pmod{x^3 + x^2 + 1}$