Report

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1 Introduction

Version control systems (VCS) have been steadily becoming an ubiquitous and central tool in programming. Many big projects, with hundreds of collaborators, fundamentally rely on these kind of tools to allow collaborators to interact with one another and to keep a structured log of the units of change. At the heart of most modern VCS is the unix diff utility. This tool computes a line-by-line difference between two files of text, determining the smallest set of insertions or deletions of lines that transform one file into the other. This sequence of transformation produced by `diff3` is called an edit script. Edit scripts are used when two collaborators have modified the same source file in independent ways and the VCS wants to reconcile the two independent changes into a single one. This operation is clearly not always possible: if two collaborators have modified the same line the two resulting edit scripts will "overlap", it is not clear which one of the two modifications should be picked.

In general, some conflicts will always require a manual intervention: when two people change the same thing in two different ways there is no general way of deciding which should be the resulting transformation; however the diff tool makes a big assumption in considering lines to be the basic units in which change is observable.

According to diff, two unrelated changes on the same line would still give rise to a conflict that requires manual intervention. The main idea of this research is to design an alternative to diff which offers a finer grained control over the units of change. By attempting to extend the diff algorithm to operate on an AST that represents the parsed program we are able to focus on smaller units of change, this allows us to produce more accurate patches.

The drawback of this approach is that it moves the problem from the "flat" world of strings to the "layered" world of trees, and the problem of computing a patch between two elements suddenly becomes much more computationally expensive. Approaches similar to this one have already been explored in previous literature [5], [4] and [6]. In these works, the problem of computing the difference between two trees was always reduced to the problem of computing the difference of a flattened representation of the tree. While the flattened representation makes the problem of computing a difference easier, it makes reconstructing a valid tree from the representation more complex, and patches are more likely to generate ill-structured data. The novelty in the work of Miraldo, Dagand and Swierstra [7] lies in the idea of enforcing a structure-preserving, type-directed approach. On one hand structural information is directly encoded in the patches, so that applying a patch will always produce well formed code. On the other hand, the type information encoded in the grammar of the AST is exploited in the creation of a patch, making the process more efficient.

The paper introduces a theoretical and practical framework to define and compute patches between structured data. This framework is generic and makes extensive use of dependent types in order to guarantee structure preserving transformations.

2 Overview

In the remainder of this proposal we will show a full Haskell implementation of the algorithm presented by Miraldo et al. [7], which is presented in Agda in the original paper. The algorithm is implemented generically and described with dependent types; in order to perform experiments I ported the implementation to Haskell. Generic programming and dependent types are both supported by Haskell, albeit in a less streamlined way compared to Agda. Both these tools require some language extensions and machinery which are non trivial in Haskell: for this reason some time is spent introducing the techniques and setting up the building blocks for the implementation.

The following section is an introduction to dependent types in Haskell as they will be crucial in encoding the structure we want to express in our data types and their transformations. Essentially we want to characterise patches by the transformation they operate on the source code (e.g. the patch that adds an extra argument to a function) as such we need dependent types to reflect onto the type system the action of a patch on a certain value. This will assure that any code produced by the algorithm is structurally valid by construction.

Following dependent types we will need to introduce sums of products: these give us a general way to view types and will allow us to define an algorithm that is independent of the representation of the AST for the language we are treating. In this regard there is a fine balance between having a core algorithm which is generic and can be applied to any language, and the use of domain-specific strategies to guide the generation of patches by using knowledge specific to the language in question. One of the goals of this thesis is to investigate this balance: how necessary are domain-specific differencing strategies in order to keep the combinatorial explosion in check? Despite the generality of the algorithm, we have instantiated it for the Clojure programming language [9]; this choice is motivated by the general simplicity of parsing languages that derive from LISP and by the need to select a language that is popular enough to have large active projects, available on Github, that will provide us a good sample of data to test.

In the last sections we will review some of the possible future directions that can be explored with this framework both in terms of gathering concrete evidence about the performance of the algorithm and extending it in order to explore the different design choices that were made along the way.

3 Dependent types in Haskell

With time, Haskell's type system has kept evolving from its humble Hindley-Miller origins and through the use of different language extensions it has gained the ability to express more complex types. In particular, many efforts have gone to add support for dependently typed programming in the latest years.

One major stepping stone in this direction is the DataKinds ([3]) extension which duplicates an ordinary data type, such as

```
data Nat = Z | S Nat
```

at the kind level, this means that from this declaration we automatically get two new types, namely `Z` of kind `Nat` and `S` of kind `Nat -> Nat`.

We can use the `Nat` kind to index generalised algebraic data types (GADTs) in a way that allows us to create an analogue of a dependent type. In the case of `Nat`, we can use it to define a GADT for vectors of a given length.

```
data Vec :: * -> Nat -> * where
  Vn ::                    Vec x Z
  Vc :: x -> Vec x n -> Vec x (S n)
```

Such a vector is either the empty vector, which is indexed by `Z`, or a vector which is built by adding an element of type `x` in front of a vector of `xs` of length `n`, yielding a vector of `xs` of length `S n`.

This allows us to define principled analogues of some functions which operate on lists. The infamous `head` function will crash our program when passed an empty list; equipped with these `Vec`, we can rule this out by construction.

This is how we can define a `head` function on vectors.

```
head :: Vec x (S n) -> x
head (Vc h t) = h
```

Informally, we are saying that the `head` function takes as argument a vector with length strictly greater than 0. In this way, if we try to pass an empty vector to `head` we will get a compile time error instead of the usual runtime one.

Another extension which plays a crucial role in dependent types is `TypeFamilies`: informally it allows us to write functions which operate on types, we will use this to define concatenation between `Vec`s.

The following type family can be seen as a function that takes two types of kind `Nat` and returns another type of kind `Nat` representing the result of adding those two types.

```
type family (m :: Nat) :+ (n :: Nat) :: Nat where
  Z      :+ n = n
  (S m) :+ n = S (m :+ n)
```

Equipped with this type family we can now define concatenation between vectors.

```
vappend :: Vec x n -> Vec x m -> Vec x (n :+ m)
vappend Vn            ys = ys
vappend (x `Vc` xs) ys = x `Vc` (vappend xs ys)
```

One interesting thing to note is that up to this point, we never use the `Nat` part of a vector at runtime. That information is only used at compile time to check that everything "lines up" the way it should be, but it could actually be erased at runtime.

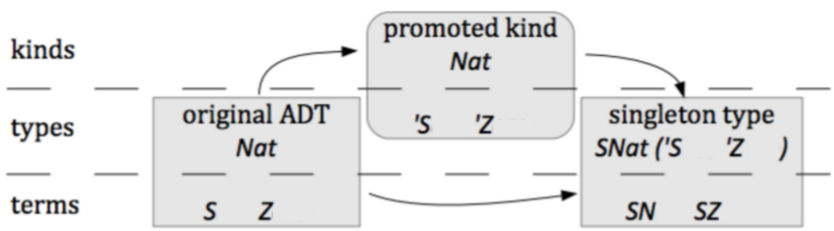
Suppose we want to write a `split` function, this function takes an `n` of kind `Nat`, a vector of length `n :+ m` and splits it into a pair of vectors, with respectively `n` and `m` elements.

The first problem we incur in is that we can not pass something of kind `Nat` to our `split` function, in fact the types `Z` and `S` have no inhabitants, so we can not construct any term of those types. Furthermore, we want to express that this `n` of kind `Nat` that we pass as a first argument, is the same `n` as in the vector length. The idea is to wrap this type into a singleton data type, giving us a dynamic container of the static information.

```
data SNat :: Nat -> * where
  SZ ::            SNat Z
  SN :: SNat n -> SNat (S n)
```

The name singleton comes from the fact that each type level value of kind `Nat` (namely `Z` or `S` applied to another type of kind `Nat`) has a single representative in the `SNat` type.

The following figure gives a good representation of this process: the DataKinds extension promotes things of type `Nat` to things of kind `Nat`. The singleton allows us to take one step back in this ladder, and associates to every thing of kind `Nat` a term from the singleton type `SNat`. The following picture, borrowed from Eisenberg et al. [2], gives a good representation of this process.



We can think of the DataKinds extension as a way of embedding dynamic information into the static fragment of the language. Singletons, on the other hand, are a way to reflect this static information back to the dynamic level, and make runtime decisions based on the types we obtain.

Singletons solve the two problems outlined above: they have kind `*` and contain a `Nat` that we can later refer to in our function definition. We can now define `split` as follows:

```
split :: SNat n -> Vec x (n :+ m) -> (Vec x n, Vec x m)
split SZ     xs          = (Vn, xs)
split (Sn n) (x `Vc` xs) = (x `Vc` ys, zs)
  where
    (ys, zs) = split n xs
```

With these three tricks up our sleeve (data kind promotion, type level functions and singletons) we can emulate some of the features that are present in dependently typed languages such as Agda. These features allow us to emulate explicit dependent quantification. We can actually go even further in Haskell: Lindley et McBride [1] show us how to emulate implicit types via type classes and ultimately how all kinds of quantification, modulo some boilerplate, are possible in Haskell.

4 Sum of Products

The basic idea of the Sum of Products (SOP) approach is to define a generic representation of data types and to define functions that act on this representation. The bulk idea is to view each type as a choice between a constructor and all the arguments that are passed to that constructor. It is useful to think about the two different levels: on the first level we make a choice about which constructor to pick; this corresponds to a sum over all the constructors of the type. On the second level, we are choosing a certain number of arguments to feed to that constructor and this can be viewed as a list, or product, of those arguments. Clearly the products will depend on the choice of constructor, each of which could take a different number of arguments of possibly different types. This motivates the need to represent the product as some sort of heterogeneous list. A constructor can also take no arguments, in which case we can simply use an empty list to represent that, but can also take an argument of the same type it is trying to construct (like the `S` constructor from the previous section). In this case, the recursive argument can itself be encoded as an SOP and the same encoding can be used all the way down to the leaves.

Since every simple data type can be encoded as an SOP, we can write functions that act on this representation and regard them as generics. For every data type we can first convert it to the SOP representation and then pass it to the desired function. Since the SOP representation is isomorphic to the original type, we can eventually reconstruct the desired term after we are done working with the representation. This encoding is presented by Loh et al. [8] and represents one of the numerous different ways to approach generic programming in Haskell. In the remainder of this section we will try to formalise the intuition presented above by building the SOP representation of a simple data type.

An AST consists of a family of datatypes, with a main one which represents the outer structure of the language, and a number of other possibly mutually recursive datatypes appearing as arguments to the constructors of the main datatype. We can define a very simple language which consists of only one datatype, `SExpr` which is isomorphic to binary trees of integers. This choice is just for ease of presentation, the following construction can be applied to a family of mutually recursive datatypes.

```
data SExpr = Operation SExpr SExpr | Value Int
  deriving (Eq, Show)
```

For each type that appears as an argument to a constructor in our family of datatypes, we will construct an atom representing that type. Following the example above, we will define

```
data U = KInt | KSExpr
```

Now, we can group all the constructors appearing in our original datatypes under the `Constr` type in a similar way as we did for the atoms.

```
data Constr = Operation | Value
```

We have now deconstructed our original datatype into two levels: `Constr` corresponding to the level of sums (the constructor) and `U` corresponding to the level of products (the atoms).

Since we had to define two additional datatypes that have no relation between each other we need a way to tie them together on the type level. To achieve this, we define the `ConstrFor` data type, which can be viewed as a proof that a certain constructor builds element of a certain family. In general an AST consists of a family of possibly recursive datatypes, hence we will need the additional information about what family the constructor `Constr` belongs to.

```
data ConstrFor :: U -> Constr -> * where
  OperationProof :: ConstrFor KSExpr Operation
  ValueProof     :: ConstrFor KSExpr Value
```

Finally we must encode one last bit of information: the "shape" of each constructor. To do so, we can use a closed type family which can be viewed as a function on types. This function takes a `Constr` and returns a list of atoms representing the arguments the constructor accepts.

```
type family TypeOf (c :: Constr) :: [U] where
  TypeOf Operation = '[KSExpr, KSExpr]
  TypeOf Value = '[KInt]
```

We will also need to associate each atom with a singleton, which will allow us to relate terms of our language to their type level representation.

```
data Usingl :: U -> * where
  Uint   :: Int -> Usingl KInt
  Usexpr :: SExpr -> Usingl KSExpr
```

Since `TypeOf` returns something of kind `[U]` we will define another GADT named `All`, that maps a type constructor `k -> *` over an argument of kind `[k]` giving us something of kind `*` to quantify over a list of singletons.

The definition for `All` is straightforward:

```
data All (k -> *) :: [k] -> * where
  An :: All p '[]
  Ac :: p x -> All p xs -> All p (x ': xs)
```

With this setup we can finally construct the `View` datatype; this loosely corresponds to a generic view as sum of products of a datatype, and simply deconstructs each term of a type into a constructor and a list of arguments applied to that constructor

```
data View u where
 Tag :: ConstrFor u c -> All UsingI (TypeOf c) -> View u
```

An element of type `View u` represents an element of type `u` deconstructed into its SOP view; the first argument records the choice of the constructor for that datatype and the second is the heterogenous list of arguments that are required to build that constructor, note that we are dependently relating the shape of the product to the actual choice of constructor (the c).

To conclude the section we can attempt to construct the `View` of a simple expression representing an operation between two values.

```
expr = Operation (Value 1) (Value 1)
```

The outmost constructor (`Operation`) gets mapped to the corresponding `OperationProof`; the shape of the second argument ($[KSExpr, KSExpr]$) is completely determined by this choice.

```
viewExpr = Tag OperationProof (Usexpr (Value 1) `Ac` Usexpr (Value 1))
```

Constructing a view can be thought as a way to unwrap the top-most level of the AST; as shown in the example above, the values of the operation are left untouched in the resulting view. Loosely speaking this justifies the intuition that viewing a type as a sum of products gives us information about its shape but does not change any information about its values, enabling us to switch back and forth between representation without losing anything.

5 Type-directed diff

The approach presented by Miraldo and Swierstra [7] takes advantage of the structure encoded in types to define a generic type-directed diff algorithm between typed trees. The inspiration comes from the diff utility present in Unix which is at the heart of the current methodologies employed by VCS to attempt to compute a patch between two different versions of the same file. The limitations of the diff algorithm, as it currently stands, is that it does not employ any structural information between the data it is trying to merge. Files are parsed on a line by line basis and, as the authors show, this is somewhat resilient to vertical changes in the source code. On the other hand it completely breaks down when dealing with horizontal changes, which constitute a heavy chunk of the changes that are usually made to code.

Suppose we have the following innocuous looking function in clojure:

```
(defn head
  [l]
  (first l))
```

Suppose we make two independent modifications, the first of which adds a default parameter to be returned in case the list is empty, and the second one that changes the name of the function from *head* to *fst*.

```
(defn head
  [l, d]
  (if (nil? l)
    (d)
    (first l)))
```

```
(defn fst
  [l]
  (first l))
```

When attempting to reconcile these two changes with the diff algorithm we will run into problems. Despite the two are modifying disjoint pieces of the actual code, the diff algorithm employed by most VCS when trying to compute a merge patch between three objects will output a conflict. The reason is that both changes touch the same line; this is a nuisance in terms of having to manually solve the conflicts. Furthermore it also introduces some level of non-determinism, as the presence of conflicts may depend on things like indentation instead of being a fundamental property of the transformation.

The underlying idea to the approach presented in the article is to employ the generic SoP view presented in the preceding section to define a generic way to view datatypes. Once that is settled, we obtain a view of any well defined program as a structured tree of data, where each object we inspect can be represented as a choice of a certain constructor, among the available ones, and a choice of arguments to that constructor.

In the process of transforming one tree into another we need to keep track of three different things:

- Changes on the constructor level - The internal nodes of the trees

- Changes on the product level - The branches that go from a node to its children
- Changes on the atoms - The elements that are pointed to by the branches

The atoms can be either other internal nodes, in which case we recurse down the tree, or leaves in which case we record the pair of source and target leaf.

We will use the simple binary tree language presented in previous sections as a working example to show the construction of a patch. The transformation we will walk through is the following: given the following AST:

```
p1 = Operation (Value 1) (Operation (Value 1) (Value 1))
```

we want to characterise the patch that transforms it to

```
p2 = Operation (Value 1) (Operation (Value 2))
```

The first structure we will employ is the *spine*, this can be thought of some sort of common skeleton between the two trees which captures the parts that do not change under the transformation.

5.1 Spine

Calculating a spine for two trees loosely corresponds to calculating the longest common prefix between two strings. Recall that the two trees x and y are viewed as SoP, in this sense calculating the spine between x and y corresponds to capturing the common coproduct structure between them. If we think of x_1 and y_1 as two nodes of a tree we will have three cases to consider.

- $x_1 = y_1$
- x_1 and y_1 have the same constructor but not all the subtrees are equal
- x_1 and y_1 have different constructors

This gives rise to the following three different constructors for the Spine GADT, each corresponding to one of the cases described above.

```
data Spine (at :: U -> *)(al :: [U] -> [U] -> *) :: U -> * where
  Scp  :: Spine at al u
  Scns :: ConstrFor u s -> All at (TypeOf s) -> Spine at al u
  Schg :: ConstrFor u s -> ConstrFor u r
       -> al (TypeOf s) (TypeOf r)
       -> Spine at al u
```

If the two elements are the same, the spine is trivially a copy. If the top level constructors match, the spine consists of this information and a way to transform the pairs of constructor fields. Lastly, if two constructors don't match, the spine must record this and also contain a way to transform the list of source fields into the list of destination fields.

The `Scp` constructor corresponds to the first case, in which we need to record no additional information other than the fact that the two elements are equal. Before looking at the other two constructors, let's focus our attention for a moment to the three arguments that a spine takes: the third parameter of the spine (the `U`) represents the underlying type for which we are trying to compute the transformation. The other two, `al` and `at` are respectively a function between products that describes what to do with the different constructor fields and a function between atoms which describes what to do with the paired fields in case we have the same constructor. These functions are needed for the remaining constructors: the `Scns` constructor corresponds to the case where the constructor is left untouched but some of the arguments have changed. For this reason its second argument consists of the predicate `at` applied to the list of arguments which describes how to transform them.

Finally, `Schg` represents a change of constructor on the sum level: the first two arguments record the source and destination constructors, the third argument is the `al` function applied to the constructor fields of the source and destination constructor respectively.

Let's not worry about the `al` and `at` parameters for the time being, these will later be used to close the recursive knot and generate the full patch by interleaving the construction showed here and in the following sections. For now we can simply observe that if we were to calculate a spine between the two programs introduced above we would proceed by constructing their view as presented in 4 obtaining the following

```
p1 = Tag OperationProof (Usexpr (Value 1) ‘Ac‘ Usexpr (Operation (Value 1) (Value 1)))
p2 = Tag OperationProof (Usexpr (Value 1) ‘Ac‘ Usexpr (Value 2))
```

These views are not completely equal, but their first argument is. This represents the outer choice of constructor and we are after all ultimately transforming an `Operation` into another one. The spine produced by these two views will then start with an `Scns` recording the fact that the outer constructor has stayed the same but there are some changes in its arguments.

```
spine = Scns OperationProof _
```

We ignored the second argument up to this point (representing it as an underscore); let's turn our attention to that now: since we know that the constructor is unchanged in the transformation, we also know that both for the source and destination tree, the number and types of its arguments will be the same. For this reason we can simply pair up the corresponding arguments and calculate the diff between every pair. The second argument to `Scns` can be read as: the function `at` applied to a list of pairs of elements of the same type. The type of each pair is specified by `TypeOf s`, which in our working example is equal to $[Usexpr, Usexpr]$. Essentially we have a list of `Usexpr` pairs and are left with the problem of calculating patches between the elements contained in each pair. We can easily see that the first pair of our example will give rise to an `Scp`, the two subtrees are in fact the same and we can simply copy the information along the transformation. The second pair is more interesting though: this is the case where we have a change on the constructor level and the spine we produce will be the following

```
Schg OperationProof ValueProof _
```

However the remaining argument to fill is not as simple as the `Scns` case since `Operation` and `Value` expect completely different arguments. In the case where the constructor had remained the same, we could pair up the arguments and proceed from there; however, when the external constructor has changed, there is no obvious way of pairing up the arguments. Indeed they might be completely in different numbers and types, which motivates the following definition of alignments.

5.2 Alignment

As mentioned in the previous paragraph, the spine takes care of matching the constructors of two trees, beyond that we still need to define a way to proceed with the diff between the products of data stored in the constructors. Recall that this alignment has to work between two heterogeneous lists corresponding to the fields associated with two distinct constructors. The approach presented below is inspired by the existing algorithms based on the edit distance between two strings. The problem of finding an alignment of two lists of constructor fields can be viewed as the problem of finding an edit script between them. An edit script is simply a sequence of operations which describes how to change the source list into the destination. To compute an edit script we simply traverse the lists, from left to right, considering one element from each list. At each step we are presented with three choices:

- We can match the two elements (Amod) of the list and continue recursively aligning the rest
- We can insert the destination element before the current element in the source list (Ains) and recursively compute an alignment between whatever we have in the source list and the tail of the destination.
- We can delete the element from the source list (Adel) and recursively compute the alignment between the rest of the source and the destination.

This approach is similar to the one used to compute edit scripts; there is one major difference though. In the case of strings we can assume deletions and insertions to be somewhat equivalent in cost thus we can safely try to maximise one of the two. However, in our case, the elements we are inserting or deleting are subtrees of arbitrary size, therefore it is not obvious if we should try and maximise insertions or deletions.

This poses the problem that when enumerating alignments we have no guiding heuristic to cut the number of solutions, for the time being we will simply ignore the problem and resort to enumerate all possible alignments, avoiding to skew the algorithm into preferring insertions over deletions or vice versa.

The following GADT models the sequence of operations that represent an alignment.

```
data Al (at :: U -> *) :: [U] -> [U] -> * where
  A0   :: Al at '[] '[]
  Ains :: Usingl u -> Al at xs ys -> Al at xs (u ': ys)
  Adel :: Usingl u -> Al at xs ys -> Al at (u ': xs) ys
  Amod :: at u -> Al at xs ys -> Al at (u ': xs) (u ': ys)
```

`Al` is parametrised by the same function on pairs of introduced in the spine. `A0` represents the empty alignment, `Ains` and `Adel` take as first argument a singleton representing a runtime instance of the type `u`. These two, together with an alignment for the rest of the list, give us the alignment with an insertion (resp. deletion) as explained in the section above. In the `Amod` case the first argument is the predicate on the underlying atom that describes how to transform it and the second one, as for the case of insertions and deletions, represents an alignment between the rest of the lists.

Let's walk through calculating the alignment for our running example: we have to produce an alignment between `[KSExpr, KSExpr]` and `[Kint]` (recall that these are the shapes of the `Operation` and `Value` constructors).

Because of the way we define the `Amod` constructor, more precisely because of the `at` function that describes how to transform an atom into the other we restrict ourselves to only attempt an `Amod` between two singletons of the same underlying type. This means that in a case like this one, we will only be able to transform one list into the other by repeated applications of `Ains` or `Adel`. It is worth noticing that this restriction is not mandatory and we could in principle allow these transformations as well, the choice here is purely pragmatical and the underlying reason is, this will be a recurring theme, to reduce the sheer amount of combinations that must be checked. What we are left with are the three possible alignments that can be formed by inserting the `Uint` and deleting the two `Usexpr`s, we will generate all of them and keep performing the computations on each branch.

5.3 Atoms

Having figured out all the alignments between two lists of constructor fields, we still have to decide what to do in the case where we match two elements. We need to make a distinction between the possibly recursive fields and the constant ones. In the case of constant fields like `Int`s or `String`s, a transformation between two values of this type consists of a pair recording the source value and the destination value. In the case of a recursive datatype we are essentially left with the problem we started from: transforming a value of a data type into another. To do so, we simply start all over again, recursively computing a spine and an alignment between constructor fields. Note that this construction explicitly excludes pairing constant atoms to recursive ones, this choice serves to prune the search space and reduce the number of possible pairings that can be constructed.

To represent pairs of constant atoms we can introduce a helper `Contract` datatype which lifts f over a pair of xs.

```
newtype Contract (f :: k -> *) (x :: k) = Contract { unContract :: (f x , f x) }
```

To distinguish between recursive and non recursive elements of the language we define a typeclass with no additional methods, and add instances of this typeclass only for the recursive atoms.

Once again, borrowing the language definition from the previous section, we will have the following class and instances defined

```
class IsRecEl (u :: U) where
instance IsRecEl KSExpr where
```

With this we can define the following datatype to represent diffs between atoms of our language.

```
data At (recP :: U -> *) :: U -> * where
  Ai :: (IsRecEl u) => recP u -> At recP u
  As :: Contract Usingl u -> At recP u
```

Here the `At` datatype is parametrised by a predicate that describes how to transform the recursive atoms. The first constructor, `Ai`, which represents the recursive case is parametrised by this predicate, the constraint is added to ensure by construction that when we build an `Ai` we can only do so for the elements of the language that actually are recursive. The other case is covered by the `As` constructor, recall that in this case `Contract` simply lifts `Usingl` to a pair of elements of type u, so the first parameter can be read as: a pair of `Usingl u`.

In our example we have already seen the case for `Ai`, the atoms paired by the first spine were all $Kexpr$s which we recursively calculated spines on. The `As` will be produced when we match two `Value`s that contain different integers, in that case we produce a pair of $UsingKint$ that record the transformation from one Int to the other.

5.4 Recursive alignments

Starting by computing the spine is not necessarily the optimal choice, this can be seen from the following simple example between lists:

```
[ 1, 2, 3, 4 ] -> [ 2, 3, 4 ]
```

Clearly the optimal patch will proceed to delete the first element and then copy over any remaining one. Our definition, however, does not allow for such deletions. Deletions (resp. insertions) are only handled by alignments. To handle such cases we can extend our spines and alignments with the datatype `Almu` that allows insertions or deletions to happen at the top-level.

A match of constructors will be represented as a spine while insertions and deletions will record the constructor being inserted (resp. deleted) and a `Ctx` which records which fields are associated to that constructor. `Ctx`s are inspired by zippers: they can be thought as a representation of a type with a hole somewhere; the hole represents the place where we plug in the rest of the tree to continue the computation.

```
data Almu :: U -> U -> * where
  Alspn :: Spine (At Almu) (Al (At Almu)) u -> Almu u u
  Alins :: ConstrFor v s -> Ctx (Almu u) (TypeOf s) -> Almu u v
  Aldel :: ConstrFor u s -> Ctx (Almu v) (TypeOf s) -> Almu u v
```

This gives rise to another occasion for non-determinism, as `Almu` has the same shortcomings of `Al` meaning that we have no obvious choice of what operation should be maximised over the others. As for alignments we decide to proceed non-deterministically and compute every possibly choice, the next section will introduce some possible optimisations that can be carried out on these two levels to reduce the number of branches being generated at each step.

5.5 Putting everything together

With these types it is easy to write a function that computes the diff between two trees. We can write this function from the "bottom up", starting from the atoms and working our way up through spines and recursive alignments. Notice how all these functions return a list of results, as non-determinism comes into play at every step. The signatures have been slightly simplified from the actual implementation where, for example, the result is parametrised by a monad, making it more general. The diff function for atoms should have the following signature

```
diffAt :: ( forall r . IsRecEl r => Usingl r -> Usingl r -> [rec r])
          -> Usingl a -> Usingl a -> [At rec a]
```

This function is parametrised by a function that describes the treatment for recursive atoms. By inspecting the first singleton we learn whether the atom is recursive or not; if that is the case, the function that deals with the recursive elements can be used to build the corresponding `At`. In the other case, when the element is non recursive, we can simply pair up the two constant atoms with a `Contract` and build the non recursive `At`.

We can then proceed by implementing the function that computes all the spines between recursive elements.

```
diffS :: IsRecEl a => (forall r . IsRecEl r => Usingl r -> Usingl r -> [rec r])
      -> Usingl a -> Usingl a -> [Spine (At rec) (Al (At rec)) a]
```

`diffS` lifts the parameter it takes, a function to handle the recursive elements of our language, over the predicate parameters to the spine that results from the two singletons.

We finally have to define a function that computes the diff in terms of `Almus`. This will call `diffS` in case of two matching constructors from which we can compute the spine wrapping that with the corresponding `Alspn` constructor. In the other cases it will attempt the insertion (resp. deletion) at the constructor level by recording the constructor being inserted (deleted) and producing a `Ctx` which describes where the original tree is attached in respect to the added (deleted) constructor. This function will have the following signature

```
diffAlmu :: (IsRecEl u, IsRecEl v) => Usingl u -> Usingl v -> [Almu u v]
```

As is the case for the alignment between products, here we will simply proceed by enumerating all possible recursive alignments, attempting at each level the alignment of spines, insertions and deletions. One shortcoming with this approach lies in the great combinatorial explosion of possibilities that arises in computing the alignments for constructors and products. The following paragraph will describe an optimisation we can employ to prune the number of possible alignments. This optimisation will allow the program to run in an acceptable time in real world scenarios, however we are still at least an order of magnitude slower than `diff3`, this points to the necessity of further and more aggressive optimisations that may be explored in future work.

Optimisation Given the `Al` and `Almu` type defined in the previous paragraphs we are still left with the problem to computing these two levels of alignments. Since we don't know a priori which alignment is more efficient we will non-deterministically compute all the possible ones. The number of all possible alignments can grow very quickly and, to make things worse, we are dealing with alignments of arbitrarily large subtrees, which prevents us from optimising towards insertions or deletions. It is easy to see that in some cases, prioritising deletions can be more profitable and in other it may be better to do the opposite; this uncertainty stems from the fact that at the time we are calculating the alignment we have no information about the size of the subtrees we are considering.

Despite this limitation, there is one optimisation we can introduce which is to avoid performing insertions if the last step we performed was a deletion and viceversa. To implement this we must add a parameter that tracks the operation that was taken at the previous steps, we will call this the `Phase`. We can define a function with the same signature as `align` but parametrised with a `Phase`. At each step we inspect the `Phase` and if the last step was an insertion (resp. deletion) we will only proceed with a match and another insertion (resp. deletion).

The optimisation can be used on the two different levels; the same idea is used to prune the search space for both the alignment between the list of atoms and the recursive alignment between constructors.

6 Evaluation process

Having developed a general framework to compute patches between typed trees, we know want explore its performance in the context of a real programming language. To test this, we developed a parser for Clojure; the implementation of this parser can be somewhat different to one designed to interpret and run Clojure code. While it may be fruitful to encode as much domain specific knowledge into the parser, thus enabling possible further optimisations. One further consideration is that the parser should try to capture as much syntactical information as possible, in order to produce code that strives to respect any syntactical convention embraced by the authors.

In order to test the framework in a real world context we need to find some suitable data. To acquire this we explored all the Clojure repositories on Github and extracted the ones with the best combination of stars and collaborators. A high number of collaborators will possibly imply a higher chance for conflicts in the source tree, the high number of stars is a good indicator of the quality of the Clojure code and hopefully provides a selection of repository from different domains.

What we need is a way to identify merge points in a projects history; we also want to know how `diff3` performed in those merges, in essence: we want to find all merge points and record if the merge was performed automatically or a conflict had to be manually fixed.

For each file where a conflict may arise we want to find a common ancestor between the branches and calculate two sets of patches, transforming this file into the two versions that are currently present at the top of their respective tree. This operation will yield three files: the common ancestor (O) and the two versions from the conflicting branches (A and B). Before feeding this files to the algorithm we take one additional step by running diff3 on the three files and picking out just the fragments of these files that generated the merge conflicts, we then expand these fragments to the full subexpression that contains them ending up with three new files (O1, A1, B1) which contain only the Clojure expressions that turned out to be conflicting during the merge process.

We wrote some scripts to mine the data from Github and to walk through the source trees mining conflict points that can be used to test the framework. Unfortunately tests on real world data are still out of reach in the current implementation, the sheer number of patches to enumerate is still too big, even with the optimisation described above, and the vast majority of tests ran on real world data simply consume too much memory and get terminated by the OS.

While this data set turned out not to be useful in the current iteration, it still provides a good benchmark to keep testing different ideas that can make the problem more tractable. The next section will describe possible future work that can be done on the algorithm but the main line of improvement will be to optimise the algorithm enough to make it handle this data set.

7 Future Work

The optimisation described in 5.5 is probably still not suited for large real world applications. Even with the optimisation, the framework can only handle relatively small non-pathological inputs (in the order of 10-15 lines). Clearly this is still not good enough to perform experiments on real world data. The greatest unsolved issues is the one regarding performance. For this reason, future work will mainly focus on making the algorithm run in acceptable time on real world data. This will allow us to compare directly to `diff3` and gather insightful data about the tradeoffs between the two approaches.

To speed up the algorithm there are essentially two approaches. One possible direction in which to focus future work is to explore the possibility of using the standard unix `diff3` algorithm as an oracle to prune the alignment trees that are being generated. The idea is that instead of enumerating all possible alignments between two trees, we can check and see how `diff3` treats the sequence of lines in which that tree resides in the source. This can allow us to prune the search space based on the information we can derive from `diff3` and may be able to speed up the computation to handle larger inputs. This idea can be taken even further: we can generalise this approach to introduce an oracle in places where the non-determinism comes at play. The optimisation described in 5.5 can be thought of as an oracle too: this oracle has access to the history of choices that were taken on each branch and, at each step it will inspect this history and output only the branches that do not introduce duplication. The nice thing about this approach is that it will allow us to test different kinds of optimisations in a clean and flexible way; we could even imagine an oracle that interacts with the user, occasionally asking her for guidance into which branches to pursue. Furthermore, we could define a notion of composition between oracles that will give us the chance to combine different optimisations into one.

Another approach that we could take in the attempt to speed up the algorithm is to define an heuristic to score patches. This will allow us to greedily prune the search space for alignments (both recursive and not). With this approach, the question that arises is: what are the properties of patches for which we can compare and score them? The answer is not clear yet. Informally we want to prefer patches that make minimal modifications and encourage copying as much as possible. This is because if a patch consists of a copy on a certain sub-tree, we can be sure that we can safely merge this with any patch that modifies that same sub-tree. On the other hand, if a patch deletes a sub-tree, then the only patches we can merge it with are the ones that also delete the same sub-tree. These considerations suggest the definition of some notion of disjointness between patches. This should encode the fact that the two patches don't apply conflicting transformations between each other. This property is key in ensuring that two patches can be safely merged as we expect disjoint patches to commute in the order they can be applied. Experimentation may yield a counterexample to this conjecture but on the other hand could also provide good insight into the exact notion of disjointness that is needed between patches.

In addition to the possible ways to make the algorithm run faster there is still an issue of representation for the patches that are generated. The resulting patch objects that are generated are often very complex and contain deeply nested information which is not easy to parse by the human eye. Existing diff tools often prepend a plus or minus sign at the beginning of a line to signal it being deleted or inserted and have custom notation for conflicts. In our case the problem is complicated by the fact that information can not only be displayed along the different lines but also inside of them. It would be useful to have a representation of these patches which conveys the information they represent in an easy to digest way, this representation could as well be interactive (possibly in HTML) in order to facilitate the navigation through the different levels of the patch.

Ideally all these points should be explored in future work; however, both the oracle approach and the cost heuristic should be considered of primary importance, as the results from experimenting on concrete data sets (or lack of results) clearly point out. My plan is to explore the oracle approach first. This consists in modifying the algorithm presented here in a way that substitutes all the places where a non-deterministic choice was made with a call to an oracle. The definition of this oracle has to be general enough to accommodate for many different oracles, each functioning in a different way. From there I will attempt to implement different oracles, starting from the optimisation presented in this report and the one depending on `diff3` to explore their performance.

If the approach of using oracles will not yield satisfactory results I will explore with the definition of a cost heuristic for patches and extend the algorithm to prune the branches according to this heuristic. This approach could possibly be coded as an oracle too and for this reason it makes sense to explore it subsequently to the introduction of oracles. The complications of this may lie in the definition of disjointness as any cost function will implicitly rely on it to score the different patches.

A nice visual representation of patches would probably help all future work on the subject, making debugging and experimenting easier to mentally parse and follow. This is probably a low hanging fruit that will make picking the rest much easier, as such my first task will be to work on a representation of patches that is less elaborate with the possibility of expanding on it once the algorithm is running in acceptable time.

The following table attempts to fix some estimates on when each step of the future work may be completed. I prioritised the Oracle framework over the exploration of the cost function keeping in mind that the approach reveals itself to not be fruitful early on, it will be possible to switch to the other while retaining the current time estimates.

The last phase is left for exploring disjointness, this includes both the theoretical definition and the practical experimentation. These two phases can proceed in a loop as the correct definition of this notion is still unclear and will become clearer through experimentation and comparison against current diffing algorithms.

Visual Representation of Patches	10 - 10 - 2017
Oracle Framework	20 - 10 - 2017
Convert optimisations to Oracles	25 - 10 - 2017
Implement new Oracles (Diff, etc)	15 - 11 -2017
Experimentation and comparison	01 - 12 - 2017
Exploring disjointness	15 - 12 - 2017
Final writeup	30 - 12 - 2017

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