Кисленко Назар ЖС-11

Варіант №5

№ 1

а) $\lim\limits_{x\to 0} \dfrac{(8-3x-x^2)^{\frac{1}{2}} - 2 - \frac{x}{4}}{(1-x^2)(1+x^{5/2})-1} = \dfrac{8^{\frac{1}{2}}\left(1-\frac{3x}{8}-\frac{x^2}{8}\right)^{\frac{1}{2}} - 2 - \frac{x}{4}}{(1-x^2)(1+x^{5/2})-1} =$

$= \dfrac{\sqrt{8}\left(1-\frac{1}{2}\left(\frac{3x}{8}-\frac{x^2}{8}\right)+o(x^2)\right) - 2 - \frac{x}{4}}{1+x^{5/2}-x^2-x^{9/2}-1} = \dfrac{\sqrt{8}\left(1-\frac{3x}{16}+\frac{x^2}{16}+o(x^2)\right)-2-\frac{x}{4}}{x^{5/2}-x^2-x^{9/2}} =$

$= \dfrac{\sqrt{8} - \frac{3\sqrt{8}x}{16} + \frac{\sqrt{8}x^2}{16} + o(x^2) - 2 - \frac{x}{4}}{x^2(-1-x^{1/2}-x^{5/2})} = \dfrac{\cancel{x^2}\left(\frac{\sqrt{8}-2}{x^2} - \frac{3\sqrt{8}}{16x} + \frac{\sqrt{8}}{16} - \frac{1}{4x} + o(x^2)\right)}{\cancel{x^2}(-1-x^{1/2}-x^{5/2})}$

$= \dfrac{\frac{\sqrt{8}-2}{x^2} - \frac{3\sqrt{8}}{16x} + \frac{\sqrt{8}}{16} - \frac{1}{4x} + o(x^2)}{-1-x^{1/2}-x^{5/2}} \to \dfrac{\infty}{-1} = -\infty$

Формули: $(1+x)^{\mu} = 1 + \mu x + o(x)$

б) $\lim\limits_{x\to 0} \left(tg\left(\frac{\pi}{4}-x\right)\right)^{ctg x} = e^{ctg x \ln tg(\pi/4 - x)} = e^{ctg x \cdot (\ln(1-tg x) - \ln(1+tg x))} =$

$= e^{ctg x(-tg x - tg x + o(tg x))} = e^{-2}$ (Оскільки $ctg x \cdot tg x = 1$)

Формули: $\ln(1+x) = x + o(x)$

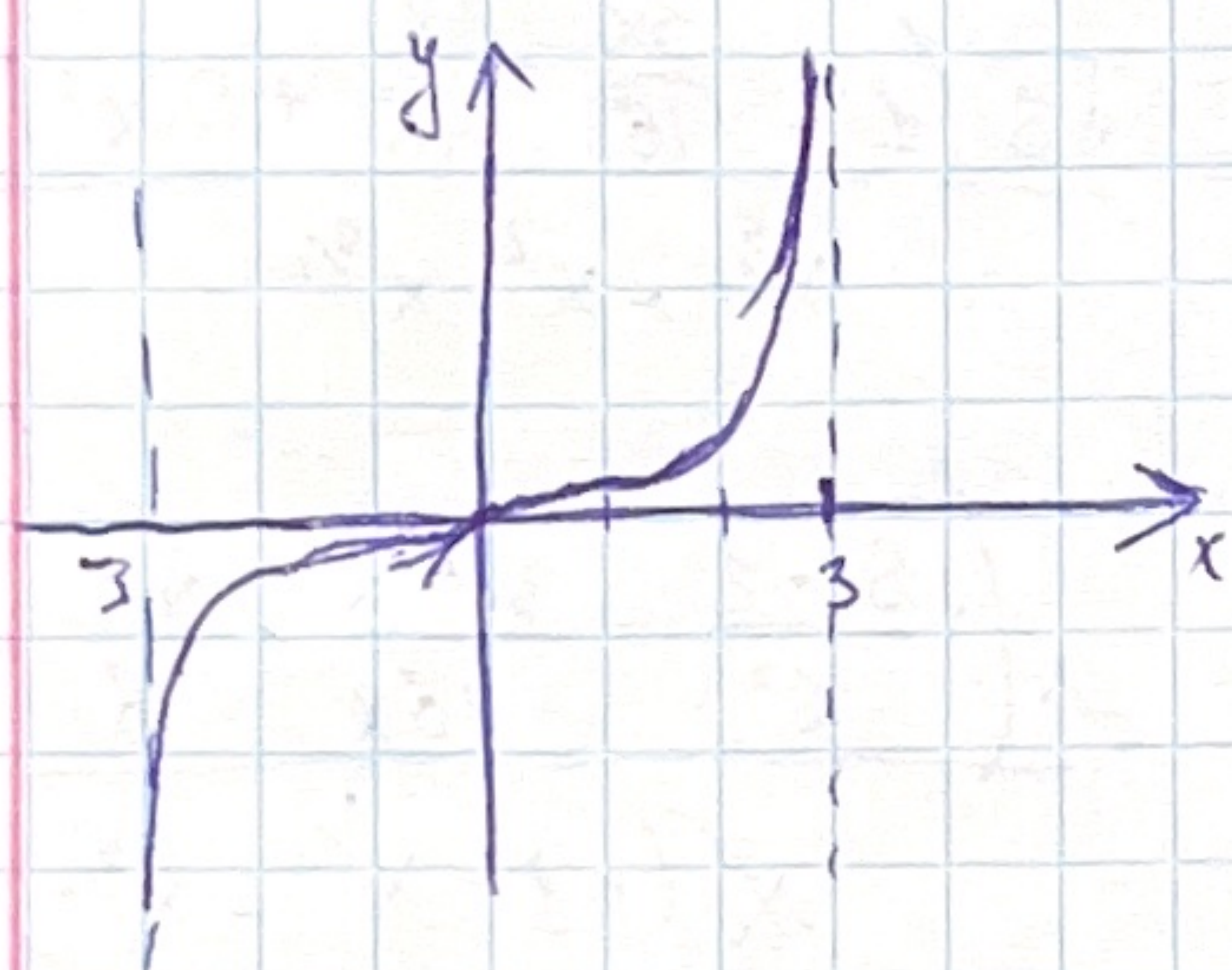
№3.

$f(x) = \frac{x}{9-x^2}$ $(f: R \to R)$ 1) $X = (0;3)$ 2) $X = (-3;0)$ 3) $X = (1;+\infty)$

1) $X = (0;3)$

$\lim\limits_{x \to 0+0} \frac{x}{9-x^2} = \frac{0+0}{(3+0)(3-0)} = \frac{0}{9} = 0$ $f(0) = 0$

$\lim\limits_{x \to 3-0} \frac{x}{9-x^2} = \frac{3-0}{9-9+0} = +\infty$



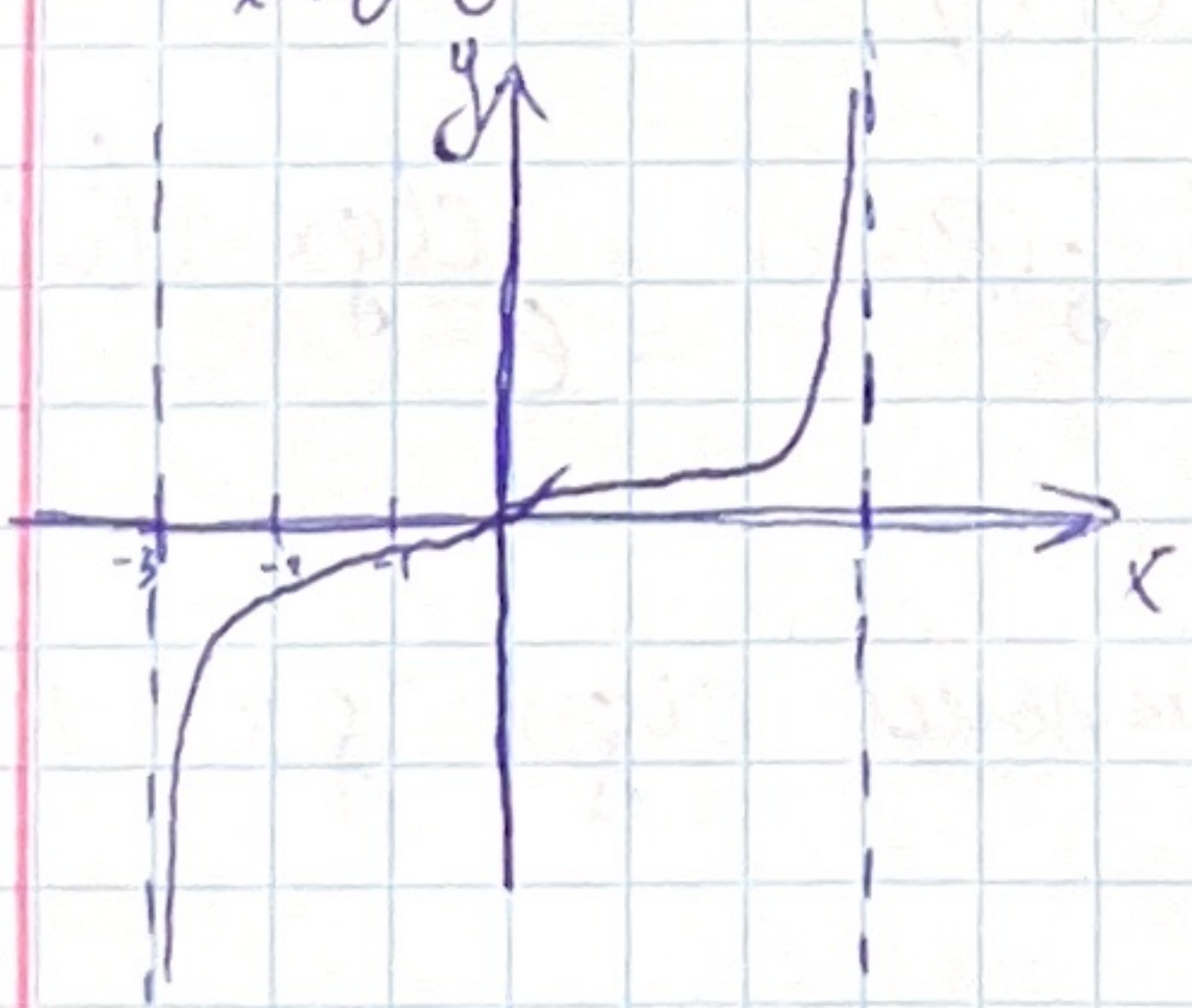
На графіку: зростає крутизна

$f(x)$ не є р.н. на $X = (0;3)$

2) $X = (-3;0)$

$\lim\limits_{x \to -3+0} \frac{x}{9-x^2} = \frac{-3+0}{(3-3+0)(3+3+0)} = \frac{-3}{0+0} = -\infty$

$\lim\limits_{x \to 0-0} \frac{x}{9-x^2} = \frac{0-0}{(3+0+0)(3-0+0)} = \frac{0}{0+0} = 0$

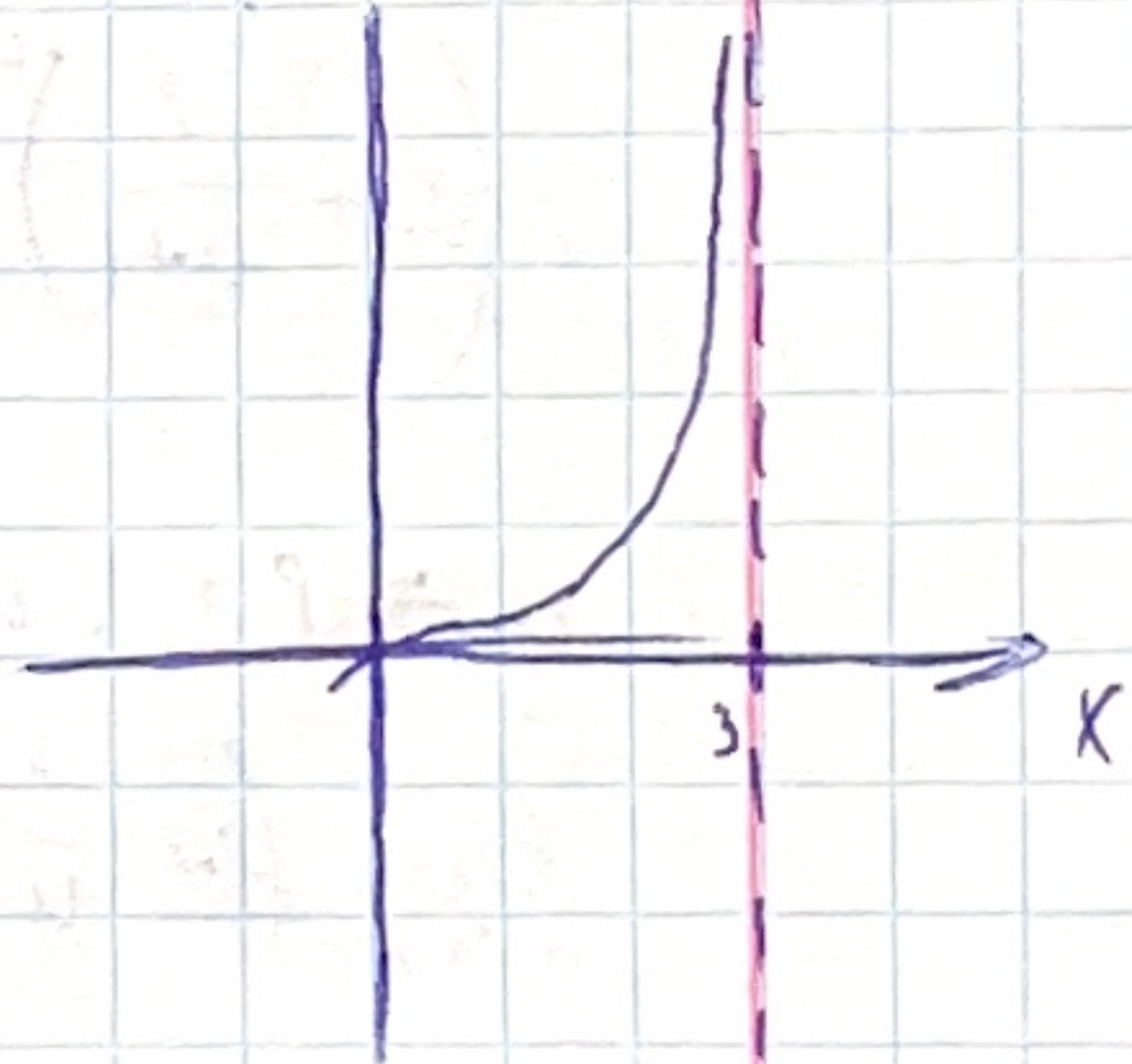


Графік: зростає крутизна

$\Rightarrow$ $f(x)$ не є р.н. ~~на~~ на $(-3;0)$ за критерієм р.н.

3) $X \in (1; +\infty)$

$x = 3$ – точка розриву II-го роду



$$\lim_{x \to 3-0} \frac{x}{9-x^2} = \frac{3-0}{9-9+0} = +\infty$$

$$\lim_{x \to 3+0} \frac{x}{9-x^2} = -\infty$$

$\Rightarrow f(x)$ не є рівномір. непер. на $X = (1; +\infty)$

№ 4

$$y = \ln tg\left(\frac{x}{2} + \frac{x\pi}{4}\right)$$

$$\left(\frac{x}{2} + \frac{x\pi}{4}\right) = x\left(\frac{1}{2} + \frac{\pi}{4}\right)$$

$$y' = \frac{1}{tg\left(\frac{x}{2} + \frac{x\pi}{4}\right)} \cdot \frac{1}{\cos^2\left(\frac{x}{2} + \frac{x\pi}{4}\right)} \cdot \left(\frac{1}{2} + \frac{\pi}{4}\right) = \frac{\cos\left(\frac{x}{2} + \frac{x\pi}{4}\right)}{\sin\left(\frac{x}{2} + \frac{x\pi}{4}\right)} \cdot \frac{1}{\cos^2\left(\frac{x}{2} + \frac{x\pi}{4}\right)} \cdot$$

$$\cdot \left(\frac{1}{2} + \frac{\pi}{4}\right) = \frac{\frac{1}{2} + \frac{\pi}{4}}{\sin\left(\frac{x}{2} + \frac{x\pi}{4}\right) \cdot \cos\left(\frac{x}{2} + \frac{x\pi}{4}\right)} \Big\backslash \cdot \frac{2}{2} = \frac{1 + \pi/2}{2\sin\left(\frac{x}{2} + \frac{x\pi}{4}\right) \cdot \cos\left(\frac{x}{2} + \frac{x\pi}{4}\right)} = \frac{1 + \pi/2}{\sin\left(x + \frac{x\pi}{2}\right)}$$

Формули: $ax' = a$; $tg x = \frac{1}{\cos^2 x}$; $\ln x = \frac{1}{x}$

№ 2 $\qquad a, \alpha, \beta$ – $f(x)$ – неперерв.

$$f(x) = \begin{cases} \dfrac{x^\alpha - a^\alpha}{x^3 - a^3}, & x \neq a \\ \beta, & x = a \end{cases}$$

$$\lim_{x \to a} \frac{x^\alpha - a^\alpha}{(x-a)(x^2 - ax + a^2)} = \left| \begin{array}{l} x - a = t \\ t \to 0 \end{array} \right| = a^\alpha \lim_{t \to 0} \frac{1 + \frac{\alpha t}{a} + o(t) - 1}{t(a+t)^2 + a(a+t) + a^2} =$$

$$= \frac{a^\alpha \cdot \frac{\alpha}{a}}{a^2 + a^2 + a^2} = \frac{\alpha \cdot a^\alpha}{3a^3} = \frac{\alpha}{3} \cdot a^{\alpha - 3} \qquad \beta = \frac{\alpha}{3} a^{\alpha - 3}$$

Формула: $(1+x)^a = 1 + ax + o(x)$

№ 6

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

Нехай $x\to+\infty$

$$n<x<n+1 \Rightarrow \frac{1}{n+1}<\frac{1}{x}<\frac{1}{n} \Rightarrow \underbrace{1+\frac{1}{n+1}<1+\frac{1}{x}<1+\frac{1}{n}}$$

Піднесемо $\underbrace{\quad}$ до степенів.

$$\left(1+\frac{1}{n+1}\right)^n<\left(1+\frac{1}{x}\right)^x<\left(1+\frac{1}{n}\right)^{n+1}$$

$n\to\infty$: перейдемо до границі

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\cdot\lim_{n\to\infty}\left(1+\frac{1}{n}\right)=e\cdot1=e$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^n=\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^{n+1}\cdot\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^{-1}=e\cdot1^{-1}=e$$

$$e<\lim_{n\to\infty}\left(1+\frac{1}{x}\right)^x<e$$

$$\lim_{n\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

Аналогічно для $x\to-\infty$