Introduction to Databases
Tutorial 5

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Problem 1 (mandatory). Consider the following set of FDs:

$$D \to AC\ , \qquad AB \to DE\ , \qquad FD \to E\ , \qquad C \to F$$

(a) Indicate which of the above FDs hold in the following relation:

A	B	C	D	E	F
1	1	2	3	0	4
2	1	2	3	1	4
1	1	3	3	0	5

Moreover, for each FD that does not hold provide two tuples (from the above relation) constituting a violation.

(b) Determine whether each of the following FDs is implied by the FDs above:

$AC \to E$	$BD \to EF$	$EF \to BC$	$BC \to BF$
$AD \to CF$	$ABC \to DF$	$DEF \to AB$	$DF \to AE$
$CD \to DE$	$BE \to AC$	$CD \to ED$	$DE \to AF$

(c) For each FD in point (b) that is implied, write a derivation using Armstrong's axioms (including union and decomposition). **Requirement for tutorial marks:** attempt at least one derivation.

Solution.

(a) Let r_1, r_2 and r_3 denote the rows of the given table (in the order in which they are shown, from top to bottom). Then, we have the following:

- $D \to AC$ does not hold because r_1 and r_2 agree on the value of D but they disagree on the values the assign to the pair of attributes A, C.
- $AB \to DE$ holds because all rows that agree on the values they assign to the pair of attributes A, B, namely r_1 and r_3, also agree on the values they assign to the pair of attributes D, E.
- $FD \to E$ does not hold because r_1 and r_2 agree on the values they assign to the pair of attributes F, D but they disagree on the value of E.
- $C \to F$ holds because all rows that agree on the value of C, namely r_1 and r_2, also agree on the value of F.

(b) Call Σ the given set of FDs. Recall that, for any FD $X \to Y$, we have $\Sigma \models X \to Y$ if and only if $Y \subseteq C_\Sigma(X)$.

- $\{E\} \not\subseteq C_\Sigma(AC) = ACF$, so $\Sigma \not\models AC \to E$
- $CF \subseteq C_\Sigma(AD) = ADCFE$, so $\Sigma \models AD \to CF$
- $DE \subseteq C_\Sigma(CD) = CDAFE$, so $\Sigma \models CD \to DE$
- $EF \subseteq C_\Sigma(BD) = BDACEF$, so $\Sigma \models BD \to EF$
- $DF \subseteq C_\Sigma(ABC) = ABCDEF$, so $\Sigma \models ABC \to DF$
- $AC \not\subseteq C_\Sigma(BE) = BE$, so $\Sigma \not\models BE \to AC$
- $BC \not\subseteq C_\Sigma(EF) = EF$, so $\Sigma \not\models EF \to BC$
- $AB \not\subseteq C_\Sigma(DEF) = DEFAC$, so $\Sigma \not\models DEF \to AB$
- $CD \to ED$ is the same as $CD \to DE$ because the l.h.s. and r.h.s. of an FD are *sets* of attributes
- $BF \subseteq C_\Sigma(BC) = BCF$, so $\Sigma \models BC \to BF$
- $AE \subseteq C_\Sigma(DF) = DFACE$, so $\Sigma \models DF \to AE$
- $AF \subseteq C_\Sigma(DE) = DEACF$, so $\Sigma \models DE \to AF$

(c)
- Derivation of $AD \to CF$

(1) $D \to AC$	[given in Σ]
(2) $AD \to AC$	[from (1) by augmentation with A]
(3) $C \to F$	[given in Σ]
(4) $AC \to ACF$	[from (3) by augmentation with AC]
(5) $AD \to ACF$	[from (2) and (4) by transitivity]
(6) $AD \to CF$	[from (5) by decomposition]

Step (6) above uses decomposition, which is a non-essential axiom. It could be replaced as follows:

(6a) $ACF \to CF$	[by reflexivity]
(6b) $AD \to CF$	[from (5) and (6a) by transitivity]

In general, the decomposition axiom:

$$\frac{X \to YZ}{X \to Y}\ \text{(decomposition)}$$

can be replaced by the following sequence that uses only essential axioms:

$$\frac{X \to YZ \qquad \dfrac{}{YZ \to Y}\ \text{(reflexivity)}}{X \to Y}\ \text{(transitivity)}$$

- Derivation of $CD \to DE$

(1) $C \to F$	[given in Σ]
(2) $CD \to FD$	[from (1) by augmentation with D]
(3) $FD \to E$	[given in Σ]
(4) $CD \to E$	[from (2) and (3) by transitivity]
(5) $CD \to DE$	[from (4) by augmentation with D]

- Derivation of $BD \to EF$

(1) $D \to AC$ [given in Σ]
(2) $DB \to ACB$ [from (1) by augmentation with B]
(3) $DB \to AB$ [from (2) by decomposition]
(4) $DB \to C$ [from (2) by decomposition]
(5) $AB \to DE$ [given in Σ]
(6) $DB \to DE$ [from (3) and (5) by transitivity]
(7) $C \to F$ [given in Σ]
(8) $DB \to F$ [from (4) and (7) by transitivity]
(9) $DB \to DEF$ [from (6) and (8) by union]
(10) $DB \to EF$ [from (9) by decomposition]

Step (9) above uses union, which is a non-essential axiom. It could be replaced as follows:

(9a) $DB \to DBE$ [from (6) by augmentation with DB]
(9b) $DBE \to DEF$ [from (8) by augmentation with DE]
(9c) $DB \to DEF$ [from (9a) and (9b) by transitivity]

In general, the union axiom:

$$\frac{X \to Y \quad X \to Z}{X \to YZ} \text{ (union)}$$

can be replaced by the following sequence that uses only essential axioms:

$$\frac{\dfrac{X \to Y}{X \to XY} \text{ (augmentation with } X) \qquad \dfrac{X \to Z}{XY \to YZ} \text{ (augmentation with } Y)}{X \to YZ} \text{ (transitivity)}$$

- Derivation of $ABC \to DF$

(1) $AB \to DE$ [given in Σ]
(2) $ABC \to DEC$ [from (1) by augmentation with C]
(3) $C \to F$ [given in Σ]
(4) $DEC \to DEF$ [from (3) by augmentation with DE]
(5) $ABC \to DEF$ [from (2) and (4) by transitivity]
(6) $ABC \to DF$ [from (5) by decomposition]

- Derivation of $CD \to ED$

(1) $C \to F$ [given in Σ]
(2) $CD \to FD$ [from (1) by augmentation with D]
(3) $FD \to E$ [given in Σ]
(4) $FD \to ED$ [from (3) by augmentation with D]
(5) $CD \to ED$ [from (2) and (4) by transitivity]

- Derivation of $BC \to BF$

(1) $C \to F$ [given in Σ]
(2) $BC \to BF$ [from (1) by augmentation]

- Derivation of $DF \to AE$

(1) $DF \to E$	[given in Σ]
(2) $DF \to DE$	[from (1) by augmentation with D]
(3) $D \to AC$	[given in Σ]
(4) $DE \to ACE$	[from (3) by augmentation with E]
(5) $DF \to ACE$	[from (2) and (4) by transitivity]
(6) $DF \to AE$	[from (5) by decomposition]

- Derivation of $DE \to AF$

(1) $D \to AC$	[given in Σ]
(2) $C \to F$	[given in Σ]
(3) $AC \to AF$	[from (2) by augmentation with A]
(4) $D \to AF$	[from (1) and (3) by transitivity]
(5) $DE \to AFE$	[from (4) by augmentation with E]
(6) $DE \to AF$	[from (5) by decomposition]

Problem 2 (optional). Let R, S and T be relations on attributes A, B, C. Given the following set of INDs:

$$R[A,B] \subseteq S[B,C] \qquad\qquad S[C,B] \subseteq T[C,A]$$

determine which of the following INDs are implied:

$$\begin{array}{ccc} R[A] \subseteq T[A] & R[B] \subseteq T[B] & R[C] \subseteq T[C] \\ R[A] \subseteq T[B] & R[B] \subseteq T[A] & R[B] \subseteq T[C] \\ R[C] \subseteq T[B] & R[A] \subseteq T[C] & R[C] \subseteq T[A] \end{array}$$

Solution. Call Σ the given set of INDs. Using the axioms for INDs we can compute the closure Σ^+ of Σ:

(1) $R[A,B] \subseteq S[B,C]$	[given in Σ]
(2) $S[C,B] \subseteq T[C,A]$	[given in Σ]
(3) $R[A] \subseteq S[B]$	[from (1) by projection]
(4) $S[C] \subseteq T[C]$	[from (2) by projection]
(5) $R[B,A] \subseteq S[C,B]$	[from (1) by permutation]
(6) $S[B,C] \subseteq T[A,C]$	[from (2) by permutation]
(7) $R[B] \subseteq S[C]$	[from (5) by projection]
(8) $S[B] \subseteq T[A]$	[from (6) by projection]
(9) $R[A,B] \subseteq T[A,C]$	[from (1) and (6) by transitivity]
(10) $R[A] \subseteq T[A]$	[from (9) by projection]
(11) $R[B,A] \subseteq T[C,A]$	[from (9) by permutation]
(12) $R[B] \subseteq T[C]$	[from (11) by projection]
(13) All trivial INDs obtained by reflexivity	

So $R[A] \subseteq T[A]$ and $R[B] \subseteq T[C]$ are implied by Σ (because they are in Σ^+).