5 - Orthogonality

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5.1 Orthogonality in R^n

- **Orthogonal Set** if all pairs of distinct vectors in the set are orthoganal $v_i \cdot v_j = 0$
- Thm 5.1 - vectors of an orthogonal set are LI
- **Orthogonal Basis** - for a subspace W, is a basis for W that is an orthoganal set.
- Thm 5.2 - let $\{v_1, v_2 \dots v_k\}$ be an orthoganal basis for a subspace W of R^n and let w be any vector in W. Then the unique scalars c_1, c_2, c_k such that

  $w = c_1 v_1 + \cdots + c_k v_k$ are given by;

  $$c = \frac{w \cdot v_i}{v_i \cdot v_i} \quad \text{for } i = 1 \dots k$$

- **Orthonormal Set** if it is an orthogonal set of unit vectors
- **Orthonormal Basis** is a basis which is an orthonormal set

Thm 5.3 - let $\{q_1, q_2 \dots q_k\}$ be an orthonormal basis for subspace W, let w be any vector in W

$$w = (w \cdot q_1)q_1 + (w \cdot q_2)q_2 + \cdots + (w \cdot q_k)q_k$$

- Thm 5.4 - Colums of an $m \times n$ matrix Q form an orthonormal set iff $Q^T Q = I$

→ **Orthoganal Matrix**

- Thm 5.5 - A square matrix is orthogonal iff $Q^{-1} = Q^T$
- Thm 5.6 - let Q be an $n \times n$ matrix. The following are equivalent;

  a) Q is orthoganal

  b) $\|Qx\| = \|x\|$ for every x in R^n

  c) $Qx \cdot Qy = x \cdot y$ for every x & y in R^n

- Thm 5.7 - If Q is an orthoganal matrix, then its rows form an orthonormal set.
- Thm 5.8 - Let Q be an orthoganal matrix;

  a) Q^{-1} is orthogonal

  b) $\det Q = \pm 1$

  c) If λ is an eigenvalue of Q, then $|\lambda| = 1$

  d) If Q_1 & Q_2 are orthogonal matrices, so is $Q_1 Q_2$

5.2 Orthogonal Complements & Projections

- **Orthogonal Projection** Let W be a subspace & $\{u_1, \dots u_n\}$ be an orthogonal basis for W, the orthogonal projection of v onto W is defined as:

  $$\text{proj}_W(v) = \left(\frac{u_1 \cdot v}{u_1 u_1}\right)u_1 + \cdots + \left(\frac{u_k \cdot v}{u_k \cdot u_k}\right)u_k$$, and the component orthogonal $\text{perp}_W(v) = v - \text{proj}_W v$

→ Showing v can be decomposed with respect to a subspace & its orthogonal complement