Chapter 6 - Counting

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6.1 THE BASICS

A finite set, S, has $2^{|S|}$ distict subsets
- There is a bijection between subsets of S and bitstring length $|S|$

Number of Functions: For all finite sets A & B, the number of distinct functions $f\ A \to B$

$|B|^{|A|}$

Number of One-to-One functions for $A \to B$, $\frac{|B|!}{|A|!}$
- if $|A| > |B|$, there are no such functions
- else; there are $|B|(|B|-1)(|B|-2)\dots(|B|-|A|+1)$

Sum Rule if A & B are finite sets that are disjoint ($A \cap B = \emptyset$), then;
$|A \cup B| = |A| + |B|$

Subtraction Rule For any finite sets A & B, $|A \cup B| = |A| + |B| - |A \cap B|$
- This is just subtracting the number of ways that are counted twice.

Division Rule There are n/d ways to do a task if it can be done using a procedure that can be carried out in n ways, and for every way w exactly d of the n ways correspond to way w.

The Pigeonhole Principle for any positive integer k, if $k+1$ objects (pigeons) are placed in k boxes, then at least one box contains 2 or more options.
> Generalised: if $N \geq 0$ objects are placed in $K \geq 1$ boxes, then at least one box contains at least $\lceil \frac{N}{K} \rceil$ objects

6.3 PERMUTATIONS & COMBINATIONS

Permutations

Permutation of set S is an ordered arangment of the elements of S

Permutations

Permutation of set S is an ordered arrangement of the elements of S
- AKA a sequence containing every element of S exactly once
- alternatively; **Permutation** bijection $S \to S$

r-Permutation a permutation of r distinct elements of S. The number of r permutations of a set with n elements is denoted $P(n,r)$.

$$P(n,r) = n \cdot (n-1)(n-2)\dots(n-r+1) = \frac{n!}{(n-r)!}$$

↳ simplifies to $n!$ if $r=n$.

Stirlings Formula aproximates the size of $n!$ given; $2^n \leq n! \leq n^n$.

$$n! \sim \sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n$$

r-Combinations of set S is an unordered collection of r elements of S, (eg a subset of S size r)

$C(n,r)$, $\binom{n}{r}$, n chose r denotes the number of r-combinations of an n element set and is denoted the **Binomial Coefficient** → as it gives coefficients for a binominal expansion

2 For all integers $n \geq 1$, and all integers r such that $0 \leq r \leq n$:

$$C(n,r) = \binom{n}{r} = \frac{n!}{r! \cdot (n-r)!}$$

also; $\binom{n}{r} = \binom{n}{n-r}$

6.4 BINOMIAL PISH

1 for $n \geq 0$, $(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$

BINOMIAL THEOREM

also; Total Subsets $\sum_{k=0}^{n} \binom{n}{k} = 2^n$ and, Symetry $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$

2 For all integers $n \geq 0$, $0 \leq r \leq n+1$; $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$

PASCALS IDENTITY

and, $\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$

3 $\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{r-k} \binom{n}{k}$

VANDEMONDE'S IDENTITY

implies; $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$

4 $\binom{n+1}{r+1} = \sum_{j=r}^{n} \binom{j}{r}$

dont really understand.

Multiset over S is an unordered set with possible repitition — Placing r indistinguishable objects into n boxes
- Number of r combinations w/ repitition; $\binom{n+r-1}{r} = \binom{n+r-1}{n-1}$

proof each r-combination can be

Number of r combinations w/ repitition; $\binom{n+r-1}{r} = \binom{n+r-1}{n-1}$

proof each r-combination can be associated with a string of n-1 bars and r stars,

- The bars partition the string into different segments
- The stars in each segment represent the number of copies of the element.

Permutations with indistinguishable objects

1 The number of r permutations with repitition is n^r

TABLE 1	Combinations & permutations Formulae	
Type	Reps	
r-permutation	No	$\frac{n!}{(n-r)!}$
r-combination	No	$\frac{n!}{r!(n-r)!}$
r-permutation	Yes	n^r
r-combination	Yes	$\frac{(n+r-1)}{r!(n-1)!}$

2 The number of permutations of n objects, with n_1 indistinguishable objects of type 1, n_2 of type 2, and n_k of type k is;

$$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}$$

Multinomial Coefficients