4 - Eigenstuff

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4.1 Eigenvalues & Eigenvectors

Q For any square matrix A, does there exist non-zero vectors x such that Ax is just a scalar multiple of x

- **Eigenvalues** let A be an $n \times n$ matrix a scalar λ is called an eigenvalue of A if there is a non-zero vector such that: $Ax = \lambda x$
- **Eigenvector** such a vector x is called an eigenvalue corresponding to λ
- **Eigenspace** is the collection of all eigenvectors corresponding to λ, along with the zero vector, denoted E_λ

- the geometric interpretation $(\mathbb{R}^2)$ implies that in $Ax = \lambda x$, Ax & x are parallell.
- λ is an Evalue of A iff the nullspace of $A - \lambda I$ is nontrivial. Since A is invertable iff $\det A \neq 0$ & FTIM, A has a non-trivial nullspace iff A is non-invertable. $\rightarrow$ Thus λ is an Evalue of A iff $\det(A - \lambda I) = 0$

? In Z_n (modulo) the characteristic equation is converted

4.2 Determinants

Determinant: let A be an $n \times n$ matrix with entries a_{ij}

$$\det A = |A| = a_{11}\det A_{11} - a_{12}\det A_{12} + \cdots + (-1)^{1+n} a_{1n}\det A_{1n}$$
$$= \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det A_{1j}, \quad \det A_{ij} \text{ is called the } (i,j) \text{ minor of } A.$$

- Thrm 4.2 - the determinant of a triangular matrix is the product of its entries
- Thrm 4.3 - let $[a_{ij}]$ be a square matrix;
  a) if A has a zero r/c $\rightarrow \det A = 0$
  b) if B is obtained by interchanging 2 r/c $\rightarrow \det B = -\det A$
  c) if A has 2 identical r/c $\rightarrow \det A = 0$
  d) if B is obtained by multiplying a r/c of A by K $\rightarrow \det B = K \det A$
  e) if A, B & C are Identical, except the i^{th} r/c of C is the sum of A & B $\rightarrow \det C = \det A + \det B$
  f) if B is obtained by adding a multiple of one row to another $\rightarrow \det B = \det A$
- Thrm 4.4 - Let E be an elementary matrix.
  a) if E results from interchanging 2 rows of I_n then $\det E = -1$
  b) if E results from multiplying a row of I_n by k then $\det E = k$
  c) if E results from adding a multiple of one row to another then $\det E = 1$
- Thrm 4.7 - If A is an $n \times n$ matrix $\det(kA) = k^n \det A$
- Thrm 4.8 - If A & B are $n \times n$ matrices $\det(AB) = (\det A)(\det B)$
- Thrm 4.9 - If A is invertible, $\det A^{-1} = 1/\det A$
- Thrm 4.10 - for any square matrix A $\det A = \det A^T$

- **Thm 4.10** - for any square matrix A, $\det A = \det A^{T}$

- Thrm 1.1 - Crammars rule
- Thrm 4.12 - $A^{-1} = (1/\det A)\,\mathrm{Adj}\,A$, where the 'Adjoi

4.3 Evalues & Evectors of n×n

The eigenvalues of a square matrix are precisely the sol
$\det(A - \lambda I) = 0$

* To find Evalues, Evectors, Espaces & bases for A, a
  1) calculate the characteristic polynomial $\det(A-\lambda I)$ of
  2) Find the Evalues of A solving the characteristic equatio
  3) For each Evalue λ, find the null space of the matrix (A-
  4) Find a basis for each Espace.

- **Algebraic Multiplicity** - of an Evalue is the number o
- **Geometric Multiplicity** - of an Evalue is $\dim E_\lambda$,

- Thrm 4.16 - A square matrix is invertable iff 0 is no
- Thrm 4.17 - ++ FTIM ;
  n) $\det A \neq 0$
  o) 0 is not an Evalue of A.

nt matrix is a transposed matrix of cofactors

Matrices

lutions λ of the equation

n $n \times n$ matrix:

A

on $=0$

$-\lambda I)$ This is the Espace, E_λ, with vectors as Evectors

f times it is the root of the characteristic equation

the dimension of the Espace

t an Evalue of it.

b) 0 is not an Evalue of A.

- **Thrm 4.18** - Let A be a square matrix with Evalue λ
  a) For any positive interger n, λ^n is an eigenvalue
  b) If A is invertable then $\frac{1}{\lambda}$ is an eigenvalue of A

- **Thrm 4.19** - Suppose the nxn matrix has Evectors
R^n that can be expressed as a LC of these Eve

$$x = c_1 v_1 + c_2 v_2 + \cdots + c_m v_m$$, then for any integer k

$$A^k x = c_1 \lambda_1^k v_1 + c_2 \lambda_2^k v_2 + \cdots + c_m \lambda^k v_m$$

- **Thrm 4.2** - Eigenvectors corresponding to destinc

4.4 Similarity & Diagonalisatio

- **Similar Matrices** - let A & B be nxn matric
P such that $P^{-1}AP = B \longrightarrow AP = AB$ If A is si

- **Thrm 4.21** - c) If $A \sim B$ & $B \sim C$ then $A \sim C$
- **Thrm 4.22** - Let A & B be nxn matrices. Then;
  a) det A = det B
  b) A is invertable iff B is invertable
  c) A & B have the same rank

λ corresponding to Evector x;
of A with corresponding Evector x (any integer if invertal
with corresponding Evector x
$v_1, v_2 \ldots v_m$ with corresponding Evalues $\lambda_1, \lambda_2 \ldots \lambda_m$. IF x is a vecto
ctors -
x;

t Evalues are LI.

on

es, A is similar to B if there is an invertable $n \times n$ mat
imilar to B we write $A \sim B$

ble)

r in

rix

c) A & B have the same rank
d) A & B have the same characteristic polynomical
e) A & B have the same eigenvalues
f) $A^m \sim B^m$ for all intergers $m \geq 0$ (for all inter

- Two matrices can have these properties in commo
- Thrm 4.22 better used to show 2 matrices are n

- **Diagonisable** if there is a diagonal matrix D su

- **Thrm 4.23 - Sovlving Diagon Alley**

A is diagonisable iff A has LI eigenvectors;
Specificaly, $P^{-1}AP = D$ is satisfied iff columns of
and the diagonal entries of D are the ei

- **Thrm 2.42** - the total colection of basis vectors for al
- **Thrm 4.25** - If a is an nxn matrix with n distinct

* **Thrm 4.27 - The Diagonalisation Theorem**

Let A be an nxn matrix whose distinct Evalues are

a) A is diagonisable
b) The union B of the bases of the Espace of A
c) The algebraic multiplicity of each Evalue equ

gers if invertable)
and still not be similar...
ot similar

ch that A is similar to D. $(P^{-1}AP = D)$

$\longrightarrow P = \begin{bmatrix} [\varepsilon_1] & [\varepsilon_2] & [\varepsilon_n] \end{bmatrix}, \quad D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$

p are n linearly independant eigen vectors of A
genvalues of A corresponding in the same order
l eigenspaces are LI
Evalues, then a is diagonisable

e $\lambda_1, \lambda_2 \ldots \lambda_k$, the following statments are equivalent;

contains n vectors
uals is geometric multiplicity