3 - Matrices

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3.1 Matrix Operations

- **Multiplication**: If a is an m×n matrix and B is an n×r matrix, the (i,j) entry =

  $C_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in} + b_{nj}$

  $= \sum_{k=1}^{n} a_{ik}b_{kj}$

  $m \times \underbrace{n \quad n}_{\text{Same}} \times r$ — Size of AB: $m \times r$

- $A^k = \underbrace{AAA}_{k}$ | $A^rA^s = A^{r+s}$ | $(A^r)^s = A^{rs}$ | $A^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$

Thrm 3.4 -
a) $(A^T)^T = A$
b) $(A+B)^T = A^T + B^T$
c) $(kA)^T = k(A)^T$
d) $(AB)^T = B^TA^T$
e) $(A^r)^T = (A^T)^r$

- **Symetric** if $A^T = A$

3.2 Matrix Algebra

- **Linear Combinations** apply to matrices, as does **span**

Q) Is $B = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$ a linear combination of $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ & $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

→ $c_1\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c_2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dots = \begin{aligned} c_2 + c_3 &= 1 \\ c_1 + c_3 &= 4 \\ -c_1 + c_3 &= 2 \\ c_2 + c_3 &= 1 \end{aligned}$ ∴ same as asking, is $\begin{bmatrix} 1 \\ 4 \\ 2 \\ 1 \end{bmatrix}$ a LC of $\begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Thrm 3.5 For any square matrix, $A + A^T$ is symetric, and A^TA/AA^T are also symetric

3.3 Inverse Matrix

- **Inverse** $AA^{-1} = I$ · if A^{-1} exists, A is invertable
- For a 2×2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
  → if $\det A = 0$, not invertable.

Thrm 3.9
a) $(A^{-1})^{-1} = A$
b) $c(A)^{-1} = (\frac{1}{c}A)^{-1}$
c) $AB^{-1} = B^{-1}A^{-1}$
d) $(A^T)^{-1} = (A^{-1})^T$
e) $(A^n)^{-1} = (A^{-1})^n$

- **Elementary Matrix** is any matrix that can be obtained by performing elem. row ops on an identity matrix
- Solving for Inverse $[A | I] \rightarrow [I | A^{-1}]$
  → Row of zeros, → non-invertable