# Algorithms and Data Structures 2020 Exam–Question 1

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15-12-2020

## Question 1

(a).

Asymptotic upper and lower bounds for $T(n) = T(n-2) + \log(n)$.

Lets Substitute $n = n - 1$,

$T(n-1) = T(n-3) + \log(n-1)$

$T(n-2) = T(n-4) + \log(n-2)$

$T(n-3) = T(n-5) + \log(n-3)$

$T(n-4) = T(n-6) + \log(n-4)$

Substituting this pattern to the limit in the original equation:

$T(n) = logn + log(n-1) + log(n-2) + log(n-3) + \cdots + log(3) + 0 + 0$

Using $\log(m) + \log(n) = \log(mn)$:

$T(n) = log[n(n-1)(n-2)(n-3)...3 + 0 + 0]$

Hence,

$$\underline{T(n) = \Theta(\log(n!))}(= \Theta(n\log(n)))$$

(b).

i.

The statement is false. Proof by counter example.

Consider the graph ABCD with weights AB=4,AD=2,AC=3,BC=4,CD=2

In the example (A,B,C,A) is a cycle with minimum weight AC = 3. Using prims algorithm from B we get MST {BC,CD,AD}.

ii.

Ignoring the edge case where the largest edge weight is not the exclusive largest edge weight. Proof by contradiction.

Assume that the largest weight ($v_i$,$v_{i+1}$) is in the MST.

There must be an vertex connecting the edges of the max-vertex to the rest of the MST as an MST is connected.

Hence if we cut the vertex, a maximum of 1 edge can be estranged from the MST.

We know we only need to make 1 connection to return it to the MST.

We know this connection exists by virtue of it being in a cycle.

And we know this reconnection is of lower weight by virtue of the problem.

Hence we have a contradiction a la minimum weight.

(c).

$n \sum_{i=0}^{n-1} a_i^2 = \sum_{i=0}^{n-1} |b_i|^2$

Expanding the LHS with the equation given in Lecture 5:

$\sum_{i=0}^{n-1} |b_i|^2 = n \sum_{i=0}^{n-1} \sum_{k=0}^{n-1} (a_k \cdot \omega_n^{ik}) \sum_{j=0}^{n-1} (a_j \cdot \omega_n^{ij})$

$\sum_{i=1}^{n-1} |b_i|^2 = \sum_{i,j,k} (a_k \cdot a_j \cdot \omega_n^{i(k-j)})$

... incomplete