Dimension Intuitive idea about size

- **Dimension** - If S is a subspace, then $\dim(A)$ is the nº of vectors in the basis of S
- **Rank** - of a matrix is the dimension of its row/col space.
- **Nullity** - of a matrix is the dimention of its null space, $\text{nullity}(A)$

- Thrm (3.21/3.27) - Fundamental Theorem of Invertable Matrices
Let A be an $n \times m$ matrix, the following are equivalent;

a. A is invertable
b. $Ax = b$ has a unique solution
c. $Ax = 0$ has only the trivial solution
d. The REF of A is I_n
e. A is the product of elementary matrices
f. $\text{rank}(A) = n$
g. $\text{nullity}\, A = 0$
h. col/row vectors are LI
i. col/row vectors span R^n
j. col/row vectors form a basis for R^n

3.6 Linear Transformations

- Matrices can transform vectors as a type of function; $w = Tv$
- **Linear Transformation** A transformation $T: R^n \to R^m$ is a linear transformation iff
  1) $T(u+v) = Tu + Tv$ $\forall$ u & v in R^n and;
  2) $T(cv) = cT(v)$ $\forall$ v in R^n

- **Transformation Matrices**

Reflection in x-axis	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	Reflection in $y=-x$	$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
Reflection in y-axis	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	Rotation θ anticlockwise	$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
Reflection in line $x=y$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Dialation, scale factor k	$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

- Thrm 3.30 - All matrix transformations are linear transformations
- Thrm 3.31 - Standard matrix of a linear transformation. Let $T: R^n \to R^m$ be a linear transformation, then T is a matrix transformation, Specifically, $T = T_A$, where A is the $m \times n$ matrix;

$$A = [T(e_1) \vdots T(e_2) \vdots \cdots \vdots T(e_k)]$$

→ AKA we can find a transformation matrix of a LT by determining its effect on the basis vectors

- Thrm 3.32 - $(S \circ T)v = S(T(v))$ where $S \circ T = [S][T]$