5.3 Gram Schmit Process

Constructing an orthonormal basis for any subspace

The Gram-Schmidt process - begin with an arbitary basis & orthoganalise 1 vector at a time

* GS Process (Thrm 5.15)

Let $\{x_1 \dots x_k\}$ be a subspace W of R^n and define the following;

$v_1 = x_1$ | $v_2 = x_2 - \left(\frac{v_1 \cdot x_2}{v_1 \cdot v_1}\right)v_1$ | $v_3 = x_3 - \left(\frac{v_1 \cdot x_3}{v_1 \cdot v_1}\right)v_1 - \left(\frac{v_2 \cdot x_3}{v_2 \cdot v_2}\right)v_2$ | $v_k = x_k - \left(\frac{v_1 \cdot x_k}{v_1 \cdot v_1}\right)v_1 - \dots - \left(\frac{v_{k-1} \cdot x_k}{v_{k-1} \cdot v_{k-1}}\right)v_{k-1}$

5.4 Orthogonal Diagonalisation of Symetric Matrices

- **Orthogonaly Diagonisable** - if there exists an orthogonal matrix Q and diagonal matrix D; such that; $Q^TAQ = D$
- Thrm 5.17 - If A is orthogonaly diagonisable then A is symetric
- Thrm 5.19 - If A is a symetric matrix then any two eigenvectors (of distinct Evalues) are or
- Thrm 5.20 - Spectral Theorem - Every real symetric square matrix can be diagonali

5.5 Applications: Graphing Quadratics

- We can represent quadratic forms using matrices;

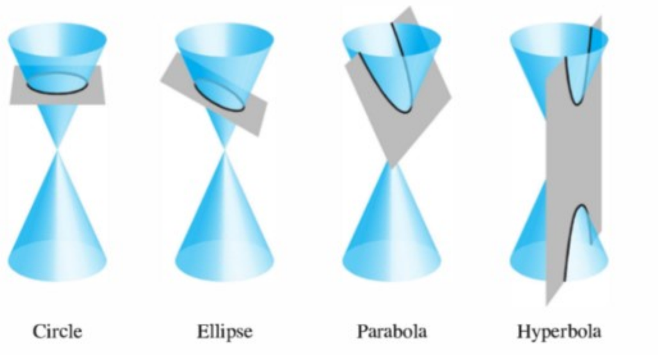
$x^2+by^2+cxy = [x\ y]\begin{bmatrix} a & c/2 \\ c/2 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ and; $ax^2+by^2+cz^2+dxy+exz+fyz = [x\ y\ z]\begin{bmatrix} a & d/2 & e/2 \\ d/2 & b & f/2 \\ e/2 & f/2 & c \end{bmatrix}$

→ these are symetric, making; $f(x) = x^TAx$, where A is the matrix accociated with f.

- **general form** of a quadratic equation in two variables x and y;

$$ax^2+by^2+cxy+dx+ey+f=0$$

- the graphs of such equations are called 'conic sections' or conics
- **Non degenerate conics**; ellipses (circles), hyperbolas & parabola's
- **Standard position** represented in the following forms;



Ellipse or Circle

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$