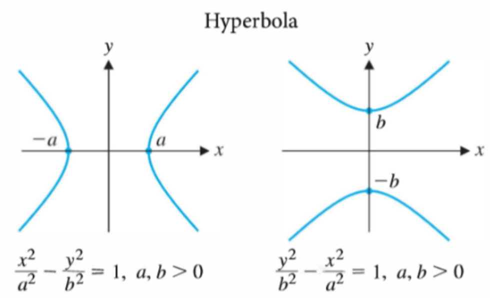
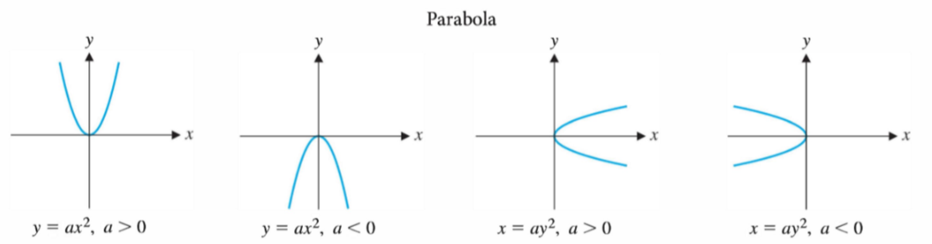
Hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$



Parabola $y=ax^2$



· If a quadratic contains too many terms to be written in one of these forms, its graph is not in standard p.

1) When there are additional terms but no xy term, the conic has been translated.

* · Factorise, complete the square, Substitute (to get x', y' coordinate system)

Q) Identify and graph the conic section whose equation is;

$x^2+2y^2-6x+8y+9=0$

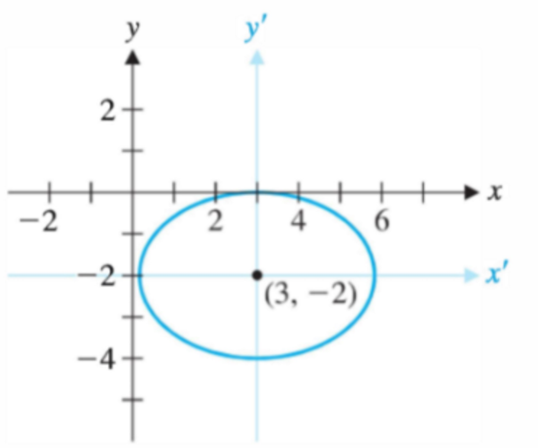
→ $(x^2-6x)+(2y^2+8y)=-9$; grouping x & y terms

$(x^2-6x+9)+2(y^2+4y+2)=-9+9+8$; completing the square.

$(x-3)^2+2(y-2)^2=8$

subbing $x'=x-3$, $y'=y+2$

$$\frac{(x')^2}{8}+\frac{(y')^2}{4}=1$$



→ this is an ellipse in standard position in the x', y' coordinate system

2) If the conic contains a cross product term then it represents a conic that has been rotated

* · write in matrix form $f(x)=x^TAx$,

Q) Identify the conics whose equation is; $5x^2+4xy+2y^2=6$

$A=\begin{bmatrix}5&2\\2&2\end{bmatrix}$; LHS in quadratic form so can rewrite in matrix form as x^TAx

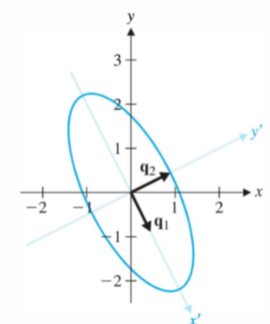
$Q=\begin{bmatrix}\frac{2}{\sqrt5}&\frac{1}{\sqrt5}\\\frac{1}{\sqrt5}&\frac{-2}{\sqrt5}\end{bmatrix}$; Q orthogonally diagonalises A

· Notice $\det Q=-1$, rotate columns to get +1 & a positive axis

$x^TAx=(x')Dx'=6$

$(x')^2+6(y')^2=6$

$$=\frac{(x')^2}{6}+y=1$$



representing an elipse in the $x'y'$ coordinate system

· See page 4.19 for an example entailing a translation & Rotation.