2b - Systems of Linear Equations

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Q) Show that $R^2 = \text{span}\left(\begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}\right)$

We need to any arbitary vector can be written as an LC of the vectors;

$$x\begin{bmatrix}2\\1\end{bmatrix} + y\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix} \rightarrow \left[\begin{array}{cc|c}2 & 1 & a\\1 & 3 & b\end{array}\right] \rightarrow \left[\begin{array}{cc|c}-1 & 0 & b-3a/7\\0 & 1 & a+2b/7\end{array}\right]$$

$$\rightarrow \left(\frac{3a-b}{7}\right)\begin{bmatrix}2\\1\end{bmatrix} + \left(\frac{a+2b}{7}\right)\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$$

* · to find the span of a set of vectors, solve by GE (plane if 2 vectors in R^3)

Linear Dependence - Thrm 2.5 - if a vector can be written as a LC of others

LD: A set of vectors $v_1, v_2 \dots v_k$ are LD if there are scalars c_1, c_2, c_k, at least 1 of which is non-zero, such that,

$$c_1 v_1 + c_2 v_2 + \dots + c_k v_k = 0$$

- ∴ a set of vectors which are not linearly dependant are called **Linearly Independent**
- to determine if a system is LD sub into definition.

Thrm 2.6 - a set of vectors are LD if the matrix created from its column vectors (A) give $[A|0]$ a non trivial solution

Thrm 2.7 - let $v_1, v_2 \dots v_k$ be row vectors in R^n, let A be the $m \times n$ matrix $\begin{bmatrix}v_1\\v_2\\\vdots\\v_k\end{bmatrix}$. the set is LD if rank(A) < m

Thrm 2.8 - Any set of vectors in R^n is linearly dependant if m > n