Deductive Databases

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Datalog rules (1)

$$\underbrace{H(\bar{x})}_{\text{head}} :\!- \underbrace{S_1(\bar{x}_1), \ldots, S_n(\bar{x}_n)}_{\text{body}}$$

Head predicate over variables and constants

Body conjunction of possibly negated atoms (**subgoals**)
- relational atoms (predicates)
- comparisons between variables/constants

- Head variables are implicitly **universally quantified**
- Body variables not in head are **existentially quantified**
- Rule: Body $\rightarrow$ Head (if the body is true, the head is true)

Datalog rules (2)

Example

$$\text{Lucky}(x) \;{:}{-}\; \text{Customer}(x, y, z), \text{Account}(u, z, x, w), w > 10000$$

In relational calculus we could write:

$$\begin{aligned}\text{Lucky} = \{\, x \mid \exists y, z, u, w \; & \text{Customer}(x, y, z) \\ & \wedge \text{Account}(u, z, x, w) \wedge w > 10000 \,\}\end{aligned}$$

Safety

Every variable (in head or body)
appears in at least one non-negated relational atom

Not safe: $\text{BigNumber}(x) \;{:}{-}\; x > 1000000000$

Datalog programs

Program = set of Datalog rules

Example

$$\text{Parent}(x, y) \;{:}{-}\; \text{Mother}(x, y)$$
$$\text{Parent}(x, y) \;{:}{-}\; \text{Father}(x, y)$$

edb (extensional database) relations **stored** in the database

- can appear only in the body of rules

idb (intensional database) derived relations

- can appear both in the head or body of rules

From relational algebra to Datalog

Every relational algebra expression can be translated into Datalog

Projection $\pi_{\#2}(R)$

$$E(x) \coloneq R(y, x)$$

Selection $\sigma_{\#1\,\mathbf{op}\,c}(R)$

$$E(x, y) \coloneq R(x, y), x\ \mathbf{op}\ c$$

Product $R \times S$

$$E(x, y, w, z) \coloneq R(x, y), S(w, z)$$

Difference $R - S$

$$E(x, y) \coloneq R(x, y), \neg S(x, y)$$

Union $R \cup S$

$$E(x, y) \coloneq R(x, y)$$
$$E(x, y) \coloneq S(x, y)$$

From relational algebra to Datalog

Let R and S be relations over A, B

$$E = \underbrace{\pi_{A,D}\Big(\underbrace{\sigma_{B=C}\big(\underbrace{R \times \rho_{A\to C,\,B\to D}(S)}_{E_1}\big)}_{E_2}\Big)}_{E_3} \cup \underbrace{\rho_{B\to D}(R - S)}_{E_4}$$

$$\begin{aligned}
E_1(x, y, w, z) &\coloneq R(x, y), S(w, z) \\
E_2(x, y, w, z) &\coloneq E_1(x, y, w, z), y = w \\
E_3(x, y) &\coloneq E_2(x, u, v, y) \\
E_4(x, y) &\coloneq R(x, y), \neg S(x, y) \\
E(x, y) &\coloneq E_3(x, y) \\
E(x, y) &\coloneq E_4(x, y)
\end{aligned}$$

Limitations of relational algebra/calculus

Parent $=$ table of pairs x, y where x is the parent of y

$$\begin{aligned}
\text{Parent} &= \{x, y \mid \text{Parent}(x, y)\} \\
\text{Grandparent} &= \{x, y \mid \exists z\ \text{Parent}(x, z) \land \text{Parent}(z, y)\} \\
\text{Great-grandparent} &= \{x, y \mid \exists z\ \text{Grandparent}(x, z) \land \text{Parent}(z, y)\}
\end{aligned}$$

For a **given** k, we can express the query Ancestor^k

But
We **cannot express** the Ancestor relation itself
that is: an Ancestor^k query that works for **every** k

Recursion in Datalog

The head relation of a rule can appear in its body

$$\begin{aligned}
&\text{Ancestor}(x, y) :\!-\ \text{Parent}(x, y) \\
&\text{Ancestor}(x, y) :\!-\ \text{Ancestor}(x, z), \text{Parent}(z, y)
\end{aligned}$$

Intuition
x is an ancestor of y if

x is a parent of y

or

x is an ancestor of a parent of y

Dependency graph

IDB predicate P **depends on** (IDB) predicate Q
if there is a rule with P in the head and Q in a subgoal

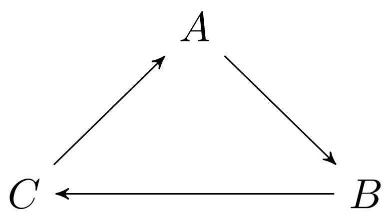
Dependency graph

nodes IDB predicates

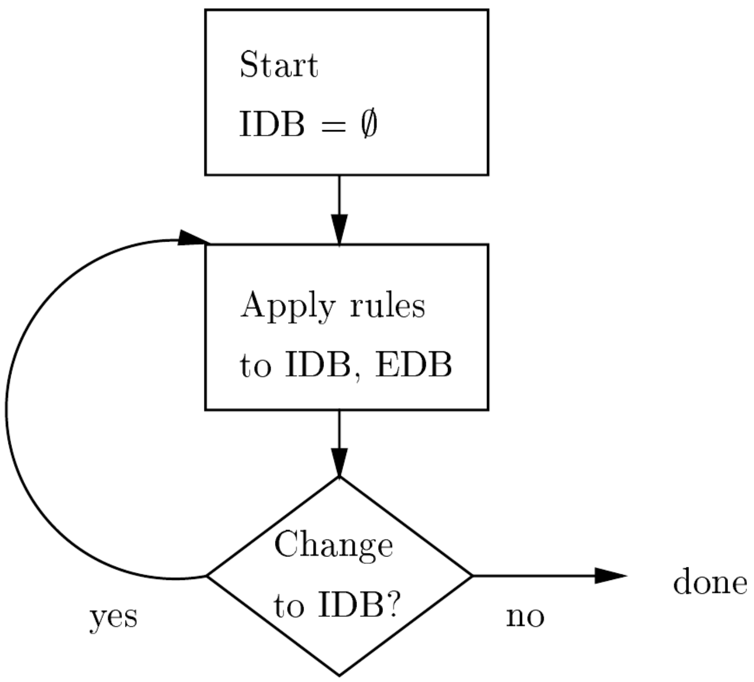
edges $P \rightarrow Q$ if P depends on Q

A **cycle** in the dependency graph means the program is recursive

$$
\begin{aligned}
C(x) &:- A(y,x) \\
C(x) &:- S(x,y), y > 1 \\
B(x,y) &:- C(x), P(x,y) \\
A(x,y) &:- B(y,x)
\end{aligned}
$$



Iterative Fixpoint Evaluation



Evaluation of recursive programs

- The **Parent** relation (EDB) never changes
- The **Ancestor** relation (IDB) is initially empty

  $\text{Ancestor}_0 = \varnothing$

- At step $i + 1$ compute:

  $\text{Ancestor}_{i+1}(x, y) \text{ :– } \text{Parent}(x, y)$
  $\text{Ancestor}_{i+1}(x, y) \text{ :– } \text{Ancestor}_i(x, z), \text{Parent}(z, y)$

- Stop when a **fixpoint** is reached

  $\text{Ancestor}_{i+1} = \text{Ancestor}_i$

Evaluation of recursive programs

IDB:

Ancestor	
John	Mary
John	Jane
Jane	Louis
Mary	Linda
Louis	Mark
John	Linda
John	Louis
Jane	Mark
John	Mark

EDB:

Parent	
John	Mary
John	Jane
Jane	Louis
Mary	Linda
Louis	Mark

$$\text{Ancestor}(x, y) \text{ :– } \text{Parent}(x, y)$$
$$\text{Ancestor}(x, y) \text{ :– } \text{Ancestor}(x, z), \text{Parent}(z, y)$$

Recursion in SQL

Suppose we have a table Parent with attributes name, child

```
WITH RECURSIVE Ancestor(name, descendant) AS (
    SELECT *
    FROM   Parent
    UNION
    SELECT A.name, P.child
    FROM   Ancestor A, Parent P
    WHERE  A.descendant = P.name
)
SELECT * FROM Ancestor ;
```

The definition mimics the structure of the Datalog program

Nonlinear recursion

The head relation can appear **more than once** in its body

$$\text{Ancestor}(x,y) \;{:}{-}\; \text{Parent}(x,y)$$
$$\text{Ancestor}(x,y) \;{:}{-}\; \text{Ancestor}(x,z), \text{Ancestor}(z,y)$$

Intuition

x is an ancestor of y if

x is a parent of y

or

x is an **ancestor of an ancestor** of y

Nonlinear recursion in SQL

Not supported

```
WITH RECURSIVE Ancestor(name, descendant) AS (
    SELECT *
    FROM   Parent
    UNION
    SELECT A.name, A2.desc
    FROM   Ancestor A1, Ancestor A2
    WHERE  A1.descendant = A2.name
)
SELECT * FROM Ancestor ;
```

```
ERROR: recursive reference to query "ancestor" must not appear
more than once
```

Recursive programs with negation

Consider the program $P = \{R(x) :\!-\, S(x), \neg R(x)\}$

IDB:

R

EDB:

S
1
2

Step 0 IDB relation R is empty

Step 1 $R = \{1, 2\}$

Step 2 $R = \varnothing$

Step 3 $R = \{1, 2\}$

Step 4 $R = \varnothing$

… **No fixpoint!**

Iteration never ends

Stratification

Partition a program P into a sequence of subprograms $P_1, \ldots, P_n$

- Each subprogram defines one or more IDB relations
- If a relation S is used positively in the definition of R
  then S must be defined **earlier or simultaneously** with R
- If a relation S is used negatively in the definition of R
  then S must be defined **strictly before** R

Stratum 0 EDB relations

Stratum i IDB relations that depend
- positively on relations in any stratum $j \leq i$
- negatively on relations in any stratum $j < i$

Stratification

Stratum graph

nodes IDB predicates

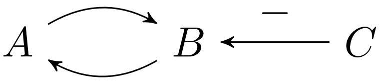
edges $P \rightarrow Q$ if P depends on Q
label the edge with "–" if Q is a negated subgoal

Stratifiable program: no cycle involving at least one negated edge

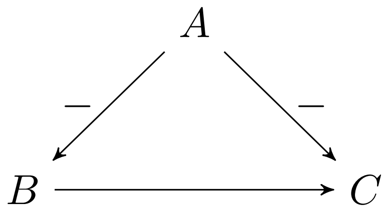
$$R(x) :- S(x), \neg R(x)$$

$$\begin{tikzcd} R \arrow[loop above, "-"] \end{tikzcd}$$

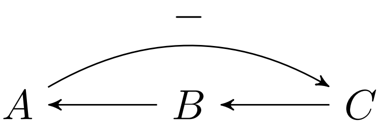
More examples



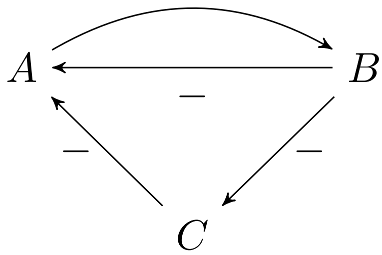
Stratifiable: $A = B < C$



Stratifiable: $C \leq B < A$



Not stratifiable:
$A \leq B \leq C < A$

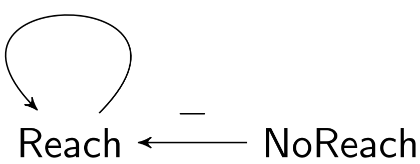


Not stratifiable: $A < B \leq A$

Stratified example

Which target nodes cannot be reached from any source node?

$$
\begin{aligned}
\text{NoReach}(x) &:- \text{Target}(x), \neg\text{Reach}(x) && \text{(rule1)}\\
\text{Reach}(x) &:- \text{Source}(x) && \text{(rule2)}\\
\text{Reach}(x) &:- \text{Reach}(y), \text{Link}(y, x) && \text{(rule3)}
\end{aligned}
$$



Stratum 0	Source, Link, Target
Stratum 1	Reach
Stratum 2	NoReach

Evaluation of stratified programs

P partitioned into a **sequence** $P_1, \ldots, P_n$

Gives us an order in which to apply (each group of) rules

At each iteration k, execute each subprogram in sequence

(1) Apply all the rules in P_1

(2) Apply all the rules in P_2

$\vdots$

(n) Apply all the rules in P_n

Evaluation of stratified programs

$$
\begin{aligned}
\text{Reach}(x) &:- \text{Source}(x) && (P_1)\\
\text{Reach}(x) &:- \text{Reach}(y), \text{Link}(y, x) && (P_1)\\
\text{NoReach}(x) &:- \text{Target}(x), \neg\text{Reach}(x) && (P_2)
\end{aligned}
$$

EDB

Source
1
2

Link	
1	2
3	4
2	4

Target
2
3
4

IDB

Reach
1
2
4

NoReach
3

Further remarks on SQL recursion

- Requires **stratified negation**
- Only **linear recursion**

Problems

Arithmetic operations
: introduce new values not present in the database

Multiset semantics
: rules must be applied several times
: **cycles in the data** must be detected