4b - Eigenstuff

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- Thrm 1.1 - Crammars rule
- Thrm 4.12 - $A^{-1} = (1/\det A)\,\mathrm{Adj}\,A$, where the 'Adjoint matrix is a transposed matrix of cofactors

4.3 Evalues & Evectors of nxn Matrices

The eigenvalues of a square matrix are precisely the solutions λ of the equation

$$\det(A - \lambda I) = 0$$

* To find Evalues, Evectors, Espaces & bases for A, an nxn matrix:

1) calculate the characteristic polynomial $\det(A - \lambda I)$ of A
2) Find the Evalues of A solving the characteristic equation $=0$
3) For each Evalue λ, find the null space of the matrix $(A - \lambda I)$. This is the Espace, E_λ, with vectors as Evectors
4) Find a basis for each Espace.

- **Algebraic Multiplicity** - of an Evalue is the number of times it is the root of the characteristic equation
- **Geometric Multiplicity** - of an Evalue is $\dim E_\lambda$, the dimension of the Espace

- Thrm 4.16 - A square matrix is invertable iff 0 is not an Evalue of it.
- Thrm 4.17 - ++ FTIM;
  n) $\det A \neq 0$
  o) 0 is not an Evalue of A.
- Thrm 4.18 - Let A be a square matrix with Evalue λ corresponding to Evector x;
  a) For any positive interger n, λ^n is an eigenvalue of A with corresponding Evector x (any integer if invertable)
  b) If A is invertable then $\frac{1}{\lambda}$ is an eigenvalue of A with corresponding Evector x
- Thrm 4.19 - Suppose the nxn matrix has Evectors $v_1, v_2 \ldots v_m$ with corresponding Evalues $\lambda_1, \lambda_2 \ldots \lambda_m$. If x is a vector in $\mathbb{R}^n$ that can be expressed as a LC of these Evectors -

  $x = c_1 v_1 + c_2 v_2 + \cdots + c_m v_m$, then for any integer k;

  $A^k x = c_1 \lambda_1^k v_1 + c_2 \lambda_2^k v_2 + \cdots + c_m \lambda^k v_m$

- Thrm 4.2 - Eigenvectors corresponding to distinct Evalues are LI.

4.4 Similarity & Diagonalisation

- **Similar Matrices** - let A & B be nxn matrices, A is similar to B if there is an invertable nxn matrix P such that $P^{-1}AP = B \rightarrow AP = AB$ If A is similar to B we write $A \sim B$

- Thrm 4.21 - c) If $A \sim B$ & $B \sim C$ then $A \sim C$

- **Thrm 4.22** - Let A & B be nxn matrices. Then;
  a) $\det A = \det B$
  b) A is invertable iff B is invertable
  c) A & B have the same rank
  d) A & B have the same characteristic polynomial
  e) A & B have the same eigenvalues
  f) $A^m \sim B^m$ for all intergers $m \geq 0$ (for all inter

- Two matrices can have these properties in commor
- Thrm 4.22 better used to show 2 matrices are n

  - **Diagonisable** if there is a diagonal matrix D su

- Thrm 4.23 - Sadurim Diagon Allu

gers if invertable)

and still not be similar...

not similar

ch that A is similar to D. ($P^{-1}AP = D$)

[[c][c][c]] [λ_1 0 0]

- **Thrm 4.23 - Solving Diagon Alley**

A is diagonisable iff A has LI eigenvectors;
Specifically, $P^{-1}AP = D$ is satisfied iff columns of p
and the diagonal entries of D are the eigen

- **Thrm 2.42** - the total collection of basis vectors for al
- **Thrm 4.25** - If a is an nxn matrix with n distinct

* **Thrm 4.27 - The Diagonalisation Theorem**

Let A be an nxn matrix whose distinct Evalues are

a) A is diagonisable
b) The union B of the bases of the Espace of A
c) The algebraic multiplicity of each Evalue equ

$\longrightarrow P = \begin{bmatrix} [E_1] & [E_2] & [E_n] \end{bmatrix} \quad D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$

are n linearly independant eigen vectors of A
values o A corresponding in the same order
eigenspaces are LI
Evalues, then a is diagonisable

$\lambda_1, \lambda_2 \ldots \lambda_k$, the following statments are equivalent;

contains n vectors
uals is geometric multiplicity