Algorithms and Data Structures 2010 May Exam

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Question 1

(a). *State the definitions of a 'spanning subgraph', 'weight' of a subgraph and 'minimum spanning tree'.*

spanning subgraph of a graph, $\mathcal{G} = VE$, is a subset of $\mathcal{G}$ containing all vertices, V and a subset of the edges E.

weight of a subgraph the weight of a subgraph is the summation of the individual weight of all its edges.

minimum spanning tree of a graph $\mathcal{G}$ is a connected spanning subgraph $\mathcal{T}$ of $\mathcal{G}$ of minimum weight.

(b). *State prim' algorithm in pseudocode. For an input graph with n vertices and m edges what is its running time?*

Algorithm *Prim*($\mathcal{G}$: (VE) , w : Weight Matrix) $\rightarrow \mathcal{T}$: MST

1. $\mathcal{T} \leftarrow$ arbitrary $v \in G$ # v is an vertex in V
2. **while** fringe edge exists in $\mathcal{T}$:
3. add fringe edge of minimum weight to $\mathcal{T}$
4. **return** $\mathcal{T}$

where a **fringe edge** is an edge of the subtree $\mathcal{T}$ with exactly one endpoint in $\mathcal{T}$.

Runtime is dependent on how the data structure is implemented, specifically, the *priority queue* used to find the minimum weight fringe edge in $\mathcal{T}$ with runtimes as follows,

Data Structure	Runtime
Array	$T(n, m) = \Theta(n^2)$
Heap	$T(n, m) = \Theta((n + m) \log(n))$
Fibonacci Heap	$T(n, m) = \Theta(n \log(n) + m)$

(c). *Illustrate how Prim's algorithm proceses the following input by indicating for each step of the main loop what edges Prim's algorithm has selected.*

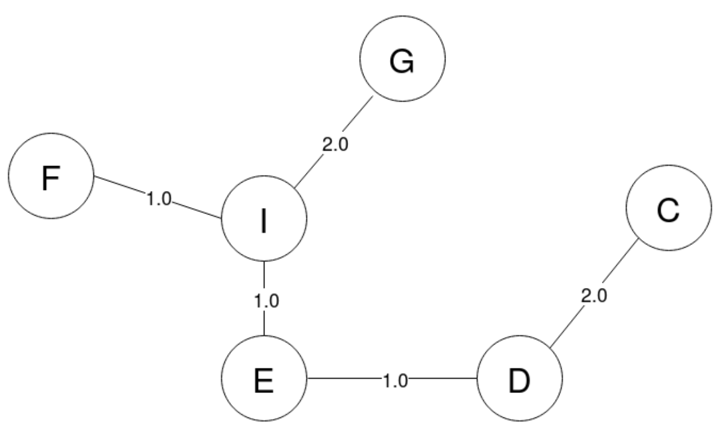
1. {AB}
2. {AB, BC}
3. {AB, BC, CD}
4. {AB, BC, CD, DE}
5. {AB, BC, CD, DE, DH}
6. {AB, BC, CD, DE, DH, EI}
7. {AB, BC, CD, DE, DH, EI, IF}
8. {AB, BC, CD, DE, DH, EI, IF, IG}
9. {AB, BC, CD, DE, DH, EI, IF, IG}

(d). *Assume that $\mathcal{G} = V E, w)$ is a connected edge-weighted graph and that $T \subset V$ is a non-empty set of vertices. A 'Steiner tree' for T is a connected (but not necessarily spanning) subgraph $H = (V_H, E_H)$ of G that is a tree and whose vertex set V_H contains T as a subset. Its weight is*

$$w(H) = \sum_{e \in E_H} w(e)$$

In the example graph from (c), find a steiner tree of minimum weight for $T = \{C, E, F, G\}$.

Solution,



(e). *Suppose that an edge weighted connected graph $G = (V, E, w)$ with n vertices* *and m edges and a set $T \subset V$ are given. Devise an algorithm that computer an optimal Steiner tree in time $(2 + o(1))^n$. Also provide an informl argument as to why your algorithm produces the correct output and why it has the desired running time.*

```
def steiner(G,w,T) -> H:
  H = T[0]  # any edge in T will do
  # initialise a priority queue of edges in T
  # sorted by path distance to H
  Q = priorityQueue(T, dist_to_H)
  while not Q.isEmpty():
    t = Q.pop()
    H += min_path(t, H)

  return H
```

The algorithm produces the correct output