3.5 Subspaces, Basis, Dimension & Rank

Subspaces are a generalisation of lines & planes through the origin

- **Subspace** of R^n is a collection of vectors S, such that;
  1) the zero vector is in S.
  2) if u & v are in S, $u+v$ are in S.
  3) if u is in S and c is a scalar, then cu is in S

- Thrm 3.1.9 - if $v_1, v_2, \ldots v_k$ are vectors in R^n then $\text{span}(v_1, v_2 \ldots v_k)$ is a subspace of R^n

- $\text{span}(v_1, v_2, v_k)$ is 'the subspace spanned by $(v_1, v_2 \ldots v_k)$
- to determine whether something is a subspace, check the theorem properties, then T / counterexample.

- **Row/Column Space** - Let A be an $n \times n$ matrix, the row/col space of A is the subspace (row(A)) spanned by the rows/cols of A

* · to find the row space, $A^T x = w^T$ - find the transpose and do the columns!

Null Space Let A be an $n \times m$ matrix, the null space of A is the subspace of R^n consisting of solutions of the homogenious linear system $Ax = 0$ & denoted by null(A)

- Thrm 3.22 - for systems of linear equations $Ax = b$, exactly one of the following is true;
  a) There is no solution
  b) There is a unique solution
  c) There are infinitly many solutions

Basis of a subspace describes the direction vectors of the lines/planes

- **Basis** for subspace S of R^n is a set of vectors in S that
  1) span S and,
  2) are linearly independant

→ $e_1, e_2 \ldots e_n$ is called the standard basis

* · to find the basis of a subspace;
  1) find the REF R of A
  2) use the col vectors of A that correspond to the columns R containing the leading LI variables to form a basis for col(A)
  3) solve for leading variables, eg $Rx = 0$ in terms of free variables ($s, t \ldots$)
  4) set free variables = to parameters, sub back into vector form (to write as linear combination)

  → these vectors form a basis for null(A)

Thrm 3.23 - Let S be a subspace of R^n, any 2 bases have the same number of vectors

... be a complement of R, any 2 bases have the same number of vectors