IDS Lecture 18: Entailment of Constraints

Implication of Constraints

A set σ of constraints **implies** (or **entails**) a constraint ϕ if *every* instance that satisfies σ also satisfies ϕ

Syntax: $\sigma \models \phi$

Question: Does σ imply ϕ?

Relevance

- do the given constraints imply bad ones
- to the given constraints look bad but imply good ones

Axiomatisation of Constraints

An axiomatisation is–

Sound if every derived constraint is implied
Complete if every implied constraint can be derived

Sound + Complete axiomatisation gives a procedure $\vdash$ such that

$$\sigma \models \phi \iff \sigma \vdash \phi$$

Intuition: if we derive there is an implicit constraint

Armstrong's Axioms (for FDs)

Essential Axioms

Reflexivity $Y \subseteq X \Rightarrow X \rightarrow Y$
Argumentation $X \rightarrow Y \Rightarrow XZ \rightarrow YZ \forall Z$
Transitivity $X \rightarrow Y \wedge Y \rightarrow Z \Rightarrow X \rightarrow Z$

Derived Axioms

Union $X \rightarrow Y \wedge Y \rightarrow Z \Rightarrow X \rightarrow YZ$
Decomposition $X \rightarrow YZ \Rightarrow X \rightarrow Y \wedge X \rightarrow Z$

Closure of a set of FDs

Let F be a set of FDs, the **Closure** (F^+) of F is the set of all FDs implied by the FDs in F.

- can be computed using Armstrong's axioms

Attribute Closure

The **Closure ($C_f(X)$) of a set X of Attributes w.r.t. a set F of FDs** is the set of attributes we can derive from X using th FDs in F

$$C_F(X) = \{A \mid F \vdash X \to A\}$$

Properties

- $X \subseteq C_F(X)$
- $X \subseteq Y \Rightarrow C_F(X) \subseteq C_F(Y)$
- $C_F(C_F(X)) = C_F(X)$

Solution to implication Problem

$$F \models Y \to Z \iff Z \subseteq C_F(Y)$$

Closure Algorithm

Input: a set F of of FDs and a set X of attributes
Output: $C_F(X)$, the closure of X with respect to F

1. unused := F
2. closure := X
3. **while** $(Y \to Z) \in$ unused **and** $Y \subseteq$ closure
4. closure := closure $\cup Z$
5. unused := unused $-\{Y \to Z\}$
6. **return** closure