IDS Lecture 10: Translating Relational Calculus to Relational Algebra

Active Domain in Relational Algebra

For a Relation R over attributes $A_1, ..., A_n$

Adom(R) given by $\rho_{A_1 \to A}(\pi_{A_1}(R)) \cup \ldots \cup \rho_{A_n \to A}(\pi_{A_n}(R))$

Adom(D) $\bigcup_{R \in D} Adom(R)$ (where D is a Database)

- *Intuition:* return all elements of database in single column
- we denote $Adom_N$ (where N is a name) to the RA expression that returns the query.

From RC to RA

Translate each FOL formula ρ to a RA expression E

Assumptions

- no universal quantifiers, implications or double negations
  - see L07 "Quantifier Duality" for conversions
- no distinct pair of quantifiers binds the same variable name
- no variable name occurs both free and bound
- no variable names repeated within a predicate
- no constants in predicates
- no atoms of the form x **op** x or c_1 **op** c_2

Predicate

$R(x_1, ..., x_n) \implies \rho_{A_1 \text{is translated to} \eta(x_1), ..., A_n \to \eta(x_n)}(R)$

Example: For R over A, B, C, $R(x, y, z)$ is translated to $\rho_{A \to A_x, B \to A_y, C \to A_z}(R)$

Existential Quantification

$\exists x\, \varphi$ is translated to $\pi_{\eta(X - \{x\})}(E)$
where

- E is the translation of φ
- X is free(φ)

Example: For φ with free variables x, y, z and translation E,, $\exists y \varphi$ is translated to $\pi_{A_x, A_z}(E)$.

Comparisons

x **op** y is translated to $\sigma_{\eta(x) \textbf{ op } \eta(y)}(\mathbf{Adom}_{\eta(x)} \times \mathbf{Adom}_{\eta(y)})$

x **op** c is translated to $\sigma_{\eta(x)\ \mathbf{op}\ c}(\mathbf{Adom}_{\eta(x)})$

- where c is a constant

Example 1: $x = y$ is translated to $\sigma_{A_x=A_y}(\mathbf{Adom}_{A_x} \times \mathbf{Adom}_{A_y})$

Example 2: $x > 1$ is translated to $\sigma_{A_x>1}(\mathbf{Adom}_{A_x})$

Negation

$\neg\varphi$ is translated to $\prod_{x\in\mathbf{free}_\varphi} \mathbf{Adom}_{\eta(x)}) - E$

- where E is the translation of ϕ
- $\prod$ is the Cartesian product

Example: For φ with free variables x, y and translation E, $\neg\varphi$ is translated to $\mathbf{Adom}_{A_x} \times \mathbf{Adom}_{A_y} - E$

Disjunction

$\varphi_1 \vee \varphi_2$ is translated to $E_1 \times (\prod_{x\in X_2-X_1} \mathbf{Adom}_{\eta(x)} \cup E_2 \times (\prod_{x\in X_1-X_2} \mathbf{Adom}_{\eta(x)})$ where

- E_i is the translation of φ_i
- $X_i = \mathbf{free}(\varphi_i)$

Conjunction

Same as disjunction, but uses $\cap$ for $\cup$

Example: For Relations,

- Customer (C) : CustID, Name
- Account (A) : Number, CustID

Translate $\exists x_4 C(x_1, x_2) \wedge A(x_3, x_4) \wedge x_1 = x_4$

Solution:
Environment $\eta = \{x_1 \mapsto A, x_2 \mapsto B, x_3 \mapsto C, x_4 \mapsto D\}$

1. $C(x_1, x_2) \Rightarrow C$
2. $A(x_3, x_4) \Rightarrow \rho_{x_1\to x_4}(A)$
    - to remove clashes
3. $x_1 = x_4 \Rightarrow \sigma_{A=D}(\mathbf{Adom}_A \times \mathbf{Adom}_D)$
4. $2 \wedge 3 \Rightarrow 2 \times \mathbf{Adom}_A \cap 3 \times \mathbf{Adom}_B$
    - note 2 (LHS) and 3 (RHS) do not have same free variables, so we need to add the missing variables to each side
5. $1 \wedge 4 \Rightarrow (C \times \mathbf{Adom}_C \times \mathbf{Adom}_D) \cap 4 \times \mathbf{Adom}_B$
6. $\exists x_4 5 \Rightarrow \pi_{A,B,C}(5)$

Expanding out:

$$\begin{aligned}\pi_{A,B,C}((&C \times \mathbf{Adom}_C \times \mathbf{Adom}_D)\cap\\ &((\rho_{x_1 \to x_4}(A) \times \mathbf{Adom}_A)\cap\\ &(\sigma_{A=D}(\mathbf{Adom}_A \times \mathbf{Adom}_D)) \times \mathbf{Adom}_B) \times \mathbf{Adom}_B)\end{aligned}$$

- suspicious of final adom?

Equivalent to (solution on slides):

$$\begin{aligned}\pi_{A,B,C}((&C \times \mathbf{Adom}_C \times \mathbf{Adom}_D) \cap\\ &(A \times \mathbf{Adom}_A \times \mathbf{Adom}_B) \cap\\ &(\sigma_{A=D}(\mathbf{Adom}_A \times \mathbf{Adom}_D)) \times \mathbf{Adom}_B \times \mathbf{Adom}_C))\end{aligned}$$

- Where $C = \rho_{CustID \to A, Name \to B}(C)$
- $A = \rho_{Number \to C, CustID \to D}(A)$