IDS Lecture 19: Entailment of Constraints 2

Keys, candidate keys and prime candidate

Let R be a relation with set of attributes U and FDs F. $X \in U$ is a **key** for R if $F \models X \to U$. Equivalently, X is a key if $C_F(X) = U$ as $C_F(X) = U \iff \{A \mid F \models X \to A\}$.

Candidate Key (minimal set of attributes) X such that $\forall Y \subset X$, Y is not a key.

Prime attribute an attribute of a candidate key.

Computing all Candidate Keys

Algorithm *CandidateKeys(U : {Attribute}, F : {FD})* $\to$ *CK : {{Attribute}}*

1. $ck := \emptyset$
2. $G(V, E) := V = \{v \mid v \in \mathcal{P}(U)\}$, $E = \{\overrightarrow{XY} \mid X \in v, Y \in V, X - Y = \{A\}\}$
3. **while** G is not empty:
4. $\quad v :=$ node without children
5. $\quad$ **if** $C_F(X) = U$:
6. $\qquad ck := ck \cup \{X\}$
7. $\qquad G := G - (X + X_{ancestors})$
8. $\quad$ **else**:
9. $\qquad G := G - X$

- more optimal variant in the tutorial (lazy expansion of graph)

Implication of Inclusion Dependencies (INDs)

Inclusion Dependency Every X is a Y, such as in a foreign key constraint. *Example:* every manager is an employee.

Axiomatisation

Reflexivity $R[X] \subseteq R[X]$

Transitivity $R[X] \subseteq S[Y] \wedge S[Y] \subseteq T[Z] \Rightarrow R[X] \subseteq R[Z]$

Projection $R[X, Y] \subseteq S[W, Z]$ with $|X| = |W| \Rightarrow R[X] \subseteq S[W]$

Permutation $R[A_1, ..., A_n] \subseteq S[B_1, ..., B_n] \Rightarrow R[A_{i_1}, ..., A_{i_n}]S[B_{i_1}, ..., B_{i_n}]$
where $i_1, ...i_n$ is a permutation of $1, ..., n$.

FDs and INDs Together

We have shown,

1. Given a set F of FDs and an FD f we can decide whether $F \models f$
2. Given a set G of INDs and an INDs g we can decide whether $G \models g$

Implication Problem: Asking $F \cup G \models f$ or $F \cup G \models g$ is UNDECIDABLE, no algorithm exists can always solve it. This holds for the case of *keys* and *foreign keys* .