Lecture 4: Relational Algebra

Relational Algebra

Relational Algrbra Expression takes an input of relation(s) (R), applies a sequence of operations and returns a relation as an output.

Operations

Projection (π) vertical operation which chooses columns. Of general form

$$\pi_{A_1,\ldots,A_n}(R)$$

taking only the values of attributes A_1 to A_n for each tuple in R.

Selection (σ) horizontal operation on rows. Of general form

$$\sigma_{condition}(R)$$

taking only the tuples in R for which the condition is satisfied.

- for $\sigma_{\theta_1}(\sigma_{\theta_2}(R)) = \sigma_{\theta_1 \wedge \theta_2}(R)$, the RHS generally has faster runtime.

Product ($\times$) cartesian product *concatenates* each tuple of R with each tuples of S. Of general form

$$R \times S.$$

- conditional on the relations having a disjoint set of atributes
- $cardinality(R \times S) = cardinality(R) \times cardinality(S)$
  - where **Cardinality** is the number of attribues.
- $arity(R \times S) = arity(R) + arity(S)$
  - where **Arity** is the number of rows.

Renaming (ρ) gives a new name to some attribute of a relation with syntax

$$\rho_{replacements}(R)$$

where a replacement has the form $A \rightarrow B$.

Union Intersection & Difference

Note: Relations must have the same attributes.

Union ($\cup$) set of all rows in R and S
Intersection ($\cap$) all rows that belong to both R and S
Difference ($-$) all rows in A that are not in B

Joining relations

Joins can be created by combining Cartesian product ($\times$) with selection (σ).

Natural Join ($\bowtie$) joins two tables on their *common attributes*
Theta-join $R \bowtie_\theta S = \sigma_\theta(R \times S)$
Equijoin $\bowtie_\theta$ where θ is a *conjunction of equalities*
Semijoin $R \ltimes_\theta S = \pi_X(R \bowtie_\theta S)$ where X is the set of attributes of R
Antijoin $R \bar{\ltimes}_\sigma S = R - (R \ltimes_\sigma S)$

Translating SQL to/from Relational Algrbra

`SELECT` $\Longleftrightarrow$ projection (π)
`FROM` $\Longleftrightarrow$ Product ($\times$)
`WHERE` $\Longleftrightarrow$ selection (σ)

`SELECT` $A_1, ..., A_n$
`FROM` $T_1, ..., T_m$
`WHERE`
$\updownarrow$
$\pi_{A_1,...,A_n}(\sigma_{<condition>}(T_1 \times ... \times T_m))$
where common attributes in $T_1, ..., T_m$ must be renamed.