IDS Lecture 6: Predicate Logic

Free variables variables that are not in the scope of any quantifier. A variable that is not free is bound.

Interpretations

A formula may be true or false w.r.t a given *interpretation*.

Interpretation defines the semantics of the language; an assignment of variables that gives meaning to a statement.

Semantics of FOL: Interpretations

First Order Structure $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

Δ non empty domain of objects (universe)
$a^{\mathcal{I}}$ function which gives meaning to constant & predicate symbols

- $a^{\mathcal{I}} \in \Delta$ – gives meaning to *constants*, "object a by means of interpretation function $\mathcal{I}$".
- $R^{\mathcal{I}} \subseteq \Delta^1 \times ... \times \Delta^n$ – gives meaning to *predicates*, "mapping it to an element in our domain (objects in the unverse)"

Variable Assignment (v) maps each variable to an object in Δ

- *Notation:* $v[x/d]$ is v with x $\rightarrow$ d

Semantics of FOL: Terms

Interpreatation of terms under $(\mathcal{I}, v)$

$$x^{\mathcal{I},v} = v(x)$$

$$a^{\mathcal{I},v} = a^x$$

Formulas

$(\mathcal{I}, v) \models \phi$ means interpretation (I, v) satiisfies formula ϕ

$$\begin{aligned}
I, v \models P(t_1, ..., t_n) &\iff (t_1^{\mathcal{I},v}, ..., t_n^{\mathcal{I},v}) \in P \\
I, v \models \neg\phi &\iff \mathcal{I}, v \not\models \phi \\
I, v \models \phi \wedge \psi &\iff \mathcal{I}, v \models \phi \text{ and } \mathcal{I}, v \models \psi \\
I, v \models \phi \vee \psi &\iff \mathcal{I}, v \models \phi \text{ or } \mathcal{I}, v \models \psi \\
I, v \models \phi \rightarrow \psi &\iff \mathcal{I}, v \models \phi \text{ then } \mathcal{I}, v \models \psi \\
I, v \models \forall x\phi &\iff \text{for every } d \in \Delta : \mathcal{I}, v[x/d] \models \phi \\
I, v \models \exists x\phi &\iff \text{there exists } d \in \Delta \text{ s.t } \mathcal{I}, v[x/d] \models \phi
\end{aligned}$$