

习题4 不定积分

§ 1.1 填空题

1. 设 $F(x)$ 是 $f(x)$ 的一个原函数, 则 $\int e^{-x} f(e^{-x}) dx = \underline{\hspace{2cm}}$ $(-F(e^{-x}) + C)$
2. 设 $\int x f(x) dx = \arcsin x + C$, 求 $\int \frac{1}{f(x)} dx = \underline{\hspace{2cm}}$ $(\frac{1}{f(x)} = x\sqrt{1-x^2}, -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C)$
3. 如果 $f(x) = e^{-x}$, 则 $\int \frac{f'(\ln x)}{x} dx = \underline{\hspace{2cm}}$ $(\frac{1}{x} + C)$
4. $\int x f(x^2) f'(x^2) dx = \underline{\hspace{2cm}}$ $(\frac{1}{4}[f(x^2)]^2 + C)$
5. 已知 $f'(e^x) = x e^{-x}$, 且 $f(1) = 0$, 则 $f(x) = \underline{\hspace{2cm}}$ $(\frac{1}{2}(\ln x)^2)$

§ 1.2 求下列不定积分

1. $\int \frac{dx}{4-9x^2}$ $(\frac{1}{12} \ln |\frac{2+3x}{2-3x}| + C)$
2. $\int \frac{1}{\sqrt{4-9x^2}} dx$ $(\frac{1}{3} \arcsin(\frac{3}{2}x) + C)$
3. $\int \frac{1}{\sqrt{5-2x-x^2}} dx$ $(\arcsin(\frac{x+1}{\sqrt{6}}) + C)$
4. $\int \frac{dx}{\sqrt{x(4-x)}}$ $(\arcsin \frac{x-2}{2} + C)$
5. $\int \frac{1}{x\sqrt{1-\ln^2 x}} dx$ $(\arcsin(\ln x) + C)$
6. $\sin x \sin 3x dx$ $(\frac{1}{8}(2 \sin 2x + \sin 4x) + C)$
7. $\int \sin^2 3x dx$ $(\frac{1}{2}x - \frac{1}{12} \sin 6x + C)$
8. $\int \frac{1-\sin x}{x+\cos x} dx$ $(\ln |x+\cos x| + C)$
9. $\int \frac{dx}{1+\sin x}$ $(\tan x - \sec x + C)$
10. $\int \frac{dx}{\sin^2 x + 5 \cos^2 x}$ $(\frac{1}{\sqrt{5}} \arctan \frac{\tan x}{\sqrt{5}} + C)$

§ 1.3 第二类换元法

1. $\int \frac{dx}{(2-x)\sqrt{1-x}}$ $(-2 \arctan \sqrt{1-x} + C)$
2. $\int x^3 \sqrt{4-x^2} dx$ $(\frac{1}{5}(4-x^2)^{\frac{5}{2}} - \frac{4}{3}(4-x^2)^{\frac{3}{2}} + C)$
3. $\int \frac{dx}{x\sqrt{x^2+1}}$ $(-\ln \frac{1+\sqrt{1+x^2}}{x} + C)$

§ 1.4 分部积分法

$$1. \int \frac{\ln x - 1}{x^2} dx = \underline{\hspace{2cm}} \quad \left(-\frac{1}{x} \ln x + C\right)$$

$$2. \int \arctan \sqrt{x} dx = \underline{\hspace{2cm}} \quad ((x+1) \cdot \arctan \sqrt{x} - \sqrt{x} + C)$$

$$3. \int (\arcsin x)^2 dx = \underline{\hspace{2cm}} \quad (x(\arcsin x)^2 + 2\sqrt{1-x^2} \arcsin x - 2x + C)$$

$$4. f'(x^2) = \ln x (x > 0), \text{ 求 } f(x) \quad \left(\frac{1}{2}x \ln x - \frac{x}{2} + C\right)$$