AIC

Cours TC5

Result of TP2

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29 sept. 2016

1 Exercise1: The translation of image by DFT

1.1 Input

We choose the picture of a palace ceiling, Tx=200.8, Ty=400.3, as the inputs. Here the size of the picture is (607,805,3) which has 607*805 pixels and 3 color channels. The constant Tx and Ty is the vector given for the translation.

Figure 1: The origin image for translation

1.2 Steps of the translation

- 2D-DFT of the picture $u \to \hat{u}$ using the command fft2();
- calculate the phase matrix to multiply for each channel;

$$Iu(x+\tau_x, y+\tau_y) := \frac{1}{M}\frac{1}{N}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}\hat{u}(k,l)e^{2i\pi(x+\tau_x)}e^{2i\pi(y+\tau_y)} \quad (1)$$

* Pay attention that the second half of the matrix have to move a phase 2π.

- Transform the picture inversely using the command ifft2();

1.3 Show the result of translation

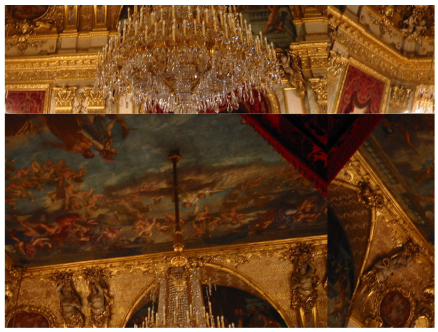

Figure 2: Translation by the vector(τ_x, τ_y)

2 Exercise2: Zooming-Zero padding

2.1 Input

We still choose the picture of the palace ceiling as the input, and the zoom factor is given 2 as example.

2.2 Steps of zooming

- 2D-DFT of the picture $u \rightarrow \hat{u}$ using the command fft2(),the size of $\hat{u}$ is (607,805,3);
- Add zeros to the high-frequency part of the matrix, namely the intern part. The object of this step is to zoom the matrix to the given size;

$$\begin{bmatrix} \hat{u}_{1,1} & \hat{u}_{1,2} & \dots & \hat{u}_{1,[N/2]} & \hat{u}_{1,[N/2]+1} & \dots & \hat{u}_{1,N-1} & \hat{u}_{1,N} \\ \hat{u}_{2,1} & \hat{u}_{2,2} & \dots & \dots & \dots & \dots & \hat{u}_{2,N-1} & \hat{u}_{2,N} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \hat{u}_{[M/2],1} & \dots & \dots & \dots & \dots & \dots & \dots & \hat{u}_{[M/2],N} \\ \hat{u}_{[M/2]+1,1} & \dots & \dots & \dots & \dots & \dots & \dots & \hat{u}_{[M/2]+1,N} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \hat{u}_{M-1,1} & \dots & \dots & \dots & \dots & \dots & \dots & \hat{u}_{M-1,N} \\ \hat{u}_{M,1} & \dots & \dots & \dots & \dots & \dots & \dots & \hat{u}_{M,N} \end{bmatrix} \tag{2}$$

$$\Downarrow$$

$$\begin{bmatrix} \hat{u}_{1,1} & \dots & \hat{u}_{1,[N/2]} & 0 & \dots & 0 & \hat{u}_{1,[N/2]+1} & \dots & \hat{u}_{1,N-1} & \hat{u}_{1,N} \\ \hat{u}_{2,1} & \dots & \dots & 0 & \dots & 0 & \dots & \dots & \hat{u}_{2,N-1} & \hat{u}_{2,N} \\ \dots & \dots & \dots & 0 & \dots & 0 & \dots & \dots & \dots & \dots \\ \hat{u}_{[M/2],1} & \dots & \dots & 0 & \dots & 0 & \dots & \dots & \dots & \hat{u}_{[M/2],N} \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ \hat{u}_{[M/2]+1,1} & \dots & \dots & 0 & \dots & 0 & \dots & \dots & \dots & \hat{u}_{[M/2]+1,N} \\ \dots & \dots & \dots & 0 & \dots & 0 & \dots & \dots & \dots & \dots \\ \hat{u}_{M-1,1} & \dots & \dots & 0 & \dots & 0 & \dots & \dots & \dots & \hat{u}_{M-1,N} \\ \hat{u}_{M,1} & \dots & \dots & 0 & \dots & 0 & \dots & \dots & \dots & \hat{u}_{M,N} \end{bmatrix} \tag{3}$$

- Transform the picture inversely using the command ifft2();

2.3 Show the result of zooming

Figure 3: zoom by factor=2(1)

The result above is a picture with size(1214,1610) which is (607*2,805*2). In this way, the zooming fact is well received. However, the picture is very dark, due to the narrowed amplitude.

We have time the signals with $factor^2 = 4$, so that the picture becomes brighter.

Figure 4: zoom by factor=2(2)

3 Theoretical Exercise: Image rotation by DFT

It is obvious to implement the rotation in the original space, but as for frequency domain, it seems a little bit complicated. As in the image translation, we implement the rotation matrix on each pixel in the inverse Fourier transform and get the result.

The simple algorithm is:

1) Calculate the 2 dimensions DFT of the original image

2) Use fftshift() to make sure the brightest part is in the center of the frequency domain

3)

$$rot\_matrix_1 = \begin{bmatrix} 1 & -tan(\theta) \\ 0 & 1 \end{bmatrix} \tag{4}$$

$$rot\_matrix_2 = \begin{bmatrix} 1 & 0 \\ sin(\theta) & 1 \end{bmatrix} \tag{5}$$

$$rot\_matrix_3 = \begin{bmatrix} 1 & -tan(\theta) \\ 0 & 1 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} X_o \\ Y_o \end{bmatrix} = rot\_matrix_1 \times rot\_matrix_2 \times rot\_matrix_3 \times \begin{bmatrix} X \\ Y \end{bmatrix} \tag{7}$$

X_o and Y_o here represent the coordinates after the rotation transform of [X,Y] and then we apply the new coordinates in the procedure of inverse Fourier transform

4) Keep the real part of the result