AIC

Cours TC5

Result of TP3

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1 Exercise: Rotating using the method of L. Yaroslavsky

1.1 Input

$$factor = 2;$$
$$theta = pi/8;$$

We choose the picture of a palace ceiling as the inputs. Here the size of the picture is (607,805,3) which has 607×805 pixels and 3 color channels. The parameter factor gives the scale of the enlarged picture, and theta is the angle of the rotation.

Figure 1: The origin image for rotation

1.2 Put the original picture in a larger background

In order to rotate the picture, we should, first of all, put the original one in a background large enough. This is because when rotating, the dimension of the picture should change in rows and columns. The enlarged size should be larger than the diagonal. Here the given parameter $factor$ indicates the enlarged size as $(factor * 607, factor * 805, 3)$.

Noted that the background is black in this case, given 0 to all the channels.

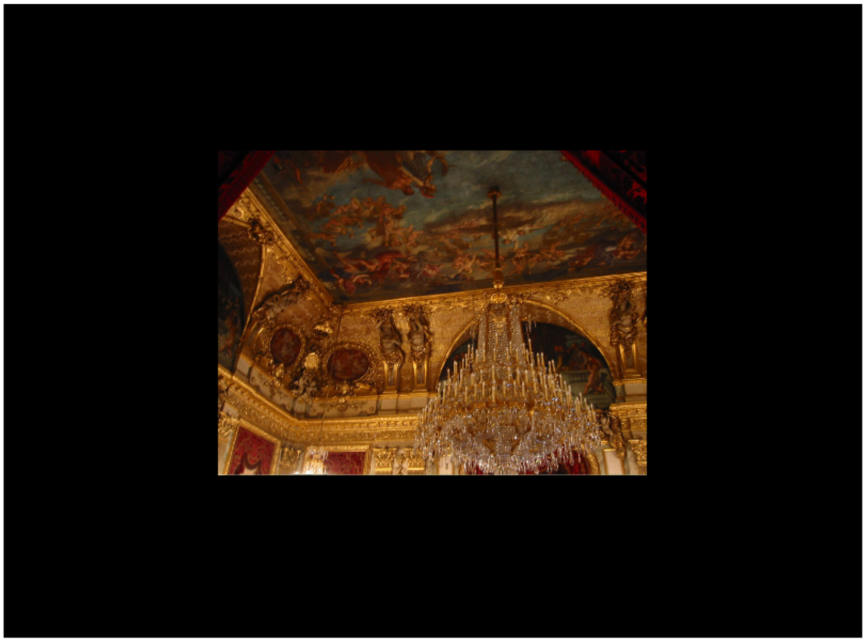

Figure 2: The picture in the background with $factor = 2$

1.3 The method of L.Yaroslavsky

It is obvious to implement the rotation in the original space, but as for frequency domain, it seems a little bit complicated. As in the image translation, we implement the rotation matrix on each pixel in the inverse Fourier transform and get the result.

The method of L. Yaroslavsky illustrated that the original rotation matrix can be converted into three matrices which are much more practical in an engineering environment. Since each translation only aim to manipulate lines or columns, we transfer the rotation operation into there translation operations.

$$\begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} = \begin{bmatrix} 1 & -tan(\theta/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sin(\theta) & 1 \end{bmatrix} \begin{bmatrix} 1 & -tan(\theta/2) \\ 0 & 1 \end{bmatrix} \tag{1}$$

The simple algorithm is rotate the picture by realising three tranlations seperately, namely matrix by matrix.

For each of the translation, the main steps is as follows:

1) Calculate the 1 dimensions DFT of the picture (in rows or colomns)
2) Use fftshift() to make sure the brightest part in the center of the frequency domain and calculate the phases or calculate the phases in two parts
3) Add the phases of each pixel
4) Transform the picture inversely using the command ifft()
5) Keep the real part of the result

For example, in the first matrix:

$$\begin{bmatrix} M' \\ N' \end{bmatrix} = \begin{bmatrix} 1 & -tan(\theta/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} M - Ntan(\theta/2) \\ N \end{bmatrix} \tag{2}$$

where $\begin{bmatrix} M \\ N \end{bmatrix}$ is the row index and column index of a pixel. This translation has no change in column index, however, a translation of $-Ntan(\theta/2)$ in row index. In the other word, for column(N), the pixel will have a $-Ntan(\theta/2)$ translation: the first column doesn't move; the second move $-tan(\theta/2)$;the third move $-2tan(\theta/2)$;...

1.4 Results of three translations

1.4.1 Matrix 1

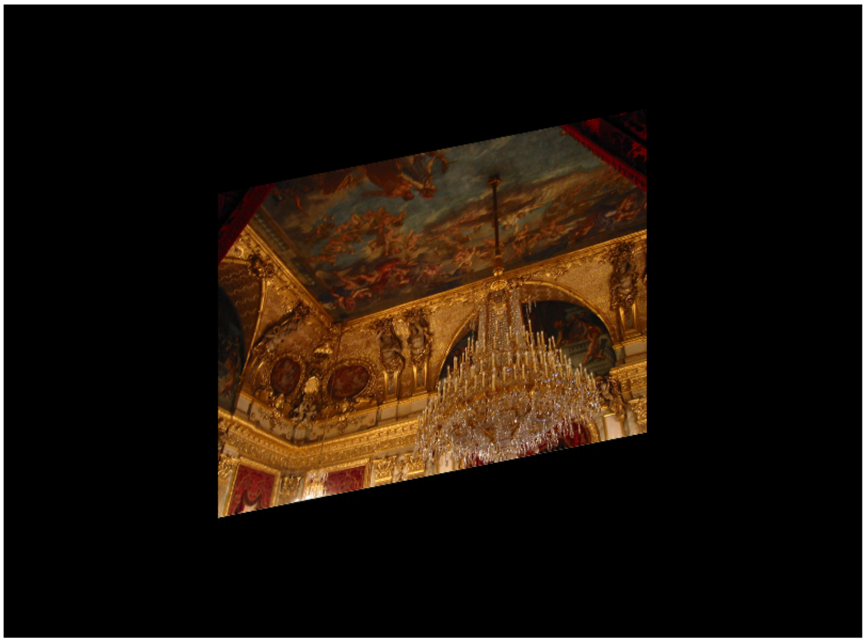

Figure 3: First translation

The result showed that each column moves correspondingly certain distance according to the first matrix.

1.4.2 Matrix 2

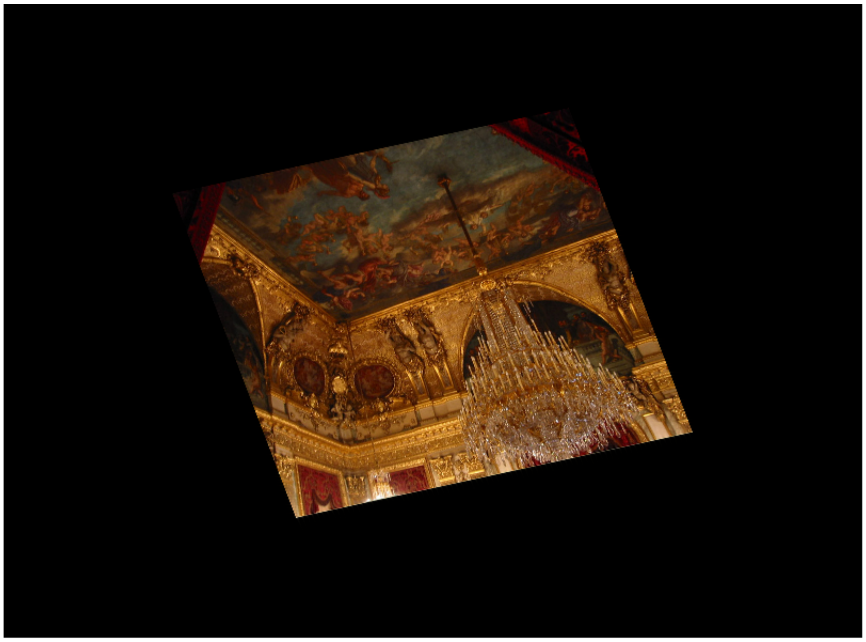

Figure 4: Second translation

We implemented the second matrix on the result of the first translation, which manipulated the rows correspondingly and got the result

1.4.3 Matrix 3

Figure 5: Third translation (final version)

We repeated the operation of the first translation to get the final version of the rotation.

2 Exercise: Exercise convolution

Please refer to scanned version below.

Exercice convolution — Binôme: Dong FEI & Xiaoxiao CHEN

Exercice 1 (ω — calcul de convolution)

1. On a $\forall n \in \mathbb{Z}$, $V(n) = U(n) - U(n-1) + 3U(n+1)$

Supposons la réponse impulsionnelle est $h(n)$

$$\omega(U(n)) = U * h(n) = \sum_{m=-\infty}^{+\infty} U(n-m)h(m)$$
$$= \cdots + U(n)h(0) + U(n-1)h(1) + U(n+1)h(-1) + \cdots$$
$$= U(n) - U(n-1) + 3U(n+1)$$

$$\Rightarrow h(n) = \begin{cases} 3 & n=-1 \\ 1 & n=0 \\ -1 & n=1 \end{cases}$$

Supposons $U(n) = \alpha U_1(n) + \beta U_2(n)$

$$\omega(U(n)) = \omega(\alpha U_1(n) + \beta U_2(n)) = \alpha U_1(n) + \beta U_2(n) - (\alpha U_1(n-1) + \beta U_2(n-1)) + 3(\alpha U_1(n+1) + \beta U_2(n+1))$$
$$= \alpha(U_1(n) - U_1(n-1) + 3U_1(n+1)) + \beta(U_2(n) - U_2(n-1) + 3U_2(n+1))$$
$$= \alpha V_1(n) + \beta V_2(n) \Rightarrow \text{linéaire}$$

$$\omega(U(n-\tau)) = \sum_{m=-\infty}^{+\infty} U(n-\tau-m)h(m) = U(n-\tau)h(0) + U(n-1-\tau)h(1) + U(n+1-\tau)h(-1)$$
$$= U(n-\tau) - U(n-1-\tau) + 3U(n+1-\tau)$$
$$= V(n-\tau) \Rightarrow \text{Invariante par translation}$$

2. $\forall n \in \mathbb{Z}$, $V(n) = U(2n)$

Supposons la réponse impulsionnelle est $h(n) \Rightarrow \omega(U(n)) = U * h(n) = V(n)$

$U(n) = \alpha U_1(n) + \beta U_2(n)$

$$\omega(U(n)) = \omega(\alpha U_1(n) + \beta U_2(n)) = \alpha U_1(2n) + \beta U_2(2n)$$
$$= \alpha V_1(n) + \beta V_2(n) \Rightarrow \text{linéaire}$$

$$\omega(U(n-\tau)) = U(2(n-\tau)) = U(2n-2\tau) \neq U(2n-\tau)$$

Donc Elle est linéaire mais pas invariant par translation

3. $\forall n \in \mathbb{Z}$, $V(n) = \max(U(n), U(n-1), U(n+1))$

$$\omega(U(n-\tau)) = \max(U(n-\tau), U(n-1-\tau), U(n+1-\tau)) = V(n-\tau)$$

$\Rightarrow$ Invariante par translation

On peut utiliser un contre exemple pour justifier elle n'est pas linéaire

si $U(n) = \delta(n) = \begin{cases} 1 & n=0 \\ 0 & \text{sinon} \end{cases}$ ou $\delta'(n) = \begin{cases} 1 & n=1 \\ 0 & \text{sinon} \end{cases}$

$$\omega(U(n)) = V(n) = \max(U(n), U(n-1), U(n+1))$$
$$\omega(\alpha U_1(n) + \beta U_2(n)) = \max(\alpha U_1(n) + \beta U_2(n), \alpha U_1(n-1) + \beta U_2(n-1), \alpha U_1(n+1) + \beta U_2(n+1))$$

Si $U_1(n) = \delta(n)$, $U_2(n) = \delta'(n)$

quand $n=0$ $\quad \omega(\alpha\delta(n) + \beta\delta'(n)) = \max(\alpha, \beta) \neq \alpha V_1(n) + \beta V_2(n) = \alpha + \beta$

4. $\forall n \in \mathbb{Z}, V_n = U_{n-1}$

Supposons la réponse impulsionnelle est $h(n) \Rightarrow W(U(n)) = U * h(n) = V(n)$

$U(n) = \alpha U_1(n) + \beta U_2(n)$

$W(U(n)) = W(\alpha U_1(n) + \beta U_2(n)) = \alpha U_1(n-1) + \beta U_2(n-1)$

$= \alpha V_1(n) + \beta V_2(n) \Rightarrow$ linéaire

$W(U(n-\tau)) = U(n-\tau-1) = V(n-\tau) \Rightarrow$ Invariante par translation

$W(U(n)) = U * h(n) = \sum_{m=-\infty}^{+\infty} U(n-m) h(m)$

$= \cdots + U(n-1) h(1) + \cdots = U(n-1)$

$\Rightarrow h(n) = \begin{cases} 1 & n = 1 \\ 0 & \text{sinon} \end{cases}$

5. $\underline{g(x) = \int_{x-\frac{1}{2}}^{x+\frac{1}{2}} f(x)\, dx\ x - 1/2\ f(t)\, dt\ ?}$

On prend $g(x) = \int_{x-\frac{1}{2}}^{x+\frac{1}{2}} f(t)\, dt$

$f(t) = \alpha f_1(t) + \beta f_2(t)$

$g(f(t)) = \int_{x-\frac{1}{2}}^{x+\frac{1}{2}} (\alpha f_1(t) + \beta f_2(t))\, dt = \alpha \int_{x-\frac{1}{2}}^{x+\frac{1}{2}} f_1(t)\, dt + \beta \int_{x-\frac{1}{2}}^{x+\frac{1}{2}} f_2(t)\, dt$

$= \alpha g(f_1(t)) + \beta g(f_2(t)) \Rightarrow$ linéaire

$g(f(t-\tau)) = \int_{x-\frac{1}{2}}^{x+\frac{1}{2}} f(t-\tau)\, dt \Rightarrow$ Invariant par translation

$f * h(x) = \int_{-\infty}^{+\infty} f(t) h(x-t)\, dt = g(x) = \int_{x-\frac{1}{2}}^{x+\frac{1}{2}} f(t)\, d(t)$

donc on a $h(x-t) = 1$ quand $x - t \in (x - \frac{1}{2}, x + \frac{1}{2})$

$\Rightarrow h(x) = \mathbb{1}_{[-\frac{1}{2}, \frac{1}{2}]}(x)$

6. $g(x) = \max\{f(t), t \in [x-1, x+1]\}$

$g(f(t-\tau)) = \max\{f(t-\tau), t-\tau \in [x-1, x+1]\} \Rightarrow$ Invariant par translation

$\cancel{f(t) = \alpha f_1(t) + \beta f_2(t)}$

Si $f_1(t) = \delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{Sinon} \end{cases}$ ~~et~~ $f_2(t) = \delta'(n) = \begin{cases} 1 & n = 1 \\ 0 & \text{sino} \end{cases}$

Si $t \in [-1, 1]$, $g(\alpha f_1(t) + \beta f_2(t)) = \max\{\alpha f_1(t) + \beta f_2(t), t \in [-1, 1]\}$

$t = 0$ ou $t = 1$

$= \max\{\alpha, \beta\} \neq \alpha + \beta$

Donc c'est invariant par translation mais pas linéaire

Exercice 2

1. $W(n) = U * V(n) = \sum_{m=-\infty}^{+\infty} U(m)V(n-m) = U(0)\,V(n) = V(n) = \sqrt{\log(\cos(3n)+2)}$

2. $W(n) = U * V(n) = \sum_{m=-\infty}^{+\infty} U(m)V(n-m) = U(0)V(n) + U(1)V(n-1)$

quand $n=0$ $\quad W(0) = U(0)V(0) = 2\times 5 = 10$

$n=1$ $\quad W(1) = U(0)V(1) + U(1)V(0) = 2\times 3 + (-\frac{1}{2})\times 5 = 3.5$

$n=2$ $\quad W(2) = U(0)V(2) + U(1)V(1) = 2\times 4 + (-\frac{1}{2})\times 3 = 6.5$

$n=3$ $\quad W(3) = U(1)V(2) = (-\frac{1}{2})\times 4 = -2$

$$W(n) = \begin{cases} 10 & n=0 \\ 3.5 & n=1 \\ 6.5 & n=2 \\ -2 & n=3 \\ 0 & \text{sinon} \end{cases}$$

3. On définit la convolution sur question (2) est $W'(n)$, $U'(n)$

ici On a $U_{-1} = U'_0 \quad U_0 = U'_1 \Rightarrow W(n) = W'(n+1)$

$$\left.\begin{aligned} \text{Comme } W(n) &= U * V(n) = \sum_{m=-\infty}^{+\infty} U(n-m)V(m) \\ &= U(n)V(0) + U(n-1)V(1) + U(n-2)V(2) \\ &= 5U(n) + 3U(n-1) + 4U(n-2) \end{aligned}\right\}$$

en pareil avec exercice 1

C'est un système a) Linéaire

b) Invariante par translation

donc
$$W(n) = \begin{cases} 10 & n=-1 \\ 3.5 & n=0 \\ 6.5 & n=1 \\ -2 & n=2 \\ 0 & \text{sinon} \end{cases}$$

4. Suite U est la somme des deux suites U précédentes

On définit les suites U précédentes commes U_1 et U_2

donc $U(n) = U_1(n) + U_2(n)$

parce que c'est un système linéaire et invariante par translation

On a $W(n) = W_1(n) + W_2(n)$

$$\Rightarrow \quad W(n) = \begin{cases} 10 & n=-1 \\ 13.5 & n=0 \\ 10 & n=1 \\ 4.5 & n=2 \\ -2 & n=3 \\ 0 & \text{sinon} \end{cases}$$

5. $U_n = (-\frac{1}{2})^n \quad (n \geqslant 0) \qquad V(n) = \begin{cases} 1 & n=0 \\ \frac{1}{2} & n=1 \\ 0 & \text{sinon} \end{cases}$

$W(n) = U * V(n) = \sum_{m=-\infty}^{+\infty} U(n-m)V(m)$

$= U(n)V(0) + U(n-1)V(1)$

$\cancel{= (-\frac{1}{2})^n + \frac{1}{2}(-\frac{1}{2})^{n-1}}$

$= U(n) + \frac{1}{2}U(n-1)$

$$\Rightarrow \quad W(n) = \begin{cases} 0 & n<0 \\ 1 & n=0 \\ 0 & n>0 \end{cases}$$