Exercise érit Tp6. Binôme Xiaoxiao Chen et Dong Fei

(1) $\hat{f}(\xi) = \int_{\mathbb{R}} f(x) e^{-ix\xi} dx$

$$= \int_{\mathbb{R}} \frac{\mathbb{1}_{[0,k]}(x)}{k} e^{-ix\xi} dx$$

$$= \frac{1}{k} \int_0^k e^{-ix\xi} dx$$

$$= -\frac{1}{i\xi k} [e^{-ix\xi}]_0^k$$

$$= \frac{1}{i\xi k} (1 - e^{-ik\xi})$$

$$= \frac{1}{i\xi k} e^{-\frac{1}{2}ik\xi} (e^{\frac{1}{2}ik\xi} - e^{-\frac{1}{2}ik\xi})$$

$$= \frac{1}{i\xi k} e^{-\frac{1}{2}ik\xi} \cdot 2i\sin(\tfrac{1}{2}k\xi)$$

$$= e^{-\frac{1}{2}k\xi} \cdot \operatorname{sinc}\left(\frac{k\xi}{2\pi}\right)$$ (in the note, it's writen $\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$)

(2) when a convolution is inversible. the transform of. $\hat{f}(\xi)$ should not be zero $\Rightarrow$ $0 < k \le \pi$. when $k > \pi$, the convolution is null.

~~so.~~ $\cancel{\frac{k\xi}{2\pi} \ne n\pi \quad n \in \mathbb{N}}$

$\Rightarrow$ so the convolution is inversible when $0 < k \le \pi$.

1. $\text{obs}(x) = \int_0^{\Delta t} u(x - vt)\, dt$

$$= \frac{1}{v} \int_0^{\Delta t} u(x - vt)\, d(vt)$$

$$= \frac{1}{v} \int_0^{v\Delta t} u(x - vt) \times \mathbb{1}\, d(vt)$$

we have f_k ~~take~~ give $k = v\Delta t$ $\quad f_{v\Delta t}(x) = \frac{\mathbb{1}_{[0, v\Delta t]}(x)}{v\Delta t}$

$$\Rightarrow \text{obs}(x) = \frac{1}{v} \int_0^{\Delta t} u(x - vt)\, v\Delta t\, f_{v\Delta t}(vt)\, d(vt)$$

$$= \Delta t \cdot u * f_{v\Delta t}(x).$$

(2) when $\Delta t \uparrow$, the image is more blurring.
(plus flou que l'image u).

3. i) – calculate the TFD of the picture u.

ii) – ~~calculate the TFD~~
multiply the TFD(u) with TFD(f_k)

iii) – use ifft2() the received the picture with blur.