Universitatea Tehnica din Cluj Napoca
Facultatea de Automatică și Calculatoare
Specializarea Automatică și Informatică Aplicată

Proiect

Teoria Sistemelor I

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Contents

1.1 Prezentare proces(vibration absorber):

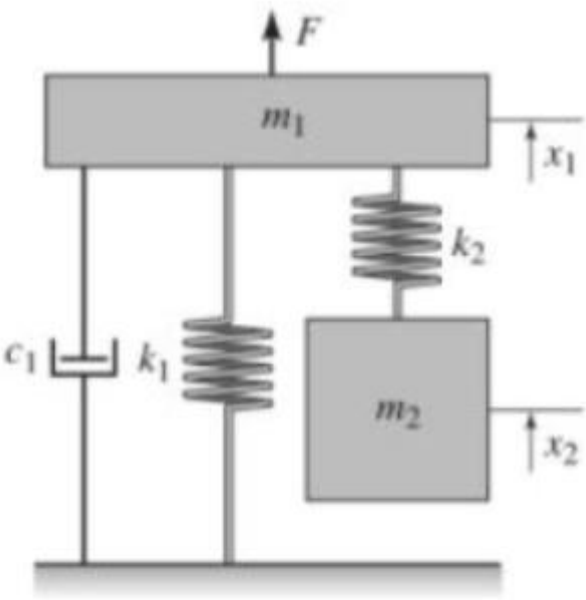


Figure 1: Schema procesului

Componente:

- 2 corpuri de masa m1, respectiv m2
- 2 resorturi elastice cu constanta elastica k1, respectiv k2
- un dumper cu coefficient de frecare b1

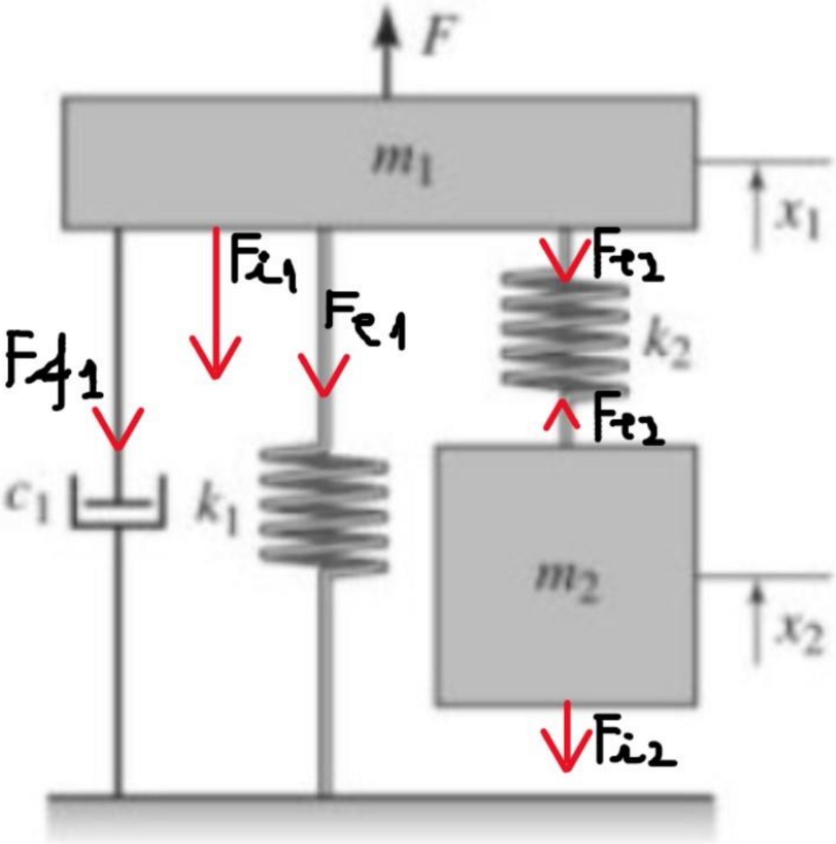
Semnalele de intrare:

F=u

Valorile numerice:

- $m_1 = 20kg$
- $m_2 = 15kg$
- $b_1 = 1000$
- $k_1 = 10^4$
- $k_2 = 15^4$

1.2 Modelul matematic u/x/y



Aleg variabilele de stare

$$\begin{pmatrix} x_1 \\ x_2 \\ \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

Scriu legea a II-a a lui Newton:

$$\begin{cases} (1): F = u = Ff_1 + Fe_1 + Fi_1 + Fe_2 \\ (2): Fi_2 = Fe_2 \end{cases}$$

$$\xrightarrow{(1)} Fi_1 = u - Ff_1 - Fe_1 - Fe_2$$

$$\rightarrow m_1\ddot{x_1} = u - b_1\dot{x_1} - k_1\dot{x_1} - k_2x_2 \ \ (3)$$

a$\xrightarrow{(2)} m_2\ddot{x_2} = k_2x_2$

Stim ca:

$$\begin{cases} \dot{x_1} = x_3 \rightarrow \ddot{x_1} = \dot{x_3} \\ \dot{x_2} = x_4 \rightarrow \ddot{x_2} = \dot{x_4} \end{cases}$$

$\xrightarrow{(1),(2),(3)}$:

$$\begin{cases} \dot{x_1} = x_3 \\ \dot{x_2} = x_4 \\ \dot{x_3} = -\dfrac{k_1}{m_1}x_1 - \dfrac{k_2}{m_1}x_2 - \dfrac{b_1}{m_1}x_3 + \dfrac{u}{m_1} \\ \dot{x_4} = \dfrac{k_2}{m_2}x_2 \\ y = x_1 \end{cases}$$

Asadar,realizarea de stare devine:

Simbolic:

$$\left(\begin{array}{cccc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\dfrac{k_1}{m_1} & -\dfrac{k_2}{m_1} & -\dfrac{b_1}{m_1} & 0 & -\dfrac{1}{m_1} \\ 0 & \dfrac{k_2}{m_2} & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right)$$

Valoric:

$$\left(\begin{array}{cccc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -500 & -2.5313e+03 & -50 & 0 & -0.05 \\ 0 & 3375 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right)$$

2. 1 Modelul intrare-iesire u/y

$$H(s) = \frac{L\{y(t)\}(s)}{L\{u(t)\}(s)} = \frac{Y(s)}{U(s)}$$

$$\rightarrow \frac{\frac{1}{m_1}s^2 - \frac{k_2}{m_1 m_2}}{s^4 + \frac{b_1}{m_1}s^3 - \left(\frac{k_2}{m_2} - \frac{k_1}{m_1}\right)s^2 - \frac{b_1 k_2}{m_1 m_2}s - \frac{k_1 k_2}{m_1 m_2}} = \frac{Y(s)}{U(s)}$$

$$\frac{1}{m_1}s^2 U(s) - \frac{k_2}{m_1 m_2}U(s) = s^4 Y(s) + \frac{b_1}{m_1}s^3 Y(s) - \left(\frac{k_2}{m_2} - \frac{k_1}{m_1}\right)s^2 Y(s)$$

$$sY - \frac{b_1 k_2}{m_1 m_2}(s) - \frac{k_1 k_2}{m_1 m_2}Y(s)$$

$$\frac{1}{m_1}\ddot{u}(t) - \frac{k_2}{m_1 m_2}u(t) = y^{(4)}(t) + \frac{b_1}{m_1}\dddot{y}(t) - \left(\frac{k_2}{m_2} - \frac{k_1}{m_1}\right)\ddot{y}(t) - \frac{b_1 k_2}{m_1 m_2}\dot{y}(t)$$

$$-\frac{k_1 k_2}{m_1 m_2}y(t)$$

2.2 Functia de transfer prin realizarea de stare

Pentru a calcula functia de transfer, voi folosi relatia:

(4)

$$\boxed{H(s) = \frac{Y(s)}{U(s)} = C\,(sI - A)^{-1}\,B + D.}$$

De unde stim ca:

$$(sI-A)^{-1} = \frac{1}{det(sI-A)^{-1}}(sI-A)^{*}$$

$$det(sI-A) = \begin{vmatrix} s & 0 & -1 & 0 \\ 0 & s & 0 & -1 \\ \frac{k_1}{m_1} & \frac{k_2}{m_2} & s+\frac{b_1}{m_1} & 0 \\ 0 & -\frac{k_2}{m_2} & 0 & s \end{vmatrix} \xrightarrow{sC_3+C_1}$$

$$= \begin{vmatrix} s & 0 & -1 & 0 \\ 0 & s & 0 & -1 \\ s^2+\frac{sb_1}{m_1}+\frac{k_1}{m_1} & \frac{k_2}{m_2} & s+\frac{b_1}{m_1} & 0 \\ 0 & -\frac{k_2}{m_2} & 0 & s \end{vmatrix} \rightarrow$$

$$\xrightarrow{(-1)^5} \begin{vmatrix} s & 0 & -1 & 0 \\ 0 & s & 0 & -1 \\ s^2+\frac{sb_1}{m_1}+\frac{k_1}{m_1} & \frac{k_2}{m_2} & s+\frac{b_1}{m_1} & 0 \\ 0 & -\frac{k_2}{m_2} & 0 & s \end{vmatrix}$$

$$-\begin{vmatrix} 0 & s & -1 \\ s^2+\frac{sb_1}{m_1}+\frac{k_1}{m_1} & \frac{k_2}{m_2} & 0 \\ 0 & -\frac{k_2}{m_2} & s \end{vmatrix} \xrightarrow{C_2+sC_3}$$

$$-\begin{vmatrix} 0 & 0 & -1 \\ s^2+\frac{sb_1}{m_1}+\frac{k_1}{m_1} & \frac{k_2}{m_2} & 0 \\ 0 & s^2-\frac{k_2}{m_2} & s \end{vmatrix}$$

Calculand determinantul, obtin:

$$det(sI-A) = \frac{1}{m_1m_2}[m_1m_2s^4+m_2b_1s^3-(k_2m_1-k_1m_2)s^2-b_1k_2s - k_1k_2]$$

$$\xrightarrow{(4)} H(s) = (1 \quad 0 \quad 0 \quad 0)\frac{(sI-A)^*}{\det(sI-A)}\begin{pmatrix} 0 \\ 0 \\ 1 \\ \frac{}{m_1} \\ 0 \end{pmatrix}$$

In urma unei analize a ecuatiei, observ ca doar pozitia (1,3) va fi diferita de 0,in urma inmultirii, asadar voi ignora restul elementelor din matrice.

Deci voi avea: (5)

$$(1 \quad 0 \quad 0 \quad 0)\frac{\begin{pmatrix} s & 0 & R & 0 \\ 0 & s & \frac{k_2}{m_2} & -\frac{k_2}{m_2} \\ -1 & 0 & s+\frac{b_1}{m_1} & 0 \\ 0 & -1 & 0 & s \end{pmatrix}}{\det(sI-A)}\begin{pmatrix} 0 \\ 0 \\ 1 \\ \frac{}{m_1} \\ 0 \end{pmatrix}$$

Deci:

$$R = (-1)^4 \begin{vmatrix} 0 & s & \frac{k_2}{m_2} \\ -1 & 0 & 0 \\ 0 & -1 & s \end{vmatrix} = \begin{vmatrix} s & \frac{k_2}{m_2} \\ -1 & s \end{vmatrix} = s^2 - \frac{k_2}{m_2}$$

$$\xrightarrow{(5)} H(s) = \frac{R * \frac{1}{m}}{\det(sI - A)}$$

Deci:

$$H(s)_{sim} = \frac{\frac{1}{m_1}s^2 - \frac{k_2}{m_1 m_2}}{s^4 + \frac{b_1}{m_1}s^3 - \left(\frac{k_2}{m_2} - \frac{k_1}{m_1}\right)s^2 - \frac{b_1 k_2}{m_1 m_1}s - \frac{k_1 k_2}{m_1 m_2}}$$

$$H(s)_{val} = \frac{0.05s^2 - 168.75}{s^4 + 50s^3 - 2875s^2 - 168752s - 1687500}$$

->Pentru verificare, voi introduce matricea A B C D in Matlab si voi folosi functia ss2tf(A,B,C,D), prin care voi obtine un numitor si un numarator

->Dupa aceea voi folosi functia tf(numarator,numitor), pe care o egalez cu H si obtin functia de transfer.

->Am folosit si functia ss() pentru a vedea mai bine spatiul starilor

```
A = [0 0 1 0; 0 0 0 1; -500 -2.5313e+03 -50 0; 0 3375 0 0];
B = [0;0;0.05;0];
C = [1 0 0 0 ];
D = [0];
sys = ss(A,B,C,D)
```

```
sys =

  A =
           x1     x2     x3     x4
   x1       0      0      1      0
   x2       0      0      0      1
   x3    -500  -2531    -50      0
   x4       0   3375      0      0

  B =
         u1
   x1     0
   x2     0
   x3  0.05
   x4     0

  C =
       x1  x2  x3  x4
   y1   1   0   0   0

  D =
       u1
   y1   0

Continuous-time state-space model.
```

```
[numarator,numitor]=ss2tf(A,B,C,D);
H = tf(numarator,numitor)
```

```
H =

                 0.05 s^2 - 168.8
  ----------------------------------------------
  s^4 + 50 s^3 - 2875 s^2 - 1.687e05 s - 1.688e06

Continuous-time transfer function.
```

3.Singularitatile sistemului

Pentru a afla singularitatile sistemului, voi folosii functiile pole(H) si zero(H)

```
zpk(H)
```

```
ans =

        0.05 (s-58.09) (s+58.09)
  ---------------------------------------
  (s-58.09) (s+58.09) (s+36.18) (s+13.82)

Continuous-time zero/pole/gain model.
```

```
Hm = minreal(H);
pole(H)
```

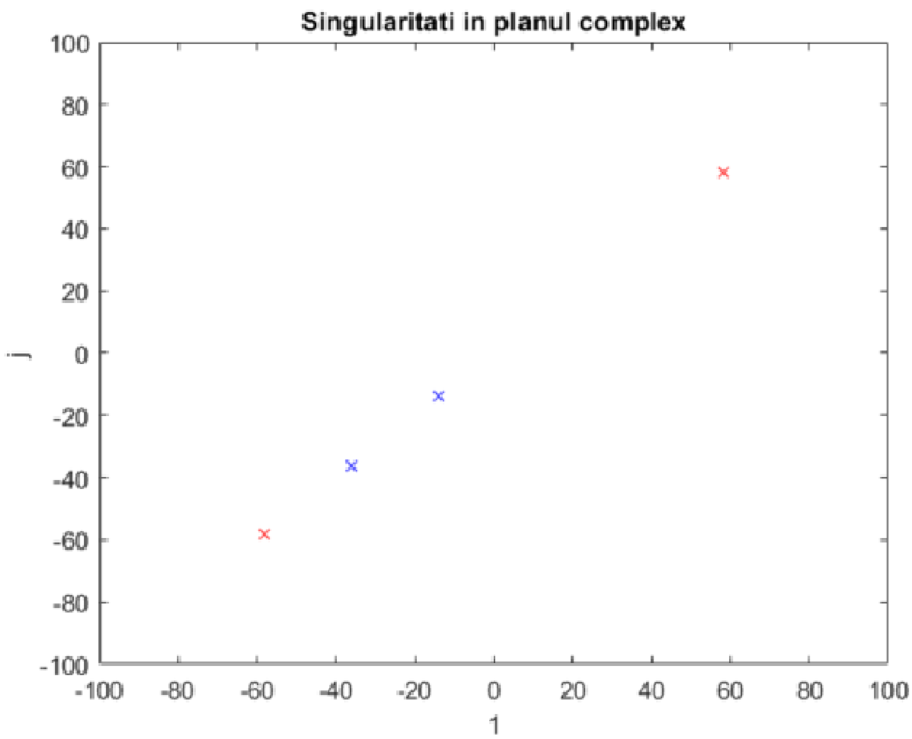
```
ans = 4x1
     58.0948
    -58.0948
    -36.1803
    -13.8197
```

```
zero(H)
```

```
ans = 2x1
     58.0948
    -58.0948
```

Mai departe, voi reprezenta singularitatile in planul complex

```
%Singularitati
figure,
plot(58.048,58.0948,'rx')
axis([-100 100 -100 100])
title('Singularitati in planul complex')
xlabel('1')
ylabel('j')
hold on
plot(-58.048,-58.0948,'rx')
hold on
plot(-36.1803,-36.1803,'bx')
hold on
plot(-13.8197, -13.8197, 'bx')
hold on
```



4.1 Forma canonica de control(FCC)

Se consideră sistemul LTI descris prin funcţia de transfer:

$$H(s) = d + \frac{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}.$$

Forma canonică de control este:

$$\left(\begin{array}{c|c} A_{FCC} & B_{FCC} \\ \hline C_{FCC} & D \end{array}\right) = \left(\begin{array}{cccccc|c} -a_{n-1} & -a_{n-2} & \dots & -a_1 & -a_0 & & 1 \\ 1 & 0 & \dots & 0 & 0 & & 0 \\ 0 & 1 & \dots & 0 & 0 & & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1 & 0 & & 0 \\ \hline b_{n-1} & b_{n-2} & \dots & b_1 & b_0 & & d \end{array}\right).$$

Aplicand teoria asupra sistemului ales, obtin:

$$\left(\begin{array}{cccc|c} -\dfrac{b_1}{m_1} & \dfrac{k_2}{m_2} - \dfrac{k_1}{m_1} & \dfrac{b_1 k_2}{m_1 m_2} & \dfrac{k_1 k_2}{m_1 m_2} & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & \dfrac{1}{m_1} & 0 & -\dfrac{k_2}{m_1 m_2} & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} -50 & 2875 & 168750 & 37968 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0.05 & 0 & -168.75 & 0 \end{array}\right)$$

Obtin ecuatiile de stare:

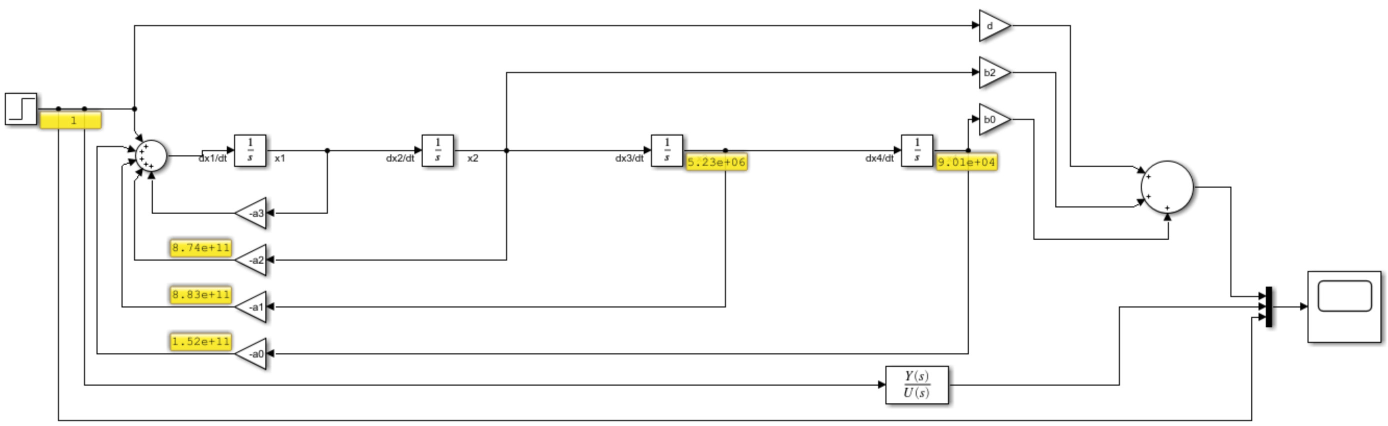
$$\begin{cases} \dot{x_1} = -a_3 x_1 - a_2 x_2 - a_1 x_3 + a_0 x_4 + u \\ \dot{x_2} = x_1 \\ \dot{x_3} = x_2 \\ \dot{x_4} = x_3 \\ \dot{y} = b_2\, x_2 + b_0\, x_4 + du \end{cases}$$

Pentru verificare, voi folosi functia ft2ss() in Matlab:

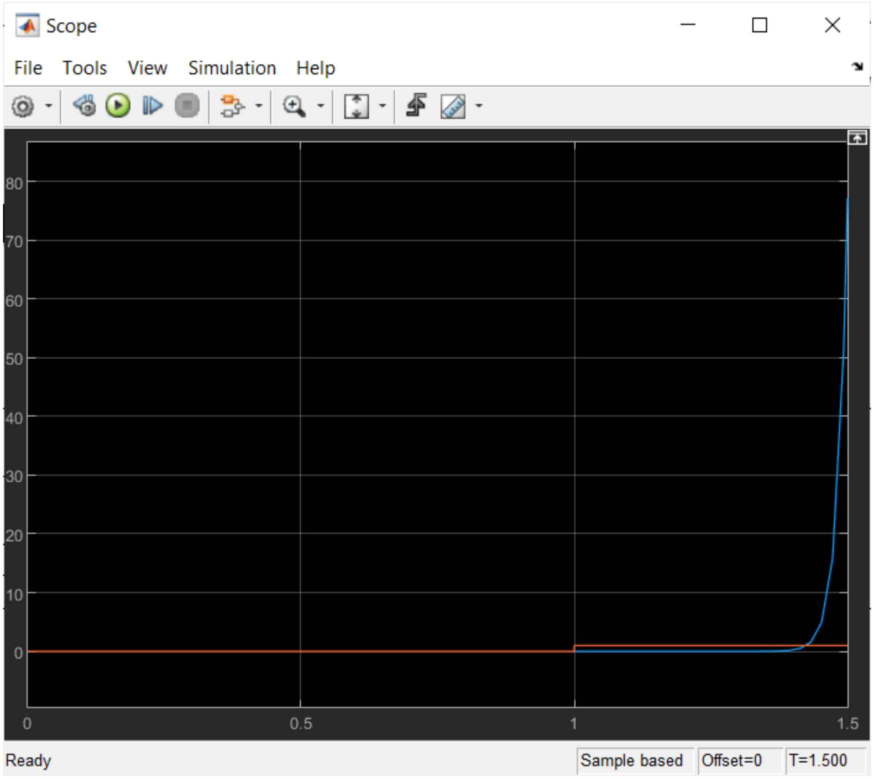
```
%FCC Functia tf2ss ret FCC din ABCD
H1 = tf(num,den);
[AFCC, BFCC, CFCC,D] = tf2ss(num,den)
```

```
AFCC = 4×4
        -50      2875    168752    1687500
          1         0         0          0
          0         1         0          0
          0         0         1          0

BFCC = 4×1
     1
     0
     0
     0

CFCC = 1×4
        0    0.0500         0  -168.7500

D = 0
```

Realizare schema Simulink



Scope:


Scope
File Tools View Simulation Help
80
70
60
50
40
30
20
10
0
0
0.5
1
1.5
Ready
Sample based
Offset=0
T=1.500

4.2 Forma canonica de observare (FCO)

Se consideră sistemul LTI descris prin funcţia de transfer:

$$H(s) = d + \frac{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}.$$

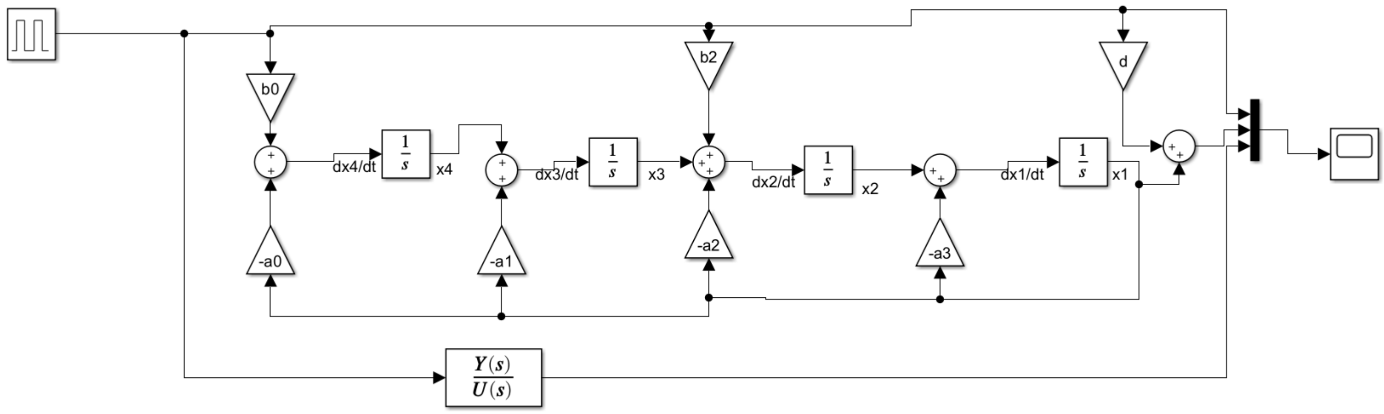
Forma canonică de observare este:

$$\left(\begin{array}{c|c} A_{FCO} & B_{FCO} \\ \hline C_{FCO} & D \end{array}\right) = \left(\begin{array}{ccccc|c} -a_{n-1} & 1 & \dots & 0 & 0 & b_{n-1} \\ -a_{n-2} & 0 & \dots & 0 & 0 & b_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -a_1 & 0 & \dots & 0 & 1 & b_1 \\ -a_0 & 0 & \dots & 0 & 0 & b_0 \\ \hline 1 & 0 & \dots & 0 & 0 & d \end{array}\right).$$

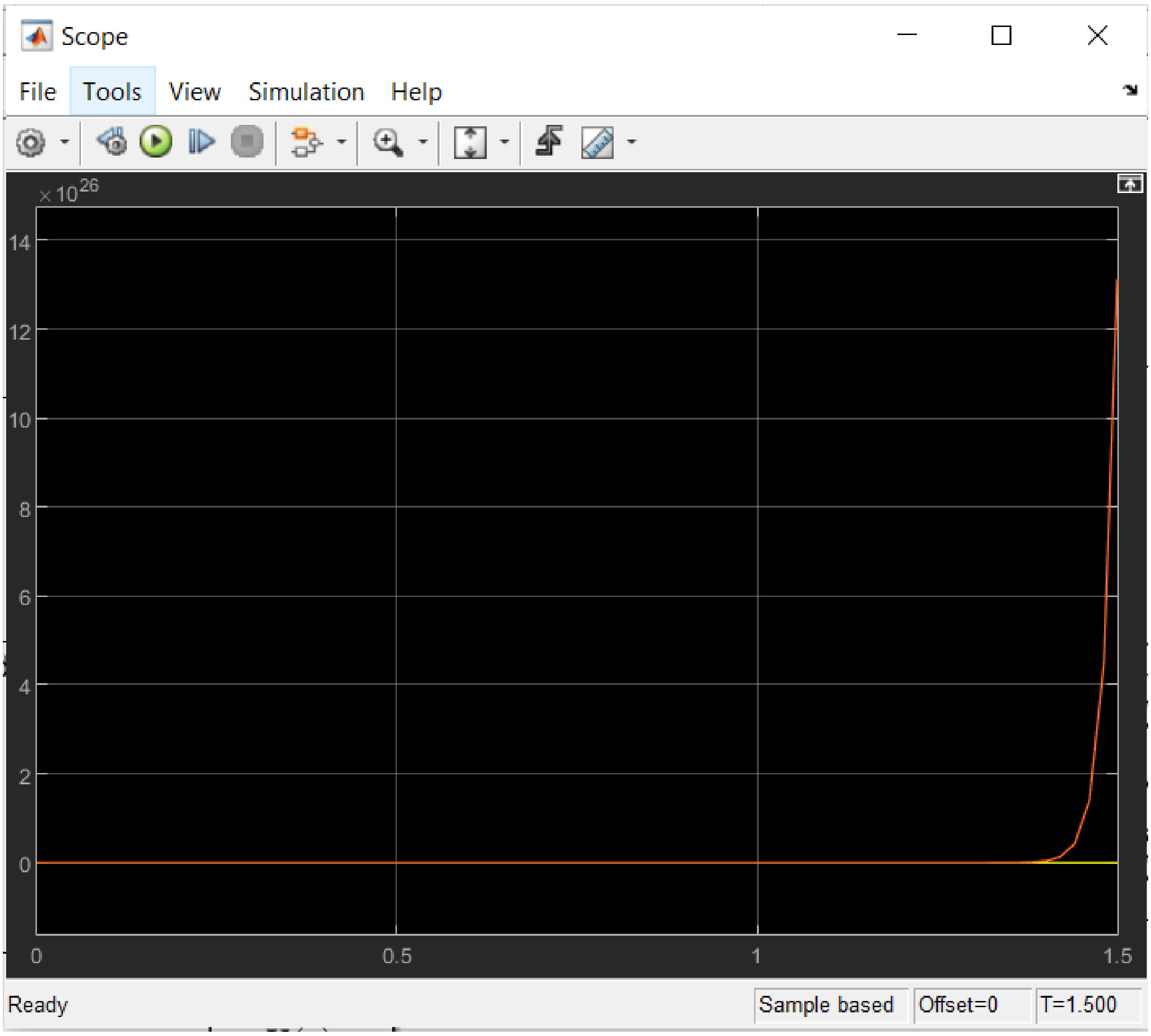
Pentru a obtine FCO, se realizeaza conversia din functia de transfer in FCC prin tf2ss si apoi se foloseste proprietatea de dualitate a celor doua forme canonice:

$$A_{FCO} = A_{FCC}^T, \qquad B_{FCO} = C_{FCC}^T, \qquad C_{FCO} = B_{FCC}^T$$

Realizare schema Simulink:



Scope:


Scope
File
Tools
View
Simulation
Help
× 10^26
14
12
10
8
6
4
2
0
0
0.5
1
1.5
Ready
Sample based
Offset=0
T=1.500

5. Deducere forma minimala functie de transfer

Conform teoriei, pentru a obtine forma minimala, trebuie sa impartim cele doua polinoame ce alcatuiesc functia de transfer.Dupa impartire vom obtine parametrii Markov cu ajutorul carora alcatuim matricea Hankel, daca rangul acesteia este egal cu gradul maxim al polului , atunci matricea este in forma minimala, altfel se va continua cu algoritmul.De cate ori repetam algoritmul, de atatea ori vom putea face reduceri.

Impartire polinoame:

$\cancel{\frac{1}{m_1}s^2} - \frac{k_2}{m_1 m_2}$	$s^4 + \frac{b_1}{m_1}s^3 - \left(\frac{k_2}{m_2} - \frac{k_1}{m_1}\right)s^2 - \frac{b_1 k_2}{m_1 m_2}s - \frac{k_1 k_2}{m_1 m_2}$
$\cancel{\frac{1}{m_1}s^2} + \frac{b_1}{m_1 m_1}$	$\frac{1}{m_1}s^{-2}$
$-\frac{1}{m_1}\left(\frac{k_2}{m_2} - \frac{k_1}{m_1}\right)$	● ● ●
$-\frac{1}{m_1}\frac{b_1 k_2}{m_1 m_2}s^{-1}$	● ● ●
$-\frac{1}{m_1}\frac{k_1 k_2}{m_1 m_2}s^{-2}$	
● ● ● $-$	

Analog mai departe...

Pentru cazul procesului ales, am nevoie de 7 parametrii Markov.Ii voi deduce din Matlab folosind functia deconv() si un N = 7 intr-un zeros(), pentru afisarea tuturor celor 7 parametrii:

```
%Parametrii Markov
num = [0,0,0.05,0,-168.75];
den = [1,50,-2875,-168752,-1687500];
N = 7;
format longg;
gv = deconv([num, zeros(1,N)],den)
```

gv = 1×8

	1	2	3	4	5
1	0	0	0.05	-2.5	100

Asadar, parametrii Markov sunt:

$\gamma_0 = 0,\ \gamma_1 = 0,\ \gamma_2 = 0.05,\ \gamma_3 = -2.5,\ \gamma_4 = 100,\ \gamma_5 = -3749.9,$

$\gamma_6 = 137490\ si$

$\gamma_7 = -49990125$

Matricea Hakel devine:

$$\begin{pmatrix} 0 & 0.05 & -2.5 & 100 \\ 0.05 & -2.5 & 100 & -3749.9 \\ -2.5 & 100 & -3749.9 & 137490 \\ 100 & -3749.9 & 137490 & -49990125 \end{pmatrix} \rightarrow \det(H_{44}) = 0$$

Umatorul pas este sa calculez determinantul acestei matrici.Observ ca este egal cu 0, deci functia mea de transfer nu este in forma minimala.

$$\begin{pmatrix} 0 & 0.05 & -2.5 \\ 0.05 & -2.5 & 100 \\ -2.5 & 100 & -3749.9 \end{pmatrix} \rightarrow \det(H_{33}) = 0$$

Reduc matricea la ordin 3 si calculez iar determinantul, observ ca tot 0 imi da.Deci voi avea 2 reduceri intre poli si zerouri in functia de transfer.

$$\begin{pmatrix} 0 & 0.05 \\ 0.05 & -2.5 \end{pmatrix} \rightarrow \det(H_{22}) \neq 0$$

Reduc matricea la ordinul 2 si observ ca determinantul este diferit de 0.Asadar, rangul matricei este egal cu 2.Dupa cele doua reduceri, functia mea de transfer va capata forma ei minimala.

Aplic mai departe algoritmul, formez ecuatia cu parametrii Markov

$$\begin{pmatrix} \gamma_3 \\ \gamma_4 \end{pmatrix} = -\begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_3 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

Obtin:

$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = -\begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_3 \end{pmatrix}^{-1}\begin{pmatrix} \gamma_3 \\ \gamma_4 \end{pmatrix}$$

Introduc in Matlab ecuatia pentru un rezultat corect si rapid:

```
H22 = [0,0.05;0.05,-2.5];
bv = [-2.5;100];
av = -inv(H22)*bv
```

```
av = 2×1
   500
    50
```

Asadar: $a_0 = 500\ si\ \ a_1 = 50, \rightarrow \alpha^{`}(s) = s^2 + 50s + 50$

Analog aplic si pentru Beta:

$$\begin{pmatrix} b_2 \\ b_1 \\ b_0 \end{pmatrix} = -\begin{pmatrix} \gamma_0 & 0 & 0 \\ \gamma_1 & \gamma_0 & 0 \\ \gamma_2 & \gamma_1 & \gamma_0 \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \\ a_0 \end{pmatrix}$$

Introduc in Matlab:

```
a1 = av(2);
a0 = av(1);
b_coef = [0,0,0;0,0,0;0.05,0,0] *[1;a1;a2]
```

```
b_coef = 3×1
     0
     0
  0.05
```

Asadar: $b_2 = 0$, $b_1 = 0$ si $b_0 = 0.05 \rightarrow \beta^{`}(s) = 0.05$

Intr-un final obtin:

$$H(S) = \frac{\beta^{`}(s)}{\alpha^{`}(s)} = \frac{0.05}{s^2 + 50s + 50}$$

Verific in Matlab pentru corectitudine:

```
zpk(H)
```

```
ans =

          0.05 (s-58.09) (s+58.09)
  ---------------------------------------
  (s-58.09) (s+58.09) (s+36.18) (s+13.82)

Continuous-time zero/pole/gain model.
```

```
Hm = minreal(H)
```

```
Hm =

         0.05
  ----------------
  s^2 + 50 s + 500

Continuous-time transfer function.
```

6.1 Stabilitate interna (Routh-Hurwitz)

Routh-Hurwitz este un criteriu matematic ce ne ajuta sa determinam numarul radacinilor din semiplanul drept al unui polinom.

Pentru a determina numărul rădăcinilor din semiplanul drept al unui polinom

$$P_c(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0,\ a_n \neq 0,$$

construim tabelul Routh-Hurwitz astfel:

$$\begin{array}{c|ccccc} s^n & a_n & a_{n-2} & a_{n-4} & \cdots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\ \hline s^{n-2} & b_1 & b_2 & b_3 & \cdots \\ s^{n-3} & c_1 & c_2 & c_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s^0 & m_1 & 0 & 0 & \cdots \end{array}$$

Reguli de completare:

- dacă nu există suficienţi coeficienţi pentru a completa o linie, aceasta se completează cu 0;

Realizez tabelul Routh-Hurwitz cu ajutorul polinomului caracteristic

$$Pc(\lambda) = \lambda^4 + 50\lambda^3 - 2875\lambda^2 - 50 * 3375\lambda \quad - 50 * 3375$$

$$\begin{array}{c|ccc} \lambda^4 & 1 & -2875 & -50 * 3375 \\ \lambda^3 & 50 & -50 * 3375 & 0 \\ \hline \lambda^2 & b_1 & b_2 & b_3 \\ \lambda^1 & c_1 & c_2 & 0 \\ \lambda^0 & d_1 & 0 & 0 \end{array}$$

$$b_1 = -\frac{\begin{vmatrix} 1 & -2875 \\ 50 & -50 * 3375 \end{vmatrix}}{50} = 500$$

$$b_2 = -\frac{\begin{vmatrix} 1 & -50 * 3375 \\ 50 & 0 \end{vmatrix}}{50} = -50 * 3375, \qquad b_{3=0}$$

$$c_1 = -\frac{\begin{vmatrix} 50 & -50 * 3375 \\ 500 & -50 * 3375 \end{vmatrix}}{500} = -45 * 3375, \quad c_2 = 0$$

$$d_1 = -\frac{\begin{vmatrix} 500 & -50 * 3375 \\ -45 * 3375 & 0 \end{vmatrix}}{-45 * 3375} = -\frac{95}{45}, \quad c_2 = 0$$

Asadar, obtin:

$$\begin{pmatrix} 1 \\ 50 \\ 500 \\ -45 * 3375 \\ -\frac{95}{45} \end{pmatrix}$$

de unde rezulta ca exista o singura schimbare de semn, deci exista o singura radacina in semiplanul drept al polinomului.

Pentru corectitudine, verific si in Matlab utilizand functia eig(A)

```
%stabilitate


%stabilitate interna metoda 1
eig(A)%->NU este pozitiv definita(nu toate v.p sunt pozitive)->sistemul NU este intern asimptotic stabil
```

```
ans = 4x1
          -13.8196601125011
          -36.1803398874989
           58.0947501931113
          -58.0947501931113
```

Se poate observa ca exista o singura valoare pozitiva, asadar avem o singura radacina in semiplanul drept

6.1 Stabilitate externa

Pentru a determina stabilitatea externa, voi folosi forma minimala a functiei de transfer Hm, daca polii acestuia sunt toti negativi, atunci functia va fi stabil externa, altfel instabil externa

Pentru corectitudine voi folosi Matlab:

```
%stabilitate externa
Hm
```

```
Hm =

       0.05
  ----------------
  s^2 + 50 s + 500

Continuous-time transfer function.
```

```
zpk(Hm)
```

```
ans =

        0.05
  -------------------
  (s+36.18) (s+13.82)

Continuous-time zero/pole/gain model.
```

```
pole(Hm)
```

```
ans = 2×1
      -36.180339887499
      -13.8196601125011
```

Ambii poli sunt negativi, asadar sistemul este extern stabil

7.Stabilitate interna cu Lyapunov

Matricea A este instabila, are valori proprii negative, deci functia de energie nu va converge la 0 sau la o valoare constanta.
Voi deduce un P in Matlab cu functia lyap.
Functia de energie este prin definitie o functie pozitiv definita:

$$V(x) = x^T P x$$

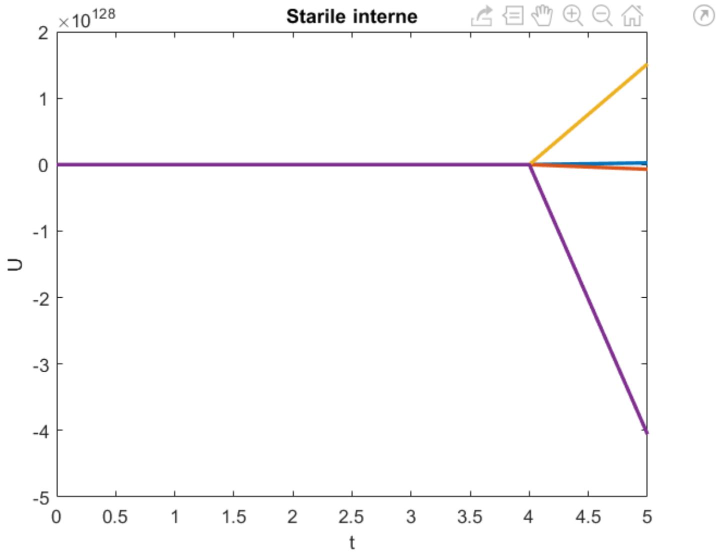
$$V\big(x(t)\big) = V(t) = x^T(t) P x(t)$$

Voi obtine un vector V, de scalari, ce vor reprezenta energiile la fiecare moment de timp.

```
Q = 20*eye(4);
P = lyap(A',Q);
eig(P);
```

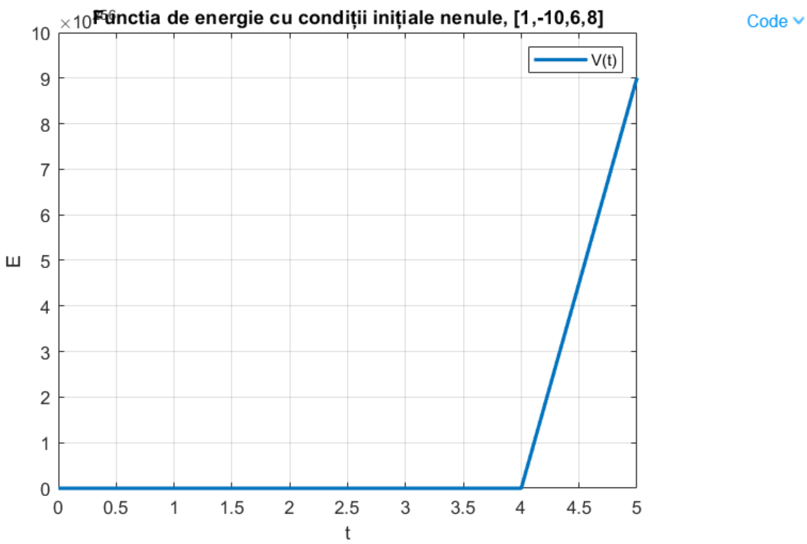
```
ans = 4×1
    -4.68689187721851e+18
         -23.857177734375
         111.433231375314
     1.38870870436104e+15
```

```
%%
t = 0:1:5;
%u = zeros(1,length(t));
u = ones(1,length(t));
[y,t,x]=lsim(sys,u,t,[1,-10,6,8]);

figure,
plot(t,x,'LineWidth',2),
xlabel('t'), ylabel('U'), title('Starile interne');
```




```
V = zeros(1,length(t));

for i=1:length(t)
   V(i) = x(i,:)*P*x(i,:)';
end
V(i) = x(i,:)*P*x(i,:)'
```

```
figure,
plot(t,V,'LineWidth',2), shg, legend('V(t)'), grid;
xlabel('t'), ylabel('E'),title('Functia de energie cu condiții inițiale nenule, [1,-10,6,8]');
```




```
%instabil intern, nu ne atingem de el dar tot tinde la infinit
```

8.1 Functia pondere

Raspunsul la impuls al unui sistem este dat de:

$$h(t) = L^{-1}\{H(s)\}(functia\ pondere)$$

Functia de transfer in forma minimala este:

$$H(s) = \frac{0.05}{s^2 + 50s + 50}$$

In cazul rezolvarii ecuatiei de gradul II de la numitor, voi obtine un delta>0.

Asadar, polii vor fi reali, si voi folosi urmatoarea relatie predata la ora de laborator:

$$H_4(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)}$$

Functia H(s) se va putea scrie :

$$H(s) = \frac{0.05}{(s + 36.18)(s + 13.82)}$$

Voi da factor fortat valorile polilor pentru a obtine forma specifica pentru sistemele de gradul II

$$H(s) = \frac{0.05}{36.18 * 13.82(\frac{s}{36.18} + 1)(\frac{s}{13.82} + 1)}$$

$$\rightarrow H(s) = \frac{5 * 10^{-4}}{(\frac{s}{36.18} + 1)(\frac{s}{13.82} + 1)}$$

Dupa ce am obtinut forma specifica, voi putea deduce:

- constantele de timp: $T_1 = 0.02\ si\ T_2 = 0.07$
- factor de amortizare ζ nu am
- factorul de proportionalitate: $K = 5 * 10^{-4}$

- pulsatie naturala ω_n nu am

Aplic Laplace invers si obtin conform indrumatorului de laborator relatia:

$$h_4(t) = \mathcal{L}^{-1}\left\{\frac{K}{(T_1 s+1)(T_2 s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{KT_1}{T_1-T_2}}{T_1 s+1} + \frac{\frac{KT_2}{T_2-T_1}}{T_2 s+1}\right\} =$$

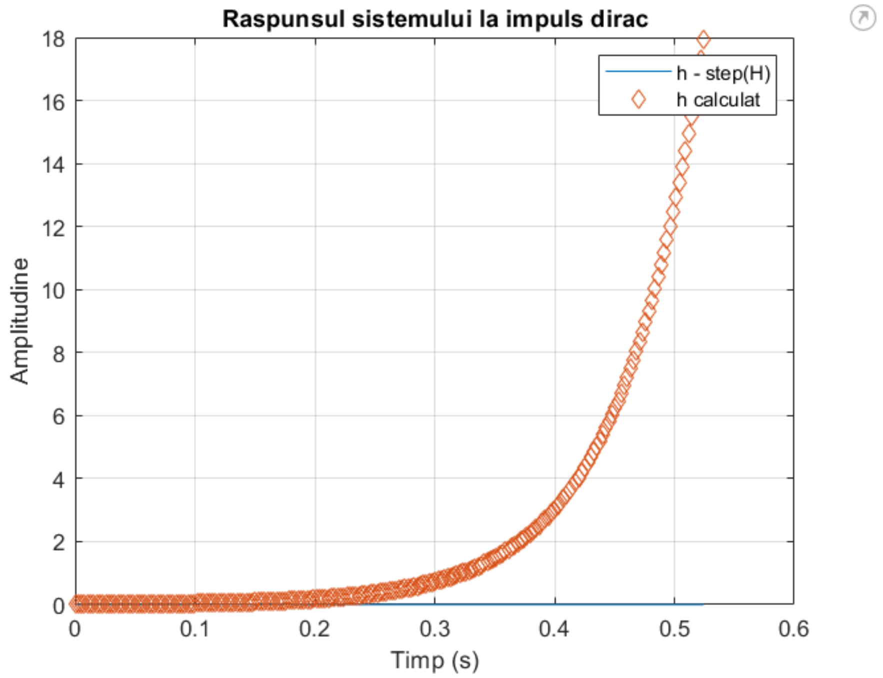
$$= \frac{K}{T_1-T_2}\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{T_1}} - \frac{1}{s+\frac{1}{T_2}}\right\} = \frac{K}{T_1-T_2}\left(e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}}\right), \quad t \geq 0, \text{ cu } T_1 \neq T_2,$$

Valoarea numerica:

$$h_4(t) = \frac{K}{T_1 - T_2}\left(e^{-\frac{t}{T1}} - e^{-\frac{t}{T2}}\right) = -0.01 * \left(e^{-\frac{t}{0.02}} - e^{-\frac{t}{0.07}}\right)$$

Reprezentare Matlab:

```
%%Functia pondere(raspunsul la impuls)
[y2,t2]=impulse(Hm);
y_calc2 = -0.01*(exp(-t2/0.02)-exp(t2/0.07)) ;
figure, plot(t2,y2,t2,y_calc2,'d'), title('Raspunsul sistemului la impuls dirac');
xlabel('Timp (s)'), ylabel('Amplitudine'), grid, legend('h - step(H)','h calculat')
```



Se poate observa ca cele doua grafice nu se suprapun, asadar am gresit.

Am incercat sa folosesc functiile residue() si ilaplace() pentru a putea implementa rezolvarea cu ajutorul functiei de transfer H(s)

```
%Obtinere factori din functia de transfer
B = [0,0,0.05,0,-168.8];
A = [1,50,-2875,-1.687e05,-1.688e06]
```

```
A = 1×5
            1          50       -2875     -168700    -1688000
```

```
[R,P,K] = residue(B,A)
```

```
R = 4×1
    -1.10568928941369e-06
     -8.5998968956898e-08
     -0.00224083961270017
      0.00224203130095854

P = 4×1
        -58.1248951368228
         58.0916973516086
        -36.1303873000081
        -13.8364149147776

K =

     []
```

Am obtinut fiecare termen descompus si i-am aplicat $L^{-1}\{\}$

```
syms s t w x y f(x)
%ilaplace(0.015/(s+58.12))
ilaplace(0.015/(s+58.12))
```

ans =

$$\frac{3\,e^{-\frac{1453\,t}{25}}}{200}$$

```
ilaplace(0.0002/(s-58.09))
```

ans =

$$\frac{e^{\frac{5809\,t}{100}}}{5000}$$

```
ilaplace(0.0036/(s+36.13))
```

ans =

$$\frac{9\,e^{-\frac{3613\,t}{100}}}{2500}$$

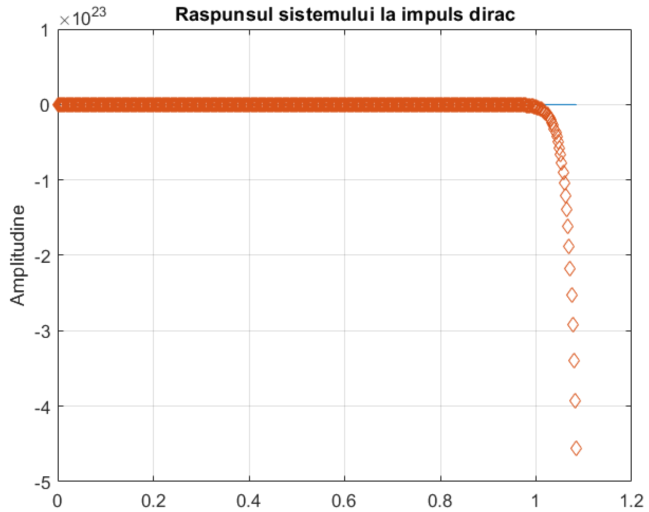
```
ilaplace(0.00238/(s+13.83))
```

ans =

$$\frac{119\,e^{-\frac{1383\,t}{100}}}{50000}$$

Dupa care le-am adunat in y_calc2 si am reprezentat grafic a doua oara

```
%%Functia pondere(raspunsul la impuls)
[y2,t2]=impulse(H1);
y_calc2 = 3*exp(-1453*t2/25)/200 -exp(5809*t2/100)/5000 - 9*exp(-3613*t2/100)/2500 + 119*exp(-1383*t2/100)/50000 ;
y_calc22 = -0.01*(exp(-t2/0.02)-exp(-t2/0.07));
figure, plot(t2,y2,t2,y_calc2,'d'), title('Raspunsul sistemului la impuls dirac');
xlabel('Timp (s)'), ylabel('Amplitudine'), grid;
```



Observ ca tot nu mi se suprapun, deci iar am gresit.

8.2 Raspunsul indicial

Raspunsul la treapta al unui sistem este dat de:

$$y(t) = L^{-1}\left\{H(s)\frac{1}{s}\right\}(raspuns\ oficial)$$

$$H(s) = \frac{0.05}{s^2 + 50s + 50} * \frac{1}{s} = \frac{5 * 10^{-4}}{\left(\frac{s}{36.18} + 1\right)\left(\frac{s}{13.82} + 1\right)} * \frac{1}{s}$$

```
numi = [0,0,0,0.5];
deni = [1,50,50,0];
Hi = tf(numi,deni);

pole(Hi)
```

```
ans = 3×1

                   0
 -48.9791576165636
  -1.0208423834364
```

Rezolvand ecuatia de gradul 3 de la numitor in Matlab, obtin tot 3 valori proprii reale, asadar voi putea folosi relatia de la laborator:

$$y_4(t) = \mathcal{L}^{-1}\left\{\frac{K}{(T_1 s + 1)(T_2 s + 1)} \cdot \frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{K}{s} + \frac{\frac{KT_1}{T_2 - T_1}}{s + \frac{1}{T_1}} + \frac{\frac{KT_2}{T_1 - T_2}}{s + \frac{1}{T_2}}\right\} \Rightarrow$$

$$\Rightarrow y_4(t) = K\left(1(t) + \frac{T_1}{T_2 - T_1}e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2}e^{-\frac{t}{T_2}}\right),\quad t \geq 0,\ \text{cu}\ T_1 \neq T_2,$$

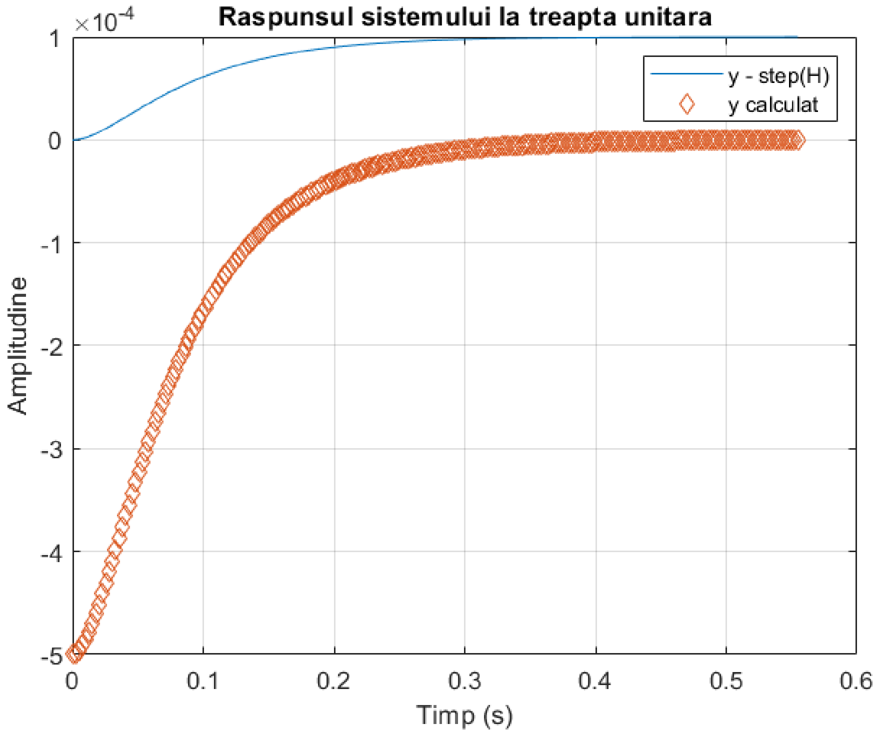
Obtin forma numerica:

$$y(t) = k\left(1(t) + \frac{T_1}{T_2 - T_1}e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2}e^{-\frac{t}{T_2}}\right)$$

$$= 5 * 10^{-4}\left(0.4\, e^{-\frac{t}{0.02}} - 1.4\, e^{-\frac{t}{0.07}}\right)$$

Obtin urmatorul grafic in Matlab:

```
%raspunsul la treapta
[y1,t1]=step(Hm);
y_calculat1 = 0.0005*(0.4*exp(-t1/0.02)-1.4*exp(-t1/0.07));
figure,
plot(t1,y1,t1,y_calculat1,'d'),title('Raspunsul sistemului la treapta unitara');
xlabel('Timp (s)'), ylabel('Amplitudine'), grid, legend('y - step(H)','y calculat')
```



Nu se suprapun, iar am gresit.

8.3 Raspunsul la rampa

Raspunsul la rampa al unui sistem este dat de:

$$y(t) = L^{-1}\left\{H(s)\frac{1}{s^2}\right\}$$

$$H(s) = \frac{0.05}{s^2 + 50s + 50} * \frac{1}{s^2} = \frac{5 * 10^{-4}}{\left(\frac{s}{36.18} + 1\right)\left(\frac{s}{13.82} + 1\right)} * \frac{1}{s^2}$$

Rezolvand ecuatia de gradul 4 de la numitor in Matlab, obtin tot 4 valori proprii reale, asadar voi putea folosi relatia de la laborator:

```
pole(Hi);
numr = [0,0,0,0,0.5];
denr = [1,50,50,0,0];
Hr = tf(numr,denr);

pole(Hr)
```

```
ans = 4×1
                    0
                    0
  -48.9791576165636
   -1.0208423834364
```

$$y_{4v}(t) = \mathcal{L}^{-1}\left\{\frac{K}{(T_1 s + 1)(T_2 s + 1)} \cdot \frac{1}{s^2}\right\} = \mathcal{L}^{-1}\left\{\frac{K}{s^2} - \frac{K(T_1 + T_2)}{s} + \frac{\frac{KT_1^3}{T_1 - T_2}}{T_1 s + 1} + \frac{\frac{KT_2^3}{T_2 - T_1}}{T_2 s + 1}\right\} \Rightarrow$$

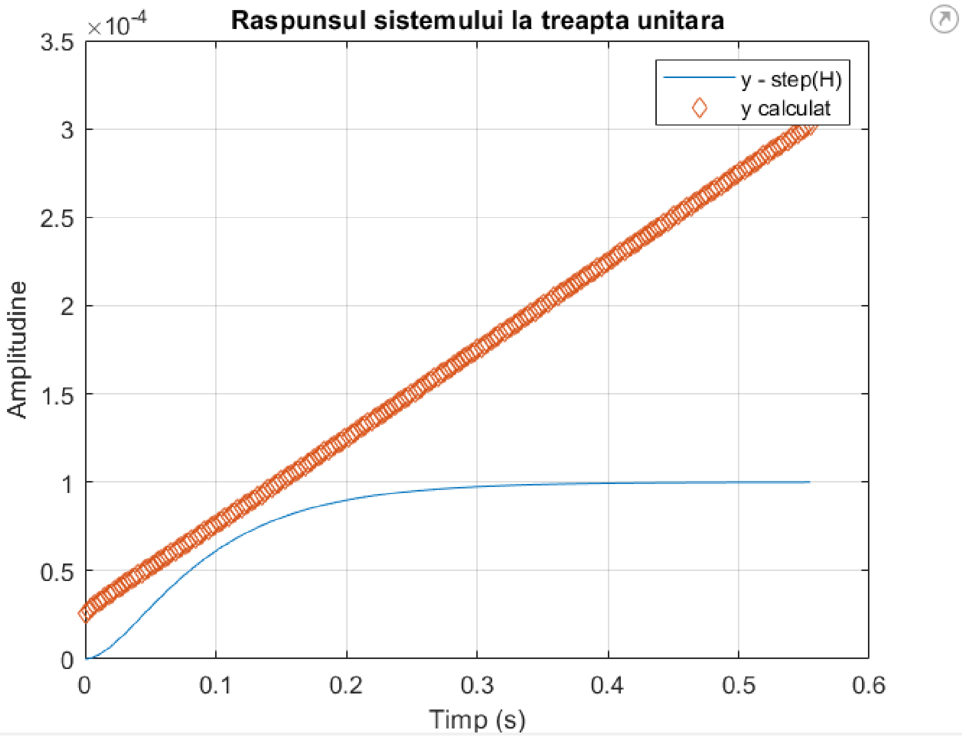
$$\Rightarrow y_{4v}(t) = K\left(t - (T_1 + T_2) + \frac{T_1^2}{T_1 - T_2}e^{-\frac{t}{T_1}} + \frac{T_2^2}{T_2 - T_1}e^{-\frac{t}{T_2}}\right), \quad t \geq 0, \text{ cu } T_1 \neq T_2,$$

Obtin forma numerica:

$$y_{4v}(t) = k\left(t - (T_1 - T_2) + \frac{{T_1}^2}{T_1 - T_2}e^{-\frac{t}{T_1}} + \frac{{T_2}^2}{T_2 - T_1}e^{-\frac{t}{T_2}}\right)$$

$$= 5 * 10^{-4}\left(t + 0.05 - 0.008 * e^{-\frac{t}{0.02}} + 0.01\, e^{-\frac{t}{0.07}}\right)$$

```
%raspunsul la treapta
[y1,t1]=step(Hm);
y_calculat1 = 0.0005*(t1+0.05-0.008*exp(-t1/0.02)+0.01*exp(-t1/0.07));
figure,
plot(t1,y1,t1,y_calculat1,'d'),title('Raspunsul sistemului la treapta unitara');
xlabel('Timp (s)'), ylabel('Amplitudine'), grid, legend('y - step(H)','y calculat')
```

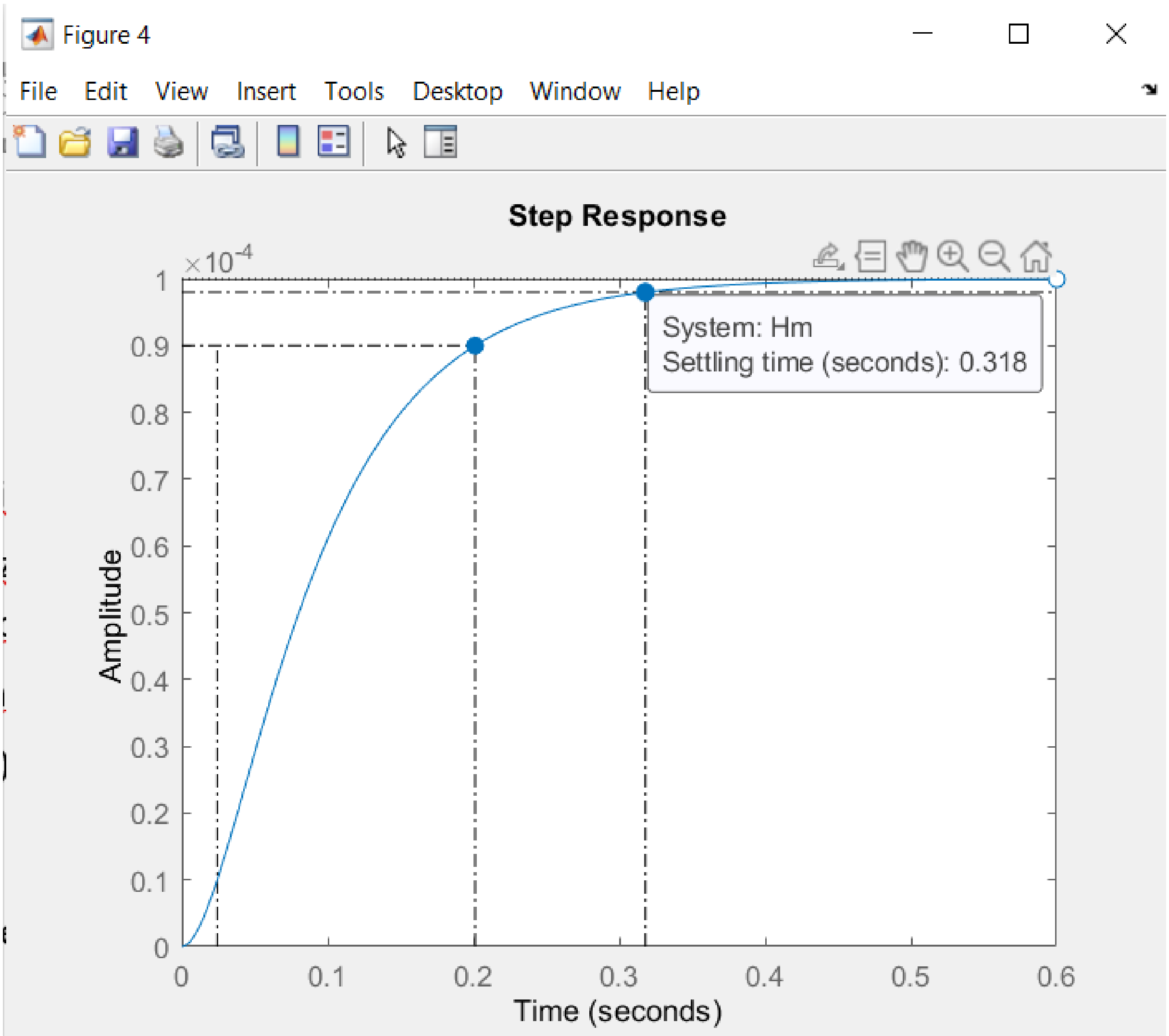


Se poate observa ca cele doua grafice nu coincid, deci sigur am gresit.

9. Performante

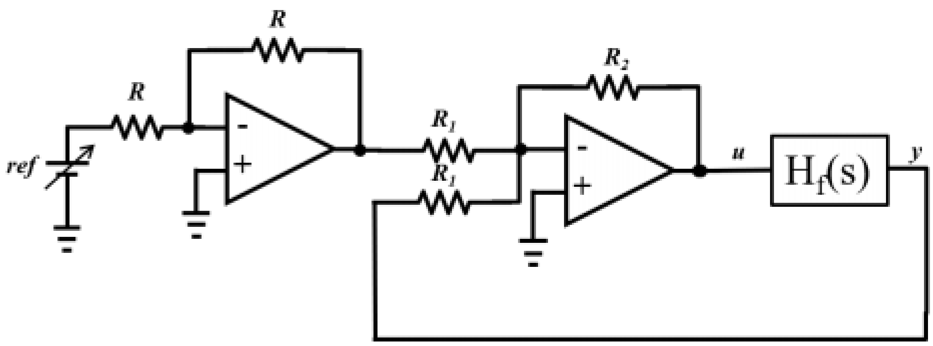
(8.1) ->

- constantele de timp: $T_1 = 0.02\ si\ T_2 = 0.07$
- factor de amortizare ζ nu am
- factorul de proportionalitate: $K = 5 * 10^{-4}$
- pulsatie naturala ω_n nu am
- t_r = 4*$(T_1 + T_2) = 0.36s$



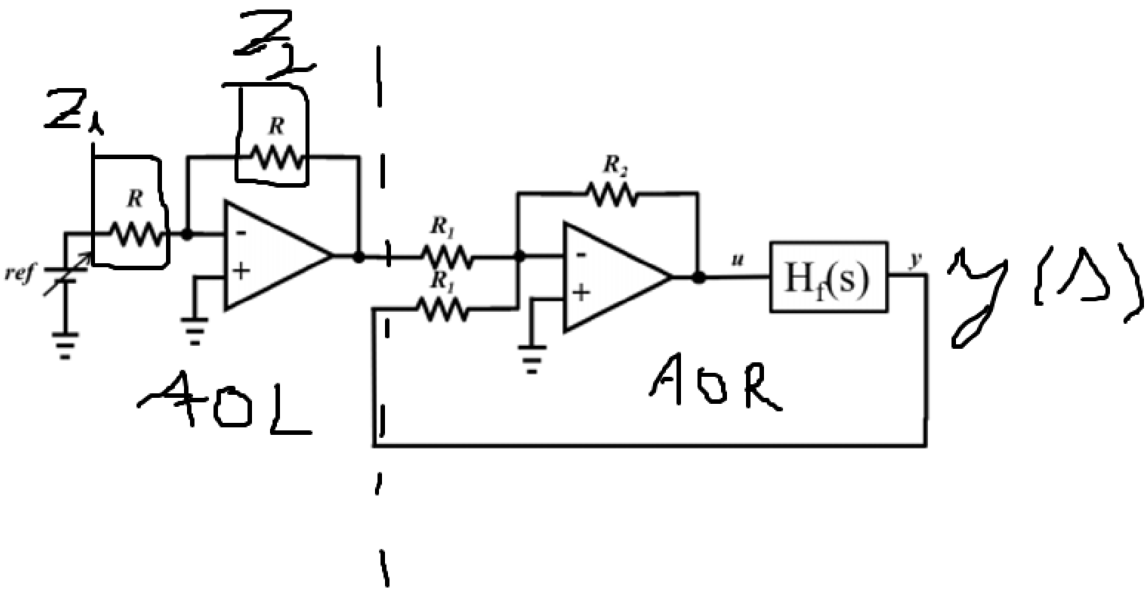
->Sistemul are un regim aperiodic amortizat

10. Aplicarea completa a algoritmului de trasare a locului radacinilora



In circuitul acesta se poate observa ca AO-ul din dreapta are un rol de sumator, iar AO-ul din stanga formeaza un 1,care este adunat prin sumator la functia de transfer a sistemului, obtinuta la punctele anterioare.

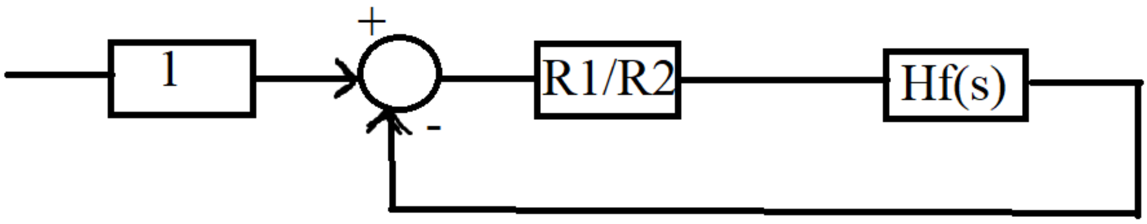
Voi imparti circuitul in doua pentru a putea obtine cele doua functii de transfer.



$$H_{AOL} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R}{R} = -1 \rightarrow H_{AOL} = -1$$

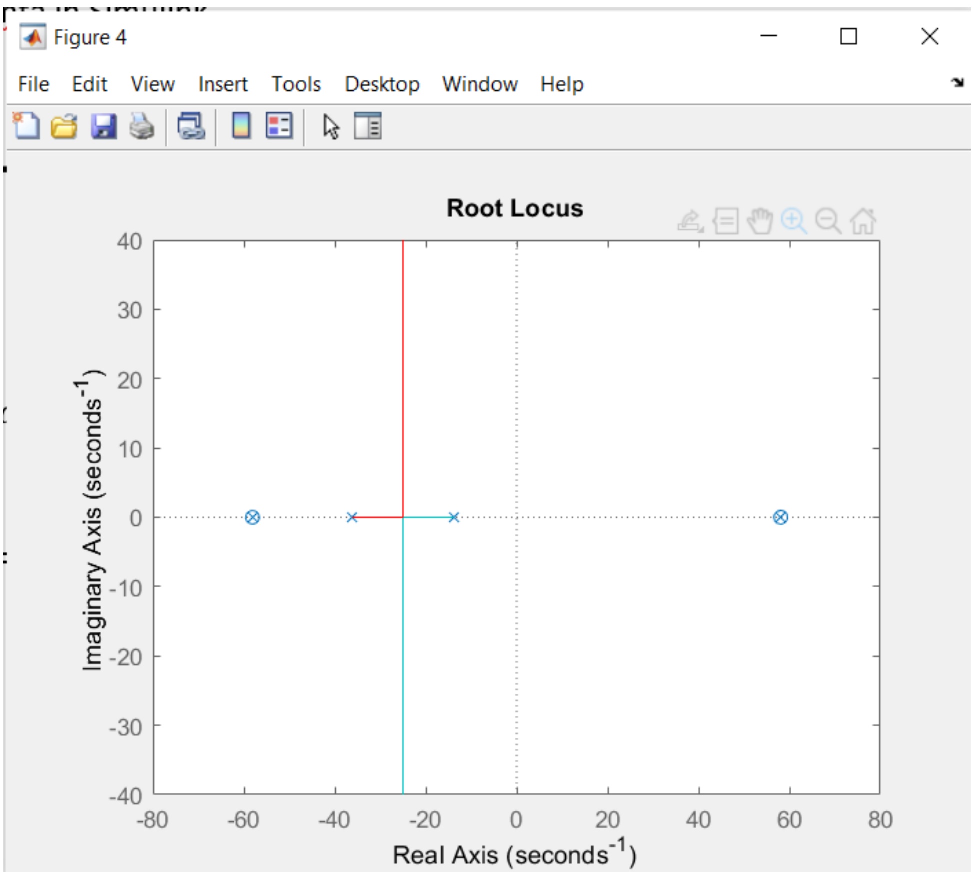
$$H_{AOR} = -\frac{Z_R}{R_1(-Y(s) - H_{AOL}(s)} = \frac{R_2}{R_1(-H_f(s) - H_{AOL}(s))}$$

Obtin schema echivalenta in Simulink:

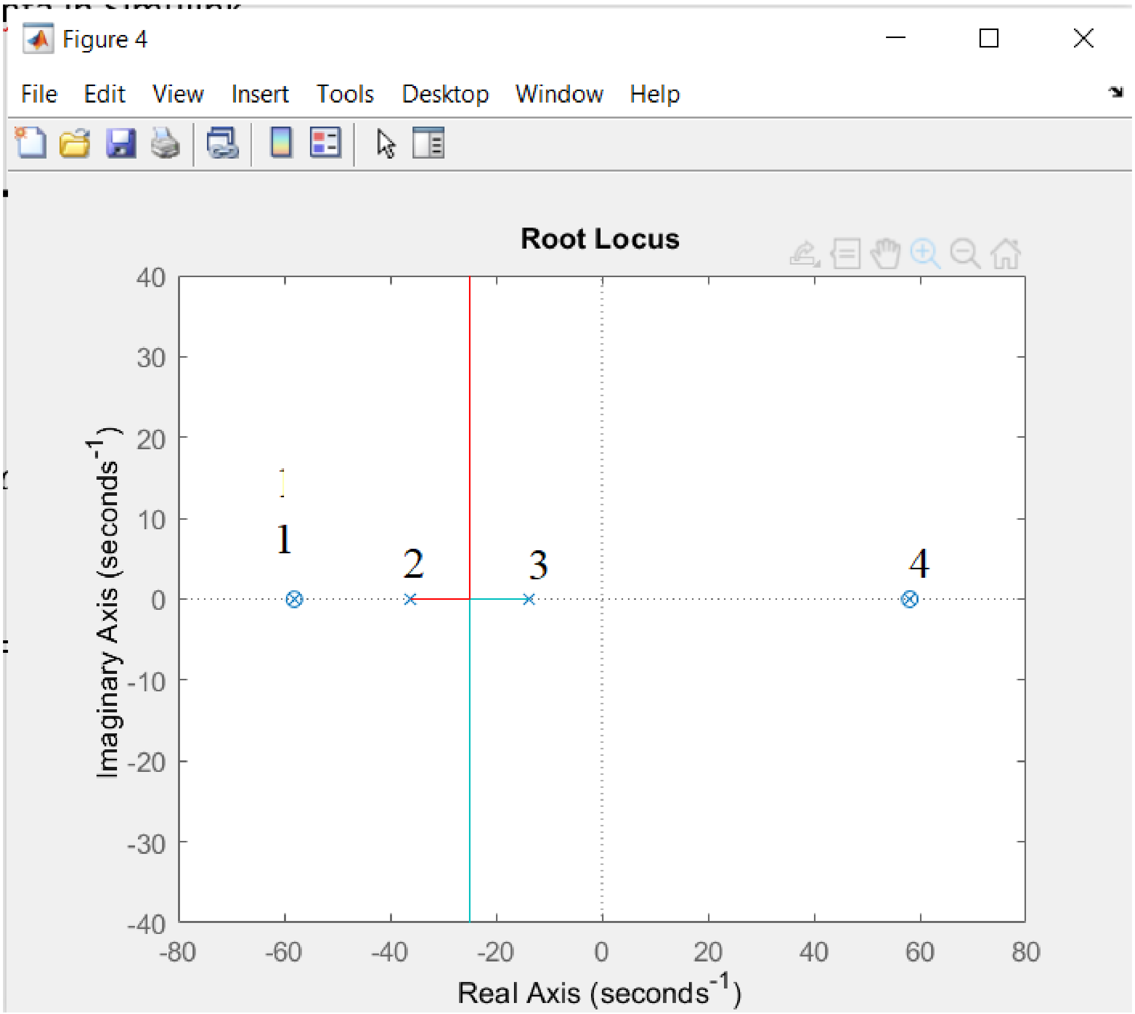


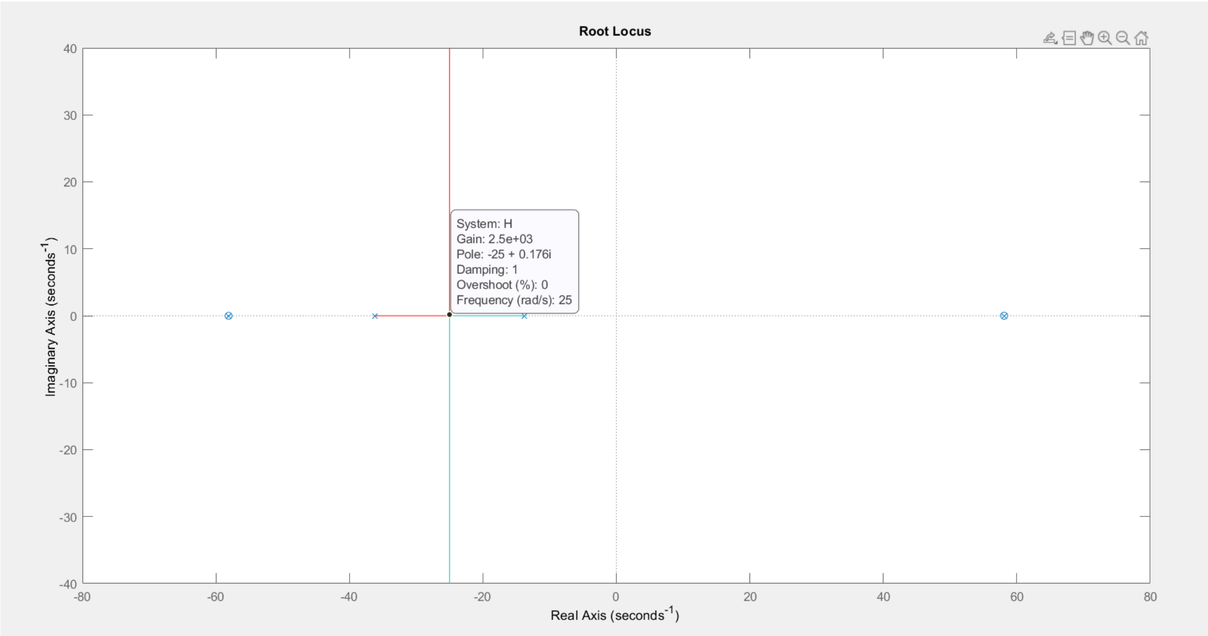
$$\frac{R_2}{R_1} = K = o\ constanta$$

$$H_o(s) = \frac{KH_f(s)}{1 + KH_f(s)} = \frac{K\frac{0.05(s-58.09)(s+58.29)}{(s-58.09)(s+58.09)(s+36.18)(s+13.82)}}{1 + K\frac{0.05(s-58.09)(s+58.29)}{(s-58.09)(s+58.09)(s+36.18)(s+13.82)}}$$



Interpretarea graficului:





In punctul k desprindere = 2.5e+03, cele doua ramuri se ciocnesc si se despart in poli complex conjugati.

Daca k este intre [0;2.5e+03], atunci sistemul are poli reali si regimul este unul aperiodic amortizat

Daca k este intre [2.5e+03,infinit], atunci sistemul are poli complecsi conjugati si regimul este oscilant amortizat.