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AMS 147

Exercise 3

1. Fixed points

$\phi(\alpha) = \alpha$

$\Phi = x + \sin x \Rightarrow x = x + \sin(x) \Rightarrow \sin(x) = 0 \Rightarrow \pi$

0

$0 + \sin(0) = 0 + 0 = 0$

Start with $x_0 = 1$

$x_0 = 1$

$x_1 = 1 + \sin(1) = 1.84147$

$x_2 = 1.84147 + \sin(1.84147) = 2.80506$

$x_3 = 2.80506 + \sin(2.80506) = 3.13528$

$x_4 = 3.13528 + \sin(3.13528) = 3.14159$

$x_5 = 3.14159 + \sin(3.14159) = 3.14159$

Converges to π

Fixed points: 0 and $x\pi$ where x is a scalar (integer)

2. Show that (2) converges to π for any initial x^0

For $x^0 < \pi$

- As shown in part 1, iterating with $x^0 < \pi$, will result in x^k slowly incrementing to π. Once π is reached, x^k will no longer increment.

For $x^0 > \pi$

- Similar to condition $x^0 < \pi$, except x^k will slowly decrement to π. Take $x^0 = 4$ for example:

$x_0 = 4$

$x_1 = 4 + \sin(4) = 3.24320$

$x_2 = 3.24320 + \sin(3.24320) = 3.14177$

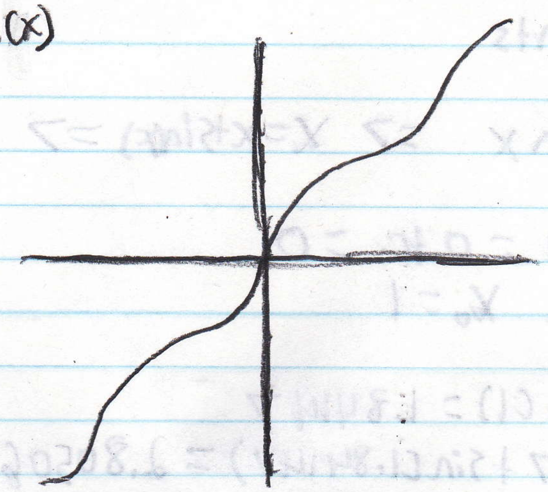
$x_3 = 3.14177 + \sin(3.14177) = 3.14159$

$x_4 = 3.14159 + \sin(3.14159) = 3.14159$

No matter the initial starting point in $(\pi/2, 3\pi/2)$ the sequence will converge to π

3. Convergence order of the sequence

$x + \sin(x)$



$$\lim_{k \to \infty} \frac{x_{k+1}}{x_k} \approx \frac{\pi}{\pi} = 1$$

Convergence order $= 1$