Homework Assignment 2

1. $f(n) \in \Theta(g(n))$ iff

$$c_1 g(n) \le f(n) \le c_2 g(n) \quad n \ge n_0$$

Prove $c_1 \max\{f(n), g(n)\} \le f(n) + g(n) \le c_2 \max\{f(n), g(n)\}$

Let $c_1 = 1$ and $c_2 = 2$

$\max(f(n), g(n)) \le f(n) + g(n)$ and $(f(n)+g(n))/2 \le \max(f(n), g(n))$

$\max(f(n), g(n)) \le f(n)+g(n)$ and $f(n)+g(n) \le 2\max(f(n), g(n))$

$\max(f(n), g(n)) \le f(n)+g(n) \le 2\max(f(n), g(n))$

$\therefore \max(f(n), g(n)) \in \Theta(f(n)+g(n))$

$\therefore f(n)+g(n) = \Theta(\max(f(n), g(n)))$

2. a. true

$2^{n+1} = c \cdot 2^n = 2 \cdot 2^n \quad c = 2$

b. false

$2^{2n} = c \cdot 2^n \Rightarrow c = 2^n$ * not a constant value

3.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ε	yes	yes	no	no	no
b.	n^k	c^n	yes	yes	no	no	no
c.	$\sqrt{n}$	$n^{\sin n}$	no	no	no	no	no
d.	2^n	$2^{n/2}$	no	no	yes	yes	no
e	$n^{\lg c}$	$c^{\lg n}$	no	no	yes	no	yes
f.	$\lg(n!)$	$\lg(n^n)$	no	no	yes	no	yes

4. d. $f(n) = O(g(n)) \Rightarrow 2^{f(n)} = O(2^{g(n)})$

Let $f(n) = 2n$ and $g(n) = n$

$f(n) = O(g(n))$

$2^{f(n)} = O(2^{g(n)})$

$2^{2n} \ne O(2^n)$ False

e. $f(n) = O((f(n))^2)$

$0 \le f(n) \le c(f(n))^2$

True as long as $n \ge 1$

h. $f(n) + o(f(n)) = \Theta(f(n))$

* This is proven similarly to how it is in Prob #1

$g(n) = o(f(n))$

$$c_1 f(n) \le f(n) + g(n) \le c_2 f(n)$$

Let $f(n) = f(n)$ $\quad g(n) = o(f(n))$

Let $c_1 = 1$ $\quad c_2 = 2$

$f(n) \le f(n) + o(f(n))$ and $(f(n) + o(f(n)))/2 \le f(n)$

$f(n) \le f(n) + o(f(n))$ and $f(n) + o(f(n)) \le 2f(n)$

$$f(n) \le f(n) + o(f(n)) \le 2f(n)$$

$$\therefore f(n) + o(f(n)) = \Theta(f(n))$$

5. Assume $o(g(n)) = f_1(n)$ and $\Omega(g(n)) = f_2(n)$

$0 \le f_1(n) < c_1 g(n)$ $\quad - o(g(n))$

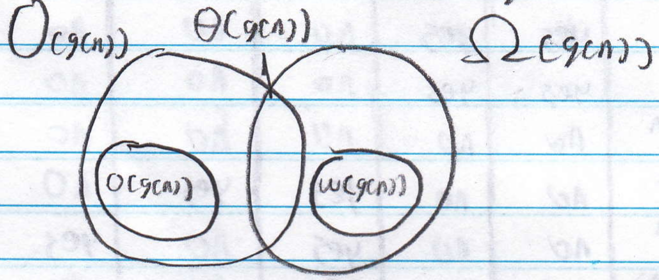
$0 \le c_2 g(n) < f_2(n)$ $\quad - \Omega(g(n))$

Assume $o(g(n)) \cap \Omega(g(n)) \ne \emptyset$

$o(g(n)) \cap \Omega(g(n)) = f_3(n)$ $(f_1(n) \in o(g(n)) \land f_2(n) \in \Omega(g(n)))$

$c_3 g(n) \le f_3(n) < c_3 g(n)$ $\quad$ - False

$o(g(n)) \cap \Omega(g(n)) = \emptyset$



6. $3^{2^n} = o(2^{3^n})$.

$3^{2n} = o(2^{3n})$

$= \lim_{n\to\infty} \left(\frac{3^{2n}}{2^{3n}}\right)$

$= \lim_{n\to\infty} \left(\frac{2^{(\log_2 3)^{2n}}}{2^{3n}}\right)$ *Changing base

$= \lim_{n\to\infty} \left(\frac{2^{2^n \log_2 3}}{2^{3n}}\right)$

$= \lim_{n\to\infty} \left(2^{2^n \log_2 3 - 3^n}\right)$

$= 0$