Homework 7

1.

	White	Gray	Black
White	Yes / tree, back, forward, cross	Yes / back, cross	Yes / Cross
Gray	Yes / tree, back	Yes / tree, back, forward	Yes / tree, forward, cross
Black	No	Yes / back	Yes / tree, back, forward, cross

2.
```
DFS(G)
    for each vertex u ∈ G.V
        Color[u] = WHITE
        p[u] = NIL
    time = 0
    for each vertex u ∈ G.V
        if color[u] == WHITE
            DFS_Visit(u)
DFS_Visit
    Color[u] = GRAY
    d[u] = time++
    for all v in Adj[u]
        if Color[v] == WHITE
            print u v "tree edge"
            p[v] = u
            DFS_Visit(v)
        else if Color[v] == GRAY
            print u v "back edge"
        else if d[u] > d[v]
            print uv "cross edge"
        else
            print u v "forward edge"
    Color[u] == black
    finish[u] = time++
```

3.
```
DFS(G)
    for all u ∈ G.V
        Color[u] = WHITE
```

$P[u] = NIL$
time = 0
k = 0
for all $u \in G.V$
if color[u] == WHITE
k++
Visit(u)

Visit(u)
color[u] = GRAY
d[u] = time++
cc[u] = k
for all $v \in Adj[u]$
if color[v] == WHITE
P[v] = u
Visit(v)
color[u] = BLACK
finish[u] = time++

* k will be increased when DFS finds a new root, so that root and its desendants will have the same cc value. Also, by nature of an undirected graph there's no cross edges. So cc[x] = cc[y] only if x and y are on the same component.

4. I. Base Case

Let $n(T) = 1$, then $h(T) = 0$

$h(T) \geq \lfloor \lg n(T) \rfloor$

$0 \geq \lfloor \lg 1 \rfloor$

$0 \geq 0$ ✓

II. Assume $h(T') \geq \lfloor \lg n(T') \rfloor$

we must prove $h(T) \geq \lfloor \lg n(T) \rfloor$

$h(T) \geq 1 + \lfloor \lg n(L) \rfloor$

$\geq \lfloor 1 + \lg n(L) \rfloor$

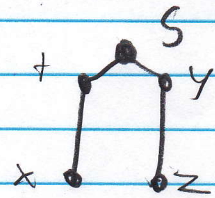
$\geq \lfloor \lg 2 + \lg n(L) \rfloor$

$\geq \lfloor \lg(2 \cdot n(L)) \rfloor$
$\geq \lfloor \lg(2 \cdot n(L) + 1) \rfloor$ * additional hint
$\geq \lfloor \lg n(T) \rfloor$

5. Dijkstra's (d/π values)

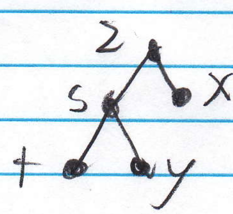
s as source vertex

	0	1	2	3	4	5
s	0/NIL	0/NIL	0/NIL	0/NIL	0/NIL	0/NIL
t	∞/NIL	3/s	3/s	3/s	3/s	3/s
x	∞/NIL	∞/NIL	9/t	9/t	9/t	9/t
y	∞/NIL	5/s	5/s	5/s	5/s	5/s
z	∞/NIL	∞/NIL	∞/NIL	11/y	11/y	11/y
Set S	∅	{s}	{s,t}	stx	stxy	stxyz



z as source

	0	1	2	3	4	5
s	∞/NIL	3/z	3/z	3/z	3/z	3/z
t	∞/NIL	∞/NIL	6/s	6/s	6/s	6/s
x	∞/NIL	7/z	7/z	7/z	7/z	7/z
y	∞/NIL	∞/NIL	8/s	8/s	8/s	8/s
z	0/NIL	0/NIL	0/NIL	0/NIL	0/NIL	0/NIL
set S	∅	z	s	zst	zstx	zstxy



6. To find the most reliable path, we utilize Dijkstra's and apply a weight on each of the edges. This weight is found by solving

the summation of the negative log of the value of the summation pairs. So we have:

Most_Reliable_Path

fore all $(u,v) \in B.E$

$w(u,v) = -\log r(u,v)$

Dijkstra (G, w, x)

Print path