DESIGN REPORT

Title:	**Aft Clamp Nut**
Part number:	
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CONTENTS

INTRODUCTION

The aft clamp nut is desired to lock the components of the shaft within the turbopump housing whilst facilitating the shaft's free rotation. This part has been designed and the details of the design process are provided herein.

REVISION HISTORY

0 Initial release

1 Added calculation for conservative estimate of seal collapse due to compression as Appendix A

APPLICABLE DOCUMENTS

For more information regarding the design process of this component, as well as modular calculation tables, please go to this Google Sheets file.

NOMENCLATURE

α	Angle of teeth slopes, [°]
λ	Lead angle of thread, [°]
σ	Tensile stress, [lbf in^{-2}]
τ	Shear stress, [lbf in^{-2}]
A	Area, [in^2]
a	Parabolic buckling formula constant
B	Width (axially), [in]
b	Parabolic buckling formula constant
C	End-condition factor
D	Major (outer) diameter, [in]
d	Inner diameter, [in]
F_i	Bearing failure force, [lbf]
F	Force, [lbf]
f	Coefficient of friction
H	Fundamental height, [in]
I	Second moment of inertia
J	Ratio of internal and external thread strengths
K	Torque factor
k	Radius of gyration, [in]
L	Length of shaft seal, [in]
L_e	Length of engagement, [in]
n_t	Number of threads in contact
P	Critical buckling load, [lbf]
p	Pitch, [in]
T	Torque, [ft$\cdot$ lbf]
t	Thickness (difference of inner and outer diameter), [in]
SS	Stainless steel
USS	Ultimate shear strength
UTS	Ultimate tensile strength

SUBSCRIPTS

2	Related to the pitch diameter
3	Related to the minor diameter
b	Related to the bearing failure
m	Mean quantity
s	Related to shearing failure
t	Related to tensile failure
x	Related to the inertial axis
y	Related to the yield strength of a material or the inertial axis
max	Maximum quantity
min	Minimum quantity

1 OVERVIEW AND REQUIREMENTS

The aft clamp nut must thread into the housing of the turbopump unit, holding the components forwards of it in place on the shaft of the turbopump. The aft clamp nut is a thin and strong component capable of bearing the load of said components forwards of its position without risk of shear failure of the threads, which would be difficult to detect ahead of failure.

REQUIREMENTS

The aft clamp nut *shall* withstand the holding requirement of the bearings further along the shaft and said holding load *shall* not exceed between 50 to 75% of the shear failure load of the part. Furthermore, the first failure mode *should* be, preferentially, in tension so as to facilitate earlier detection of failure. If this is not possible, then the failure load to generate shearing of the threads *should* not be a likely scenario. To ensure this is the case, the failure load in shear *should* far surpass the failure load of all forwards components on the shaft.

ASSUMPTIONS

The maximum number of useful threads, in sharing the load in shear, is limited by the unequal distribution of loading along the screw and will, hence, decrease to a negligible contribution beyond approximately seven threads [1]. Furthermore, the inner diameter of the aft clamp nut must be 1” (25.4 mm) to accommodate the locking component aft of the aft clamp nut. The aft clamp nut is assumed to be loaded only in the axial direction.

INTERFACE

The aft clamp nut will thread into the turbopump housing at the aftmost position, as indicated by its name, and will not, therefore, be exposed to any fluids or thermal conditions of concern. The threaded materials may experience corrosion due to galling, depending on the materials present, so this shall be considered before making any final design decisions.

2 CALCULATIONS

INTRODUCTION

The aft clamp nut shall be located in the turbopump assembly as shown in Fig. 1.

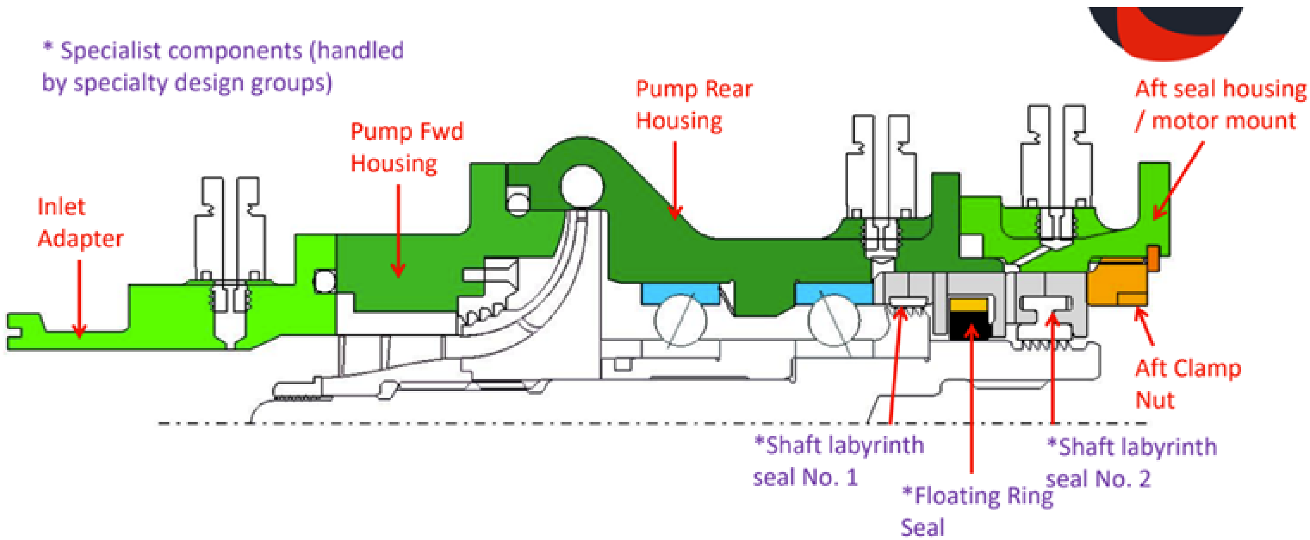

Figure 1: The cross-sectional diagram of the turbopump highlighting key components. The aft clamp nut can be found on the rightmost edge of the diagram

The bearings that are to be clamped in place by this locking nut, are Schaeffler XC7002C-T-P45-UL/FAG Super Precision Bearings. According to the manufacturer, they should not be axially loaded beyond $F_i = 1.7$ kN (approximately 382.2 lbf) [2]. Thus, the threads of the aft clamp nut must not fail in shear before an axial load of between 573.3 and 668.8 lbf (approximately 2.3 and 3.4 kN, respectively), in accordance with the previous section (Sect. 1). It should be noted that the bearings are being considered as the failure point in the components forward of the aft clamp nut, details for why the shaft seals are not going to be the failure point under compression can be found in Appendix A.

The wedge method is a commonly used and industry standard [3], but shall be substituted for the length of engagement method [4], which makes use of the thread shear area [5] approximation. This choice was made as the former requires the use of a coefficient of friction to be able to determine the torque required to rotate the nut, however, this value cannot be known at this time.

Dimensions

To incorporate with the existing housing design, the initial sizing of the aft clamp nut is as follows:

- Outer diameter, $D = 1.50$" (38.10 mm),
- Inner diameter, $d = 1.00$" (25.40 mm), and
- Width, $B = 0.25$" (6.35 mm).

To ensure at least five threads are engaged on this part, the pitch, p, should be such that $B > 5p$. Thus, the number of threads per inch (TPI) used for analysis herein ranges from 20–32. At TPI = 18, $5p = 0.39$" (9.9 mm) and, at TPI > 32, p will be so small that it becomes increasingly infeasible to manufacture the part.

There are some essential dimensions when designing a thread and attempting to compute the loading present. This includes the fundamental height [6],

$$H = \frac{p}{2}\tan(2\alpha),$$

the pitch and minor (for an external thread) diameters [6, 7],

$$d_2 = D - \frac{3}{4}H \quad \text{and}$$
$$d_3 = D - \frac{17}{12}H,$$

respectively, where all are given in inches. Furthermore, the areal formulae of interest (given in square inches) include the tensile stress area [7],

$$A_t = \frac{\pi}{16}(d_2 + d_3)^2 - \frac{\pi}{4}d^2,$$

where the internal area that is hollow has been excluded from the final area, and the approximate area of shear across the threads (or thread shear area) [5, 1],

$$A_s = \pi D \cdot 2\frac{L_e}{p}\left(\frac{p}{4} + \frac{D - d_2}{2}\tan(30)\right).$$

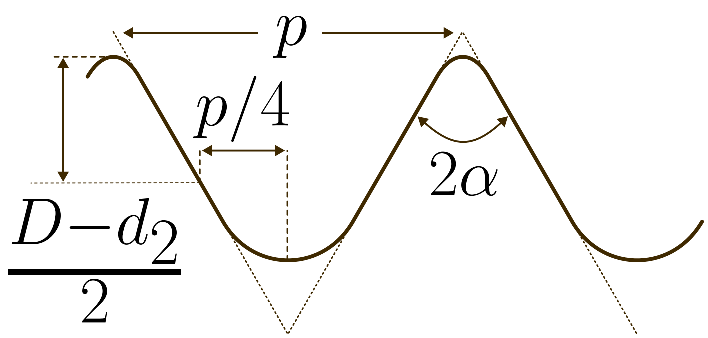

Figure 2: Schematic representation of thread and the dimensions used to approximate the area in shear

The above definition for A_s is computed by taking the area of the face of the threads in contact for the number of threads engaged (see Fig. 2) [5].

To arrive at this equation, the horizontal distance between peak and trough is multiplied by two to cover each side of the tooth and then by the number of teeth engaged. Thus, the area is found by multiplying this distance by the circumference of a single thread tip. This is, evidently, an approximation that does not account for the curves of the teeth profile, the fact that the teeth spiral and do not make concentric rings, and that the horizontal distance between peaks and troughs will be slightly shorter than the distance traversed by the climb and fall of each segment. However, this area is sufficient for the scope of this analysis.

Materials Selection

It is desirable to ensure that the part be made of a material strong enough to handle the cyclical loading expected but it is also important to consider the corrosion compatibility. In particular, because the part shall be under high stress at the thread interface, galling effects [8].

It is already known that the housing for this section of the turbopump will be made of Titanium (grade V). Thus, a reference for material compatibility is consulted and a few candidates are selected for further compatibility analysis [8].

Table 1: Galling compatibility of the proposed materials for use in making the aft clamp nut. P—Poor, F—Fair, and S—Satisfactory. Adapted from [8]

Materials	Interaction with Titanium
17-4 PH SS	F
416 SS	F
440C SS	F
Titanium	P
Inconel 600	P
Inconel 625	P

A short-list of materials expected to be available and reliable enough for use is as follows: 17-4 PH stainless steel, Type 416 stainless steel, Type 440C stainless steel, Titanium (grade V), Inconel 600, and Inconel 625. Referring to Table 1, it is immediately clear that Titanium cannot be used for the part as it is very likely to cold-weld with itself. Both of the Inconel subtances similarly would fair poorly. The remaining substances in the list are all at least passable and can be considered for the material used to make the aft clamp nut. The calculations to determine which is most eligible shall be discussed in the next section.

INITIAL LENGTH OF ENGAGEMENT

The length of engagement, L_e, is the number of threads that interface and actively support the thrust load between the internal and external threads. It is difficult to exactly compute this value and, as such, many approximations exist. A simplified version of L_e is selected [4]

$$L_e = \frac{4A_t}{\pi d_2}.$$

This is simplified from the L_e used in the formula for A_s, discussed previously, and the area of focus is $2A_t$, ensuring more contact area in shear than in tension. The basis for this assumption is that

1. It is desirable that the shear strength of the externally threaded component exceeds that of the internally threaded one so that there is a reduced risk of a pull-out event as they are difficult to detect ahead of time compared to tensile failure [1] and
2. The tensile strength of metals is usually less than double the shear strength value meaning that to get a part to fail in tension before it fails in shear one should try to achieve $A_s \geq 2A_t$

for the same force applied.

The values computed for the dimensions and areas discussed in Sect. 2 are given in Table 2 alongside the length of thread needed to engage with seven teeth, $L_{\max}$, and L_e.

Table 2: Computed dimensions for various TPI selections

TPI [in^{-1}]	p [in]	H [in]	d_2 [in]	d_3 [in]	A_t [in^2]	$L_{\max}$ [in]	L_e [in]
20	0.050	0.043	1.468	1.439	0.873	0.350	0.757
22	0.045	0.039	1.470	1.444	0.883	0.318	0.764
24	0.042	0.036	1.473	1.449	0.891	0.292	0.770
26	0.038	0.033	1.475	1.453	0.898	0.269	0.775
28	0.036	0.031	1.477	1.456	0.904	0.250	0.779
30	0.033	0.029	1.478	1.459	0.909	0.233	0.783
32	0.031	0.027	1.480	1.462	0.913	0.219	0.786

Engagement with seven teeth is the maximum useful spread of the thread loading across the part, using more teeth than this in the interface has rapidly diminishing returned beyond this point with the most loaded thread being the first and the least loaded the last [1]. Evidently, however, from the chart above it appears that $L_e > L_{\max}$ for all of the selected TPI. To understand this, the tensile and shear stress present in the threaded part will be approximated and compared with each other so that $L_e \leq L_{\max}$ could be achieved.

The tensile stress (or axial stress) is given by,

$$\sigma = \frac{F_i}{A_t} = \frac{4F_i}{\pi(D^2 - d^2)}$$

and the shear stress (transverse shear stress) is given by,

$$\tau = \frac{3F_i}{\pi D n_t p}$$

where n_t is the number of threads under stress. By setting the transverse shear stress to be at least half of the axial stress (following from the two assumptions listed previously), one can make the following equivalency,

$$
\begin{aligned}
\sigma &\geq 2\tau \\
\frac{4F_i}{\pi(D^2-d^2)} &\geq \frac{6F_i}{\pi D n_t p} \\
2Dn_t p &\geq 3(D^2-d^2) \\
&\geq 3\left(D^2-(D-2t)^2\right) \\
&\geq 3\left(D^2-D^2-4tD+4t^2\right) \\
&\geq 3\left(4t^2-4tD\right) \\
&\geq 12t\left(t-D\right) \\
n_t p &\geq 6t\left(\frac{t}{D}-1\right).
\end{aligned} \tag{1}
$$

The t is given as the thickness of the part (i.e., half of the difference between D and d). From Eq. 1, it is clear that one may make tensile failure more likely than shearing the threads by either increasing n_t or p or, conversely, by decreasing t (assuming that D is a fixed quantity). There are limits in each case, such as, increasing the number of threads engaged does not equal an inversely proportional decrease in the transverse shear stress (as discussed previously) and the pitch can only be so long whilst maintaining effectiveness as a fastener. Furthermore, the thickness of the part can only decrease so much (i.e., $t > 0$). The most feasible approach, therefore, is to decrease the thickness slightly but this will be considered later, after the material interface has been factored into the analysis in the following section.

MODIFIED LENGTH OF ENGAGEMENT

There are three cases to consider for the strengths of the materials in the internal and external threads [1]:

1. The housing (internal thread) is made of a stronger material than the aft clamp nut (external thread) meaning that, under high load, the housing threads would strip first and the aft clamp nut would be unable to support a thrust load
2. The aft clamp nut is made of a stronger material than the housing meaning that, under high load, the aft clamp nut will strip out of the housing
3. The components are made out of the same or an equally strong material meaning that, under

high load, the failure will be simultaneous.

Due to the reasons listed in Sect. 2, the third option is immediately undesirable. The first option is acceptable, given that the housing will have an L_e larger than that of the aft clamp nut and so should sufficiently avoid stripping before the tensile failure of the aft clamp nut. The second option is less desirable, as replacing the housing would be much more expensive than replacing the aft clamp nut. However, with proper attention to the risk of stripping, either material being stronger than the other is acceptable. The ratio of the ultimate tensile strength (UTS) of the external to that of the internal thread is set as [4],

$$J = \frac{\text{UTS of the external thread}}{\text{UTS of the internal thread}}.$$

If $J < 1$ then tensile failure of the aft clamp nut is more likely and there should be no need to make any adjustments [1]. If $J \geq 1$, however, then it would be prudent to increase the length of L_e by a factor of J such that a new L_e can be defined [4] as

$$L_e^* = JL_e.$$

The value of J for the selected materials are shown in Table 3.

Table 3: The value of J for the three select materials for use in the external thread for a housing of Titanium [9]

Material	Ratio of External to Internal UTS, J
17-4 PH SS [10]	1.23
416 SS [11]	0.54
440C SS [12]	0.72

From the table above, it is clear that the L_e in the case of using 17-4 PH SS should be modified as is shown in Table 4

Table 4: The new value for L_e if 17-4 PH SS is selected to account for $J > 1$

TPI [in^{-1}]	L_e [in]	L_e^*
20	0.757	0.935
22	0.764	0.943
24	0.770	0.950
26	0.775	0.956
28	0.779	0.961
30	0.783	0.966
32	0.786	0.970

This makes it appear that 17-4 PH SS may not be the most efficient selection for the aft clamp nut, however, it should be noted that it is generally the case that an internally threaded part be of lower UTS than the externally threaded part to allow a controlled level of yielding to better distribute the loading along the threads [1]. Ideally, this would not be the ideal approach in this case as the housing is, again, more expensive to replace than the aft clamp nut. Now that the values for the length of engagement have been thoroughly discussed, the following section will consider the stress placed on the part relative to the allowable for the entire system.

FAILURE TORQUE LOAD

As was discussed in Sect. 2, the threads of the aft clamp nut must not fail in shear before an axial load of between 573.3 and 668.8 lbf, as specified by the requirements in Sect. 1 of the bearing failure load. If the maximum load in this range were to be taken as the force experienced in the threads of the aft clamp nut and a worst case scenario of just five threads are engaged, then the stress for the select materials may be compared to the shear strength of each material (see Table 5).

Table 5: Applying twice the failure load, F_i, of 668.8 lbf through five threads of the aft clamp nut for various candidate materials

TPI [in^{-1}]	L_e [in]	A_s [in^{-2}]	Shear Stress, τ [psi]	Shear Stress per Shear Strength, τ^* [%]		
				17-4 PH SS [10]	416 SS [11]	440C SS [12]
20	0.250	1.080	353.9	0.336	0.722	0.562
22	0.227	1.026	372.4	0.353	0.760	0.591
24	0.208	0.982	389.3	0.369	0.794	0.618
26	0.192	0.944	404.9	0.384	0.826	0.643
28	0.179	0.912	419.2	0.398	0.856	0.665
30	0.167	0.884	432.5	0.410	0.883	0.687
32	0.156	0.859	444.9	0.422	0.908	0.706

Clearly, the shear stress expected by the maximal and conservative loading case is far below the expected shear failure stress of any of the materials. As such, an L_e of less than the computed optimal value, albeit resulting in shear failure before the desired tensile failure mode, is not a major concern for this component.

To compute the torque that would induce the failure load condition of the bearings, the following equation may be applied [13]

$$T = KF_iD,$$

where T is the applied torque to the aft clamp nut to achieve a loading of F_i and K is a torque factor that will vary based on the thread contact conditions (including lubrication or lack thereof). K can be computed using an equation found in Bundyas and Nisbett's textbook [13],

$$K = \frac{d_m}{2D}\left(\frac{\tan(\lambda) + f\sec(\alpha)}{1 - f\tan(\lambda)\sec(\alpha)}\right),$$

where d_m is the mean thread diameter, f is the coefficient of friction (set at 0.15 [13]), λ is the lead angle of the thread, and α is the thread angle (which is always 30° for a UNF type thread). The quantities of d_m and λ are given as,

$$d_m = d - \frac{\sqrt{3}p}{2} \text{ and}$$

$$\lambda = \arctan\left(\frac{p}{\pi d_m}\right).$$

Applying the above torque formula to the internal and external threads for the desired materials can be computed in tension. In shear, it is a very conservative estimation, however, for the scope of this analysis it will be applied as such with the explicit assumption that the entire force applied through the aft clamp nut is bore by the shear area only. The case of tensile failure and shear failure as well as the bearing failure condition are computed in Table 6 for the various TPI and materials if $L_e = 0.25$ [in] or the value needed for $n_t = 7$ [1].

[1]This is because of the constraints of the axial space available in the housing and the limit of seven threads relates to the diminishing support of each additional thread [1]. Also note that none of the L_e^* values computed for 17-4 PH SS were used as they were all larger than these constraints.

Table 6: T for different TPI if the axial load is equal to F_i, $L_e \leq 0.25$ in, and $n_t \leq 7$

TPI [in^{-1}]	L_e^* [in]	K	T_b [ft· lbf]	T_s [ft· lbf]			T_t [ft· lbf]		
				17-4 PH SS	416 SS	440C SS	17-4 PH SS	416 SS	440C SS
20	0.250	0.183	17.52	2490	1158	1488	3407	1500	2000
22	0.250	0.183	17.49	2486	1156	1486	3440	1515	2020
24	0.250	0.183	17.47	2483	1154	1484	3467	1527	2036
26	0.250	0.183	17.45	2481	1153	1483	3490	1537	2050
28	0.250	0.182	17.43	2479	1152	1481	3510	1546	2061
30	0.233	0.182	17.42	2312	1074	1381	3528	1554	2071
32	0.219	0.182	17.41	2166	1007	1294	3543	1560	2080

From Table 6, it is clear that the aft clamp nut is not the limiting factor in the system and should not present a risk of failure in either mode. However, should one wish to ensure that tensile failure does precede shear failure, then the ratio of shear to tensile failure torque can be optimized (similarly to Eq. 1) as follows,

$$\begin{aligned}
\frac{T_s}{T_t} &\geq 1 \\
\frac{K\tau_{\text{USS}}A_sD}{K\sigma_{\text{UTS}}A_tD} &\geq 1 \\
\tau_{\text{USS}}A_s &\geq \sigma_{\text{UTS}}A_t \\
\tau_{\text{USS}}\left[2\pi D\frac{L_e}{p}\left(\frac{p}{4}+\frac{D-d_2}{2}\tan(30)\right)\right] &\geq \sigma_{\text{UTS}}\left[\frac{\pi}{16}(d_2+d_3)^2-\frac{\pi}{4}d^2\right] \\
\tau_{\text{USS}}\left[2\pi D\frac{L_e}{p}\left(\frac{p}{4}+\frac{3H}{8}\frac{\sqrt{3}}{3}\right)\right] &\geq \sigma_{\text{UTS}}\left[\frac{\pi}{16}\left((D-\frac{3}{4}H)+(D-\frac{17}{12}H)\right)^2-\frac{\pi}{4}\left(D-2t\right)^2\right] \\
\tau_{\text{USS}}\left[2\pi D\frac{L_e}{p}\left(\frac{p}{4}+\frac{3p}{16}\right)\right] &\geq \sigma_{\text{UTS}}\left[\frac{\pi}{16}\left(2D-\frac{13}{6}H\right)^2-\frac{\pi}{4}\left(D-2t\right)^2\right] \\
\frac{14\pi}{16}\tau_{\text{USS}}DL_e &\geq \frac{\pi}{16}\sigma_{\text{UTS}}\left[\left(2D-\frac{13}{6}H\right)^2-4\left(D-2t\right)^2\right] \\
14\tau_{\text{USS}}DL_e &\geq \sigma_{\text{UTS}}\left[\left(2D-\frac{13}{6}H\right)^2-4\left(D-2t\right)^2\right] \\
14\frac{\tau_{\text{USS}}}{\sigma_{\text{UTS}}}DL_e-\left(2D-\frac{13}{6}H\right)^2+4\left(D-2t\right)^2 \geq 0.&
\end{aligned} \tag{2}$$

Note that τ_{USS} and σ_{UTS} are the ultimate shear and tensile strengths of a material. Solving the above equation, by iteration, to satisfy the equality for each TPI results in the maximum possible thickness that ensures a tensile failure for each of the materials (see Table 7).

The ability to make use of the results shown in Table 7 depend on other constraints, however, so it should only be used in conjunction with the other findings that the aft clamp nut is unlikely to be responsible for failure of any kind in this application.

Table 7: Maximum allowable thickness, t, such that the failure mode of the aft clamp nut is tensile rather than shear for each of the candidate materials

TPI [in^{-1}]	Maximum Allowable Thickness, t [in]		
	17-4 PH SS [10]	416 SS [11]	440C SS [12]
20	0.1801	0.1900	0.1832
22	0.1774	0.1873	0.1805
24	0.1751	0.1850	0.1782
26	0.1732	0.1830	0.1763
28	0.1715	0.1814	0.1747
30	0.1586	0.1675	0.1614
32	0.1474	0.1557	0.1501

RESULTS AND SIZING

The results of the analysis show that any of the three candidate materials (17-4 PH SS, 416 SS, and 440C SS) are sufficiently able to interact with the likely housing material (Titanium) and perform more than adequately under the prescribed loads. To the extent that reducing the thickness to induce a tensile failure is unnecessary and would complicate the selection of, for example, the snap ring used to lock the aft clamp nut. The feasibility of machining the part is also a factor and by reducing the thickness it may complicate the profile needed to adequately hold against the seals ahead of the aft clamp nut. Therefore, to ensure the most number of threads in contact, whilst still being functional, and fitting within the width constraint of $B = 0.25$ in, the TPI should be set at 28. If, however, $n_t = 6$ is allowable, then the TPI may be set at 24. Furthermore, to ensure that the standard practise of having a weaker material for the internal thread than the external, the aft clamp nut should be made of 17-4 PH stainless steel.

3 MANUFACTURING

This component cannot be purchased readily off-the-shelf and, as such, will need to be manufactured in house. The thread and internal exclusion can be simply machined on a lathe (as long as it has fine enough thread cutting). However, the thread locking clips will require more complex machining steps, to the point that it perhaps necessitates the use of a metal 3D printer. This is to be determined, however.

A proper, engineering drawing of the component should be prepared, for the moment the part should resemble that shown in Fig. 3

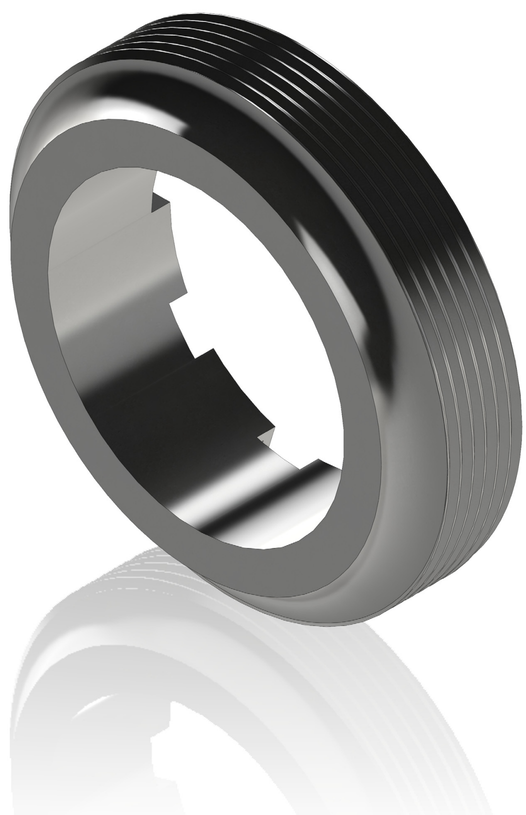

Figure 3: Mock-up of the aft clamp nut design

4 CONCLUSION

In conclusion, the aft clamp nut has been designed on a theoretical basis only, informed by industry "Rules of Thumb" as opposed to validated testing results or finite element analysis methods. The characteristics of the design are as follows:

- The thread will be a UNF type thread of 24 TPI or, if seven threads are needed for engagement to be more conservative, then let the TPI be 28
- The length of the part shall be such that six threads are engaged (0.25” at 24 TPI)
- The outer diameter of the part shall be, nominally, 1.5”
- The inner diameter of the part shall be, nominally, 1.0” (to accommodate the snap ring)
- The part is noted to be insufficiently sized to ensure tensile failure ahead of shearing of the threads due to unnecessary additional costs and the lack of concern for a failure of the part at all
- The part material shall be 17-4 PH steel, inserted into a housing of (likely) Titanium (grade V)

Though it may be desirable to test this component, to ensure the shear failure risk is low enough to subvert the commonly held concept of tensile failure before shear to better detect it, it is not practical nor is it likely to matter given that the expected loading of the part will be significantly (value here) lower than the estimated shear failure axial load.

A LABYRINTH SEALS FAILURE

If one were to assume the worst case scenario for the failure of the seals in the turbopump due to compression applied axially, then it must be determined if the column will simply yield or buckle and, in such cases, will this be less than the failure load of the bearings, F_i.

First, the failure force in the case of yielding can be calculated using the following equation,

$$F = \sigma_{\mathrm{UTS}} A_{\mathrm{min}},$$

where A_{min} is the smallest cross-sectional area, along the shaft, of the seals. In this case, the material is $15-5$ PH stainless steel so $\sigma_{\mathrm{UTS}} = 155$ ksi [14, 15] and the seal has a cross-sectional area of $A_{\mathrm{min}} = 0.25\pi\left(D^2 - d_{\mathrm{max}}^2\right)$. For the seals, the inner diameter is a maximum (for the worst case computation) of 1.28 in, with a constant outer diameter of 1.35 in. Putting these together gives $F = 2240$ lbf, a factor of $5.86F_i$. In the case of simply yielding, then one could substitute the σ_{UTS} for the yielding stress, which is $\sigma_{\mathrm{y}} = 145$ ksi [14, 15] and results in a mild reduction of the failure force to $F = 2100$ lbf or, as a factor of the bearing failure force, $5.49F_i$.

Second, for the case of buckling, it is first necessary to determine whether or not the Euler formula is suitable. This first requires determining the slenderness ratio and the minimum slenderness ratio to utilise the Euler formula for buckling. The slenderness ratio is simply the length of the "column", which in this case is the seal length, $L = 1.35$ in over the radius of gyration, k, which is determined by the column cross-section. In this case, k is given as [13],

$$k = \sqrt{\frac{I_x}{A}},$$

where I_x (or I_y depending on the axis of buckling, but in this case it will be axisymmetric about the shaft axis so there is no difference) is the second moment of inertia of the cross-sectional area. If one assumes that the shaft seals are a simple hollow cylinder of outer diameter, D, and a constant [2] inner diameter, d, then k becomes,

$$k = \sqrt{\frac{D^2 + d^2}{16}},$$

[2] A very conservative approximation

which equals $k = 0.465$ in and in turn results in a slenderness ratio of 3.44. The Euler formula for buckling is given as [13],

$$\frac{P}{A} = \frac{C\pi^2 E}{(L/k)^2},$$

where P is the buckling load, C is the end-condition factor (determined to be 1 in this case [13]), and E is the Young's modulus of the material (for $15 - 5$ PH SS, this is 28500 ksi). Therefore, the minimum slenderness ratio to use the Euler formula is for $P/A = \sigma_{\mathrm{y}}/2$ [13],

$$\frac{L}{k}_{\mathrm{min}} = \sqrt{\frac{C\pi^2 E}{\sigma_{\mathrm{y}}/2}},$$

which gives a value of 62.3. The conservative value for the slenderness ratio of the seal is 3.44, clearly far below the threshold value to use the Euler formula. Instead, the parabolic formula shall be used, which is given as [13],

$$\frac{P}{A} = a - b\left(\frac{L}{k}\right)^2,$$

where a and b are constants determined by the placement of a parabola for the plot of P/A versus the slenderness ratio. Typically [13], $a = \sigma_{\mathrm{y}}$ and

$$b = \left(\frac{\sigma_{\mathrm{y}}}{2\pi}\right)^2 \frac{1}{CE}.$$

Therefore, the parabolic formula becomes,

$$\frac{P}{A} = \sigma_{\mathrm{y}} - \left(\frac{\sigma_{\mathrm{y}}}{2\pi}\frac{L}{k}\right)^2 \frac{1}{CE},$$

and, in this case, evaluates to a buckling load of $P = 20.9$ kip. This value is clearly far higher than the simple yielding failure value and, as such, buckling should not be expected for the shaft seals, especially given that the inner diameter will not be constant. Thus, failure of the seals is not of concern and they should be in nominal condition long after the bearings are expected to have failed.

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