ME310 – Instrumentation and Theory of Experiments

Project: Design and Implementation of a Transduction System for the Measurement of a Weakly Nonlinear Mechanical Oscillator

Concepts: Dynamic data acquisition, A/D conversion, GUI data acquisition software, signal conditioning, linear and non linear oscillations

Deliverables: Group Report, Group Presentation, Peer/Individual Assessment. See syllabus for dates.

1. Introduction

Your design project will be to plan and implement a measurement system, from transducer(s) to digitization, which will measure the temporal response and frequency response of a mechanical 2nd-order oscillator. The oscillator consists of two masses which can slide vertically on a rod joined by a spring (see Fig. 1). The lower mass (m_b) is driven by a motor (through a device called a scotch yoke) to oscillate vertically at a range of user-adjustable frequencies (the RPM of the motor can be controlled by controlling the motor current). The spring transmits the force from the lower mass to the upper mass (m), which causes it to move. A plate can be attached to the top mass to add mass to it and to provide a mounting surface for a transducer (use of the plate in your measurement system is optional). Since it is a second order system, the motion of the upper mass due to the lower mass will be dependent on the frequency of the motion of the lower mass. The output from your transducer/signal conditioning stage must be capable of being acquired on a computer. This project has long had a place in ME310 – originally it was a lab entitled "Response of a Spring-mass-damper System." The data acquisition system used was instructive, but perhaps not optimal. It consisted of taping a pen to the upper mass, and allowing the pen to scribe lines on a moving screen as the system oscillated. The board and pen are shown in Fig. 1; however that does not mean you will have to use them in your measurement system (note that I frown on use of such a method as an acceptable calibration method!). By designing a computer-based data acquisition system you will able to characterize the response of the oscillator in a more systematic manner.

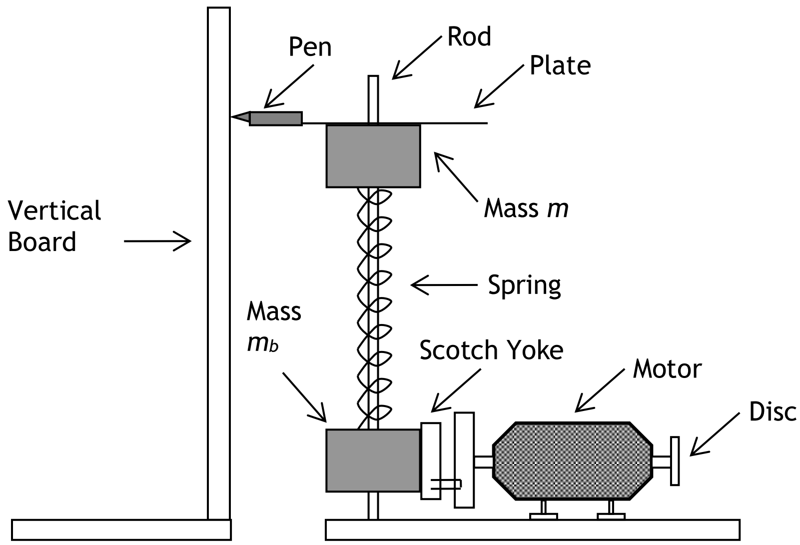

Figure 1: ***Original*** **apparatus used for studying a mechanical second order system.**

1.1. Objective

Measure the "frequency response", i.e., the amplitude response of the spring-mass-damper system as a function of varying driving frequency. Data must be digitally acquired. To meet this objective, you must design and use a transduction scheme that will measure the oscillations of the top mass *m* (plus the optional plate, should you decide to use one).

1.2. Approach

Since time is short, 4 pre-selected transduction schemes have been developed**.** You are not restricted to these 4, but if you choose to research another transduction scheme of your own, keep in mind that there is only a limited budget on which to draw to purchase new items. This project REQUIRES that you IMPLEMENT your scheme, so if it can't be bought you've wasted your time. Therefore, don't suggest using a scanning laser vibrometer, since those cost upwards of $25,000. A TOTAL price tag (transducer, electronics, other necessary stuff) of around $200.00 would be an upper bound.

We have purchased at least 2 setups for each of the 4 pre-selected systems. In the 4-group sections each group must use a unique measurement scheme and transducer. Once your group has chosen a particular system, contact the GSTs so they can ensure that only 1 group is assigned any single system in any given lab section. If you wish, the GSTs will simply assign you a particular transducer scheme. COMPLETE THIS STEP IN YOUR FIRST LAB PERIOD. In the 5-group sections we recommend 2 groups both use the optical transducer.

In the next section, a model is presented that can be used to predict the motion of the top mass provided the response is linear, and the motion of the bottom mass is known. Since each of you will use a different measurement system, there will be no detailed procedures provided for you for this project: you must devise them yourselves;

however, in Section 3, a set of general procedures are outlined that should guide you to a successful conclusion of the project. The four pre-selected transduction schemes are described in Section 4. Finally, Section 5 contains questions that you must answer using the data you collected.

2. Theory

Consider the system of two masses connected by a spring shown in Fig. 2. A displacement of the lower mass m_b will cause a corresponding displacement of the upper mass m. The displacements would be equal to each other only if the motion of mass m_b takes place very slowly (i.e. at a low frequency). As the frequency of motion of mass m_b increases, the displacement of mass m becomes increasingly different from that of mass m_b. This is because of interaction between the spring force and inertia force on the upper mass m.

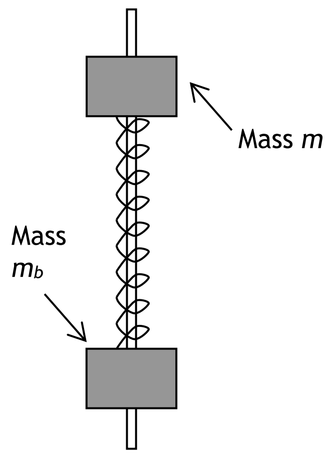

Figure 2: System of coupled masses arranged to slide on a rod.

The equation of motion can be derived by applying Newton's Second law to the top mass. Consider a free body diagram of the top mass, here F_s is the force due to the spring, and F_d is the force due to the damper.

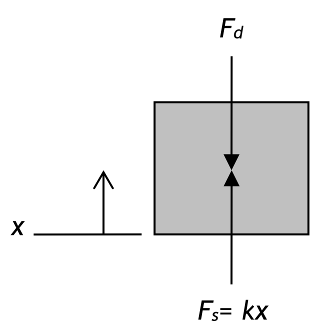

Figure 3: Free body diagram of mass *m*, neglecting gravity and nonlinearity.

The forces can be approximated as linear: $F_s = -k(x - x_b)$, and $F_d = -c\dot{x}$, with k *the spring constant,* x *the position of the top mass relative to the lab,* x_b *the position of the bottom mass relative to the lab, and* c the effective damping constant. Note that the spring force depends on the compression and expansion of the spring which in turn depends on the relative motion of the top and bottom mass. The damping force, on the other hand, depends on the absolute motion of the top mass.

Newton's second law states that the sum of the forces on the mass is equal to its mass times its acceleration (here recall that x has been defined as the vertical direction):

$$\sum F_x = m\ddot{x} = -c\dot{x} - k(x - x_b)$$

Rearranging terms gives:

$$m\ddot{x} + c\dot{x} + kx = kx_b$$

Substituting for x_b yields:

$$m\ddot{x} + c\dot{x} + kx = kX_b \sin(\omega t) \tag{1}$$

Where we incorporate the fact that the bottom mass will move sinusoidally with a fixed amplitude X_b, and a variable frequency ω, $x_b = X_b \sin(\omega t)$.

In order to solve equation (1) to find $x(t)$ explicitly as a function of $x_b(t)$, we work with complex exponentials. Observe that since both k and X_b are constants the product kX_b is also a constant; the units of this product will show that it is a force. Therefore, let

$$kX_b = F_o$$

and since

$$\mathrm{Im}\{e^{i\omega t}\} = \sin(\omega t)$$

then

$$m\ddot{x} + c\dot{x} + kx = F_o e^{i\omega t}$$

and we will look at just the imaginary part of the solution when we get it. (why?)
We assume linearity and thus pure harmonic time dependence:

$$x(t) = Xe^{i\omega t}$$

in which X is a constant, the output amplitude. Differentiating with respect to time gives

$$\dot{x}(t) = i\omega X e^{i\omega t}$$

$$\ddot{x}(t) = i^2\omega^2 X e^{i\omega t} = -\omega^2 x(t)$$

Substitution gives:

$$\left(-m\omega^2 + i\omega c + k\right)X_b e^{i\omega t} = F_o e^{i\omega t}$$

Canceling exponentials and rearranging gives:

$$X = \frac{F_o}{k - m\omega^2 + ic\omega}$$

Multiply top and bottom by the complex conjugate of the denominator to get:

$$X = \frac{k - m\omega^2}{\left(k - m\omega^2\right)^2 + \left(c\omega\right)^2} + i\frac{c\omega}{\left(k - m\omega^2\right)^2 + \left(c\omega\right)^2}$$

At this point, we recall that for this type of expression:

$$a + ib = Ae^{i\varphi}$$

Where:

$$A = \sqrt{a^2 + b^2}$$

and:

$$\frac{a}{b} = \tan\varphi$$

so:

$$X = \frac{F_o}{\left[\left(k - m\omega^2\right)^2 + \left(c\omega\right)^2\right]^{1/2}} e^{i\varphi}$$

Divide top and bottom by k, and recall that $kX_b = F_o$ or, $F_o/k = X_b$, then letting:

$$\omega_n = \sqrt{k/m},$$

$$c_c = 2\sqrt{mk},$$

and

$$\zeta = c/c_c$$

X can be written as:

$$X = \frac{X_b}{\left[\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right]^{1/2}} e^{i\varphi}$$

If we are interested in the ratio of the magnitude of input and output amplitudes at a fixed frequency ω, we have:

$$\frac{X(\omega)}{X_b(\omega)} = M(\omega) = \frac{1}{\left[\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right]^{1/2}}$$

where X(ω) is the output amplitude. A measurement of these amplitudes would result in an *experimental* value for M at that frequency (equation on the left), whereas knowledge of ω_n and ζ at a particular ω would provide a *theoretical* value for M at that frequency (equation on the right).

Also:

$$\varphi = \tan^{-1}\left(\frac{a}{b}\right)$$

so:

$$\phi(\omega) = \tan^{-1}\left(\frac{-2\zeta\dfrac{\omega}{\omega_n}}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}\right) = -\tan^{-1}\left(\frac{2\zeta\dfrac{\omega}{\omega_n}}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}\right)$$

Therefore, the complete time-dependent steady-state solution is (taking the imaginary part of the complex solution):

$$x(t) = \mathrm{Im}\left\{Xe^{i\omega t}\right\}$$

$$x(t) = \mathrm{Im}\left\{M(\omega)X_b e^{i(\omega t+\varphi(\omega))}\right\}$$

$$x(t) = M(\omega)X_b \sin(\omega t + \varphi(\omega))$$

Summarizing the steady state response of the displacement of the top mass as a function of the displacement of the bottom mass we get:

$$x(t) = X(\omega)\sin(\omega t + \varphi(\omega))$$

in which

$$X(\omega) = M(\omega)X_b$$

$$M(\omega) = 1/\left[\left[1-\left[\frac{\omega}{\omega_n}\right]^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right]^{1/2} \tag{2}$$

is the "amplitude magnification ratio," "amplitude ratio," or "magnitude ratio," in which

$\omega_n = \sqrt{k/m}$, is the natural frequency

$c_c = 2\sqrt{mk}$, is the critical damping coefficient (non-oscillatory solutions)

$\zeta = c/c_c$, is the damping ratio

and

$$\varphi(\omega) = -\arctan\left[\left[2\zeta\frac{\omega}{\omega_n}\right] / \left[1-\frac{\omega^2}{\omega_n^2}\right]\right] \qquad (3)$$

is the phase lag.

It can be seen that the motion of mass m is dependent not only on the motion of mass m_b but also on the system parameters ω_n and ζ. The behavior of M and φ as functions of ω/ω_n and ζ (i.e. Eqns. 2 and 3) are shown in Figs. 4 and 5.

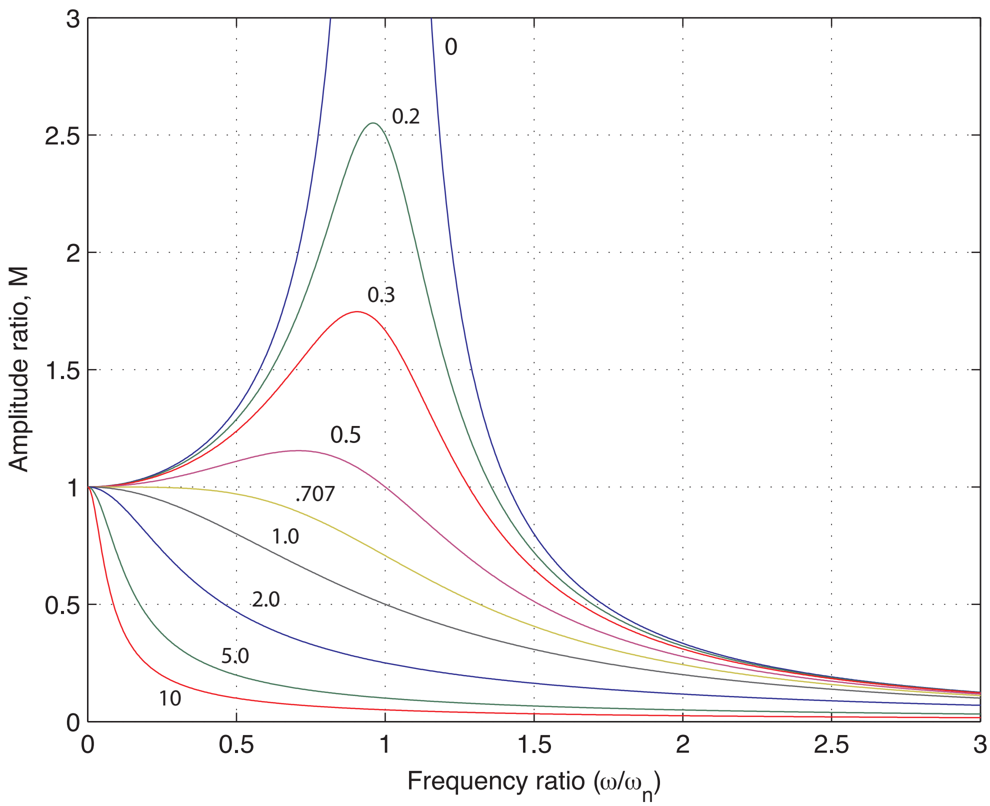

Figure 4: Frequency Response (Eq. 2) of second order system for various values of c/c_c (the damping ratio ζ).

Here it should be stressed that the forgoing theory was developed for a 2nd order *linear* oscillator. It is very difficult to assemble a truly linear 2nd order oscillator, i.e. one that does not deviate at all from linear theory: all real systems have some small amount of non-linearity inherent to them, including the system you will study (Discussion question: what aspect(s) about this setup lend themselves to non-linearity?). You should be aware of the fact that you may encounter *measurable* deviations from the linear theory developed in this section. Since the mathematics involved is beyond the scope of this course, you do not have to incorporate nonlinear terms into the model – just be sure to understand the linear model well enough so you can recognize deviations from it.

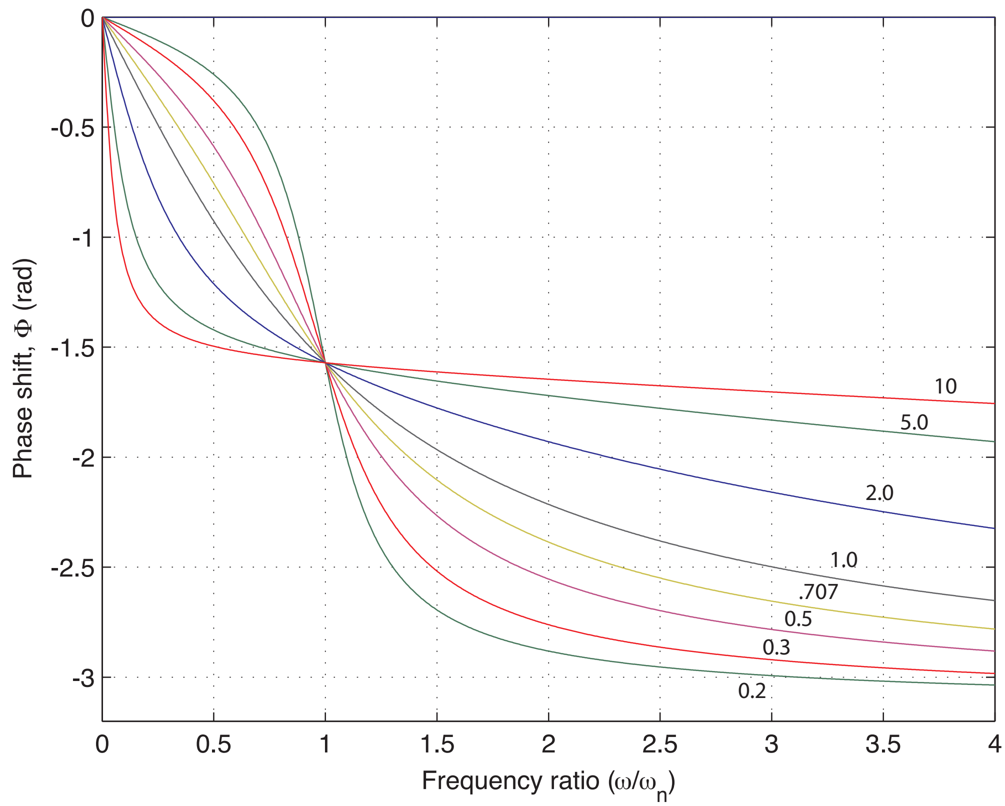

Figure 5: Phase-shift response (Eq. 3) for a second-order system.

3. Experimental Procedures

NOTE: SINCE THIS EXPERIMENT INVOLVES EXPOSED MOVING PARTS, THE WEARING OF SAFETY GOGGLES IS REQUIRED.

Here a set of general procedures is described. It is written in a general sense because each of your measurement systems will be different, so you will have to individually determine the best procedures to follow to complete the project. In order to accomplish some of these tasks you will likely need to perform basic experimental tasks that are not specifically described on this list. See me if you have questions.

- Determine the spring constant of the spring used in the oscillator directly via static deflection (ie, without removing the spring from the shaft). Note that the value of the top collar's mass is inscribed on the collar itself, in units of grams. Please do *not* remove the collar for any aspect of the project, since the bearings that support the collar against the shaft are easy to fall out, changing the damping characteristics.

- Determine the natural frequency of the upper mass (you should use multiple methods as a sanity check).

- Determine the damping ratio of your system (use multiple methods here as well).

- Determine the driving frequency for a particular setting of the knob. You can use your measurement system to do this if it is capable of doing so, if not, use a stroboscope.

- Measure the total linear displacement of the lower mass. This is the peak-to-peak amplitude of the input amplitude X_b.

- Measure x(t), the amplitude vs time of the top mass. Determine the peak amplitude X and the frequency(ies??) of the oscillating mass. How fast do you have to sample (acquire your data) to resolve the oscillations?

- Measure the frequency response of the oscillator (this can be simply repeating the amplitude vs time step above for different input frequencies). Take data for **both** increasing and decreasing drive frequencies. The data should be taken at enough frequencies to form a well-resolved (i.e. 4 frequency steps is **not** sufficient, shoot for more – around 25 if possible) set of response data that spans from below the resonance frequency, defined as $\omega_R = \omega_n \sqrt{1 - 2(c/c_c)^2}$, to above it. Each measurement should be made only after the system has attained a steady state at the new frequency (Why?). Ideally the data you report should be the average of several runs at each frequency so you can estimate the precision uncertainty of your measurement system.

- *EXTRA CREDIT: The only way to determine the phase lag of the top mass will be if your setup allows (or if you have utilized more than 1 transducer) simultaneous measurement of the displacement vs time of the bottom mass for each frequency.*

While measuring the dynamic response of the oscillator, you should note whether the oscillator bottoms out (are these data useful?). Also, note the DC offset of your signal, as you may need it for the analysis of your data. Contact your instructor if the spring noticeably wears against the rod.

4. Description of Pre-Selected Measurement Schemes

The table below contains a brief description of each of the pre-selected measurement schemes. **Note that the website has data sheets and information posted on most of these transducers and the data acquisition board.** The oscillator collars are tapped for **10-32 screws**. Do not remove the collars or spring connections at any point during this project!

PRE-SELECTED SCHEMES	Ultrasonic air-ranging transducers	Optical proximity sensor
Method/principles	Short tone bursts of ultrasound are emitted from a focused air transducer, which is periodically switched between send ("pulse") and receive ("echo") mode. Pulses are reflected from the object to be measured. By knowing the time between emission and reception of a pulse and the sound speed in air, the distance can be calculated.	Continuous, modulated broadband LED light is emitted and collected from a fiber bundle. The light emitted has a known spreading angle. The light is reflected from the object to be measured. By obtaining the reflected intensity, the distance to the object can be calculated. This sensor **requires** your group to use the correct bracket to **safely** support the sensor.
Quantity measured	Acoustic pressure pulse, in phase	Broadband continuous light intensity
Output of transducer/ embedded electronics	Analog AC voltage pulse (toneburst) with center frequency at transducer resonance frequency, variable start time, pulse length, and repetition frequency.	Analog DC current (4-20 mA) proportional to received light intensity.
Spatial Resolution	Dependent on product specs and implementation: potentially 100 microns	Dependent on product specs & implementation: potentially 1 mm or better
Temporal resolution	Dependent on repetition frequency: 3 Hz to 25 Hz, or user supplied?	Potentially 100 Hz
Range of motion detected	Dependent on transducer selected: 8 cm to 30 ft	Model dependent: 24 – 48 in. nominal

PRE-SELECTED SCHEMES	Linear potentiometer	Accelerometers
Method/principles	A sliding contact connected to an extensible wire moves between two endpoints of a third resistive wire (or resistor) which drops an applied voltage. The voltage read from the sliding contact is some fraction of the applied voltage, and is linearly proportional to the length the extensible wire is extended. Thus, suitably calibrated, a measurement of the voltage is a measurement of the distance.	An internal oscillatory mechanism is mounted to the walls of container, which is then mounted to the device in question. The acceleration of the base of the container is the quantity sensed.
Quantity measured	DC voltage proportional to resistance of changing length of wire.	Acceleration
Output of transducer/embedded electronics	DC voltage.	Analog DC voltage signal proportional to the acceleration.
Spatial Resolution	Dependent on product specs and implementation: potentially 100 microns or better	Indirect, proportional to acceleration.
Temporal resolution	Dependent on acquisition, unknown lifetime issues	DC – 100 Hz
Range of motion detected	0.5 to 25 in	Unlimited (except by cable length), but acceleration is limited by bandwidth

As you can see from this partial listing of features, there are potential advantages and disadvantages to each of the pre-selected transducers. Each group will be required to choose either one of the transducer types from the table, or implement another scheme/transducer not listed in the table. For example, you might (but probably won't) elect to measure the Doppler-shifted acoustic signal, by either reflecting a source from the plate, or mounting a source to the plate. You would then need to get a source and receiver, function generator, and analog electronics to detect the Doppler shift (FM demodulation is the usual way). The resulting analog signal could then be digitized.

Safety

Because of the rapidly-moving parts in the apparatus, this project has the most safety concerns out of all the labs in ME310. As usual, 'safety' refers both to your safety (don't get your hair or jewelry caught in the apparatus) **AND** to the equipment that you're using. You will likely be building circuits and in some instances, attaching equipment to the mass (accelerometer, potentiometer). Be **very** careful to properly wire up your circuits and to keep all instrumentation and cabling safe from entanglement with the mass. We have gone through several sensors in past years due to careless setup issues – don't add your group's name to the list!

Basic guideline: Several of these sensors require an external power supply, and here you'll want to use the DC power supplies that are on the equipment racks. **Make sure** that you set the proper voltage on the power supply **BEFORE** connecting the voltage wires to your sensor. This may seem obvious, but it is easy to accidentally dial in the wrong voltage, and sensors have been destroyed in previous years due to this issue.

Computer Acquisition

For all transducers, the output is an analog voltage or current signal. This signal needs to be suitably conditioned and then digitized. Each of the racks is equipped with an A/D board installed in one of the PCI slots. These boards have 16 bit resolution, maximum 250 kSample/s sampling rate, and multiple input range options. The PCs have MATLAB and LabView installed to perform software control of the A/D conversion and data analysis. You will have to learn some MATLAB or LabView to acquire data but template data acquisition files will be made available on the course website (for MATLAB) or via the LabView files you've used in lab (don't overwrite these files! Save them under a new name for your group and then modify as necessary).

5. Results

Note: ALL plots in your report should be **original** (including the theory section)! It is not difficult to reproduce Figs. 4 and 5. A full uncertainty analysis is expected for all results and the relevant intermediate steps necessary to generate them.

1. Demonstrate knowledge of the time-dependent **position** of the top collar as measured by your sensor. Plot $x(t)$ vs time for **a few** representative frequencies spanning the range.

2. Plot the experimental amplitude ratio X/X_b as a function of ω, with appropriate error bars from your uncertainty analysis. Be sure to indicate which points correspond to increasing frequency and which correspond to decreasing frequency. As a guide the plot

should look something like the theoretical curves in Fig. 4. Keep in mind that for this frequency and amplitude range, a log scale likely isn't the best option.

3. On the same plot, plot the theoretical magnitude ratio M using your experimentally determined values for ω_n and ζ. Clearly label the theoretical curve in order to distinguish it from the experimental curves. Does the model correspond to the data collected while increasing the frequency, or to the data collected while decreasing the frequency, or neither? Explain why or why not.

4. Plot the theoretical phase shift as a function of ω using the experimentally-determined values for ω_n and ζ. If you are able to measure it (for extra credit), plot the phase shift as a function of frequency as well. If you cannot measure it be sure to note its qualitative behavior. Discuss the behavior of the phase in terms of the theoretical model.

5. Produce a table that shows the system characteristics & uncertainty (k, ω_n, ζ, c, and m).

6. Project Report

The report for the design project should generally follow the format that you've used for the regular weekly labs. It should include a full Analysis & Uncertainty Analysis section. The Theory section should describe both the theory behind a second-order system **and** the theory of the nature of your transduction system. Describe the basis for **how** it makes its measurement (see Figliola & Beasley as well as references below), since understanding how your measurement was made is equally important as understanding the response of a second-order system. Instead of a Procedure checklist, describe the experimental steps that were involved in the course of making all the relevant measurements. Make sure you keep a detailed lab notebook to track everything that you did! Note that the Equipment List should be especially detailed, with oscillator number, model numbers, resolution, voltage/current range, and error parameters.

7. Further Reading

Figliola, Richard S. and Donald E. Beasley. Theory and Design for Mechanical Measurements. 5th ed. New York: Wiley, 2011.

Beckwith, Thomas G., Marangoni, Roy D., Lienhard, John H. Mechanical Measurements, 6th ed. New Jersey: Pearson Prentice Hall, 2007.

Wheeler, Anthony J. and Ahmad R. Ganji. Introduction to Engineering Experimentation. 2nd ed. Upper Saddle River, New Jersey: Pearson Prentice Hall, 2004.

8. Who to blame:

Written by B. Bharadvaj (1984). Revisions: Bharadvaj (1985); M Isaacson (1985); Isaacson, M. D'Angelo (1987); Isaacson (1992, 1996, 1997); C. Thomas, RG Holt (2003); itty bitty changes by Holt (2005, 2007, 2009); C Farny (2010, 2011, 2013, 2015, 2016).