WHY–5 — Four-Bit Receipts: Sufficiency, Escalation to 8/64, and the Hot■Zone “Qubit■Cap”

CQE Textbook • Phase II — Applied Foundations

Abstract. Claim. In CQE, a 4■bit receipt per sidecar is sufficient for stable governance and replay in routine workflows. When domains require greater discriminative power (collision resistance, multi■actor provenance, or cross■site reconciliation), the system lifts monotonically to 8 or 64 bits without changing the reasoning spine. We prove sufficiency, define escalation triggers, and show how “hot■zone qubit■caps” bundle four bits into local corrective actors that resolve contention while preserving global parity.

1. Why four bits are enough (most of the time)

1.1 Role of a receipt in CQE A receipt is a compact fingerprint of a stabilized sidecar run: it certifies palindromic rest (mirror pass), octet coverage, ∆■lifts applied, and strict thresholds achieved. The receipt must be: • Small (to travel everywhere with the object), • Deterministic (replay regenerates the same code under the same gates), • Monotone (once earned, later replays can only keep it or improve to a stricter code), • Composable (multiple receipts can be fused without ambiguity).

1.2 Information carried by the 4■bit code Let b3 b2 b1 b0 be the four bits. A canonical assignment that matches CQE practice is: • b3 — Mirror integrity: forward■inverse ≈ identity within tolerance (idempotent rest). • b2 — Octet coverage: ≥K of 8 views passed (K configurable; default 6 or 7). • b1 — ∆■lift monotonicity: all local repairs reduced debt and caused no regression (non■increasing debt vector). • b0 — Strict ratchet: thresholds successfully tightened at least once with no rollback. This layout encodes sufficiency for most “go/no■go + replay” decisions: 0000 means unearned; 1xx1 is a strong working state; 1111 is a gold■standard pass. Because each bit corresponds to an independent gate, the code cleanly composes across sidecars (bitwise AND for consensus, OR for permissive union).

1.3 Hamming spacing and why 4 bits work With 4 bits there are 16 codes. CQE avoids adjacent codes for materially different states by enforcing a minimum Hamming distance of 2 for “named” classes. That leaves at most 8 usable named classes (exactly the octad). This is not an accident: the n=4→5 hinge forces eight inequivalent insertion classes; our 16■code space cleanly hosts them with one bit of slack per axis (mirror/∆ vs octet/strict). In routine operations, actors only need to know which of those eight legal states they are in; finer partitioning lives in the receipts’ annex (hashes, debts, votes). Hence four bits suffice to route work and replay deterministically.

2. The annex: receipts beyond 4 bits, without changing the code

A receipt is (code4, annex). The annex carries high■entropy details: • Merkle root over (rest_hash, ∆■script, thresholds, view votes), • OPE/FCE debt vectors (operation / fit■conditioning error), • Environment knobs (versions, calibration IDs).

Crucially, annex content never alters the 4■bit code; it refines it. If two annexes disagree, the mirror gate will fail on replay and the 4■bit code will refuse to certify. Thus 4 bits govern, the annex explains. This separation keeps transport cheap while protecting truth.

3. Escalation to 8 or 64 bits: when and how

3.1 Triggers Escalate the code width when any of the following hold: T1 — Cross■site reconciliation needs >8 named classes with Hamming≥2 (e.g., federation of labs). T2 — Adversarial collision budget rises (e.g., public marketplaces of commits). T3 — Multi■rail provenance must be encoded directly in the code (e.g., joint OPTICS+THERMAL+BIO pass/fail patterns).

3.2 Construction • 8■bit lift: concatenate two independent 4■bit receipts (e.g., compute/code lanes or two octet blocks). Enforce Hamming≥3 on named classes. • 64■bit lift: pack eight 4■bit receipts ordered by a fixed sidecar palette (S0..S63). Reserve a parity nibble for the album. All lifts are monotone: the low 4■bit projection remains unchanged. Existing tooling that only understands 4■bit codes keeps working (graceful degradation).

3.3 Collision math (sketch) Let the set of live named classes be C with |C|≤8 at 4■bit. The birthday bound for accidental collisions in annex hashes dominates practical risk; the code collision risk is controlled combinatorially by the Hamming spacing. When |C| grows, promote to 8■bit with spacing≥3, increasing the minimal error needed for any code■substitution attack. Annex Merkle roots should be ≥128 bits to push birthday collisions out of scope.

4. The hot■zone “qubit■cap”: local four■bit actors for repair

A qubit■cap is a tiny, bounded actor that attaches to a hot zone (a locus where gates disagree) and carries a 4■bit agenda: • b3: restore mirror locally (find a reversible identity), • b2: improve octet coverage (add the missing views or simulate stand■ins), • b1: propose a ∆ that reduces local debt without raising any sibling debt, • b0: retighten a threshold that was loosened during triage.

Two caps are deployed symmetrically on opposite lanes (parity), guaranteeing that local fixes remain globally palindromic. Because each cap speaks only four bits, many can run concurrently without coordination blow■up; the ledger later fuses their receipts (bitwise AND + annex merge). In practice, this behaves like “on■demand qubits”: micro■agents that collapse ambiguity at a site while keeping superposition (alternative paths) alive elsewhere.

5. Worked examples (pen■and■paper)

5.1 Optics bench: segmented mirror phasing Initial receipts: S2(1011), S0(1001). THERMAL missing octet views ⇒ S7(1 0 1 0). A qubit■cap on S7 adds a dark■sky vent simulation (virtual view), finds ∆■lift (shorter vent window), strict passes ⇒ S7 becomes 1101. Bitwise AND across sidecars: 1 0 0 1 AND 1 1 0 1 AND 1 0 1 1 → 1001 ⇒ “ship with caution” until POLAR joins. Annex shows OPE debt fell 18→14 nm WFE; Merkle root binds the ∆■script. No change to 4■bit semantics required.

5.2 DNA learning gate (wet■lab) Experiment vs. ODE overlay fails mirror (drift in late kinetics). Cap enforces a timing identity (rescale inhibitor clock), raising b3; ∆ tweaks stem length (+1 bp) lowers residuals (b1), octet expands to include salt/temperature sweeps (b2), strict recovers crosstalk ceiling (b0). Receipt flips from 0010 to 1111 with annex documenting exact sequences and buffers.

5.3 Finance stress ensemble (non■scientific but illustrative) Octet = {vol windowing, liquidity shock, scenario tree, microstructure noise, cross■asset coupling, regime change, slippage, latency}. A market replay fails strict (thresholds loosened during panic). Cap proposes a Δ (robust loss) and mirror replay (train↔eval swap), restoring 1xx1 while annex logs drawdown paths. Four bits route the pipeline; annex audits it.

6. Formal properties

P1 (Determinism). Given fixed gates and annex, code4 is deterministic under replay. P2 (Monotonicity). Any accepted Δ■lift cannot lower a previously earned bit; strict ratchet cannot relax after pass. P3 (Composability). Receipts from disjoint sidecars compose via commutative, associative bitwise operations; annexes merge via Merkle■tree union. P4 (Refinement). 8/64■bit lifts refine code4 by embedding it as the low■order projection; low■capacity agents can still make correct decisions. P5 (Safety). If annexes disagree materially, mirror fails and b3=0 forces non■commit; thus no silent divergence with equal code4 can persist under replay.

7. Implementation sketch (API)

POST /commit Body: { "sidecar": "S7.THERMAL", "code4": "1101", "annex": { "merkle": "c0a5…", "debts": {"ope":0.029,"fce":0.031}, "votes": {"mirror":"19/24","views":"44/64"}, "deltas": ["vent_window:-1 tick","emissivity:0.85"] }, "thresholds": {"strict":{"ΔT_tick_max":0.8}} }

POST /fuse # bitwise AND across receipts, annex merged by Merkle forest GET /replay # deterministically recompute code4 from annex and gates POST /escalate?width=8 # monotone lift; returns code8 with embedded code4

8. Falsifiers and health checks

F1: Two non■equivalent states share the same code4 under enforced Hamming spacing (show a concrete counterexample). F2: Replay determinism breaks: same annex/gates yield different code4. F3: A Δ■lift reduced local debt but increased sibling debt without b1 falling — violates monotonicity detection. F4: An 8/64■bit lift changes the 4■bit projection — violates refinement. Keep these in CI; they guard the spine.

9. Takeaways

• Four bits are not a compromise; they match the minimal legal partition forced by the n=4→5 hinge (the octad) and route 99% of decisions. • The annex carries the richness without bloating routing state. • Escalation paths are monotone and backward compatible. • Qubit■caps localize contention resolution while preserving global palindromy. This keeps CQE fast, auditable, and mechanically grounded.