Abstract

We present Cartan–Quadratic Equivalence (CQE), a modular, audit-first framework: E_8 geometry as a hard substrate; Alena operators as discrete guidance; and MORSR—a Middle-Out Ripple Shape Reader—for staged exploration with strict provenance. CQE unifies overlays, a Weyl-invariant quadratic objective Φ, and a monotone acceptance rule ($\Delta\Phi \le 0$; Midpoint requires strict parity improvement). The $E_8 \times 3$ comparative projection pattern merges evidence (Left/Right read-only) into a parity-governed Center solve-frame. Initial ablations show: operators drive progress; parity guards correctness; geometry constrains drift. This suite documents axioms, policy, and a shell expansion protocol ($\times 2/\times 4/\times 8$) with thresholded stopping and fully reason-coded logs.

1. Axioms

Axiom A — State Space. States are overlays on E_8 with binary activations, optional weights/phase φ, and immutable pose. Domain adapters embed native objects to overlays.

Axiom B — Group Action. A Weyl/Coxeter group G acts by isometries; geometry terms are Weyl-invariant.

Axiom C — Quadratic Objective. $\Phi = \alpha\Phi\_geom + \beta\Phi\_parity + \gamma\Phi\_sparsity + \delta\Phi\_kissing + \nu \geq 0$.

Axiom D — Equivalence. $x \sim\_CQE\ y$ iff $\exists g \in G$: $y=g\cdot x$, $\Phi(y)=\Phi(x)$, and parity signature $\pi(y)=\pi(x)$.

Axiom E — Operators (Alena). $R\theta$, WeylReflect(s_i), Midpoint, ParityMirror; MORSR orchestrates.

Axiom F — Provenance. Every accepted transition logs $\Delta\Phi$, op, reason code, policy stamp, and parent IDs (signed when keys are present).

Axiom G — Compositionality. Isomorphic domain objects embed to CQE-equivalent overlays.

2. $E_8 \times 3$ Comparative Projection

Two read-only sources (Left/Right) route into Center across allowed sectors with weights w_left, w_right. Center-only parity corrections are allowed; sources remain pristine. Conflicts resolve by deterministic tiebreakers. The result is a solve-frame overlay with provenance and sector histograms for explainability.

3. Shell Protocol & Acceptance

Run MORSR inside a hard shell (radial or graph). Reject any out-of-shell moves (log reason='out_of_shell'). Expand shell by factors ×2/×4/×8 per stage. Compute stage return (accept_rate | strict_gain | novelty) and an EMA; stop when both fall below a threshold τ. Acceptance: $\Delta\Phi \leq -\varepsilon$; Midpoint may accept with $\Delta\Phi \approx 0$ only if parity syndrome strictly decreases. Plateau accepts optional with a small global cap.

4. Governance, Logging, Replay

Policy stamp (cqe_policy_v1.json) specifies strictness, weights, shell schedule, and governance ids. Every run emits region.json (policy, stages, rings, overlay_store, status), handshakes.jsonl (per attempt fields: op, φ_before/after, Δ, reason), and metrics.json (attempts, accepts, strict_gain, reasons histogram). Overlays for accepted states are stored for deterministic replay.

5. Limitations & Risks

Φ requires domain tuning; tight geometry can stall exploration (mitigated by shells). Monotone acceptance avoids divergence but may miss uphill moves (use staged shells/plateaus sparingly). Adapters must respect native symmetries. All risks are transparent due to reason-coded provenance.