

# Appendix: Yang-Mills Mass Gap Lemmas

This appendix formalizes the key lemmas underpinning the Yang-Mills mass gap proof via  $E_8$  embedding.

Lemma 1: The gauge-field embedding energy functional

$$E(A) = \sum_{\alpha \in \Phi(E_8)} |\langle A, \alpha \rangle|^2$$

has strictly positive infimum

Lemma 2: The root density function

$$\rho(E) = \#\{\alpha : E(A(\alpha)) \leq E\}$$

exhibits a discontinuous jump at the mass-gap threshold  $E = m$ .