

Worked Example 1 – Tiny Markov Chain (3 states)

Goal: encode a 3-state chain with transitions:

A→(A:1/2,B:1/4,C:1/4); B→(A:1/3,B:1/3,C:1/3); C→(C:1).

Dictionary: ranks {A=Ace,B=2,C=3}; suits carry destination bins (\clubsuit =A, \spadesuit =B, \diamondsuit =C, \heartsuit =unused).
Face-up = token present. Piles: one row per source.

Build:

- Make piles A,B,C. For A, place 4 face-up tokens: $2 \times \clubsuit, 1 \times \spadesuit, 1 \times \diamondsuit$.
- For B: 3 tokens $1 \times \clubsuit, 1 \times \spadesuit, 1 \times \diamondsuit$. For C: 1 token $1 \times \diamondsuit$.

Interaction:

- Blue deck lists source tokens row-wise. Red deck lists paired destinations; i-th Blue aligns to i-th Red suit.

Checks: P pass (row reversal keeps counts). M pass if suit bins independent of color. Δ pass (adjacent tweak). S pass if we only tighten declared bounds.

Readback: normalize per-row destination counts to probabilities.

Worked Example 2 — 3-State Controller (Idle/Heat/Cool)

Spec: States {Idle,Heat,Cool}. Input temp {Low,OK,High}. Outputs: heat_on,cool_on.
Policy: Idle→Heat if Low, Idle→Cool if High; Heat stays on Low else→Idle; Cool stays on High
else→Idle.

Dict: ranks Idle=A, Heat=2, Cool=3. Suits encode input (\clubsuit =Low, \spadesuit =OK, \diamondsuit =High). Parity:
red=heat_on, black=cool_on. Cut separates state block from I/O.

Two-deck map:

- Blue enumerates (state,input).
 - Red gives next-state with parity flag for output.
- Checks: P pass; M pass if we swap output semantics; Δ pass (local rule tweak); S: safety bound only tightens.

Readback: for each Blue row, paired Red row = target state; parity = which output bit is 1.

Worked Example 3 — Toy Reaction Network ($A + B \rightarrow C$)

Stoichiometry: $A + B \rightarrow C$; optional leak $C \rightarrow A$ (tiny).

Dict: suits species ($\clubsuit=A$, $\spadesuit=B$, $\diamondsuit=C$, $\heartsuit=solvent$). Rank encodes quantity bucket. Face-up=present.
Piles are compartments (reactor, environment). Cut separates before→after.

Before: reactor has ♣ and ♠ face-up. Move: rewrite pair (\clubsuit, \spadesuit) to one ♦ face-up (Δ -lift).
Optional leak via auxiliary token pile.

Two-deck view: Blue=pre multiset, Red=post. Row-wise alignment encodes rewrite.

Checks: P pass; M fails unless relabeled chemistry (declared); Δ pass; S: tighten max-C bound
only after evidence.

Readback: count species suits in reactor after move; conservation visible modulo leak.