

# Legality■First AI: A Unified, Geometry■Based Framework

## Abstract

We replace similarity with legality as the source of truth. Inputs are integerized and evaluated in parallel congruence channels ( $\text{mod } 2, 4, 8, \dots$ ) on a  $k+2$  dimensional toroidal control manifold. A state is admissible iff it satisfies

(i) Type■II lattice legality (Construction■A even/unimodular), (ii) palindromic balance in mod■4/8 histograms, and  
(iii) zero syndrome for its code constraints. Otherwise a confluent, terminating rewrite decreases a lexicographic violation vector until a normal form (REST) or REJECT. Numeric glyphs compile frames; computation is ledgered and committed once; retrieval keys are invariants. This yields determinism, auditability, and energy efficiency; undecidable claims propose explicit, priced model extensions.

## Core Objects & Axioms

A1 Integerized state. Representations are integer vectors.

A2 Frame from glyphs. Numeric glyphs compile thresholds ( $\tau$ ), a scale family ( $S \approx 4^k$ ), modulus set  $M$ , and face priorities.

A3 CRT switchboard. Work in residues and stitch by CRT.

A4 Type■II legality. Construction■A lift must land on even, unimodular lattice (e.g., E8/Leech slices).

A5 Palindromic witnesses. Histogram anti■symmetry scores P4,P8 must be within bounds.

A6 Confluent reduction. Violation vector  $\Phi = (\text{synd}, P8, P4, p)$  strictly decreases in ACTION steps.

A7 Facts=Invariants. Truth confined to frame■invariant quantities; cross■frame requires sanctioned functor.

## 10■D Toroidal Control

Default  $k=8$  residue faces + phase + epoch. Parallel transport legal subspaces to prevent re■embedding drift; pose is gauge.

## Algorithms

COMPILE\_FRAME: derive  $(\tau, S, M, FACES)$ . VERIFY: compute residues, syndrome, pal scores, Type■II flag  $\rightarrow$  REST/ACTION/REJECT.

REDUCE: smallest legal move that lowers  $\Phi$ . NORMAL\_FORM: iterate VERIFY/REDUCE; ledger steps; commit once. RETRIEVE: by invariant key ( $F$ , shell, coset).

## **Guarantees & Limits**

Determinism (confluent/terminating), explainability (channel-local reasons), robustness (ECC radius), efficiency (commit-once), safety (illegal cross-frame). Limits: frame blindness; small-n degeneracy; adversarial jamming mitigated by multi-mod checks.

## **Falsifiers & Metrics**

F1 Confluence break; F2 Type-II mirage; F3 Pal witness failure; F4 Legality hallucination; F5 Energy regression; F6 Retrieval regression.

Metrics: legality-rate, confluence-hash, ECC radius, energy/commit, time-to-REST, cross-frame rejections.