

Worked Example 1 — Tiny Markov Chain (3 states)

Goal: encode a 3-state chain with transitions:  
 $A \rightarrow (A:1/2, B:1/4, C:1/4)$ ;  $B \rightarrow (A:1/3, B:1/3, C:1/3)$ ;  $C \rightarrow (C:1)$ .

Dictionary: ranks {A=Ace,B=2,C=3}; suits carry destination bins ( $\clubsuit$ =A,  $\spadesuit$ =B,  $\diamondsuit$ =C,  $\heartsuit$ =unused).  
Face-up = token present. Piles: one row per source.

- Build:
- Make piles A,B,C. For A, place 4 face-up tokens:  $2 \times \clubsuit, 1 \times \spadesuit, 1 \times \diamondsuit$ .
  - For B: 3 tokens  $1 \times \clubsuit, 1 \times \spadesuit, 1 \times \diamondsuit$ . For C: 1 token  $1 \times \diamondsuit$ .

Interaction:

- Blue deck lists source tokens row-wise. Red deck lists paired destinations; i-th Blue aligns to i-th Red suit.

Checks: P pass (row reversal keeps counts). M pass if suit bins independent of color.  $\Delta$  pass (adjacent tweak). S pass if we only tighten declared bounds.

Readback: normalize per-row destination counts to probabilities.

Worked Example 2 — 3-State Controller (Idle/Heat/Cool)

Spec: States {Idle,Heat,Cool}. Input temp {Low,OK,High}. Outputs: heat\_on,cool\_on.  
Policy: Idle→Heat if Low, Idle→Cool if High; Heat stays on Low else→Idle; Cool stays on High else→Idle.

Dict: ranks Idle=A, Heat=2, Cool=3. Suits encode input (♣=Low, ♠=OK, ♦=High). Parity: red=heat\_on, black=cool\_on. Cut separates state block from I/O.

Two-deck map:  
• Blue enumerates (state,input). • Red gives next-state with parity flag for output.  
Checks: P pass; M pass if we swap output semantics; Δ pass (local rule tweak); S: safety bound only tightens.  
Readback: for each Blue row, paired Red row = target state; parity = which output bit is 1.

Worked Example 3 — Toy Reaction Network (A + B → C)

Stoichiometry:  $A + B \rightarrow C$ ; optional leak  $C \rightarrow A$  (tiny).

Dict: suits species ( $\clubsuit=A$ ,  $\spadesuit=B$ ,  $\diamondsuit=C$ ,  $\heartsuit=\text{solvent}$ ). Rank encodes quantity bucket. Face-up=present. Piles are compartments (reactor, environment). Cut separates before→after.

Before: reactor has  $\clubsuit$  and  $\spadesuit$  face-up. Move: rewrite pair  $(\clubsuit, \spadesuit)$  to one  $\diamondsuit$  face-up ( $\Delta$ -lift). Optional leak via auxiliary token pile.

Two-deck view: Blue=pre multiset, Red=post. Row-wise alignment encodes rewrite.

Checks: P pass; M fails unless relabeled chemistry (declared);  $\Delta$  pass; S: tighten max-C bound only after evidence.  
Readback: count species suits in reactor after move; conservation visible modulo leak.