WHY–4 — E8/Leech as the Minimal Stable Geometry (Not Lattice Flattery)

Purpose

Prove (by constructive sketches and falsifiers) that CQE's octet chamber has a canonical, minimal geometric realization: E8 via Construction A from Hamming(8,4,4), and Leech Λ24 via Construction A from extended Golay(24,12,8) with evenness/half-shift gluing. Show how these shells act as governance—fixing what counts as legal forms, while meaning remains swappable tokens. Address the main skepticism ("lattice flattery") and provide pen-and-paper tests plus a tiny API to encode receipts.

What This File Claims

- n=4→5 forces exactly eight legal insertion classes; that octad is the unique minimal fan-out for lawful extension.
- Given an octad, two synchronized 8-bit parity lanes realize E8 under Construction A; no smaller simply-laced, even, unimodular lattice fits all CQE invariants.
- At 24, extended Golay's octads/glue realize a Leech-equivalent slice (rootless, even, unimodular); octads become physical 'rooms'.
- Automorphisms (M24/Co0/Monster) act as named permutations—relabelings—preserving receipts; geometry provides closure, not fashion.
- CQE gates (Mirror → Octet → Δ-lift → Strict → 4-bit) are geometric constraints in disguise; receipts are coordinates on these shells.

Recap: The Hinge n=4 → n=5 ⇒ Octad

From a unique palindromic rest at n=4 on the 4×4 parity grid, inserting a fifth symbol while preserving determinism, lockstep, and idempotence yields exactly eight non-conjugate insertion classes under dihedral+parity symmetries. This is the octad. It is not assumed; it is forced. The octad is the minimal face set that supports mirrored replay without contradiction.

Construction A: From Code to Lattice (E8 in one page)

Input: a binary linear code C ⊂ F2^n. Construction A lifts codewords to Z^n by selecting integer vectors v whose entries reduce mod 2 to C. For E8, take n=8 and C = Hamming(8,4,4). Enforce evenness and scale so minimal norm is 2. Two root families result:

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• Type I (integer roots): all permutations of (±1, ±1, 0, 0, 0, 0, 0, 0) with even
number of minus signs. (56 vectors) • Type II (half-integer roots): all (±½, ±½, …, ±½)
with an even number of plus signs. (128 vectors) Together with sign/permutation
structure consistent with Hamming(8,4,4), these give the 240 roots of E8; the full
lattice is generated by these roots.
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CQE mapping: the octet = eight coordinate views; palindromic mirror = evenness; Δ-lift = a small root move lowering local 'debt'; strict ratchet = raising minimal acceptable inner products; the 4-bit receipt binds the chosen coset/pose. E8 is the smallest even unimodular lattice in 8D, thus minimal for a stable, replayable octet chamber.

Leech Λ24 in one page: Golay + Glue + Rootless

Take the extended binary Golay code G24 (length 24, dimension 12, distance 8). Construction A lifts F2^24 to Z^24, selects vectors consistent with G24, and applies a half-shift glue to enforce evenness and eliminate norm-2 roots—yielding the Leech lattice (minimal norm 4, even, unimodular). Golay's octads (weight-8 supports) are the canonical 'rooms'. Conway's Co0 acts as the automorphism group; Monster acts on related structures (moonshine).

CQE mapping: a 24-view slice (three octads) supports typed octets across rails; annihilating roots corresponds to forbidding fragile, short-step instabilities. Automorphism actions are 'isomorphic re-threadings'—geometry-fixed, meaning-swappable. Receipts log the action element so any clone is replayable 1:1.

Why Not Some Other Lattice?

- Evenness (mirror parity) + unimodularity (determinism under replay) + minimal fan-out (octad) constrain dimension and structure.
- In 8D, the unique even unimodular lattice is E8; in 24D rootless even unimodular is unique up to isometry: Leech.
- CQE asks for smallest shells satisfying mirror/octet/strict simultaneously; E8/Leech are minimal satisfying choices, not aesthetic choices.

Worked Micro-Example (Paper-Ready)

1) Start from an octet commit:

`Receipt bits = 1011 (Mirror✓, Octet quorum✓, Strict✓, Replay✓); pose = 'H1'.`

2) Encode as Hamming syndrome:

Let H be a standard 4×8 parity-check for Hamming(8,4,4). Map the 4-bit receipt to an 8-bit code lane via a fixed injective embedding (documented in the ledger). Check H·x^T = 0 to ensure we remain in C; if not, apply Δ-lift = add a root vector to repair parity without breaking mirror.

3) Lift to E8 by Construction A and scale:

Select v ∈ Z^8 or (Z+½)^8 consistent with x mod 2, enforce evenness, scale so min■v■^2 = 2. Record lattice coset + pose hash in the receipt.

4) Mirror/replay check:

Apply inverse transform (reduce mod 2 → parity check). The receipt must be idempotent (same 4-bit row); otherwise annihilate and log breadcrumb.

Falsifiers (What Would Break This File)

- F1: Exhibit a lawful n=5 extension (respecting determinism, lockstep, idempotence) with fewer or more than 8 inequivalent insertion classes.
- F2: Produce an octet chamber satisfying CQE gates that cannot be embedded into E8 under Construction A without violating evenness/unimodularity.

- F3: Build a 24-slice with octads but unavoidable norm-2 roots (contradicting Leech rootlessness) while keeping CQE invariants.
- F4: Show that automorphism relabelings cannot be recorded as stable receipts (i.e., replay non-determinism).

Pen-and-Paper Worksheet

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• Draw an 8-strand loom. Enumerate the 56 integer roots by placing ±1 on two strands;
enforce even number of minus signs. Then enumerate the 128 half-integer roots with an
even number of +½ signs. Verify 240 total. • Take a standard H matrix for
Hamming(8,4,4). Pick any 4-bit receipt; embed to an 8-bit lane; check parity. • For
Golay: Mark a 24-grid; choose an octad support; verify distance-8 property; sketch how
half-shift glue removes norm-2 roots.
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Tiny API Sketch

```
{ "form": "E8|Leech", "constructionA": {"code": "Hamming(8,4,4)|Golay(24,12,8)",
"glue": "even|half-shift"}, "octetMap": ["H1", "H2", "…", "H8"], "receipt4": "1011",
"pose": "rotation/reflect id", "aut": "M24 element or Co0 tag", "hash": "merkle(root)"
}
```

Glossary (One-Liners)

- Construction A: Lift a linear code over Fp to a lattice in Zn by parity constraints + scaling/glue.
- Even/Unimodular: All norms even; determinant 1; implies strong parity symmetry and perfect replay.
- Octad: Weight-8 support set in Golay; in CQE, the eight-view chamber.
- ∆-lift: Local root move that lowers debt without breaking other invariants.
- Strict ratchet: Monotone tightening of thresholds post-pass; forbids backsliding.

Closing

E8 and Leech are not decorations; they are the smallest honest geometries that satisfy CQE’s invariants. They explain—rather than assume—why the octet is necessary, why receipts are tiny yet replayable, and why automorphism cloning is provenance-clean. If a smaller or different shell could do the same job under the same rules, this file invites the counterexample.