CQE Proof Pack v0.2

Proof Pack v0.2 — CQE Core Assertions (Date: 2025-09-20)

A. Shared Objects & Invariants

Alphabet $\Sigma_n=\{1,\dots,n\}$. Covering word w contains every length-n permutation as a contiguous factor.

Chamber: 4×4 parity grid with mirrored lanes. Moves are local (cell+neighbors).

Canonicalizer C (Alena priorities): (1) palindromic fix; (2) minimize defect count; (3) minimize motion; (4) avoid Joker; (5) lowest rank. Idempotent: $C(C(x))=C(x)$.

Mirror test: forward■inverse ≈ identity in gray rest within written tolerance. Δ-lift: local repair strictly reducing debt $D\geq 0$. Strict ratchet: thresholds only tighten after pass.

B. S1 — Uniqueness at n=4 (Palindromic Rest)

Statement: Under chamber rules and Alena priorities, every lawful n=4 covering canonicalizes to a single palindromic rest class (up to dihedral pose and color swap).

Sketch: Define a well-founded measure M=(#defects, total motion, #Jokers). Any non-palindromic fork increases #defects. A palindromic local flip reduces #defects without increasing motion or Joker use. Iteration terminates at zero defects. Two terminals differ by pose/color; hence a unique ledger class.

Human check: Build any n=4 weave, tally defects, apply mirrored flips until zero; record pose.

C. S2 — Exactly Eight Legal n=5 Insertions (Octad Forcing)

Statement: From the n=4 palindromic rest, the 16 cellwise insertions of symbol 5 that (i) keep determinism, (ii) repair to rest with ≤1 Joker, (iii) end idempotently, quotient by dihedral-with-parity to exactly 8 inequivalent classes.

Sketch: The 4×4 is saturated at n=4; any 5-insertion preserving palindromy locally breaks it elsewhere. Enumerate 16 loci; run Δ-lifts bounded by one Joker; retain those stabilizing under C. Quotient by D8×{parity}. Results: four axis-fixed and four off-axis anti-fixed orbits = 8.

D. S3 — 1 Palindromic + 7 Invariant Non-palindromes

Statement: Exactly one class re-canonicalizes to a global palindrome; the other seven stabilize as non-palindromic but idempotent invariants.

Sketch: Palindromic rest requires pairwise mirrored neighbors; only one orbit allows global reconciliation post 5-lift. Others settle into minimal residual asymmetry that passes mirror tolerances but is non-palindromic.

E. S4 — Symmetry Equivariance

Statement: For any dihedral symmetry ρ, $\text{decode}(\rho\cdot x)=\rho\cdot\text{decode}(x)$; class labels permute within the octad orbit.

Sketch: Local rules are parity-aware and pose-agnostic; C is built from equivariant moves; receipts are pose-free Merkle commitments.

F. Construction A Acceptance: E8 from H8 (binary)

Claim: E8 = { $u/\sqrt{2}$: $u \in Z^8$ and u mod 2 ∈ H8 } where H8 is the [8,4,4] extended Hamming code.

Acceptance test (by hand or code):

(1) Given a candidate vector $v \in R^8$, scale to $u=\sqrt{2}\cdot v$.

(2) Check u has integer or half-integer coordinates all with the same parity (all Z or all Z+1/2).

(3) Reduce u mod 2 to a binary 8-tuple; verify it is a codeword in H8 (parity-check matrix $H\cdot(u \bmod 2)^T=0$ over F2).

(4) Norm check: roots have $||v||^2=2$; short vectors satisfy the E8 minimal length property.

Parity loom mapping: clips are syndrome ticks; code membership is parity-satisfied clips; half-shift glue realizes the even unimodularity.

G. Golay/Leech Slice (24)

Construction A with the extended Golay code G24: Λ_Leech_slice = { $u/\sqrt{2}$: $u \in Z^{24}$, u mod 2 ∈ G24 and slice has no roots }. Octads correspond to weight-8 codewords; legality = parity + glue.

H. S5 — No Spurious Rest (Monotone Strict Ratchet)

Statement: With Δ-lifts that strictly decrease debt D and strict thresholds that never loosen, any sequence either terminates at a passing rest or is recorded Non-Working; it cannot falsely stabilize.

Sketch: Use lexicographic M=(D, tolerance_index). Δ-lifts decrease M; tightening reduces tolerance_index. No infinite descent; therefore termination at pass or exhaustion (Non-Working).

I. Lean/Coq skeletons (signatures)

Lean-style: structure Chamber, Overlay, Debt; def canonicalize : Chamber → Chamber; theorem idempotent : ∀x, C (C x) = C x; theorem n4_unique : ... ; theorem n5_octad : ... ;

Coq-style: Record chamber := {cells : grid4x4; parity : ...}; Theorem canonical_idem : ∀c, C (C c) = C c. Theorem n4_unique : ... .

J. Receipts & Merkle

Leafs: normalized overlay, deltas (redactable), thresholds, token guards, glyph hash. Merkle root recorded with 4-bit code; redaction retains leaf commitments.