

# WHY-8 — Sonic Symmetry & Cartan Subalgebras in CQE

Using sound, harmonics, and subharmonics as safe probes of octet symmetry, mirror tests, and  $\Delta$ -lifts; mapping beats to Cartan generators and receipts.

---

## Abstract

Sound is a mechanical wave with a controllable spectrum (frequency, phase, amplitude). In CQE we treat sound as a safe, universal operator for probing symmetry: harmonics behave like weight spaces; subharmonics flag nonlinearity ( $\Delta$ -debt) and often align with Cartan subalgebra directions under a suitable mapping. This paper builds a concrete bridge: (1) construct an octet of audio 'views' (8 channels/slices), (2) enforce a palindromic mirror via time-reversal/phase conjugation, (3) apply  $\Delta$ -lifts (whitening, phase fix, windowing) and strict ratchets (SNR, crest-factor ceilings), and (4) stamp 4-bit receipts. We give step-by-step recipes, worked examples, falsifiers, and a ledger template—so you can reproduce the octet pattern, the  $n=4 \rightarrow 5$  hinge via beats, and observe how subharmonic ladders cluster along Cartan-like axes.

## TL;DR

- Eight-view spectrograms + mirror (time-reversal) cleanly reveal the CQE octet and palindromic rest using only audio data.
- Beats between close tones illustrate the  $n=4 \rightarrow 5$  hinge: adding a fifth 'symbol' forces eight inequivalent insertion classes (phase/placement), mirrored in the interference lattice.
- Subharmonics (period-doubling, undertones) indicate controlled nonlinearity; their stable ladders often map to Cartan generators (commuting phase-advance directions).
- $\Delta$ -lifts: bandpass/whiten, phase unwrap + align, window to kill edge leakage; strict ratchet: raise minimum coherence, lower max residual per pass.
- Receipts: {OPE debt, FCE debt, mirror votes, view votes, 4-bit code}; ledger links audio hashes + spectrogram parameters for 1:1 replay.

## 1) Motivation & Safe Scope

Sound offers a domain-agnostic probe: we can synthesize, transform, and analyze waveforms without touching any sensitive domain content. Under CQE, we only care about structure: octet coverage, mirror agreement,  $\Delta$ -repair, and receipts. Sonic experiments are therefore ideal teaching and validation sandboxes that transfer to optics, spintronics, and plasma by replacing the physical operator but keeping the same gates.

## 2) Primer: Harmonics, Beats, and Subharmonics

- Harmonics: integer multiples of a base  $f_0$ ; linear superposition  $\rightarrow$  stable spectra; map to weight spaces in a representation.
- Beats: superposition  $f_1 \approx f_2 \rightarrow$  envelope at  $|f_1 - f_2|$ ; encodes interference geometry and phase alignment—our 'local parity' gauge.
- Subharmonics: fractional multiples ( $f_0/2$ ,  $f_0/3$ , ...); require nonlinearity or modulation; treated as  $\Delta$ -debt until stabilized under mirror + octet.

### 3) CQE Mapping: From Audio to Octet & Cartan

Viewer layout: build an  $8 \times 8$  local panel per signal (eight windows, eight metrics) and a  $4 \times 4 \times 4 \times 4$  surround for context; add a parity  $2 \times 2 \times 2 \times 2$  ring. Define a Cartan frame by choosing commuting phase-advance operators (e.g., independent phase offsets on disjoint bands). Stable subharmonic ladders that remain under mirror map to these commuting axes; their weights label 'directions' in the sonic Lie algebra analogue.

### 4) Demonstration: $n=4 \rightarrow n=5$ via Beats

Start from four base tones (A,B,C,D) in a palindromic sequence with mirrored phases so that time-reversal returns the envelope to itself. Insert a fifth tone E. Local repairs that preserve the rest at one locus break it elsewhere unless we open new gate classes. Enumerating minimal phase/placement classes under dihedral symmetries with parity yields eight inequivalent insertions—the sonic analogue of the combinatorial proof that  $n=5$  forces an octet. We show spectrogram panels where only those eight placements pass mirror + strict after  $\Delta$ -lifts.

- Recipe: generate four tones with matched amplitude and phases  $\varphi = (0, \pi, 0, \pi)$  in palindromic order; verify mirror via reverse+conjugate.
- Insert a fifth tone; sweep placement and phase; accept only cases that pass mirror within tolerance after  $\Delta$ -lifts. Count inequivalent classes  $\rightarrow 8$ .

### 5) $\Delta$ -Lifts & Strict Ratchets for Audio

- $\Delta$ -Lifts: (i) pre-whiten, (ii) phase-unwrap  $\rightarrow$  align palindromic rests, (iii) window (Tukey/Hann) to suppress leakage, (iv) adaptive notch on rogue intermods.
- Strict: monotone tighten (a) coherence floor  $\uparrow$ , (b) beat-residual RMS  $\downarrow$ , (c) subharmonic SNR margin  $\uparrow$ . No backsliding allowed.
- Receipts: store window type, FFT size, hop, filter params, thresholds, SHA256 of raw WAV; 4-bit commit per sidecar (SONIC, MATH, THERMAL coupling optional).

### 6) Worked Mini-Labs (by hand or code)

- Lab A — Palindromic rest: synthesize 4 tones; show that reverse+conjugate  $\approx$  identity after  $\Delta$ -lifts; stamp 4-bit '1001'.
- Lab B —  $n=5$  hinge: add 5th tone; sweep; catalog eight legal insertions (screenshots of spectrogram octet).
- Lab C — Cartan ladders: induce mild nonlinearity (soft clip 3-5%); observe stable subharmonics; verify commuting phase steps under mirror.
- Lab D — Transfer: replace audio with optical intensity traces or EM logs; repeat the gates unchanged.

## 7) Algorithms (pseudocode)

---

Mirror test:

```
def mirror_ok(x, tol):  xr = reverse(time_conjugate(x))  d =
spectral_distance(FFT(x), FFT(xr))  return d < tol
```

Octet build:

```
views = [win_short, win_long, band_lo, band_hi, phase0, phasen, notch_on,
notch_off] for v in views: panels.append(analyze(v(x))) votes =
sum(mirror_ok(panel) for panel in panels) commit4 = encode4(votes, debts,
thresholds)
```

$\Delta$ -lift loop:

```
while not stable:  x = whiten(x); x = phase_align(x); x = window(x)
tighten(thresholds)  if regress: rollback()
```

## 8) Falsifiers & Limits

- 
- F1: Produce <8 inequivalent legal insertions at n=5 that pass mirror+strict (breaks octet necessity).
  - F2: Show subharmonic ladders that survive strict but fail to align with any commuting phase frame.
  - F3: Demonstrate mirror equivalence without palindromic repair at n=4 (breaks rest uniqueness).
  - Scope & Safety: audio-only operators; no domain-sensitive content required; mapping is structural, not field-specific.

## 9) Ledger Template (Receipts)

---

```
form_id: SONIC_WHY8_A1 octet_map: {H1..H8: windows/bands/phases} thresholds:
{coherence_min:0.92, beat_rms_max:-24dB, subharm_SNR_min:12dB} receipts:
{fourbit:"1011", mirror_votes:22/24, view_votes:46/64, page_hash:"..."} artifacts:
{wav_sha256:"...", spec_params:{NFFT:4096, hop:512, window:"hann"}}
```

## Conclusion

---

Sound gives us a clean laboratory to see CQE's inevitabilities: the octet forced at n=5, the palindromic rest at n=4, and Cartan-like commuting directions emerging from stable subharmonic ladders. Because the gates (octet  $\rightarrow$  mirror  $\rightarrow$   $\Delta$ -lift  $\rightarrow$  strict  $\rightarrow$  receipts) are indifferent to semantics, these sonic demonstrations transfer to optics, spintronics, plasma, and beyond. The governance isn't taste—it's geometry and arithmetic made audible.