A Table of Commutation Relations

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1 A table of commutation relations for operators of a single particle.

1.1 Operators to use.

- Observables (Hermitian): $x, p_x, H = \frac{p_x^2}{2m}, N = a^\dagger a, J_x, J_y, J_z, J, \Pi$
- Non-observables: $a, a^\dagger, T,$

1.2 The table.

[row, column]	x	p_x	**H**	**N**	**J**	J_x	J_y	J_z	Π	a	$a^\dagger$	T
x	0	$i\hbar$										
p_x	$-i\hbar$	0	$-i\hbar\frac{\partial}{\partial x}V(x)$									
H		$i\hbar\frac{\partial}{\partial x}V(x)$	0									
N				0						$-a$	$-a^\dagger$	
J					0							
J_x						0						
J_y							0					
J_z								0				
Π									0			
a										0	1	
$a^\dagger$										-1	0	
T												0

[row, column]	S_x	S_y	S_z
S_x	0	$i\hbar S_z$	$-i\hbar S_y$
S_y	$-i\hbar S_z$	0	$i\hbar S_x$
S_z	$i\hbar S_y$	$-i\hbar S_x$	0

1.3 Operator definitions.

- $p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ in the x-basis. Townsend p.158.
- $\Pi\,|x\rangle = |-x\rangle$ Townsend p.213.
- $H = \frac{p^2}{2m} + V(x)$ Sakurai p.97, (2.4.2).

1.4 References.

0 Any operator commutes with itself.

$-i\hbar$ See Townsend eq. 6.31, p. 155 or Griffiths eq. 2.51, p.55. This leads to the uncertainty principle $\Delta x \Delta p_x \geq \frac{\hbar}{2}$.

$-a$ See Sakurai p.90, (2.3.10).

$-a^\dagger$ See Sakurai p.90, (2.3.11).

0 By the product rule.

2 A table of commutation relations for operators of two particles.