Differential Correction

Terminology

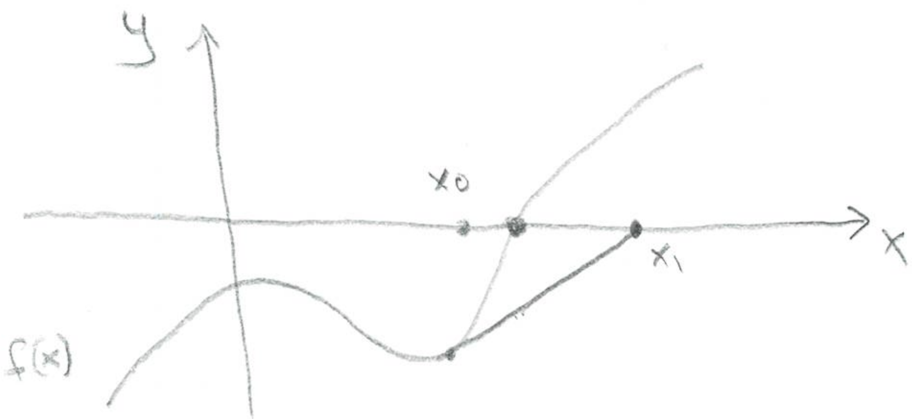
$$\dot{c} = f(c) \quad ①$$

Many unique $c(t)$ that satisfy ① $\quad c(0) = c_0$

For some T

$$c(t+T) = c(t)$$

Newton's method



$$y = mx + b$$

$$\uparrow$$

$$f'(x_0)$$

$$f(x_0) = f'(x_0)x_0 + b$$

$$b = f(x_0) - f'(x_0)x_0$$

$$x_1 = \frac{-b}{m} = \frac{f'(x_0)x_0 - f(x_0)}{f'(x_0)} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

System of equations

$$x_1 = x_0 - J_f(x_0)^{-1} f(x_0)$$

$$J_f(x_0)(x_1 - x_0) = -f(x)$$

$$q = \begin{bmatrix} c_0 \\ T \end{bmatrix}$$

$$H(q) = c(T) - c_0$$

$$J_H = \begin{bmatrix} \frac{\partial H}{\partial c_0} & \frac{\partial H}{\partial T} \end{bmatrix}$$

$$c_0 \longrightarrow c_f$$

$+$

$$[\partial x_0 \; 0 \; 0 \; 0 \; 0 \; 0]$$

$$c_{0+\partial x_0} \longrightarrow c_{f+\partial x_0}$$

$$\frac{c_{f+\partial x_0} - c_f}{\partial x_0}$$

$$c_{0+\partial y_0} \rightarrow c_{f+\partial y_0} \cdots$$

$$\frac{\partial H}{\partial c_0} = \frac{\partial c(t)}{\partial c_0} - \cancel{\frac{\partial c_0}{\partial c_0}} \nearrow I$$

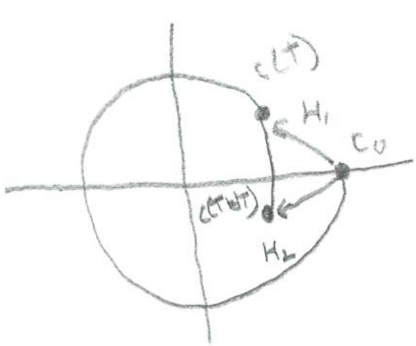
$$\dot{c} = f(c)$$

$$\frac{\partial \dot{c}}{\partial c_0} = \frac{\partial f(c)}{\partial c_0} = \underset{A}{\frac{\partial f(c)}{\partial c}} \; \underset{\Phi}{\frac{\partial c}{\partial c_0}} \qquad \frac{\partial c}{\partial c_0}(0) = I$$

State Transition Matrix (STM)

$$\frac{\partial H}{\partial c_0} = \Phi(T) - I$$

$$\frac{\partial H}{\partial T}$$



$$\partial H = H_2 - H_1$$

$$= c(T+\partial T) - c_0 - (c(T) - c_0)$$

$$= c(T+\partial T) - c(T)$$

$$\frac{\partial H}{\partial T} = \frac{c(T+\partial T) - c(T)}{\partial t}$$

$$\frac{\partial H}{\partial T} = \dot{c}(T)$$

$$\begin{bmatrix} \Phi(t)-I & \dot{c}(T) \end{bmatrix} \begin{bmatrix} \delta c_0 \\ \delta T \end{bmatrix} = -H(q)$$

6×7 7×1 6×1

1 000 000 0

$\delta X_0 = 0$

Velocity Targeter

$$c_0 = \begin{bmatrix} r_0 \\ v_0 \end{bmatrix} \Rightarrow q = \begin{bmatrix} v_0 \\ T \end{bmatrix}$$

$$J_H = \begin{bmatrix} \frac{\partial H}{\partial v_0} & \frac{\partial H}{\partial T} \end{bmatrix}$$

$$\dot{c} = f(c)$$

$$\frac{\partial \dot{c}}{\partial v_0} = \frac{\partial f(c)}{\partial v_0} = \frac{\partial f(c)}{\partial c} \cdot \overset{6\times3}{\frac{\partial c}{\partial v_0}}$$

$$STM = \frac{\partial c}{\partial c_0} = \begin{bmatrix} \cancel{\frac{\partial r}{\partial r_0}} & \frac{\partial r}{\partial v_0} \\ \cancel{\frac{\partial v}{\partial r_0}} & \frac{\partial v}{\partial v_0} \end{bmatrix}$$

$$\frac{\partial H}{\partial v_0} = \frac{\partial c(T)}{\partial v_0} - \frac{\partial c_0}{\partial v_0} \rightarrow \begin{bmatrix} 0 \\ I_{3\times3} \end{bmatrix}$$

$$J_H = \begin{bmatrix} \frac{\partial c(T)}{\partial v_0} - \begin{bmatrix} 0 \\ I_{3\times3} \end{bmatrix} & \dot{c}(T) \end{bmatrix}$$