Stationkeeping of the First Earth-Moon Libration Orbiters - The ARTEMIS Mission

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STATIONKEEPING OF THE FIRST EARTH-MOON LIBRATION ORBITERS: THE ARTEMIS MISSION

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Libration point orbits near collinear locations are inherently unstable and must be controlled. For Acceleration Reconnection and Turbulence and Electrodynamics of the Moon's Interaction with the Sun (ARTEMIS) Earth-Moon Lissajous orbit operations, stationkeeping is challenging because of short time scales, large orbital eccentricity of the secondary, and solar gravitational and radiation pressure perturbations. ARTEMIS is the first NASA mission continuously controlled at both Earth-Moon L_1 and L_2 locations and uses a balance of optimization, spacecraft implementation and constraints, and multi-body dynamics. Stationkeeping results are compared to pre-mission research including mode directions.

INTRODUCTION

Acceleration Reconnection and Turbulence and Electrodynamics of the Moon's Interaction with the Sun (ARTEMIS) is the first mission flown to and continuously maintained in orbit about both co-linear Earth-Moon libration points, EM L_1 and EM L_2.[1-6] The ARTEMIS mission transferred two of five Time History of Events and Macroscale Interactions during Substorms (THEMIS) spacecraft from their outer-most elliptical Earth orbits and, with lunar gravity assists, re-directed them to both EM L_1 and EM L_2 via transfer trajectories that exploit the Sun-Earth multi-body dynamical environment. Two identical ARTEMIS spacecraft, named P1 and P2, entered Earth-Moon Lissajous orbits on August 25th and October 22nd 2010, respectively. Once the Earth-Moon libration point orbits were achieved they were maintained there for 11 months, with the P1 spacecraft orbiting EM L_2 and P2 orbiting EM L_1. During this stationkeeping phase, P1 was transferred from L_2 to L_1. From these EM libration orbits, both spacecraft were inserted into elliptical lunar orbits on June 27th and July 17th 2011 respectively.

The challenge of ARTEMIS stationkeeping was that libration point orbits near collinear locations, including quasi-periodic Lissajous trajectories, are inherently unstable and must be controlled. For Earth-Moon applications stationkeeping is more challenging than in the Sun-Earth system, in part because of the shorter time scales, the larger orbital eccentricity of the secondary, and the fact that the Sun acts as a significant perturbing body both in terms of the gravitational force and solar radiation pressure. To accurately assess the impact of these significant differences, the orbit must be modeled as a true four-body problem. Besides these inherent issues associated with the Earth-Moon system, ARTEMIS had mission requirements to be met and spacecraft constraints on the direction of delta-velocity (Δv). Although a general trajectory was defined for the mission, there was no required reference motion. Since the ARTEMIS spacecraft were originally designed for a passive mission in Earth elliptical orbits and were already flying, fuel was extremely limited. Thus, with the unique operational constraints, accomplishment of the maintenance goals with the minimum cost in terms of fuel was the highest priority.

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Background

The ARTEMIS stationkeeping strategy commenced with our previous research, in which we investigated several methods ranging from Circular Restricted Three Body (CRTB) dynamics with shooting methods to continuation and global optimization methods. For application to ARTEMIS stationkeeping and to specifically address operational constraints a combination of operationally proven and research methods were implemented. It is noted that the ultimate stationkeeping approach was not based on control with respect to a reference orbit; rather the focus was a method that minimizes fuel use, minimizes operations requirements in terms of the frequency of the maneuvers, and permits a navigation strategy to be set in place for support as well. This philosophy influences the strategies investigated and provides observations within a general framework. The ARTEMIS stationkeeping method therefore is a blend of optimization with equality and inequality constraints for orbit and spacecraft implementation and restrictions; numerical integration; energy balance control points for the orbit; and of course use of the multi-body dynamical environment.

While a variety of stationkeeping strategies have previously been investigated for other missions, most notably for applications in the Sun-Earth system[7-16], fewer studies have considered trajectories near the Earth-Moon libration points.[17-21] In a previous paper the author researched these strategies with the intent to apply them to ARTEMIS and they serve as a basis for the selection of processes for further development to use for operational support.[17] From this research, two strategies emerged as the methods that best met the requirements for our application and these are discussed in the current paper. These two strategies include an optimal continuation scheme and the use of a global search method. Both which embrace Floquet mode directions to identify the overall dynamics. This paper also addresses the Cartesian direction in EM rotating coordinates of the Δv computed via optimization as compared to stable and unstable mode directions by integrating the State Transition Matrix (STM) from the ARTEMIS navigation solutions used to plan the maneuvers. The operational scenario uses numerical integration and incorporates the third-body perturbations.

The Goddard Space Flight Center's (GSFC) Navigation and Mission Design Branch (NMDB) operationally supports the ARTEMIS mission and provides Earth-Moon navigation, trajectory and stationkeeping maneuver design, and maneuver planning information for command generation. The ARTEMIS mission is a collaborative effort between NASA GSFC, the University of California at Berkeley (UCB), Space Science Laboratory (SSL) and the Jet Propulsion Laboratory (JPL). JPL provided the transfer trajectory concept from the elliptical orbit phase through libration orbit insertion and on to lunar orbits. The UCB SSL provides operational support for daily monitoring and maintenance of all spacecraft operations (including orbit and attitude determination) and the generation of maneuver planning for uploads using a combination of UCB and GSFC provided software.

ARTEMIS Spacecraft Overview and Maneuver Constraints

Each ARTEMIS spacecraft is spin-stabilized with a nominal spin rate of roughly 20 RPM. Spacecraft attitude and rate are determined using telemetry from a digital Sun sensor (DSS), a three-axis magnetometer (TAM), and two single-axis inertial rate units (IRUs). The propulsion system on each spacecraft is a simple monopropellant hydrazine blow-down system. The propellant is stored in two equally-sized tanks and either tank can provide propellant to any of the thrusters through a series of latch valves. Each observatory was launched with a dry mass of 77 kg and 49 kg of propellant, supplying a wet mass of 126 kg at beginning of life. At the beginning of the stationkeeping phase, the remaining fuel mass was 9.6 kg for P1and 8.6 kg for P2.

Each spacecraft has four 4.4 Newton (N) thrusters – two axial thrusters and two tangential thrusters. The two tangential thrusters are mounted on one side of the spacecraft and the two axial thrusters are mounted on the lower deck, as seen in Figure 1. The thrusters fire singly or in pairs – in continuous or pulsed mode – to provide orbit, attitude, and spin rate control. Orbit maneuvers can be implemented by firing the axial thrusters in continuous mode, the tangential thrusters in pulsed mode, or a combination of the two (beta mode). Since there are no thrusters on the upper deck, the combined thrust vector is constrained to the lower hemisphere of the spacecraft.

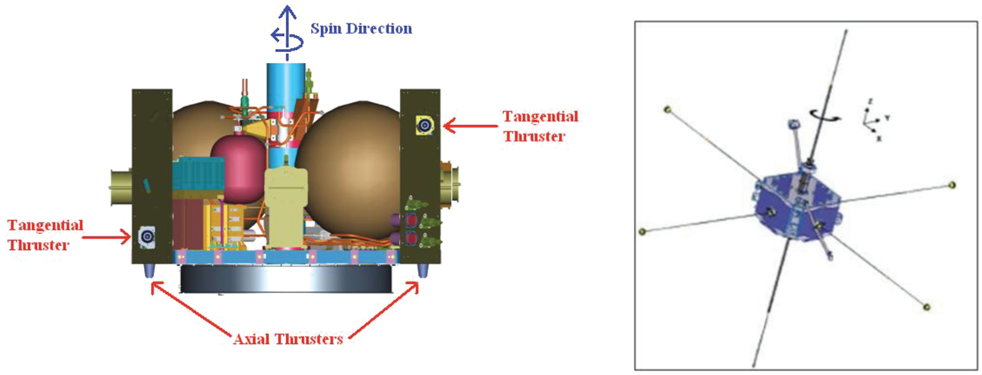

Figure 1. ARTEMIS Spacecraft Design.

The ARTEMIS spacecraft are spinning vehicles with the spin axis pointed within 5 degrees of the south ecliptic pole. These spacecraft can implement a Δv (thrust direction) along the spin axis towards the south ecliptic pole direction or in the spin plane, but cannot produce a Δv in the northern hemisphere relative to the ecliptic. Thus, most maneuvers were planned using only the radial thrusters. While the axial thrusters were used when necessary for Z-amplitude control, they were not the main control direction for stationkeeping. This constraint can limit the location of many maneuvers in the libration orbit. The trajectory was optimized incorporating a nonlinear constraint that placed the Δv in the spin plane and the epoch corresponding to the maneuver is varied to yield a radial maneuver direction.

The ARTEMIS Mission

The libration point orbits of P1 around the EM L_2 / L_1 and P2 around EM L_1 appears in Figures 2 and 3, respectively. There were no size or orientation requirements on these orbits other than to minimize the insertion and orbital maintenance requirements as both ARTEMIS spacecraft had limited combined deterministic and statistical stationkeeping Δv budgets of ~15 m/s and ~12 m/s for P1 and P2, respectively. This Δv budget included the libration point orbit stationkeeping, the transfers between libration orbits, and the transfer into lunar orbit. The P1 and P2 L_1 y-amplitudes were approximately 60,000 km with the P1 L_2 y-amplitude near 68,000 km since the overall amplitudes are determined from the use of a ballistic Sun-Earth to Earth-Moon transfer insertion. Consequently, at the end of the multi-body transfer, the final lunar libration point orbit is influenced heavily by the Moon since the transfer orbit passes relatively close to the Moon at each negative x-axis crossing with respect to the L_2 libration point. The Lissajous orbit dimensions are shown in Table 1.

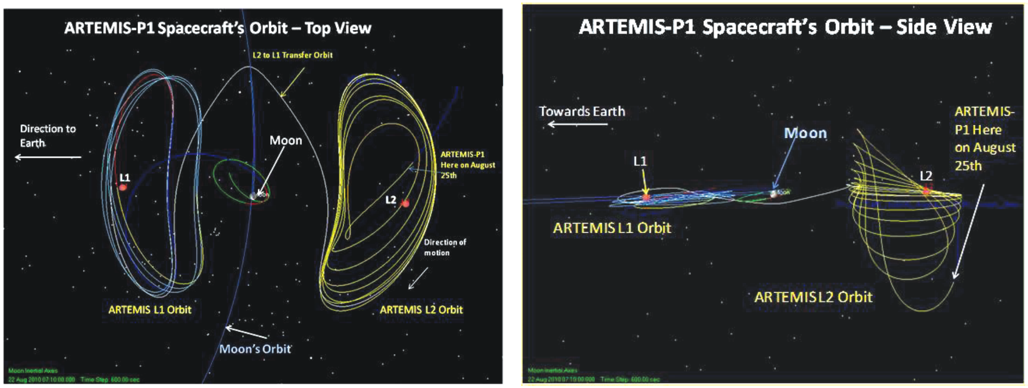

Figure 2. ARTEMIS P1 Lissajous Orbit Viewed from +Z and –Y in Earth-Moon Rotating Coordinates

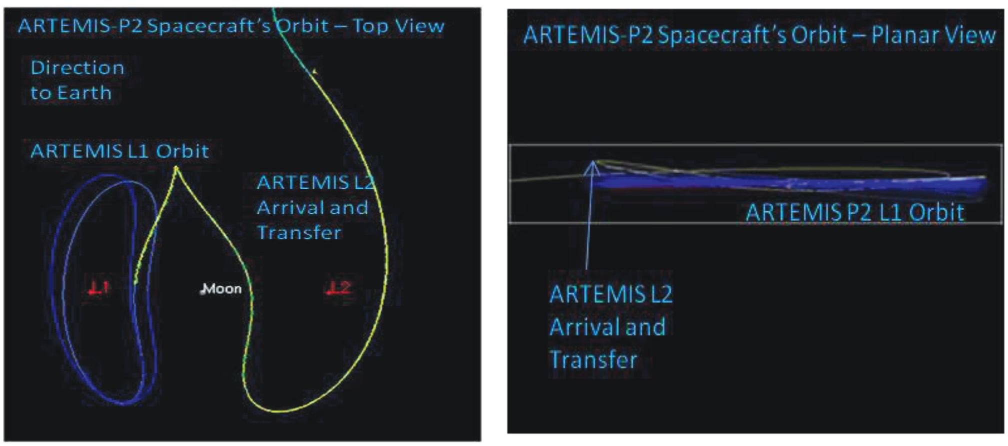

Figure 3. ARTEMIS P2 Lissajous Orbit Viewed from +Z and –Y in Earth-Moon Rotating Coordinates

Table 1. P1 and P2 Libration Orbit Dimensions

	ARTEMIS P1 @ L1	ARTEMIS P1 @ L2	ARTEMIS P2 @ L1
Max X Amplitude (km)	23656	32686	30742
Max Y Amplitude (km)	58816	63520	67710
Max Z Amplitude (km)	2387	35198	4680
Minimum Z Excursion (km)	181	n/a	246
Period (days), Average of x-axis cross to x-axis cross over 10 revs	13.51	15.47	14.19
Direction (rotation) of Z evolution (axis)	About EM X axis	About EM X axis	About EM X axis

STATIONKEEPING STRATEGY

An objective of this paper is to compare the pre-mission stationkeeping strategy to in-flight observations and experiences and to inform the reader regarding numerous operational considerations. The pre-mission stationkeeping strategy and its simulation results are presented first. Then the results of the actual mission data are shown from implementing this stationkeeping strategy. Finally, a comparison of executed Δv directions with respect to standard Floquet modes is made.

Stationkeeping Models and Software

ARTEMIS used a full ephemeris model (DE421 file) along with third body perturbations including solar radiation pressure acceleration based on the spacecraft mass and constant cross-sectional area (e.g. cannon ball model). A potential model for the Earth with degree and order eight was used. The operational plans were based on a variable step Runge-Kutta 8/9 or PrinceDormand 8/9 integrator. The libration point locations were also calculated instantaneously at the same integration interval. To compute maneuver requirements in terms of Δv, different strategies involve various numerical methods: traditional Differential Correction (DC) targeting with central or forward differencing, optimization using the VF13AD algorithm from the Harwell library and the STK Sequential Quadratic Programming (SQP) Optimizer. For the DC, equality constraints are incorporated, while for the optimization scheme, nonlinear equality and inequality constraints are employed. The software employed to met spacecraft constraints and orbit goals for our maneuver planning effort includes GSFC's General Mission Analysis Tool (GMAT) (open source s/w), AGI's STK/Astrogator, and the General Maneuver program (GMAN). GMAN has been used successfully by GSFC over 30 years to model spinning spacecraft kinematics.

Pre-Mission Stationkeeping Analysis Estimates

The pre-mission stationkeeping strategy satisfied several conditions: full ephemeris with high-fidelity models, globally optimized solutions, and methods that can be applied for any Earth-Moon orbital requirements at L_1 or L_2 and any transfer between them. Other strategies were investigated but many of the standard approaches could not be employed for various reasons, e.g., because a reference orbit is required which is not necessarily available nor desired, the strategy is based on the Circular Restricted Three Body (CRTB) model only, the process is based on linear control, or because a proposed approach cannot accommodate the ARTEMIS spacecraft constraints.[17] Numerous references in the literature offer discussion of stability and control for vehicles at both collinear and triangular libration point locations. Hoffman[18] and Farquhar[19] both provide analysis and discussion of stability and control in the Earth-Moon collinear L_1 and L_2 locations, respectively, within the context of classical control theory or linear approximations, Scheeres offers a statistical analysis approach.[12] Howell and Keeter[7] address the use of selected maneuvers to eliminate the unstable modes associated with a reference orbit; Gomez et al.[15] developed and applied the approach specifically to translunar libration point orbits. Marchand and Howell[13] discuss stability including the eigenstructures near the Sun-Earth locations. Folta and Vaughn.[21] present an analysis of stationkeeping options and transfers between the Earth-Moon locations and the use of numerical models that include discrete linear quadratic regulators and differential correctors. More recently Pavlak and Howell[22] have demonstrated maintenance using dynamical system modes. Lastly, Folta et al1[17] provided a review of all pertinent stationkeeping methods for stationkeeping in Earth-Moon libration orbits with intent of application to ARTEMIS.

From this previous research the Optimal Continuation Strategy (OCS) was chosen.[17] As shown in Table 2, this pre-mission strategy balances the orbit by meeting goals at crossing events several revolutions downstream, thereby ensuring a continuous orbit without constraining the near-term evolution or the reliance on specific orbit size or orientation specifications.

OCS maneuvers are performed to minimize the Δv requirements while ensuring the continuation of the orbit for several revolutions downstream. This method uses goals in the form of energy achieved, velocities, or time at any location along the orbit. For example, a goal might be defined in terms of the x-axis velocity component at the x-z plane crossings. While a DC scheme with Δv components was used to initialize the analysis in our previous research, for operations we switched to an SQP optimizer that uses Δv

magnitude, Δv azimuth, and maneuver epoch as controls. The orbit is continued over several revolutions by checking the conditions at each successive goal. This allows perturbations and the lunar orbit eccentricity to be modeled over multiple revolutions. Targeting is implemented with parameters assigned at the *x-z* plane crossing such that the orbit is continued and another revolution is achieved. The VF13AD1 and STK SQP optimizer were used to minimize the stationkeeping Δv by optimizing the direction of the Δv and the location (or time) of the maneuver. Included in the optimization process are the constraints required to maintain the ARTEMIS maneuvers in the spin plane. An alternative stationkeeping strategy utilizing a global search method was briefly investigated in an effort to determine the smallest Δv maneuver that maintains the spacecraft in the vicinity of the libration point for one to two additional revolutions, but not applied to ARTEMIS because of spacecraft constraint modeling.

Table 2. Pre-mission Control Strategy and Selection Criteria Applied to ARTEMIS

Strategy	Goal(s)	Advantage	Disadvantage
Orbit Continuation[8,10,11]	Velocity (or energy) is determined to deliver s/c several revs downstream (e.g., *x*-axis velocities all slightly negative)	- Guarantees a minimal Δv to achieve orbit continuation - Several control constraints can be applied - 3-D application	- Needs accurate integration and full ephemeris modeling - Logic required in s/w to check for departure trajectories - Optimization requires monitoring of process

For consideration in determining the applicability of any strategy, a unique feature of the ARTEMIS Lissajous orbit is the changing Lissajous 'inclination' or Z-axis amplitude. Over the roughly 11 months from insertion into the Lissajous orbit until the lunar orbit transfer, the P1 and P2 Lissajous trajectories evolved from a inclined (Z-amplitude) motion to one that is nearly planar (almost Lyapunov like). The impact of the Z amplitude evolution on the stationkeeping is one aspect that needs to be considered. Since the ARTEMIS Libration orbit phase was extended three months, it required a Z-axis transfer from one closing Lissajous to another closing Lissajous in order to meet the final Z amplitude and orientation for the lunar transfer.

Maneuver Locations

A consideration in the operations is the number of revolutions to be employed both for the 'targeting' as well as the placement of the maneuvers. Multiple orbit revolutions were used for the targeting goals. The maneuver location, though preferred to be near the x-z plane crossing, was dependent upon the station contact schedule. For example, maneuvers are analyzed for execution either at every *x*-z plane crossing or at every other crossing. In the previous investigation, we explored the following locations for the maneuvers: *x*-z plane crossing; maximum *y*-amplitude; and at an interval of ~3.8 days which yields 4 maneuvers per orbit. The effect of multiple maneuvers per revolution was modeled to coincide with the anticipated ARTEMIS tracking schedule. The operational execution of the maneuvers was somewhat different. We started with maneuvers near the *x*-z plane crossing, per the previous analysis, but also performed maneuvers at the y-max amplitude as well as skipping maneuvers if they became too small, less than 1 cm/s.

Stationkeeping Influenced by ARTEMIS Constraints

Using OCS, we began with estimated ARTEMIS initial conditions, and a profile generated for three maneuver locations for the aforementioned number of revolutions. Each profile varied the maneuver location and then the number of revolutions to achieve a continuation of the trajectory further downstream. Each simulation used the statistically generated navigation errors and a constant maneuver execution error of +1%. The constraint to maintain the Δv within the spin plane of the ARTEMIS spacecraft was also met. Tables 3 and 4 summarize the average pre-mission Δv results for cases that applied a 1.5-revolution and a 1-revolution continuation, respectively. These results include only 10 trials, with a trial defined as a 4-month stationkeeping simulation run with different realizations of the errors each time (see Navigation and Maneuver Errors below). Several obvious results emerge. First, maneuvers that are applied only once per revolution are approximately an order of magnitude larger than those applied at least twice per revolution. The maneuvers applied at the maximum *y*-axis amplitude are also larger than those at the *x*-z plane crossings, a result that is consistent with the preliminary results from the general stationkeeping analysis. To compare the results to a strategy that employs more frequent maneuvers, a scenario was simulated that applied maneuvers once every 3.8 days (i.e., a four-maneuvers-per-revolution sequence). This scenario was chosen based on the operational planning considerations that ARTEMIS tracking coverage and navigation solutions would be based on a three-day arc.

The overall results demonstrate that maneuvers at a frequency of at least once every seven days are desired to both minimize the Δv budget and to align with the navigation solution deliveries. A more frequent maneuver plan (3.8-day updates) is only slightly better in terms of Δv.

Table 3. Pre-Mission Continuous Method using 1.5-rev (10 Trials)*

Maneuver Location	No. of Maneuvers	Avg Δv per Maneuver (m/s)	Std Dev (m/s)	Avg Δv per Year (m/s)	Avg Time Between Maneuver (days)
X-Z plane, every crossing	**15**	**0.28**	**0.78**	**12.27**	**7.3**
X-Z plane, once per orbit	**7**	**4.88**	**7.07**	**106.51**	**15.2**
Max Y-Amp Every crossing	**15**	**0.42**	**.95**	**18.13**	**7.3**
Max Y-Amp Once per orbit	**7**	**5.46**	**6.98**	**110.91**	**14.9**
4 Pts/Rev (~3.8 days)	**33**	**0.15**	**0.33**	**13.72**	**3.8**

Table 4. Pre-Mission Continuous Method using 1-rev (10 Trials)*

Maneuver Location	No. of Maneuvers	Avg Δv per Maneuver (m/s)	Std Dev (m/s)	Avg Δv per Year (m/s)	Avg Time Between Maneuver (days)
X-Z plane, every crossing	**15**	**0.73**	**0.77**	**31.71**	**7.3**
X-Z plane, once per orbit	**7**	**14.09**	**25.06**	**285.62**	**15.2**
Max Y-Amp Every crossing	**15**	**3.36**	**3.45**	**50.4**	**7.3**
Max Y-Amp Once per orbit	**7**	**31.08**	**31.44**	**630.13**	**14.9**
4 Pts/Rev (~3.8 days)	**33**	**0.33**	**0.59**	**31.70**	**3.8**

*Note: pre-Mission Analysis uses navigation errors of 1 km and 1 cm/s (1σ), operations shows and order of magnitude less.

Navigation and Maneuver Errors

The computation of the stationkeeping Δv for an Earth-Moon libration point orbit is influenced greatly by the inclusion of both navigation and maneuver execution errors. In our pre-flight analysis, a spherical navigation error of 1-km position and 1-cm/s velocity 1σ, was generated by the use of an error covariance matrix. The maneuver errors were modeled by multiplying the computed Δv by the desired error, e.g., Δv * 1.01 for a 1% hot maneuver.

Since the navigation solution is provided by both the UCB team and the GSFC Code 595 Flight Dynamics Facility, we were able to plan maneuvers with confidence. The observed navigation uncertainty was significantly smaller than the values used in the pre-flight assessment. The tracking of P1 and P2 was accomplished using DSN, USN, and the antenna at UCB. More information on navigation can be found in reference 24. The uncertainly from the Goddard Trajectory Determination System (GTDS) least squares solution is estimated to be below 100 meters and 0.1 cm/s. It was difficult to separate the portion of the error due to the state uncertainty (OD) before maneuver execution from the maneuver execution errors because each effect was at the limit of observability.

ARTEMIS STATIONKEEPING

Stationkeeping Theory

It was already known that any change in energy from an unstable Earth-Moon libration point orbit will result in a departure from the L_1 or L_2 orbit, either towards the Moon or in an escape direction towards the Earth or the Sun-Earth regions. The Δv required to effect these changes are very small as are the accelerations from solar radiation pressure, since natural perturbations will also result in these escape trajectories. To continue the orbit downstream and maintain the path in the vicinity of the libration point, we selectively chose target goals on each side of the libration orbit. For the method applied directly to ARTEMIS these goals are directly related to the energy (velocity) at the *x*-z plane crossing to simply wrap the orbit in the proper direction, always inward and towards the libration point.

The targets used for the continuation method differed slightly between the EM L_2 orbit and the EM L_1 orbit. The continuation targets for the P1 maintenance, while in orbit about EM L_2, used two different x-axis velocities, depending on which side of the orbit P1 was on. For example, targets on the far side (away from the Moon) used an x-z plane crossing x velocity of -20 m/s with a tolerance of 1 cm/s. Targets on the close side (nearer to the Moon) used x-z plane crossing x velocity targets of +10 m/s with a tolerance of 1 cm/s. Once in orbit about the EM L_1 orbit the P1 targets were changed to meet the ongoing operations similar to P2. These targets are +/- 10 cm/s at each crossing, a much smaller velocity target. The scheme here is to continuously target the next crossing downstream, up to four crossings were used as the change in the Δv after the third crossing was usually below 0.01 cm/s and therefore unachievable by the spacecraft propulsion system.

As each crossing condition was achieved in the continuation process using multiple crossing targets, the Δv decreased to attain the next crossing. Also depending on the location of the maneuver with respect to the Moon radius, the Δv also varied from maneuver to maneuver. Table 5 provides a list of the operational constraints, conditions, or events that limited the theoretical research.

Table 5. ARTEMIS Operational Constraints and Conditions

Constraint , Condition, or Event	Stationkeeping Effect
Ground Station Contact Schedule	Tracking and Telemetry contacts sometimes limited to 1/day. Needed north/south station contacts for geometry. Most solutions converged after 3 days of tracking data
Spacecraft Spin Rate and Thruster Angle Arc	Limits Δv resolution and direction
Spacecraft attitude to 1 deg accuracy	Uncertainty in Δv direction. Need 2 sun bin transitions during the period between events.
Navigational C_r coefficient	Require tracking is >3 days, < 10 days, Cr miss-modeling is substantial effect after 1-rev in orbit.
Navigation Uncertainty	Uncertainty estimated to be on the order of 0.1 cm/s velocity and 10s meter position
Propulsion System Performance	Calibrated to <1 %
Environmental Dynamics	Earth modeled as 8x8 with lunar point mass with DE421 ephemeris models

Stationkeeping Process

As mention in the background, ARTEMIS is a team effort and the stationkeeping process demonstrates how that team process worked. Upon receipt of the daily orbit determination solution, a stationkeeping Δv was computed for several possible maneuver locations to meet the tracking and command load schedule. Therefore even if an optimal Δv was found that minimized fuel, the epoch of the maneuver was constrained to be within a scheduled tracking pass for the upload and verification in real time of the maneuver execution. This meant that there were epochs and maneuver locations with respect to the libration orbit that did not meet true optimal placement, but rather provided a minimal Δv subject to the station contact schedule. Maneuver plans were then generated as part of the optimization and the transmitted to UCB SSL for further processing within the GMAN program to target these optimal Δv. GMAN is a high fidelity propulsion modeling software that models spacecraft kinematics and dynamics, and therefore models the spinning ARTEMIS spacecraft and its attitude. GMAN output was then sent to GSFC for verification of the maneuver plan (it met the libration target conditions) and for an initial estimate of the next maneuver, since navigation and performance errors would result in the orbit eventually escaping.

Observed Stationkeeping OD Trending

The goal for ARTEMIS was to minimize the Δv and at the same time minimize operations as well. Maneuvers were mostly conducted at weekly intervals, but were also stretched out to meet the station contact schedule. As we received the daily orbit determination solution, the solution was processed to ensure that the trajectory could be met and that the navigation solution was consistent. Tables 6 and 7 and Figure 4 and 5 show the operational trending of the orbit determination solution uncertainty and the resultant stationkeeping Δv trends for P1 and P2 stationkeeping maneuvers. Note that when the X-Y RSS and X-Y-Z RSS uncertainty decreased to less than 0.1 cm/s the maneuver was planned and executed. Table 6 provides the differences in orbit determination solutions for P1 and Table 7 additionally shows the ratio and directions for P2 for two maneuver plans.

Table 6. P1 Stationkeeping #33 (SKM33) OD Uncertainty and Δv Trending

OD Epoch	CsubR	Difference from Previous Position (km)	Difference from Previous Velocity (km/s)	X Uncertainty (km/s)	Y Uncertainty (km/s)	Z Uncertainty (km/s)	XY RSS (cm/s)	XYZ RSS (cm/s)	P1 SKM33 DV May 11th @ 07:20 (cm/s)
126, May 06	1.139998	8.450E-01	3.28E-06	2.923E-07	9.188E-07	2.022E-06	0.0964	0.2240	5.616
127, May 07	1.139943	2.892E-03	1.40E-07	5.344E-08	2.806E-07	6.238E-07	0.0286	0.0686	5.150
128, May 08	1.139429	1.101E-02	8.13E-07	3.525E-08	4.985E-08	1.495E-07	0.0061	0.0162	4.389
129, May 09	1.138402	1.894E-02	-1.48E-06	4.260E-08	1.776E-07	2.121E-07	0.0183	0.0280	5.027
130, May 10	1.137285	3.949E-01	6.02E-07	8.918E-08	4.069E-07	6.452E-07	0.0417	0.0768	6.033

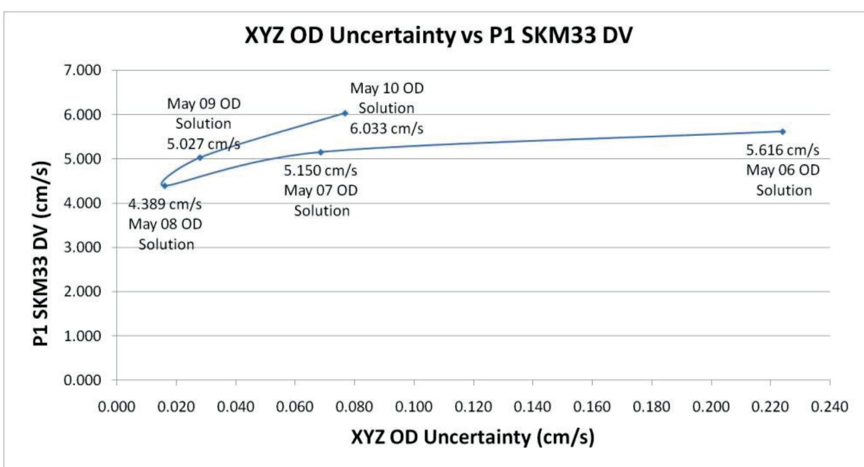

Figure 4. P1 Stationkeeping #33 (SKM33) OD Uncertainty and Δv Trending

Table 7. Sample P2 Uncertainty Tree for Stationkeeping (SKM30)

OD epoch	CsubR	X Uncertainty (km/s)	Y Uncertainty (km/s)	Z Uncertainty (km/s)	XY RSS (cm/s)	XYZ RSS (cm/s)	ratio	SKM25 DOY114 (cm/s)	TOD Az (deg)	SKM25 DOY121 (cm/s)	
108/1300	1.1200	2.17E-07	1.34E-06	2.59E-06	0.1362	0.2922	2.15	3.40		105.60	Mon. OD
109/1100	1.1200	1.13E-07	1.00E-06	2.04E-06	0.1010	0.2273	2.25	2.54		78.44	Tue. OD
110/0900	1.1200	6.05E-08	5.10E-07	1.12E-06	0.0513	0.1234	2.40	2.57		79.44	Wed. OD
111/0900	1.1201	2.66E-08	4.58E-08	1.16E-07	0.0053	0.0128	2.41	1.86		57.60	Thur. OD
112/1100	1.1200	5.58E-08	5.65E-08	1.06E-07	0.0079	0.0133	1.67	1.96		35.01	Fri. OD
112/1700	1.1199	6.56E-08	8.47E-08	1.63E-07	0.0107	0.0195	1.82	1.95	16.91	39.15	Fri. OD2
113/1100	1.1209	7.02E-08	9.19E-08	1.06E-07	0.0116	0.0157	1.36	1.20	17.12	37.18	Sat. OD
113/2100	1.1213	5.51E-08	1.17E-07	1.35E-07	0.0129	0.0187	1.44	1.36	16.86	40.00	Sun. OD
114/1800	1.1217	4.68E-09	3.44E-08	1.21E-07	0.0035	0.0126	3.63	1.33	16.66	43.25	Mon. OD

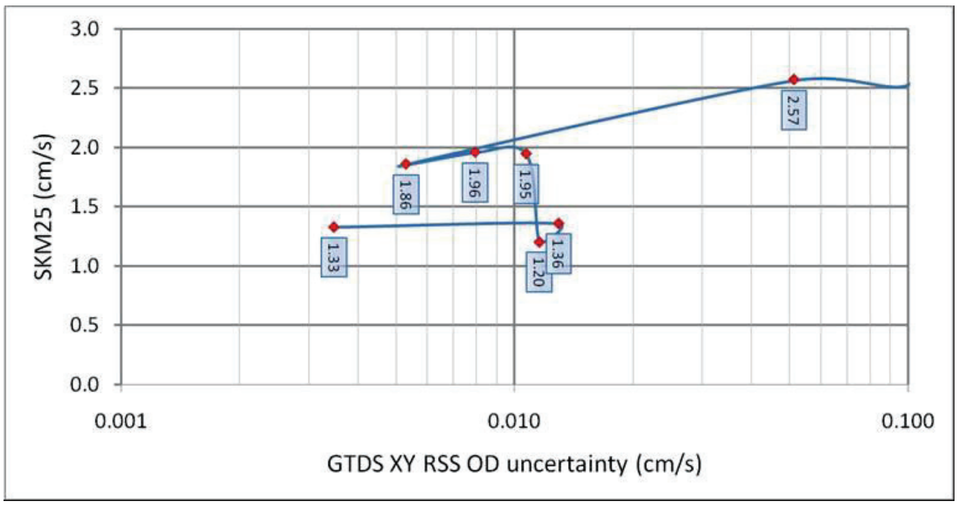

Figure 5. P2 Stationkeeping #30 (SKM30) OD Uncertainty and Δv Trending

Observed Stationkeeping Maneuver Results

Tables 8 and 9 present all the stationkeeping maneuvers for P1 and P2. The tables provide the stationkeeping number, the day of year (DOY) of the maneuver, the Δv magnitude, the cumulated Δv, the days in the Lissajous orbit and the annual cost based on the Δv and the Lissajous days. Note that maneuver 15 for P1 was an insertion Δv during the libration transfer and is not including in the stationkeeping Δv summary.

Figures 6 through 11 show the chronological Δv as each stationkeeping maneuver was executed for both P1 and P2. Figures 6, 7, and 8 show the P1 Δv for each maneuver; the annual maintenance cost for P1 in L_2 and the annual cost for P1 in the L_1 orbit. Likewise, figures 9, 10, and 11 show the P2 Δvs for each maneuver; the annual maintenance cost for P2 in L1and the annual cost for P2 in the L_1 orbit when optimal planning conditions are used. For both spacecraft, the general decrease in the stationkeeping Δvs is attributed to a change in the way the spacecraft was configured to model the arc over which the propulsion system performed and the improvement in the modeling of the environment and the Cr use from navigation solutions. Originally the arc over which the thruster is on was fixed at 60 degrees. Advanced onboard software permitted this arc to be controlled (varied) more precisely and therefore the maneuver execution was more accurate. Also, the navigation solutions provided not only the state but also a Cr value that considered the perturbation from solar radiation pressure. While P1 used the Cr provided by the navigation solution, the P2 maneuvers were originally planned with a constant Cr taken from pre libration orbit analysis to determine this value. Also the peaks are attributed to the predictions of the spin axis attitude which is accurate to only approximately 1 deg. Depending on all these values, the accuracy of the maneuver varies and therefore the subsequent maneuver to correct any errors in addition to the general continuation of the orbit could be increased.

Table 8. ARTEMIS P1 Stationkeeping Information

P1 Individual Maneuvers

SKM	Year	DOY	Day	dv (cm/s)	cum (m/s)	Liss days	annual cost (m/s/yr)
1	2010	237	Wed	256.24		0	
2	2010	251	Wed	58.40	0.58	14	15.23
3	2010	265	Wed	22.28	0.81	28	10.52
4	2010	273	Thu	34.05	1.15	36	11.63
5	2010	282	Sat	7.96	1.23	45	9.95
6	2010	291	Mon	15.84	1.39	54	9.36
7	2010	298	Mon	11.29	1.50	61	8.96
8	2010	306	Tue	11.64	1.61	69	8.54
9	2010	313	Tue	6.96	1.68	76	8.09
10	2010	321	Wed	7.13	1.76	84	7.63
11	2010	334	Tue	20.74	1.96	97	7.39
12	2010	344	Fri	22.64	2.19	107	7.47
13	2010	352	Sat	13.79	2.33	115	7.39
14	2010	361	Mon	11.57	2.44	124	7.19
15	2011	6	Mon	3.32	2.48	134	
16	2011	17	Mon	11.80	2.59	145	6.53
17	2011	24	Mon	6.38	2.66	152	6.38
18	2011	32	Tue	19.10	2.85	160	6.50
19	2011	38	Mon	22.29	3.07	166	6.75
20	2011	45	Mon	10.30	3.17	173	6.70
21	2011	49	Fri	1.17	3.19	177	6.57
22	2011	56	Fri	5.93	3.25	184	6.44
23	2011	63	Fri	1.76	3.26	191	6.24
24	2011	69	Thu	2.93	3.29	197	6.10
25	2011	76	Thu	1.74	3.31	204	5.92
26	2011	83	Thu	2.32	3.33	211	5.77
27	2011	89	Wed	2.04	3.35	217	5.64
28	2011	96	Wed	1.99	3.37	224	5.50
29	2011	103	Wed	2.17	3.40	231	5.36
30	2011	110	Wed	27.90	3.67	238	5.63
31	2011	117	Wed	2.78	3.70	245	5.52
32	2011	124	Wed	12.99	3.83	252	5.55
33	2011	131	Wed	5.03	3.88	259	5.47
34	2011	144	Tue	5.53	3.94	272	5.28
35	2011	150	Mon	1.17	3.95	278	5.19
36	2011	157	Tue	4.03	3.99	285	5.11

P2 Individual Maneuvers

SKM	Year	DOY	Day	dv (cm/s)	cum (m/s)	Liss days	annual cost (m/s/yr)
1	2010	293	Wed	11.69		0	
2	2010	300	Wed	18.38	0.18	7	9.58
3	2010	307	Wed	37.89	0.56	14	14.67
4	2010	315	Thu	24.69	0.81	22	13.43
5	2010	322	Thu	6.23	0.87	29	10.97
6	2010	333	Mon	34.85	1.22	40	11.14
7	2010	340	Mon	10.39	1.32	47	10.28
8	2010	348	Tue	6.64	1.39	55	9.23
9	2010	355	Tue	3.69	1.43	62	8.40
10	2010	362	Tue	12.13	1.55	69	8.19
11	2011	4	Tue	2.04	1.57	76	7.54
12	2011	11	Tue	11.55	1.68	83	7.41
13	2011	18	Tue	2.61	1.71	90	6.94
14	2011	25	Tue	17.85	1.89	97	7.11
15	2011	32	Tue	3.75	1.93	104	6.76
16	2011	40	Wed	29.61	2.22	112	7.24
17	2011	50	Sat	17.40	2.40	122	7.17
18	2011	56	Fri	3.63	2.43	128	6.94
19	2011	65	Sun	21.68	2.65	137	7.06
20	2011	72	Sun	20.80	2.86	144	7.24
21	2011	79	Sun	4.38	2.90	151	7.01
22	2011	86	Sun	1.99	2.92	158	6.75
23	2011	100	Sun	4.96	2.97	172	6.31
24	2011	107	Sun	4.53	3.02	179	6.15
25	2011	116	Tue	1.33	3.03	188	5.88
26	2011	123	Tue	6.85	3.10	195	5.80
27	2011	130	Tue	2.35	3.12	202	5.64
28	2011	137	Tue	1.91	3.14	209	5.49
29	2011	144	Tue	1.45	3.16	216	5.33
30	2011	152	Wed	2.43	3.18	224	5.18
31	2011	161	Mon	6.78	3.25	233	5.09

Table 9. P1 and P2 Stationkeeping Statistics

	P1 @ L_2 (cm/s)	P1 @ L_1 (cm/s)	P2 @ L_1 (cm/s)
Total Δv	**244.0**	**155.0**	**324.0**
Min Δv	**6.96**	**1.17**	**1.33**
Max Δv	**22.64**	**27.90**	**37.89**
Mean Δv	**13.51**	**7.21**	**10.85**
STD	**5.44**	**7.60**	**10.31**

Stationkeeping cost since insertion into libration orbits (w/o axial corrections to extend mission three months) gives:

- Total P1 ~ 3.99 m/s,
- Total P2 ~ 3.24 m/s,
- P1 projected yearly stationkeeping cost ~7.39 m/s per year for L_2 and 5.28 m/s per year for L_1.
- P2 projected yearly stationkeeping cost ~5.09 m/s per year.
- These Δvs per year are based on ARTEMIS maneuvers schedules and constraints

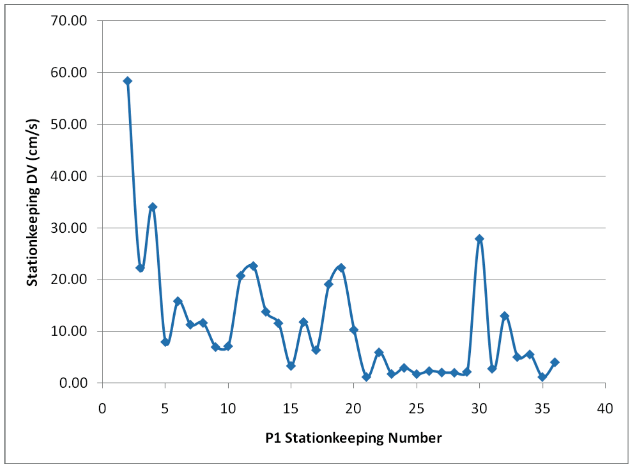

Figure 6. P1 Individual Stationkeeping Δv vs. Stationkeeping Maneuver

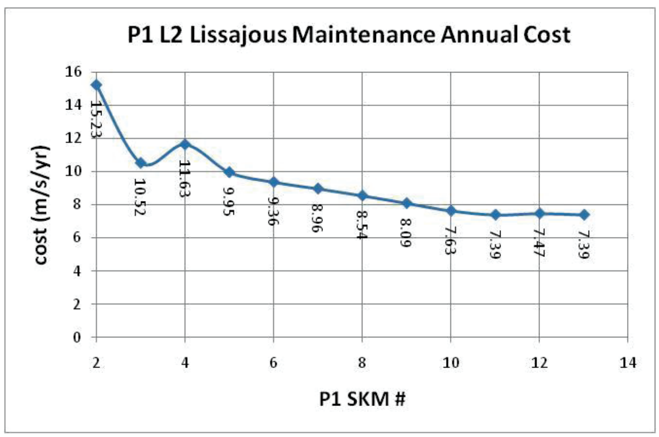

Figure 7. P1 EM L_2 Lissajous Orbit Cumulative Annual Δv Cost

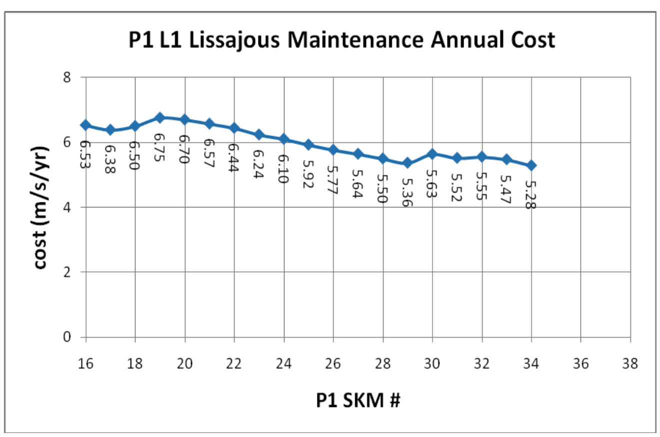

Figure 8. P1 EM L_1 Lissajous Orbit Cumulative Annual Δv Cost

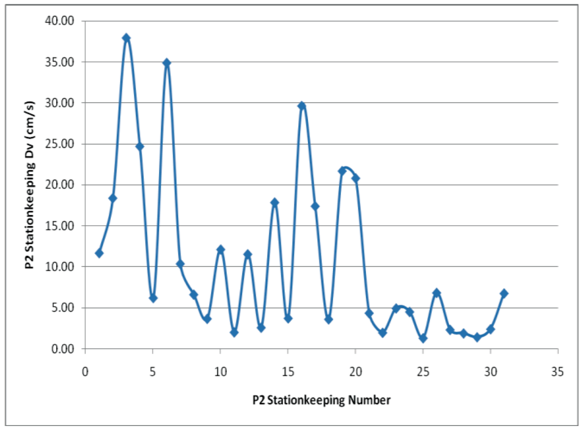

Figure 9. P2 Individual Stationkeeping Δv vs. Stationkeeping Maneuver

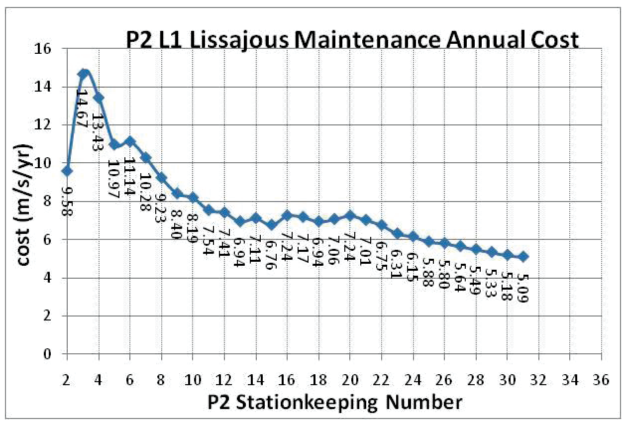

Figure 10. P2 EM L_1 Lissajous Orbit Cumulative Annual Δv Cost
(Pre Cr change)

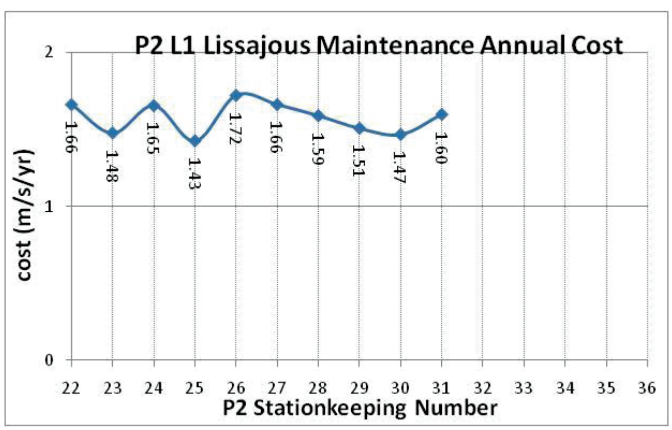

Figure 11. P2 EM L_1 Lissajous Orbit Cumulative Annual Δv Cost
(Post Cr change)

ARTEMIS Libration Mode Analysis

Research of multi-body environments has been ongoing for over a decade.[7,8,12,15,20] Working in collaboration with Purdue University, GSFC analyzed many trajectories in both Sun-Earth and Earth-Moon regimes. This analysis has shown that there could be alternate methods for stationkeeping that result in the balancing or continuation of the libration orbit over several revolutions. In general terms, this research is called Mode Analysis and analyzes the eigenstructure (eigenvectors and eigenvalues) of the libration orbit to compute information regarding the orbit stability. By proper modeling of the orbit using various methods such as CRTB and geometric means, many such studies have been completed that indicate that maneuver along the stable or unstable mode direction as represented in a Cartesian system could be used for stationkeeping. ARTEMIS permits us to validate that research and show how the optimal continuation strategy used for ARTEMIS places maneuvers along the stable mode direction.

Using the ARTEMIS orbit determination solutions along with the stationkeeping maneuvers executed using the continuous strategy; we computed an approximate monodromy matrix by generating and propagating the State Transition Matrix (STM) from an initial state. To calculate the STM, we propagate an initial state that is perturbed in each of its components (4e-4 km and 1e-4 cm/s for each position and velocity). Then in Matlab, a finite-difference STM using initial and final state information from GMAT yields an approximation of the monodromy matrix. From this information, we then compute the 6 eigenvalues, λ_i, of the STM which yields,

– $|\lambda_i| < 1 \rightarrow$ stable eigenvalue(s)
– $|\lambda_i| > 1 \rightarrow$ unstable eigenvalue(s)

Once the mode information is generated we compare actual maneuver direction with stable/unstable eigenvector information. Additional methods are being studied for computing eigenvalues/eigenvectors in less periodic portions of the orbits (i.e., the P1 L_2 Lissajous trajectory).

Shown below are three figures (Figure 12, 13, 14) that show the direction of the stable and unstable mode directions for ARTEMIS orbits of P1 in EM L_2, P1 in EM L_1 and P2 in EM L_1. Additionally, several figures are shown that for a select few stationkeeping maneuvers. Note that all the ARTEMIS maneuvers are co-aligned aloing the stable mode direction. While a full understanding of this is still being worked, a basic conclusion can be drawn that maneuver placement along the stable mode can be used to maintain the orbit.

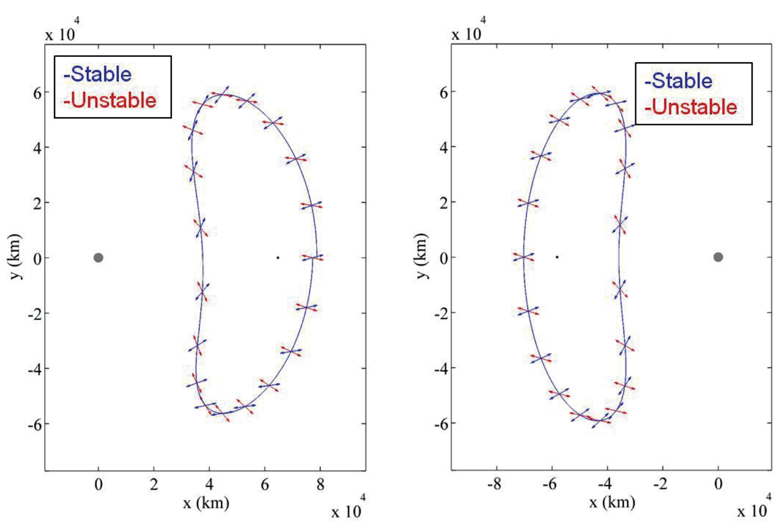

Figure 12. P1 EM L_2 and EM L_1 Stable and Unstable Directions

One may ask why all the Δvs are aligned with the stable mode rather than the cancellation of the unstable mode. At this point, we believe that the optimization process using the selected targets to 'bend' the trajectory along a continuation orbit in fact results in a maneuver that promotes the stability of the orbit, rather than taking an action to reduce the unstable direction. We also believe that the use of multiple orbits in our optimization algorithm aids in the maneuvers being in the stable direction.

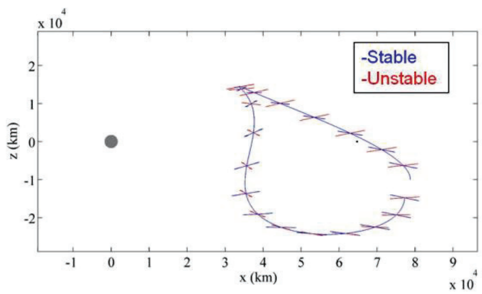

Figure 13. Sample P1 EM L_2 (side view)

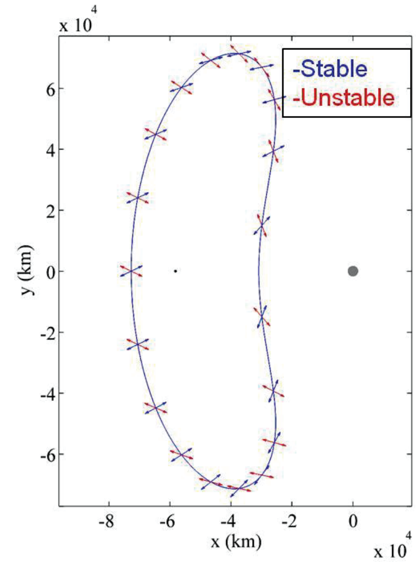

Figure 14. P2 EM L_1 Stable and Unstable Directions

Figures 15 and 16 show stationkeeping Δv directions and the stable / unstable mode directions at these maneuver epochs (location). Figures 17 and 18 present the angle between the EM rotating coordinate system Cartesian Δv vector and the stable mode directions for all stationkeeping maneuvers. As seen in these figures the Δv vector aligns closely with the stable mode for all maneuvers even with spacecraft constraints in place.

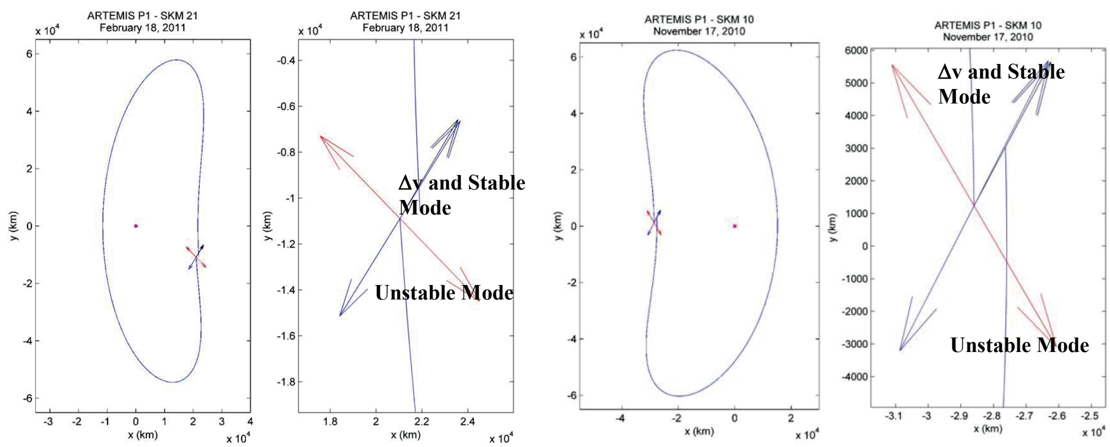

Figure 15. P1 SKM 21 (left) and SKM 10 (right) locations, Stable (blue) and Unstable (red) directions and the Δv (black) direction

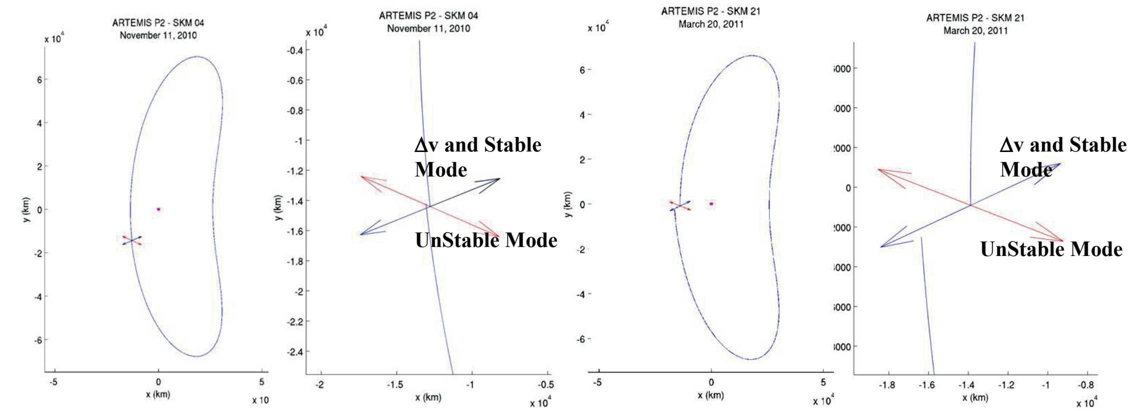

Figure 16. P2 SKM 04 (left) and SKM 21 (right) locations, Stable (blue) and Unstable (red) directions and the Δv (black) direction

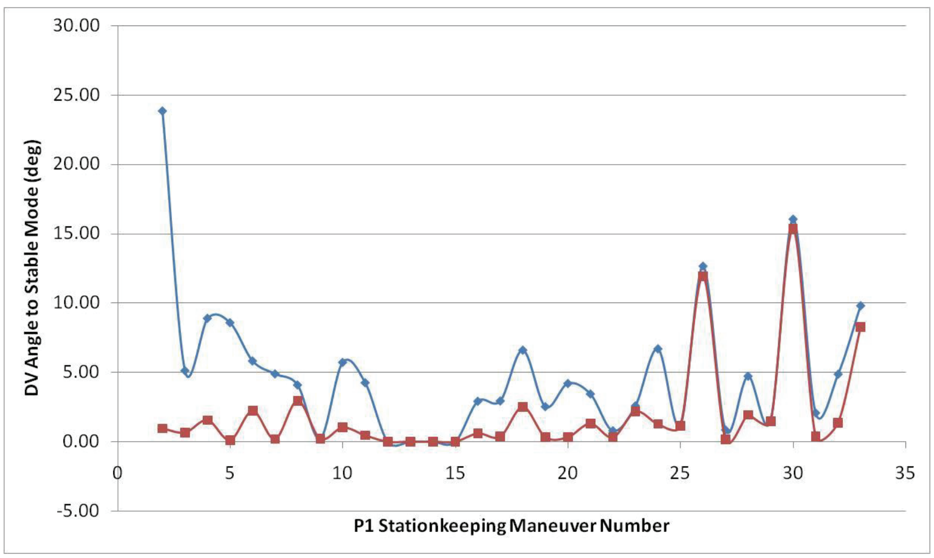

Figure 17. P1 Total (top, blue) and In-Plane (bottom, red) Angle between Δv Vector and the Associated Stable Mode Direction

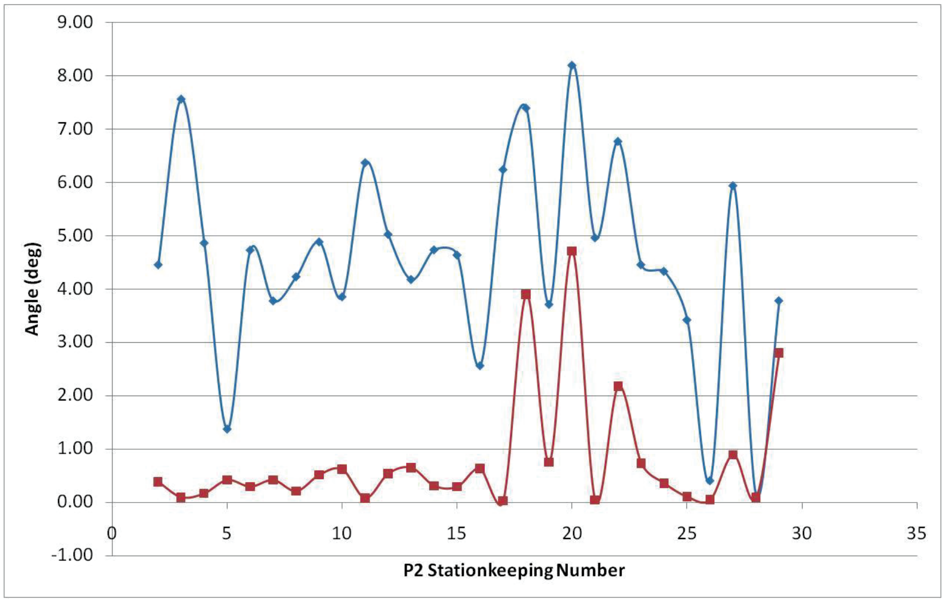

Figure 18. P2 Total (top, blue) and In-Plane (bottom, red) Angle between Δv Vector and the Associated Stable Mode Direction

Observations

The real operations have provided us with some unique and obvious observations for stationkeeping.

- Methods have been demonstrated that result in low stationkeeping Δv requirements and that meet the ARTEMIS mission requirements
- Full ephemeris model and the modeling of associated errors from navigation and maneuvers are required to accurately determine the accelerations that affect the stationkeeping Δv.
- The dynamics of the Earth-Moon environment also must be modeled over a sufficient duration (=> 21 days)
- Care must be taken in the consistent use of the Cr value of solar radiation coefficient between the maneuver planning software and the navigation software.
- Targets used for ARTEMIS EM L_2 stationkeeping were different from ARTEMIS EM L_1 stationkeeping.
- An increase in the frequency of the maneuvers tends to reduce the overall Δv requirements as does the placement of the maneuvers near the x-z plane crossing.
- Stationkeeping cost with realistically modeled navigation errors does have a floor – a rule of thumb from ARTEMIS, ~20:1 ratio of SKM Δv to nav+execution errors for ½ half rev.
- Mission applications and mission constraints must also be considered
- The methods developed allow a general application whether there is a reference orbit, spacecraft constraints on Δv direction, or orbital parameters requirements
- Maneuvers performed at the Y extrema resulted in an increase sensitivity of the orbit to the maneuver resulting in increased Δv magnitude for follow-on maneuvers
- The s/c constraint of radial maneuvers resulted in an out-of-plane component applied such that the Z-amplitude would change
- Optimized maneuver directions aligned with the dynamically stable mode direction

Two other items of interest also were observed; sensitivity of the maneuver wrt x crossing and y-extreme and the effect of the lunar eccentricity and earth perturbation on the negative y-amplitude being reduced on two week intervals. Due to station contact schedule, some stationkeeping maneuvers were placed near the y component extreme. We found that the resultant follow up stationkeeping maneuver was larger, almost a factor of 5 times larger than expected. Subsequent analysis found that the Δv directions were co-aligning with the spacecraft velocity vector direction at these locations, unlike at the x-z plane crossing where the Δv vector was almost perpendicular. Our analysis indicates that this alignment results in more uncertainty in the final velocity after the Δv was applied. We then switched back to x-z plane crossing locations when the station contact permitted. Consideration of errors in the onboard computation of the center of the spin pulse (ΔV direction) contributed to this sensitivity as well.

Additionally, as seen in Figure 2, the lower y-extreme (bottom of the EM L_1 trajectory) has variability. This effect is a result of the lunar eccentricity and the related velocity of the Moon and its perturbation on the spacecraft. The variability occurs at two week intervals and is seen when the Moon is near periapsis in its orbit.

ARTEMIS Strategy

Given the constraints of the ARTEMIS mission orbit, spacecraft maneuvers were planned at a minimum frequency of seven days and a maximum of 14 days to ensure a stable navigation solution while minimizing the Δv and staying within the ARTEMIS Δv budget. The maneuvers are also planned to occur at or near the x-z plane crossings and use a continuation method to maintain the orbit. Orbital conditions were set to permit the energy or velocity at the crossings to continue the orbit for at least 1½ revolutions. This strategy also benefits the operations by permitting a routine schedule.

SUMMARY

An Earth-Moon stationkeeping strategy has been demonstrated that results in low stationkeeping Δv requirements which meet the ARTEMIS mission requirements and spacecraft constraints. It has been demonstrated that a full ephemeris model along with accelerations from third body perturbations and the earth's potential must be modeled for accurate prediction and maneuver planning. Associated errors from navigation and maneuvers must be kept to levels below tenths of cm/s to accurately model the accelerations that affect the Δv. The dynamics of the Earth-Moon environment also must be modeled over a sufficient duration, at least 3 weeks. This duration should be equal to or greater than 21 days to account for the lunar eccentricity and to a lesser, but still important degree, the perturbation from the Sun. An increase in the frequency of the maneuvers tends to reduce the overall Δv requirements as does the placement of the maneuvers near the x-z plane crossing. In our analysis, stationkeeping cost with realistically modeled navigation errors has a floor of about 5 m/s per year, less than the Δvs from previous studies that approached 60 top 100 m/s per year.

CONCLUSIONS

While there are a number of strategies available that incorporate the Earth-Moon dynamics, the actual mission applications and mission constraints must also be considered. The methods developed here allow a general application whether there is a reference orbit, spacecraft constraints on Δv direction, or orbital parameters requirements. The required stationkeeping Δv can be minimized and has been demonstrated to be very minimal at ~ 5 m/s per year. With the ARTEMIS P1 and P2 Earth-Moon libration orbit completed, investigation of additional robust strategies and options to improve the Δv computation for stationkeeping is continuing.

ACKNOWLEDGEMENTS

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