

$$x(t) = \cancel{C_1 n_1 e^{\lambda_1 t}} + \cancel{C_2 n_2 e^{\lambda_2 t}} + C_3 n_3 e^{\lambda_3 t} + C_4 n_4 e^{\lambda_4 t}$$

$$x(0) = C_3 n_3 + C_4 n_4$$

4x2 2x1

$$\begin{bmatrix} x_0 \\ y_0 \\ \dot{x}_0 \\ \dot{y}_0 \end{bmatrix} = C_3 n_3 + C_4 n_4 = \begin{bmatrix} n_{31} & n_{41} \\ n_{32} & n_{42} \\ n_{33} & n_{43} \\ n_{34} & n_{44} \end{bmatrix} \begin{bmatrix} C_3 \\ C_4 \end{bmatrix}$$

$$\begin{cases} x_0 = C_3 n_{31} + C_4 n_{41} \\ y_0 = C_3 n_{32} + C_4 n_{42} \end{cases}$$

$$\Rightarrow \begin{bmatrix} n_{31} & n_{41} \\ n_{32} & n_{42} \end{bmatrix} \begin{bmatrix} C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\dot{x}_0 = C_3 n_{33} + C_4 n_{43}$$

$$\dot{y}_0 = C_3 n_{34} + C_4 n_{44}$$



$$\begin{bmatrix} n_{33} & n_{43} \\ n_{34} & n_{44} \end{bmatrix} \begin{bmatrix} C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \end{bmatrix}$$

$$\frac{1}{n_{31} n_{42} - n_{32} n_{41}} \begin{bmatrix} n_{42} & -n_{41} \\ -n_{32} & n_{31} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} C_3 \\ C_4 \end{bmatrix}$$

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$$\frac{1}{n_{42} - n_{32}} \begin{bmatrix} n_{42} & -1 \\ -n_{32} & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} C_3 \\ C_4 \end{bmatrix}$$

$$n_{42} = -n_{32} \Rightarrow \begin{bmatrix} \frac{1}{2} & -\frac{1}{2n_{42}} \\ \frac{1}{2} & \frac{1}{2n_{42}} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} C_3 \\ C_4 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} \lambda_3 & -\lambda_3 \\ n_{34} & n_{34} \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{n_{42}} \\ 1 & \frac{1}{n_{42}} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \end{bmatrix} \Rightarrow \frac{1}{2} \begin{bmatrix} 0 & -2\lambda_3/n_{42} \\ 2n_{34} & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \end{bmatrix}$$

$$\dot{x}_0 = \frac{-\lambda_3}{n_{42}} y_0 = \frac{+2\lambda_2}{\lambda_2 - \text{fix}} y_0$$

$$\dot{y}_0 = n_{34} x_0 = \frac{\lambda_2 - \text{fix}}{2} x_0$$

$$A\eta = \lambda\eta$$

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$$(A - \lambda I)\eta = 0$$

$$\begin{bmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ f_{xx} & f_{xy} & -\lambda & 2 \\ f_{xy} & f_{yy} & -2 & -\lambda \end{bmatrix} \begin{bmatrix} \eta_{x1} \\ \eta_{x2} \\ \eta_{x3} \\ \eta_{x4} \end{bmatrix} = 0$$

$$-\lambda\eta_{x1} + \eta_{x3} = 0$$

$$-\lambda\eta_{x2} + \eta_{x4} = 0$$

$$\eta_{x4} = \lambda\eta_{x2}$$

$$\eta_{x3} = \lambda\eta_{x1}$$

$$f_{xx}\eta_{x1} + f_{xy}\eta_{x2} - \lambda\eta_{x3} + 2\eta_{x4} = 0$$

$$f_{xy}\eta_{x1} + f_{yy}\eta_{x2} - 2\eta_{x3} - \lambda\eta_{x4} = 0$$

$$(f_{xx} - \lambda^2)\eta_{x1} + (f_{xy} + 2\lambda)\eta_{x2} = 0$$

$$(f_{xy} - 2\lambda)\eta_{x1} + (f_{yy} - \lambda^2)\eta_{x2} = 0$$

$$2\lambda\eta_{x2} = (\lambda^2 - f_{xx})\eta_{x1}$$

$$\eta_{x2} = \frac{(\lambda^2 - f_{xx})\eta_{x1}}{2\lambda}$$

$$\begin{bmatrix} \eta_{x1} \\ \eta_{x2} \\ \eta_{x3} \\ \eta_{x4} \end{bmatrix} = \begin{bmatrix} \eta_{x1} \\ \frac{(\lambda^2 - f_{xx})\eta_{x1}}{2\lambda} \\ \lambda\eta_{x1} \\ \frac{(\lambda^2 - f_{xx})\eta_{x1}}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{\lambda^2 - f_{xx}}{2\lambda} \\ \lambda \\ \frac{\lambda^2 - f_{xx}}{2} \end{bmatrix} \eta_{x1}$$

$$\eta_3 = \begin{bmatrix} 1 \\ \frac{\lambda_2 - f_{xx}}{2\lambda_3} \\ \lambda_3 \\ \frac{\lambda_2 - f_{xx}}{2} \end{bmatrix}$$

$$\eta_4 = \text{(note that } \lambda_4 = -\lambda_3)$$

$$\begin{bmatrix} 1 \\ \frac{\lambda_2 - f_{xx}}{-2\lambda_3} \\ -\lambda_3 \\ \frac{\lambda_2 - f_{xx}}{2} \end{bmatrix}$$

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