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Chapter 1

Introduction

1.1 Curve Fitting

Problem 1

This can be solved by substituting the definition of:

$$y(x, \mathbf{w}) = \sum_{j=0}^M w_j x^j$$

into the error function and then taking the derivative.

$$\begin{aligned} E(\mathbf{w}) &= \frac{1}{2} \sum_{n=1}^N (y(x, \mathbf{w}) - t_n)^2 \\ &= \frac{1}{2} \sum_{n=1}^N \left(\sum_{j=0}^M w_j x^j - t_n \right)^2 && \text{Substitute} \\ \frac{dE(\mathbf{w})}{dw_i} &= \sum_{n=1}^N \left(\left(\sum_{j=0}^M w_j x^j - t_n \right) x^i \right) && \text{Take the derivative} \\ 0 &= \sum_{n=1}^N \left(\left(\sum_{j=0}^M w_j x^j - t_n \right) x^i \right) && \text{Set derivative to 0} \\ 0 &= \sum_{n=1}^N \left(\sum_{j=0}^M w_j x^j x^i - t_n x^i \right) && \text{Set derivative to 0} \\ \sum_{n=1}^N t_n x^i &= \sum_{n=1}^N \sum_{j=0}^M w_j x^j x^i \\ \sum_{n=1}^N t_n x^i &= \sum_{n=1}^N \sum_{j=0}^M w_j x^{i+j} \end{aligned}$$

Problem 2

This is solved in almost the same way we just have one additional term for the regularization so:

$$\begin{aligned} E(\mathbf{w}) &= \frac{1}{2} \sum_{n=1}^N (y(x, \mathbf{w}) - t_n)^2 + \frac{\lambda}{2} \|\mathbf{w}\|^2 \\ &= \frac{1}{2} \sum_{n=1}^N \left(\sum_{j=0}^M w_j x^j - t_n \right)^2 + \frac{\lambda}{2} \|\mathbf{w}\|^2 \end{aligned}$$

$$\frac{dE(\mathbf{w})}{dw_i} = \sum_{n=1}^N \left(\left(\sum_{j=0}^M w_j x^j - t_n \right) x^i \right) + \lambda w_i$$

$$-\lambda w_i = \sum_{n=1}^N \left(\left(\sum_{j=0}^M w_j x^j - t_n \right) x^i \right) + \lambda w_i$$

$$-\lambda w_i = \sum_{n=1}^N \left(\sum_{j=0}^M w_j x^j x^i - t_n x^i \right)$$

$$\sum_{n=1}^N t_n x^i - \lambda w_i = \sum_{n=1}^N \sum_{j=0}^M w_j x^j x^i$$

$$\sum_{n=1}^N t_n x^i - \lambda w_i = \sum_{n=1}^N \sum_{j=0}^M w_j x^{i+j}$$

1.2 Probability Theory

Problem 3

The Probability of Selecting an Apple can be decomposed as:

$$\begin{aligned} \mathcal{P}(\text{apple}) &= \mathcal{P}(\text{apple}, \text{red}) + \mathcal{P}(\text{apple}, \text{blue}) + \mathcal{P}(\text{apple}, \text{green}) \\ &= \mathcal{P}(\text{apple}|\text{red})\mathcal{P}(\text{red}) + \mathcal{P}(\text{apple}|\text{blue})\mathcal{P}(\text{blue}) + \mathcal{P}(\text{apple}|\text{green})\mathcal{P}(\text{green}) \\ &= (.3)(.2) + (.5)(.2) + (.3)(.6) \\ &= .34 \end{aligned}$$

The probability that observing an orange came from the green box can be solved using Bayes rule:

$$\begin{aligned}\mathcal{P}(\text{green}|\text{orange}) &= \frac{\mathcal{P}(\text{orange}|\text{green})\mathcal{P}(\text{green})}{\mathcal{P}(\text{orange})} \\ &= \frac{(.3)(.6)}{.66} \\ &= .27\end{aligned}$$

Problem 5

