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Notes on Suggested Parameter Ranges for our SIR / SEIR Simulations

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1 Introduction

For our brief "primer" (Paper 1) on the sensitivity of model outputs from standard SIR and SEIR models given unavoidable uncertainty in parameter estimation, I suggest we consider doing at least one of the following two methods in order to estimate confidence intervals under more "realistic" variation of parameters:

Method 1. Draw random values of the contact rate β, the infectious period $1/\gamma$, and (for SEIR models) the latency period $1/\sigma$ from *uniform* distributions within specified ranges;

Method 2. Draw random values of β, $1/\gamma$, and $1/\sigma$ from *Gaussian* distributions with specified means $\bar{p}$ and widths σ_p for each parameter $p = \{\beta, 1/\gamma, 1/\sigma\}$.

I suggest we fix the initial conditions for each simulation and only vary the couplings β, $1/\gamma$, and $1/\sigma$. We can fix the initial conditions across all runs of the standard SIR model to be

$$I(t_0) = 10^{-5}\ ,\ \ R(t_0) = 0\ ,\ \ S(t_0) = 1 - I(t_0), \quad \text{(SIR)} \tag{1}$$

and for all runs of the standard SEIR model we can use

$$I(t_0) = 10^{-5}\ ,\ \ E(t_0) = 10\, I(t_0)\ ,\ \ R(t_0) = 0\ ,\ \ S(t_0) = 1 - I(t_0) - E(t_0). \quad \text{(SEIR)} \tag{2}$$

(It should be obvious from this formulation that I am tacitly fixing $N_{\text{total}} = 1$, so that the various compartments $S(t)$, $I(t)$, and so on represent fractions of the total population.)

For either method of drawing random values for $\{\beta, 1/\gamma, 1/\sigma\}$, I suggest we do something like 10^3 or more instantiations each (for SIR and for SEIR), so we can then construct confidence interval curves for quantities of interest. The quantities I think we should focus on include $I_{\text{peak}} = I(t_c)$, t_c, and $T(t_{\text{end}})$, since these are closest to what policymakers would need to think about when planning for peak demand for hospital beds and so on. (We should be careful in the SEIR cases to find t_c as the time when $I(t)$ reaches its maximum, $dI/dt = 0$; this will not be the same as when $\mathcal{R}_t = 1$, since, for SEIR models, $\mathcal{R}_t$ governs the growth of the combined compartments $E(t) + I(t)$, not $I(t)$ alone.)

2 Parameter Ranges for Method 1

To set reasonable ranges for the distributions of $\{\beta, 1/\gamma, 1/\sigma\}$ for Method 1, I suggest we draw the infectious period from within the range

$$1/\gamma \in \{4, 10\} \text{ days.} \tag{3}$$

This yields an average infectious period of 7.0 days, which matches what many recent papers have used as estimates in applications to COVID-19. I found a recent analysis of the spread of COVID-19 within Wuhan, China [1], in which the authors used the wide range $1/\gamma \in \{4, 14\}$ days (which yields a longer average infectious period, 9 days). The model outputs will be pretty sensitive to modest shifts in $1/\gamma$; I suggest we stick with an average value of 7 days with the somewhat-more-modest range shown in Eq. (3), and cite Ref. [1] to indicate that some recent estimates in the literature consider even broader ranges for this parameter.

In a similar way, I suggest we draw the latency period for SEIR simulations from within the range

$$1/\sigma \in \{2.2, 8.0\} \text{ days.} \tag{4}$$

This yields an average latency period of 5.1 days, and again truncates the full range compared to what has been reported in the literature (which ranges between $\{2, 14\}$ days [2, 3].)

As we know, the model outputs will be quite sensitive to small shifts in the average contact rate, β. Here I suggest we adopt a range as follows. In our preliminary report, we varied β and $1/\gamma$ such that $\mathcal{R}_0$ varied by 15%, around a mean value of $\bar{\mathcal{R}}_0 = 2.31$. That corresponds to considering a range $\mathcal{R}_0 \in \{1.96, 2.66\}$, which in turn corresponds to

$$\begin{aligned} \bar{\mathcal{R}}_0 &= \frac{1}{2.66 - 1.96} \int_{1.96}^{2.66} dx\, x = 2.31 \,, \\ \left(\mathcal{R}_0^2\right)_{\rm avg} &= \frac{1}{2.66 - 1.96} \int_{1.96}^{2.66} dx\, x^2 = 5.38 \,, \end{aligned} \tag{5}$$

from which we may infer

$$\sigma_{\mathcal{R}}^2 = \left(\mathcal{R}_0^2\right)_{\rm avg} - \bar{\mathcal{R}}_0^2 \longrightarrow \sigma_{\mathcal{R}} = 0.21. \tag{6}$$

For γ, I assume that the actual duration (i.e., $1/\gamma$) varies uniformly within the range $\{4, 10\}$ days, so for $1/\gamma$ I find

$$\begin{aligned} \frac{1}{\gamma_{\rm avg}} &= \frac{1}{10 - 4} \int_4^{10} dx\, x = 7.0 \,, \\ \left(\frac{1}{\gamma^2}\right)_{\rm avg} &= \frac{1}{10 - 4} \int_4^{10} dx\, x^2 = 52.0 \,, \end{aligned} \tag{7}$$

from which I estimate

$$\sigma_{1/\gamma}^2 \simeq \left(\frac{1}{\gamma^2}\right)_{\rm avg} - \left(\frac{1}{\gamma_{\rm avg}}\right)^2 \longrightarrow \sigma_{1/\gamma} \simeq 1.73. \tag{8}$$

The range of β we would then infer, if we consider $\mathcal{R}_0$ and γ to vary independently, would be

$$\beta = \bar{\beta} \pm \sigma_\beta, \tag{9}$$

with

$$\bar{\beta} = \frac{\bar{\mathcal{R}}_0}{(1/\gamma_{\text{avg}})} = \frac{2.31}{7.0} = 0.33 \tag{10}$$

and

$$\sigma_\beta^2 = \bar{\beta}^2 \left[\left(\frac{\sigma_{\mathcal{R}}^2}{\bar{\mathcal{R}}_0^2} \right) + \left(\frac{\sigma_{1/\gamma}^2}{(1/\gamma_{\text{avg}})^2} \right) \right] \longrightarrow \sigma_\beta = 0.09. \tag{11}$$

This suggests that we select the contact rate β from within the range

$$\beta \in \{0.24, 0.42\}. \tag{12}$$

Note that several analyses have adopted $\mathcal{R}_0 \sim 2.2$ in studies of the early spread of COVID-19, which is only 5% lower than our central value of $\bar{\mathcal{R}}_0 = 2.31$; meanwhile, studies like Ref. [1] have estimated a median value $\mathcal{R}_0 \simeq 5.7$, which is significantly greater than the upper end of the range we would be considering. Again, we can adopt the ranges in Eqs. (3), (4), and (12) for our simulations while emphasizing in the text that we have adopted rather conservative estimates for the likely range of relevant parameters.

3 Parameter Distributions for Method 2

For Method 2, we may assume that β, $1/\gamma$, and $1/\sigma$ vary independently of each other, and are drawn randomly for each simulation from Gaussian distributions with mean values

$$1/\gamma_{\text{avg}} = 7.0 \text{ days} \ , \ \ 1/\sigma_{\text{avg}} = 5.1 \text{ days} \ , \ \ \bar{\beta} = 0.33 \ , \tag{13}$$

and with widths of the distributions given by Eq. (11) for σ_β and Eq. (8) for $\sigma_{1/\gamma}$. For the latency period $1/\sigma$, we compute as above:

$$\begin{aligned} \frac{1}{\sigma_{\text{avg}}} &= \frac{1}{8.0 - 2.2} \int_{2.2}^{8.0} dx\, x = 5.1 \ , \\ \left(\frac{1}{\sigma^2} \right)_{\text{avg}} &= \frac{1}{8.0 - 2.2} \int_{2.2}^{8.0} dx\, x^2 = 28.81 \ , \end{aligned} \tag{14}$$

from which I estimate

$$\sigma_{1/\sigma}^2 \simeq \left(\frac{1}{\sigma^2} \right)_{\text{avg}} - \left(\frac{1}{\sigma_{\text{avg}}} \right)^2 \longrightarrow \sigma_{1/\sigma} \simeq 1.67. \tag{15}$$

References

[1] S. Sanche, Y. T. Lin, C. Xu, E. Romero-Severson, N. Hengartner, and R. Ke, "High contagiousness and rapid spread of Severe Acute Respiratory Syndrome Coronavirus 2," *Emerging Infectious Diseases* **26**, no. 7 (July 2020), in press, `https://wwwnc.cdc.gov/eid/article/26/7/20-0282_article`.

[2] `https://www.worldometers.info/coronavirus/coronavirus-incubation-period/` (accessed 6 April 2020.)

[3] S. A. Lauer, K. H. Grantz, Q. Bi, F. K. Jones, Q. Zheng, H. R. Meredith, A. S. Azman, N. G. Reich, and J. Lessler, "The incubation period of coronavirus disease 2019 (COVID-19) from publicly reported confirmed cases: Estimation and application," *Ann. Intern. Med.* (2020), in press, `https://www.ncbi.nlm.nih.gov/pubmed?term=32150748`.