

Financial Entanglement: A Theory of Incomplete Integration, Leverage, Crashes, and Contagion*

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June 2014

Abstract

We propose a unified model of limited market integration, asset-price determination, leveraging, and contagion. Investors and firms are located on a circle, and access to markets involves participation costs that increase with distance. Due to a complementarity between participation and leverage decisions, the market equilibrium may exhibit diverse leverage and participation choices across investors, even though investors are ex-ante identical. Small changes in market-access costs can cause a change in the type of equilibrium and lead to discontinuous price changes, de-leveraging, and portfolio-flow reversals. Moreover, the market is subject to contagion, in that an adverse shock to investors in a subset of locations affects prices everywhere.

Keywords: Financial frictions, Market fragmentation, Leverage, Crashes, Contagion

JEL Classification: G01, G12

*We are grateful for comments and suggestions from Saki Bigio, Xavier Gabaix, Douglas Gale, Michael Gallmeyer, Arvind Krishnamurthy, Anna Pavlova, Lasse Pedersen, Alp Simsek, Andrzej Skrzypacz, Wei Xiong, Adam Zawadowski, three anonymous referees, as well as participants at the AFA, NBER AP, NBER SI Macro-Finance (ME), Minnesota Macro-AP conference, Studienzentrums Gerzensee, the Cowles Foundation GE conference, LBS Four nations cup, Wharton Conference on Liquidity and Financial Crises, and the Finance Theory Group conferences, and seminar audiences at Univ. of Alberta, Univ. of Birmingham, Boston University, Chicago Booth, Columbia Business School, Erasmus University, Hanken, HEC Paris, Univ. of Hong Kong, HKUST, Imperial College, INSEAD, LBS, LSE, Univ. of North Carolina, Stanford-GSB, Tilburg, Vanderbilt, Warwick Business School, Univ. of Washington, Yale SOM.

1 Introduction

There is a substantial body of empirical literature (described in detail in the next section) concluding that market integration is limited and risk sharing is imperfect. In particular, a) capital tends to stay “close” to its origin, b) reward for risk reflects — at least partially — “local” factors that one would expect to be diversifiable, and c) reductions in capital flows and the extent of market integration tend to coincide with substantial increases in risk premia and leverage reductions.

Motivated by these findings, we propose a tractable theoretical framework that offers a unified view of asset-price determination, endogenous market fragmentation, contagion across seemingly unrelated markets, diversity of observed investment strategies, and de-leveraging.

The model features a continuum of investors and financial markets located on a circle. Investors are endowed with shares of a risky firm domiciled at their location and traded at the same location. The dividends of this security are closely correlated with dividends of securities located in nearby locations and less correlated with securities in distant locations. Even though securities in more distant locations offer greater benefits from the perspective of diversification and risk allocation, participation in such markets involves costs that grow with the distance from the current location. Such costs are reflective of the fact that informational frictions are likely to grow as investors participate in progressively more unfamiliar markets. Indeed, inside our model distance on the circle should not be viewed narrowly as geographical distance. For our purposes, distance is a broad measure parametrizing a tradeoff between the cost of overcoming frictions in accessing various asset classes and the diversification benefits offered by these asset classes. Alongside the risky markets, investors have access to a zero-net-supply bond market.

Because access to financial markets is subject to frictions, the market equilibrium features limited integration. Investors from nearby locations, who face similar cost structures, choose to participate on arcs of the circle that feature a high degree of overlap. Even though there may be many assets with no common investor, this overlap renders all markets (indirectly) “entangled.”

In such a framework, we examine four sets of issues: a) the determination of asset prices in light of limited market integration; b) the structure of optimal investment strategies, i.e., arcs of participation, leverage, and risky-asset positions chosen by investors; c) the effect of changes to the financial technology (i.e., the magnitude of market-access costs); and d) the propagation of “local” shocks to the financial technology. We summarize our findings about each of the four issues in turn.

a) Limited market integration implies that investors are over-exposed to the risks of locations in their vicinity. Consequently, risk premia are higher than they would be in a frictionless world. An important aspect of the analysis is that the magnitudes of risk premia and of portfolio flows are tightly linked and reflect the extent of market integration.

b) Although investors in any given location are identical in every respect, their investment strategies may be diverse with some investors choosing high-leverage strategies and increased extent of participation in risky markets, and others choosing unleveraged, low-participation strategies.

The intuition for this finding is that leverage and participation decisions are complements. For a given level of risk premia, participating in more markets increases the maximum Sharpe ratio attainable, which induces investors to take more leverage. Since leverage increases the overall variance of the portfolio, it increases the marginal benefit of further market participation and diversification. This complementarity between participation and leverage decisions makes the problem of choosing the optimal participation arc a non-concave problem with (possibly) multiple local optima. As is commonly the case in situations where investors maximize non-concave problems, a symmetric equilibrium (whereby all investors adopt the same policies) may fail to exist, and the only equilibrium is asymmetric. It is noteworthy that the non-concavities arise even when the participation costs are *convex*.

This implication of the model is consistent with the diversity of investment strategies offered to investors by, say, mutual funds and hedge funds. More importantly, it shows that portfolio leverage may arise endogenously in fragmented markets, without requiring inherent differences between investors.

c) An illustration of the consequences of the complementarity between the breadth of participation and leverage is provided by the introduction of borrowing limits, e.g., via

margin constraints.¹ In such a situation, different types of equilibria may coexist, generating fragility. For instance, marginal variations in participation costs may cause the type of equilibrium to change, causing discontinuous reactions in prices, capital flows, and leverage.

The intuition is as follows. Absent leverage constraints, broad-participation decisions by different investors are substitutes: The less other investors participate, the lower the prices of risky assets, and thus the higher an individual investor’s incentive to take leverage and participate broadly in markets. A binding leverage constraint changes the situation significantly. In this case, the lower risky-asset prices necessitate a lower leverage, i.e., a reduction in the extent to which one can take advantage of an access to diversified portfolios. The consequence is that the incentive to participate broadly is diminished, relative to the no-constraint case, by other investors’ non-participation. This complementarity may be so strong that multiple equilibria, featuring distinct prices, extents of integration, and aggregate leverage, can co-exist: If prices are low, then leverage is constrained to be low and there isn’t enough incentive to participate broadly, justifying low prices. If prices are high, then agents can leverage sufficiently to justify paying for broad participation, which supports high prices.

d) The fact that markets are only partially integrated may strengthen the interdependence of the prices of risky securities in different locations. We present an example where the financial technology “breaks down” (i.e., the participation costs rise to infinity) only in a subset of locations — we refer to them as the “affected” locations. We show that such an event pushes prices in almost all locations downward. This is true even for locations that are *not* connected with the affected locations through asset trade and also have negative dividend correlation with them.

The intuition for this finding is as follows. As a direct effect of the shock in the affected regions, investors in affected locations stop investing in risky markets in their vicinity. Consequently, prices in these locations fall so as to attract demand from neighboring unaffected locations. The portfolio reallocation in these neighboring locations happens away from other, farther locations, resulting in weaker demand in these farther locations, necessitating price drops in these locations as well, so as to attract demand from locations neighboring the neighboring locations, etc.

¹In Appendix D we consider alternative formulations of borrowing frictions, motivated by limited liability.

The extent of price drops required to compensate investors for tilting their portfolios to absorb local risks depends on the extent of their overall participation in risky markets. If the extent of their participation is small, so that their portfolio is heavily exposed to risks in their vicinity, then a tilt towards a nearby location requires a higher compensation. By contrast, if investors' portfolios are invested across a broad range of locations, then they are more willing to absorb risks in their vicinity. Hence, somewhat surprisingly, the higher the degree of market integration, the lower the price drop in response to a local participation shock (and vice-versa).²

In summary, the paper offers a unified framework to study several financial-market phenomena as a result of (endogenous) market segmentation. Besides the specific applications that we consider, which are meant to illustrate the structure of our framework, we believe that the paper also makes a technical contribution; it provides a particularly tractable framework to study the numerous instances in economics where frictions prevent optimal risk diversification and capital allocation across asset classes.

Our work is related to several strands of theoretical and empirical literature. We discuss connections to the theoretical literature here and postpone a discussion of related empirical literature for the next section.

Through its focus on incomplete participation by all agents, our paper is related to the seminal contribution of Merton (1987) and the studies inspired by it. We endogenize the degree of participation, and choose a specific asset-universe structure. Circular structures have been repeatedly used in a variety of applications from location models in industrial organization (Salop (1979)) to the literature on financial networks.³ From a technical perspective, our paper proposes a novel and quite tractable structure to model risky payoffs continuously distributed on a circle. The paper also differs from the financial-network literature in setup and questions addressed. Specifically, we study a relatively standard Walrasian exchange economy with (endogenously) limited participation, while that literature typically

²Interestingly, albeit in a completely different setting, Allen and Gale (2000) also finds that contagion is more severe when financial interconnectivity — between deposit-taking institutions, in that model — is low.

³An indicative and incomplete listing of papers studying networks, and in particular circular ones, in finance includes Allen and Gale (2000), Freixas et al. (2000), Allen et al. (2012), Caballero and Simsek (2012), Acemoglu et al. (2013), and Zawadowski (2013). Allen and Babus (2009) offers a survey of the literature.

concentrates on banking issues, network externalities, etc.

Allen and Gale (2004a,b) and Carletti and Leonello (2012) provide examples of mixed equilibria whereby banks — faced with the possibility of runs — may choose different strategies anticipating trading (of loans) with each other at a subsequent date. The source of asymmetric equilibria in our paper is different, and more closely related to Aumann (1966), whose analysis covers Walrasian setups (without idiosyncratic liquidity shocks) with non-concave investor-optimization problems.

There exists a vast literature analyzing the interaction between declining prices and tightening collateral constraints — we do not attempt to summarize this literature and simply refer to Kiyotaki and Moore (1997) for a seminal contribution. That literature commonly assumes an exogenous motivation for trading (e.g., differences in productivity, beliefs, or risk aversion); the role of the leverage constraint is to amplify the effect of exogenous shocks, by limiting the set of feasible trades between heterogeneous agents. In our setup, market participants are ex-ante identical, with heterogeneous strategies arising endogenously. As a result, an investor’s ability to leverage determines whether an asymmetric equilibrium can arise in the first place: The constraint doesn’t only impact the set of feasible trades, but also affects the investors’ incentive to pay the costs required for broad-participation, high-Sharpe-ratio strategies. This additional feedback effect can be very strong, rendering participation decisions complements rather than substitutes and leading to multiple equilibria, thus fragility.^{4,5}

Finally, the domino effect produced by our model relates it to the vast literature on contagion. We do not attempt to summarize this literature. Instead, we single out as a natural counterpoint a particular mechanism proposed in many models as an explanation of

⁴Krishnamurthy (2010) discusses two important aspects of a financial crisis: balance-sheet effects and the disengagement of investors from markets (specifically, due to an increase in Knightian uncertainty). Balance-sheet effects and declines in participation are also present in our framework, where the feasibility and the attractiveness of alternative leverage-participation combinations are jointly determined.

⁵The discontinuous dependence of the price on underlying parameters relates our paper to the literature modeling abrupt price changes (“crashes”) through changes in the type of equilibrium. These papers typically feature backward-bending demand curves that generate multiple equilibria (see, e.g., Gennotte and Leland (1990), Barlevy and Veronesi (2003), Yuan (2005), Brunnermeier and Pedersen (2009), and Garcia and Strobl (2011)). The mechanism responsible for multiple equilibria in our paper is different, though, since it relies on the complementarity between participation and leverage.

contagion. Specifically, many papers⁶ posit the existence of some agents, frequently of limited risk-bearing capacity, who price all the assets and whose marginal utility therefore transmits shocks from one asset to the price of another. In contrast, contagion in our model obtains even though no agent participates in all markets. A shock in one area is transmitted to a distant one by affecting first the immediately neighboring areas, which in turn affect their neighboring areas, etc. A practical implication is that there can be positive interdependence between the prices in locations by negatively correlated dividends and no common traders.⁷

The paper is organized as follows. Section 2 presents in greater detail the empirical evidence underlying and motivating this paper. Section 3 presents the baseline model. Section 4 presents the solution and the results. Sections 5 and 6 study crashes, respectively contagion. Finally, extensions, proofs and further discussion on some of the model assumptions are contained in the Appendix.

2 Motivating Facts

As a motivation for the assumptions of the model, we summarize some well-documented facts concerning the allocation of capital. Table 1 summarizes the evidence on so-called “gravity” equations in international finance. These gravity equations typically specify a log-linear relation between bilateral flows in various forms of asset trade (equities, bonds, foreign direct investment, etc.) and the sizes of the countries and the geographical distance between them.

A striking and robust finding of this literature is that bilateral capital flows and stocks decay substantially with geographical distance. This finding is surprising, since countries that are geographically distant would seem to offer greater diversification benefits; hence one would expect distance to have the opposite sign from the one found in regressions. The literature typically interprets this surprising finding as evidentiary of informational asymmetries that increase with geographical distance — a crude proxy for familiarity and

⁶See, e.g., Kyle and Xiong (2001), Cochrane et al. (2008), or Pavlova and Rigobon (2008).

⁷The fact that contagion between two markets can arise even if there are no common traders and even if the underlying dividends have no correlation makes our explanation of contagion also distinct from rebalancing approaches, such as Kodres and Pritsker (2002).

Table 1: Gravity Equations in International Trade and Finance.

The table reports a survey of the literature on gravity equations in international trade and finance. Portes and Rey (2005) uses bilateral equity flow data among 14 countries from 1989 to 1996. Buch (2005) uses the stock of assets and liabilities of banks from 1983 to 1999. Head and Ries (2008) uses bilateral FDI stocks with 30 OECD reporters and 32 partners in year 2001. Talamo (2007) uses FDI flow data from 1980-2001. Aviat and Coeurdacier (2007) uses bilateral trade data and bank asset holding data in year 2001. Ahrend and Schwellnus (2012) uses IMF’s Consolidated Portfolio Investment Survey (CPIS) in 2005-2006, which reports bilateral debt investment for 74 reporting countries and 231 partner countries. Typically, the regression performed is

$$\log(X_{i,j}) = \alpha + \beta_1 \log(GDP_i) + \beta_2 \log(GDP_j) + \beta_3 \log(Distance_{i,j}) + controls + \epsilon_{i,j}$$

where $X_{i,j}$ is the equity flow, portfolio holding, FDI flow, FDI stock, trade, or bank asset holdings.

Source	Dependent variable	Distance	t-stat
Portes and Rey (2005)	Equity flows	-0.881	-28.419
Buch (2005)	Bank asset holdings	-0.650	-12.020
Talamo (2007)	FDI flow	-0.643	-9.319
Head and Ries (2008)	FDI stock	-1.250	-17.361
Aviat and Coeurdacier (2007)	Trade	-0.750	-10.000
Aviat and Coeurdacier (2007)	Bank asset holdings	-0.756	-8.043
Ahrend and Schwellnus (2012)	Bond holdings	-0.513	-4.886

similarity in social, political, legal, cultural, and economic structures. Supportive of this interpretation is the literature that finds a similar relation between distance and portfolio allocations in domestic portfolio allocations.⁸

Further supportive evidence is provided by literature that documents how partial market integration affects the pricing of securities. Bekaert and Harvey (1995) finds that local factors affect the pricing of securities and are not driven out by global factors. An important additional finding of Bekaert and Harvey (1995) is that the relative importance of global and local factors is time varying, suggesting time-varying integration between markets.

Indeed, crisis periods offer an opportunity to visualize the extent of variation in market

⁸For instance, Coval and Moskowitz (1999) shows that US mutual fund managers tend to overweight locally headquartered firms. Coval and Moskowitz (2001) shows that mutual fund managers earn higher abnormal returns in nearby investments, suggesting an informational advantage of local investors. Using Finnish data, Grinblatt and Keloharju (2001) shows that investors are more likely to hold and trade the stocks of Finnish firms that are located close to the investors. Similar evidence is presented in Chan et al. (2005): Using mutual fund data from 26 countries, and using distance as a proxy for familiarity, this paper finds that a version of the gravity equation holds for mutual fund holdings. That is, the bias against foreign stocks is stronger when the foreign country is more distant. In a similar vein, Huberman (2001) documents familiarity-related biases in portfolio holdings.

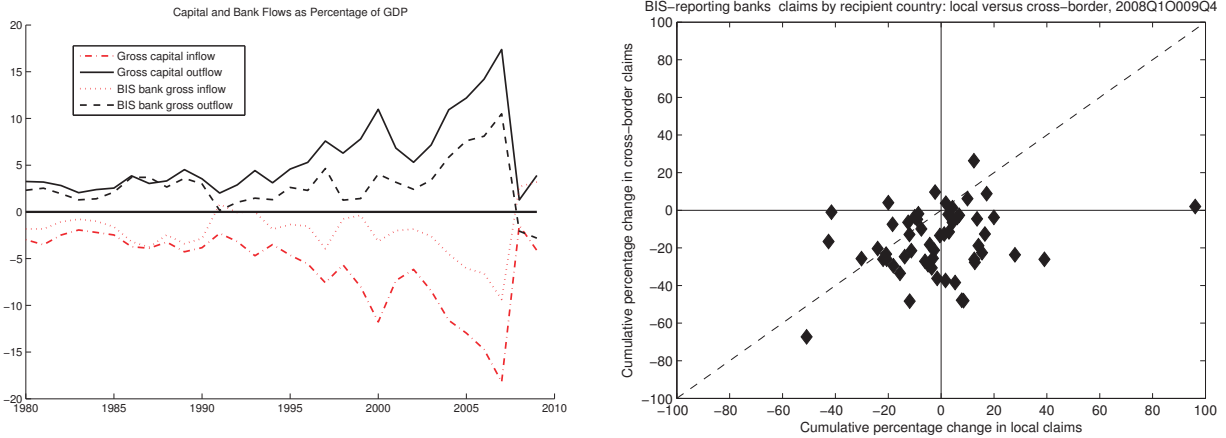


Figure 1: Reversals in global capital flows. Source: Hoggarth et al. (2010). The left plot depicts the sum of global net purchases of foreign assets by residents (labeled “Gross capital inflows”) and the sum of global net purchases of domestic assets by foreigners (labeled “Gross capital outflows”). The figure also reports cross-border bank inflows and outflows based on BIS data. Right plot: Cumulative percentage in local claims held by banks against cumulative percentage change in cross-border claims during the 2008Q1 - 2009Q4 period. Source: Hoggarth et al. (2010). based on BIS Data on the 50 largest debtor countries by foreign liabilities. For a detailed description of the sample of reporting countries, see Hoggarth et al. (2010).

integration. The left plot of Figure 1 reports the sum of global net purchases of foreign assets by residents (labeled “Gross capital inflows”) and the sum of global net purchases of domestic assets by foreigners (labeled “Gross capital outflows”). The figure also reports cross-border bank inflows and outflows based on BIS data. The picture helps visualize that, in the years preceding the financial crisis of 2008, there was a large increase in gross capital flows. This expansion in capital flows came to a sudden stop in the first quarter of 2008, as the financial crisis took hold. In line with the patterns illustrated in the left plot of the figure, Ahrend and Schweltnus (2012) documents a significantly stronger coefficient on distance in cross-border gravity equations during 2008-2009. This evidence suggests that capital becomes more concentrated “locally” during times of crisis.

The right plot of Figure 1 provides a further illustration of capital concentration during the crisis by focusing on readily available BIS data. It depicts the cumulative percentage change in banks’ cross-border claims against the respective cumulative change in their local claims for various countries between 2008Q1 - 2009Q4. Most points on the graph are below the 45-degree line, suggesting that the liquidation of the foreign holdings of banks

was disproportionately larger than the respective liquidation of their local holdings. Direct evidence of a “flight to home” effect is provided by Giannetti and Laeven (2012), which shows that the home bias of lenders’ loan origination increases by approximately 20% if the bank’s home country experiences a banking crisis. Giannetti and Laeven (2012) also argues that this flight to home effect is distinct from flight to quality, since borrowers of different quality are equally affected.

We note in passing that the increased portfolio concentration during times of crisis is not limited to international data. For instance, the collapse of the (non-agency) securitization market during the financial crisis implied increased limitations to the ability of local issuers to diversify local real-estate risk.

In summary, the empirical evidence supports the following broad conclusions: a) Capital stays “close” to its origin, which implies a “local” concentration of risk in investors’ portfolios. b) The extent of market integration is time-varying. Furthermore crises are times when financial integration diminishes quite abruptly.

3 Model

3.1 Investors and firms

Time is discrete and there are two dates, $t = 0$ and $t = 1$. All trading takes place at time $t = 0$, while at $t = 1$ all payments are made and contracts are settled. Investors are price takers, located at different points on a circle with circumference normalized to one. We index these locations by $i \in [0, 1)$. Investors have exponential utilities and maximize expected utility of time-1 wealth

$$E[U(W_{1,i})] = -E[e^{-\gamma W_{1,i}}], \quad (1)$$

where $W_{1,i}$ is the time-1 wealth of an investor in location i . The assumptions that investors only care about terminal wealth and have exponential utility are made for tractability and in order to expedite the presentation of the results. In Appendix A.1 we extend the model

to allow for consumption over an infinite horizon.

Besides having identical preferences, investors at any given location are also identical in terms of their endowments and their information sets. Specifically, at time $t = 0$ the investors in location i are equally endowed with the total supply of shares (normalized to one) of a competitive, representative firm, which is domiciled at the same location i . Each firm pays a stochastic dividend equal to D_i in period 1.

We specify the joint distribution of the dividends D_i for $i \in [0, 1]$ so as to obtain several properties. Specifically, we wish that 1) firms be ex-ante symmetric, that is, the marginal distribution of D_i be independent of i ; 2) the total dividends paid $\int_0^1 D_i di$ be constant (normalized to one); 3) firms with indices close to each other (in terms of their shortest distance on the circle) experience a higher dividend correlation than firms farther apart; and 4) dividends at different locations be normally distributed.

To formalize these notions, we let Z_i denote a standard Brownian motion for $i \in [0, 1]$, and we also introduce the quantity

$$B_i \equiv Z_i - iZ_1 \text{ for } i \in [0, 1], \quad (2)$$

known as a Brownian bridge. Two immediate implications of the construction (2) are that the Brownian bridge satisfies $B_0 = B_1 = 0$ and has continuous paths (a.s.).⁹ Using the definition of B_i , we let D_i be defined as

$$D_i \equiv 1 + \sigma \left(B_i - \int_0^1 B_j dj \right), \quad (3)$$

where $\sigma > 0$ is a constant controlling the volatility of the dividend process. Since the specification (3) plays a central role in our analysis, we first derive the statistical properties of D_i , and then we provide a graphical illustration.

Lemma 1 *D_i satisfies the following properties.*

1. (*Symmetry and univariate normality*) *The marginal distribution of D_i is the same for*

⁹Equivalently, a Brownian bridge B_t on $[0, 1]$ is a process distributed as the Brownian motion Z_t conditional on $Z_1 = 0$.

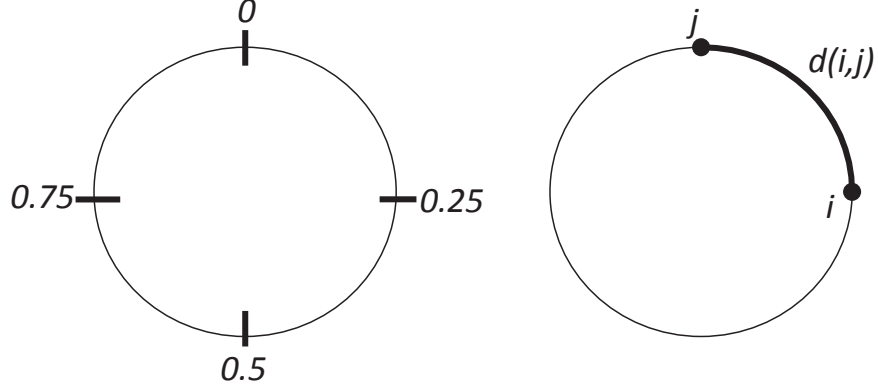


Figure 2: The left plot depicts a circle with circumference 1. The bold arc on the right plot depicts the notion of the shortest distance on the circle $d(i, j)$ between points i and j that we use throughout.

all i . Specifically, D_i is normally distributed with mean 1 and variance $\frac{\sigma^2}{12}$.

2. (No aggregate risk) $\int_0^1 D_i di = 1$.
3. (Continuity on the circle) Let $d(i, j) \equiv \min(|i - j|, 1 - |i - j|)$ denote a metric on the interval $[0, 1)$. Then D_i is continuous (a.s.) on $[0, 1)$ if the interval $[0, 1)$ is endowed with the metric d .
4. (Joint normality and distance-dependent covariance structure) For any vector of locations $i = (i_1, i_2, \dots, i_N)$ in $[0, 1)$, the dividends $(D_{i_1}, \dots, D_{i_N})$ are joint normal, with covariances given by

$$\text{cov}(D_{i_n}, D_{i_k}) = \sigma^2 \left(\frac{1}{12} - \frac{d(i_n, i_k)(1 - d(i_n, i_k))}{2} \right). \quad (4)$$

It is easiest to understand the properties of D_i by using a graphical illustration. The left plot of Figure 2 provides an illustration of the interval $[0, 1)$ “wrapped” around as a circle with circumference one. The metric $d(i, j)$ can be thought of as the length of the shortest arc connecting i and j , as the right plot of Figure 2 illustrates. Figure 3 illustrates a path of Z_i and the associated paths of B_i and D_i .

A remarkable property of the dividend structure (3) is that the covariance, and therefore correlation, of dividends in any locations i and j depend exclusively on the distance

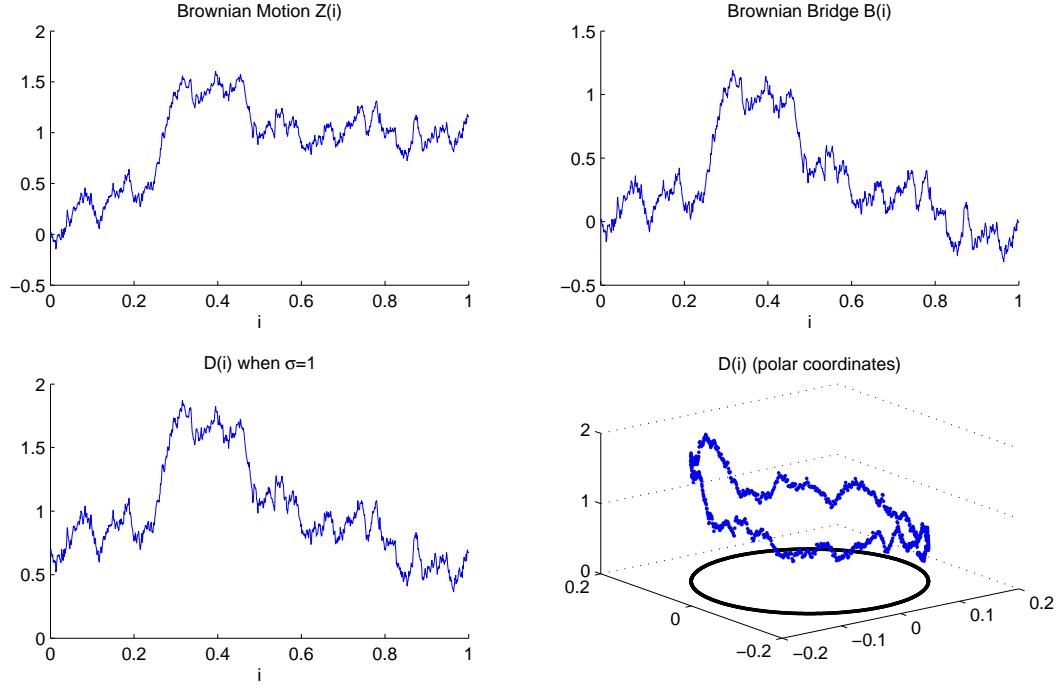


Figure 3: An illustration of the construction of the dividend process D_i . The two plots at the top depict a sample Brownian path Z_i , and the associated path of a Brownian bridge B_i . The two bottom plots depict the associated sample path of D_i , when the indices $i \in [0, 1)$ are aligned on a line and when the same interval is depicted as a circle with circumference one.

$d(i, j)$ between the two locations, but not the locations themselves. Equation (4) gives an explicit expression for the covariance of the dividends at different locations. The associated correlations follow immediately as

$$\text{corr}(D_i, D_j) = 1 - 6d(i, j)(1 - d(i, j)). \quad (5)$$

Equation (5) implies that the correlation between D_i and D_j approaches one as the distance $d(i, j)$ approaches zero, and is minimized when $d(i, j) = \frac{1}{2}$, i.e., when the two firms are located diametrically opposite each other on the circle.

An implication of Lemma 1 (property 2) is that all risk in the economy is diversifiable. It is straightforward to introduce systematic risk by simply adding a common, normally distributed shock to all dividends D_i . Such a shock would require an additional risk premium (equal across all risky assets), but would not alter any of our results otherwise. In the interest

of simplicity and to emphasize that our focus is on frictions that assign a positive risk price to *diversifiable* risk, we choose to not include aggregate risk.

3.2 Financial markets and participation costs

Investors in location i can trade claims to the stock of firms located in every location $j \in [0, 1)$ on the circle. Motivated by the evidence presented in Section 2 — specifically the fact that capital tends to stay close to its origin — we assume that participation in financial markets is costly, and the more so the farther away a financial market is from an investor's location. In Appendix C we provide one possible motivation for such costs, namely as information-acquisition costs. Specifically we present an expanded version of the model with multiple securities of unobserved quality in every location; investors have to incur a distance-dependent cost to overcome informational asymmetries that lower their returns when investing in distant locations. Indeed, the appendix shows that the comparatively higher returns obtained by an investor in locations where she is informed can render uninformed participation suboptimal; alternatively phrased, investors participate only in locations where they choose to pay the cost to become informed. We present the details of this model extension in Appendix C, and proceed with our analysis taking the participation costs as given.

We next propose a mathematical structure for the participation costs. The participation decision of investors consists of choosing a subset of all markets $[0, 1)$ in which to invest. To avoid unnecessary complications, we restrict attention to subsets of $[0, 1)$ that can be represented as a finite union of disjoint intervals with midpoints $a_{i,n}$ and lengths $\Delta_{i,n}$. Hence, the participation-cost function is a mapping from such subsets of $[0, 1)$ to the set of positive real numbers. For instance, if an investor in location i chooses to participate in N_i such intervals with midpoints $a_{i,n}$ and total length $\Delta_i \equiv \sum_{n=1}^{N_i} \Delta_{i,n}$, she incurs costs equal to

$$F_i(\{a_{i,1}, \dots, a_{i,N_i}\}, \Delta_i) = \kappa \left(b_{N_i} \sum_{n=1}^{N_i} f(d(a_{i,n}, i)) + g(\Delta_i) \right), \quad (6)$$

where $\kappa > 0$, and $f : (0, \frac{1}{2}] \rightarrow \mathbb{R}$ and $g : [0, 1) \rightarrow \mathbb{R}$ are positive, non-decreasing, differentiable, and convex functions. We assume that f has a discontinuity at zero in the sense that

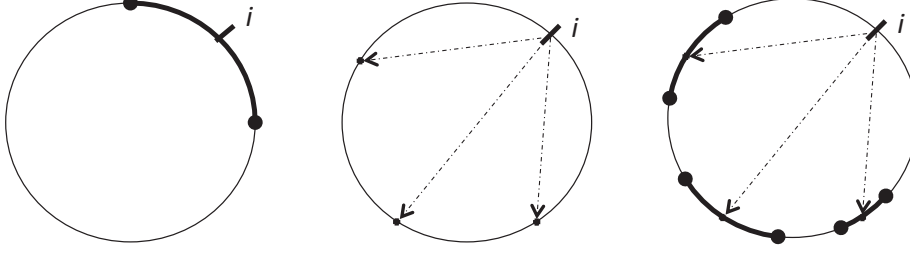


Figure 4: Illustration of participation choices under different assumptions on the participation costs $F_i(\{a_{i,1}; \dots; a_{i,N_i}\}, \Delta_i)$.

$\lim_{x \rightarrow 0} f(x) > 0$, while $f(0) = 0$. Also, $g(0) = 0$ and $g'(0) = 0$. Finally, b_{N_i} is positive and non-decreasing in N_i ; we set $b_1 = b_2 = 1$ without loss of generality.¹⁰

We make several remarks on specification (6). First we note that $F_i(\{i\}, 0) = 0$, so that participating only in the local market is costless. Second, the fact that f is increasing implies that investing in markets that are farther away (in the sense that the distance $d(a_{i,n}, i)$ from one's location is larger) is more costly than participating in markets that are close by. Third, increasing the total mass of markets in which the investor participates (Δ_i), while keeping the number (and midpoints) of intervals the same, incurs incremental rather than fixed costs. This captures the idea that expanding participation to contiguous markets is substantially less costly than participating in a market that is not adjacent to any of the markets where the investor has already decided to participate. Fourth, the fact that participation costs depend on the location of the investor implies that investors in different locations on the circle face different costs of participating in a given market j , depending on their proximity to that market. Finally, for technical convenience and ease of exposition, the costs of expanding the total measure of the interval are specified as independent of the structure of the midpoints and the lengths of the subintervals. This assumption is not crucial and can be easily relaxed.¹¹

Even though one can specify more elaborate participation-cost structures, the one given by (6) is sufficiently flexible for our purposes. To better understand this cost structure it is useful to consider some special cases. In the first special case, the investor can only

¹⁰Since $f(0) = 0$, the first mid-point that an investor chooses is always her home location. Hence b_1 can be chosen arbitrarily, as $b_1 f(0) = 0$. Similarly, $b_2 = 1$ is without loss of generality, since we can always re-define $f^* = b_2 f$ and $b_N^* = \frac{b_N}{b_2}$ without changing the total cost F_i .

¹¹Indeed, if one were to replace $g(\Delta_i)$ with $\sum_{n=1}^{N_i} g(\Delta_{i,n})$, that would introduce additional reasons for the cost structure to be non-convex, strengthening some of the conclusions of the paper.

participate in an arc centered at her home location (depicted in the left plot of Figure 4). This outcome follows from (6) if one sets $b_N f(x) = \infty$ and $g(y) < \infty$ for any $x > 0$, $y > 0$, and $N > 1$, so that the participation cost reduces to $F(\Delta_i) = \kappa g(\Delta_i)$. We note that in this special case the participation-cost structure is convex (unlike when the investor participates in disjoint intervals, which carries fixed costs). This convexity is quite attractive for illustrating some of the results of the paper, and therefore this special case will play an important role in our analysis. A second special case corresponds to situations in which an investor only participates on discrete points (middle plot of Figure 4). This outcome obtains from (6) if $b_N f(x) < \infty$ and $g(y) = \infty$ for any $y > 0$. In the general case $b_N f(x) < \infty$ and $g(y) < \infty$, the participation decision involves choosing the number of points N_i , the location of the midpoints $a_{i,n}$, and the length of the intervals $\Delta_{i,n}$ at each location. This situation is depicted in the right plot of Figure 4.

Besides the markets for risky shares, there is a market for zero-net-supply riskless bonds that pay one unit of wealth at time 1. Participation in the bond market is costless for everyone.

The participation costs act as deadweight costs that are paid out (i.e., reduce consumption) at time 1.

We remark in passing that our results would not be affected if, instead of making the participation decisions themselves, agents invested through frictionless and competitive intermediation. That is, we could introduce a competitive sector of intermediaries who incur the participation costs, choose optimal portfolios for their clients, and then charge them competitive fees to cover the participation costs.¹²

¹²In a previous version of the paper, we consider such a model. Specifically, in this model a) investors don't have direct access to markets, but rather have to hire (competitive) intermediaries in their location to gain access to distant markets, b) the intermediaries attract clients by offering portfolios that maximize their welfare, and c) intermediaries charge clients fees to cover the participation costs that they incur. Not surprisingly, the assumption of perfect competition and the lack of frictions between investors and intermediaries (no moral hazard, unobservable, idiosyncratic liquidity shocks. etc.) implies that the allocations and the prices in the two models are identical.

3.3 Individual maximization and equilibrium

To formalize an investor's decision problem we let P_i denote the price of a share in a market i , $P_B \equiv \frac{1}{1+r}$ the price of a bond, $dX_j^{(i)}$ the mass of shares in market j bought by an investor located in i ,¹³ and $X_B^{(i)}$ the respective number of bonds. Then, letting

$$W_{0,i} \equiv \int_0^1 P_j dX_j^{(i)} + P_B X_B^{(i)}$$

denote the total financial wealth of an investor in location i , the budget constraint of an investor can be expressed as $W_{0,i} = P_i$.

We are now in a position to formulate the investor's maximization problem as

$$\max_{w_i^f, G^{(i)}, N_i, \vec{a}_i, \vec{\Delta}_i} E[U(W_{1,i})], \quad (7)$$

subject to the budget constraint $W_{0,i} = P_i$, and

$$W_{1,i} = W_{0,i} \left(w_i^f (1+r) + \left(1 - w_i^f \right) \int_0^1 R_j dG_j^{(i)} \right) - F_i, \quad (8)$$

where w_i^f is the fraction of $W_{0,i}$ invested in the risk-free security by an agent in location i , $G_j^{(i)}$ is a bounded-variation function with $\int_0^1 dG_j^{(i)} = 1$, which is constant in locations where the investor does not participate (i.e., $dG_j^{(i)} = 0$ in these locations), so that $dG_j^{(i)}$ captures the fraction of the risky component of the portfolio $\left(1 - w_i^f \right) W_{0,i}$ invested in the share of stock j by a consumer located in i . Finally, $R_i \equiv \frac{D_i}{P_i}$ is the realized gross return on security i at time 1. We do not restrict G to be continuous, that is, we allow investors to invest mass points of wealth in some locations.

The definition of equilibrium is standard. An equilibrium is a set of prices P_i , a real interest rate r , and participation and portfolio decisions w_i^f , $G^{(i)}$, N_i , and $\{a_{i,1}, \dots, a_{i,N_i}, \Delta_{i,1}, \dots, \Delta_{i,N_i}\}$ for all $i \in [0, 1]$ such that: 1) w_i^f , $G^{(i)}$, N_i , and $\{a_{i,1}, \dots, a_{i,N_i}, \Delta_{i,1}, \dots, \Delta_{i,N_i}\}$ solve the optimization problem of equation (7), 2) financial markets for all stocks clear: $P_j = \int_{i \in [0,1]} \left(1 - w_i^f \right) W_{0,i} dG_j^{(i)}$,

¹³The function $X_j^{(i)}$ has finite variation. We adopt the natural convention that $X_j^{(i)}$ is continuous from the right and has left limits.

and 3) the bond market clears, i.e., $\int_{i \in [0,1]} W_{0,i} w_i^f di = 0$.

By Walras' law, we need to normalize the price in one market. Since in the baseline model we abstract from consumption at time zero for parsimony, we normalize the price of the bond to be unity (i.e., we choose $r = 0$). We discuss consumption at more dates than time one and an endogenously determined interest rate in Appendix A.1.

4 Solution and Its Properties

We solve the model and illustrate its properties in a sequence of steps. First, we discuss a frictionless benchmark, where participation costs are absent. Second, in order to highlight the core intuition, we discuss the special case where the investor can only participate in an arc centered at her location, so that her participation decision amounts to choosing the length of that arc (the special case depicted on the left panel of Figure 4). We consider the general case, which allows for both contiguous and non-contiguous participation choices, in Appendix A.2.

4.1 A frictionless benchmark

As a benchmark, we consider first the case without participation costs: $F_i = 0$. In this case, the solution to the model is trivial. Every investor i participates in every market j . The first order condition for portfolio choice is

$$E[U'(W_{1,i})(R_j - (1 + r))] = 0. \quad (9)$$

With the above first-order condition in hand, one can verify the validity of the following (symmetric) equilibrium: $w_i^f = 0$, and $G_j^{(i)} = G_j = j$ for all $(i, j) \in [0, 1) \times [0, 1)$ — i.e., investors in all locations i choose an equally weighted portfolio of every share $j \in [0, 1]$. Accordingly, $W_{1,i} = \int_0^1 D_j dj = 1$. Since in this equilibrium $W_{1,i} = 1$, the Euler equation (9) implies that

$$E(R_j) = 1 + r. \quad (10)$$

Combining (10) with the definition $R_j = \frac{D_j}{P_j}$ implies

$$P_j = \frac{E(D_j)}{1+r} = \frac{1}{1+r} = 1, \quad (11)$$

where the last equation follows from the normalization $r = 0$.

The equilibrium in the frictionless case is intuitive. Since there are no participation costs, investors hold an equally weighted portfolio across all locations. Since — by assumption — there is no risk in the aggregate, the risk of any individual security is not priced.

4.2 Symmetric equilibria with participation costs

To facilitate the presentation of some of the key results, we focus on the special case depicted in the left plot of Figure 4, i.e., the case where the investor's participation choice amounts to choosing the length Δ of an arc centered at her location. The associated participation cost is $\kappa g(\Delta)$.

To make the investor's problem well-defined and interesting, we assume that the costs of participation increase sufficiently fast (and thus become infinite) as Δ approaches one:

Assumption 1 $\lim_{\Delta \rightarrow 1} g'(\Delta)(1 - \Delta)^4 = \infty$.

Assumption 1 helps ensure that an optimal Δ less than one exists for every investor.¹⁴

Before presenting results, we introduce a convention to simplify notation.

Convention 1 *For any real number x , let $\lfloor x \rfloor$ denote the floor of x , i.e., the largest integer weakly smaller than x . We henceforth use the term “location x ” (on the circle) to refer to the unique point in $[0, 1)$ given by $x \bmod 1 \equiv x - \lfloor x \rfloor$.*

With this convention we can map any real number to a unique location on the circle with circumference one. For example, this convention implies that the real numbers -0.8, 0.2,

¹⁴If $P < 1$, so that there is a risk premium, the limiting case $\Delta = 1$ is not well defined, since both the costs and the benefits of participation diverge to infinity as $\Delta \rightarrow 1$. Assumption 1 along with an application of L'Hôpital's rule ensures that the cost component dominates the benefit component as $\Delta \rightarrow 1$, resulting in an optimal (interior) choice of $\Delta \in [0, 1)$. We also note that, if we introduce aggregate risk, then Assumption 1 can be replaced with the weaker $\lim_{\Delta \rightarrow 1} g(\Delta) = \infty$. Finally, no assumption is necessary in the presence of a leverage constraint, as in Section 5 (except that $g(\Delta) > 0$ for some $\Delta \in (0, 1)$).

and 1.2 correspond to the same location on the circle with circumference one, namely 0.2. An implication of this convention is that any function h defined on the circle extends to a function \hat{h} on the real line that is periodic with period one, i.e., $\hat{h}(x) = \hat{h}(x + 1) = h(x \bmod 1)$. From now on, we treat functions on the circle also as functions defined on the entire real line.

We next introduce two definitions.

Definition 1 *The standardized portfolio associated with $G_j^{(i)}$ is the function $L : \mathbb{R} \rightarrow [0, 1]$ defined by $L_j = G_{i+j}^{(i)}$.*

The notion of a standardized portfolio allows us to compare portfolios of investors at different locations on the circle. For example, if all investors choose portfolios with weights that only depend on the distance d between their domicile and the location of investment, and these distance-dependent weights are the same for all investors, then these investors hold the same standardized portfolio.

Definition 2 *A symmetric equilibrium is an equilibrium in which all agents choose the same participation interval Δ , the same leverage w^f , and the same standardized portfolio.*

Due to the symmetry of the problem, it is natural to start by attempting to construct a symmetric equilibrium.

Proposition 1 *For any $\Delta \in (0, 1)$ define*

$$L_j^* \equiv \begin{cases} 0 & \text{if } j \in [-\frac{1}{2}, -\frac{\Delta}{2}) \\ j + \frac{1}{2} & \text{if } j \in [-\frac{\Delta}{2}, \frac{\Delta}{2}) \\ 1 & \text{if } j \in [\frac{\Delta}{2}, \frac{1}{2}) \end{cases} \quad (12)$$

and

$$\omega(\Delta) \equiv Var \left(\int_0^1 D_j dL_j^* \right) = \frac{\sigma^2}{12} (1 - \Delta)^3. \quad (13)$$

Finally, let Δ^* denote the (unique) solution to the equation

$$\kappa g'(\Delta^*) = -\frac{\gamma}{2}\omega'(\Delta^*) \quad (14)$$

and also consider the set of prices

$$P_i = P \equiv 1 - \gamma\omega(\Delta^*). \quad (15)$$

Then, assuming that a symmetric equilibrium exists, the choices $\Delta^{(i)} = \Delta^*$, $w_i^f = 0$, $dG_j^{(i)} = dL_{j-i}^*$, and prices $P_i = P$ constitute the unique symmetric equilibrium.

Proposition 1 gives simple, explicit expressions for both the optimal portfolios and participation intervals. To understand how these quantities are derived, we take an individual agent's wealth $W_{1,i}$ from (8) and we assume that prices for risky assets are the same in all locations. Then, using $W_{0,i} = P_i = P$, $r = 0$, $R_j = \frac{D_j}{P}$, and $F_i = \kappa g(\Delta)$, we express $W_{1,i}$ as

$$\begin{aligned} W_{1,i} &= P \left(w_i^f + \left(1 - w_i^f\right) \int_0^1 R_j dG_j^{(i)} \right) - \kappa g(\Delta) \\ &= \left(P w_i^f + \left(1 - w_i^f\right) \int_0^1 D_j dG_j^{(i)} \right) - \kappa g(\Delta). \end{aligned}$$

Because of exponential utilities and normally distributed returns, maximizing $EU(W_{1,i})$ is equivalent to solving

$$\max_{dG_j^{(i)}, \Delta, w_i^f} P w_i^f + \left(1 - w_i^f\right) \int_0^1 E D_j dG_j^{(i)} - \frac{\gamma}{2} \left(1 - w_i^f\right)^2 \text{Var} \left(\int_0^1 D_j dG_j^{(i)} \right) - \kappa g(\Delta). \quad (16)$$

Noting that $ED_j = 1$, inspection of equation (16) shows that (for any w^f and Δ) the optimal portfolio is the one that minimizes the variance of dividends in the participation interval Δ . Since the covariance matrix of dividends is location invariant, the standardized variance-minimizing portfolio is the same at all locations. Solving for this variance-minimizing portfolio L_j is an infinite-dimensional optimization problem. However, because of the symmetry of the setup we are able to solve it explicitly, and equation (12) provides the solution. The optimal portfolio, L_j^* , corresponds to the distribution that minimizes the sum of the vertical

distances to the uniform distribution on $[-\frac{1}{2}, \frac{1}{2})$, subject to the constraints that $L_j = 0$ for $j \in [-\frac{1}{2}, -\frac{\Delta}{2})$ and $L_j = 1$ if $j \in [\frac{\Delta}{2}, \frac{1}{2})$. The resulting optimized variance is given by $\omega(\Delta)$, defined in (13).

Accordingly, the agent's problem can be written more compactly as

$$V = \max_{\Delta, w_i^f} Pw_i^f + \left(1 - w_i^f\right) - \frac{\gamma}{2} \left(1 - w_i^f\right)^2 \omega(\Delta) - \kappa g(\Delta). \quad (17)$$

The first-order conditions with respect to w_i^f , respectively Δ , are

$$1 - P = \gamma \left(1 - w_i^f\right) \omega(\Delta) \quad (18)$$

$$\kappa g'(\Delta) = -\frac{\gamma}{2} \left(1 - w_i^f\right)^2 \omega'(\Delta). \quad (19)$$

Since in a symmetric equilibrium market clearing requires $w_i^f = 0$, equation (18) becomes identical to (15) and (19) becomes equivalent to (14).

Equation (14) makes explicit the resolution of the tradeoff between participation costs and risk taking. Specifically, the participation interval is determined as the point where the marginal cost of participation, $\kappa g'(\Delta)$, is equal to the marginal benefit of participation, $-\frac{\gamma}{2}\omega'(\Delta)$.

Figure 5 illustrates this tradeoff by plotting the marginal cost from increasing Δ , namely $\kappa g'(\Delta)$, against the respective marginal benefit $-\frac{\gamma}{2}\omega'(\Delta)$. Since $g(\Delta)$ is convex, $g'(\Delta)$ is upward sloping. By contrast, the marginal benefit is declining, since $-\omega''(\Delta) = -\frac{\sigma^2}{2}(1 - \Delta) < 0$. Since $g'(0) = 0$, $\lim_{\Delta \rightarrow 1} g'(\Delta) = \infty$, $-\omega'(0) > 0$, and $\omega'(1) = 0$, the two curves intersect at a unique point $\Delta^* \in [0, 1)$.

Proposition 1 helps capture the economic mechanisms that underlie our model. Consider, for instance, its implications for a reduction in the cost of accessing markets (i.e., a reduction in κ). As Figure 5 illustrates, such a reduction increases the degree of participation Δ and promotes portfolio flows across different locations. In turn, this increased participation improves risk sharing across different locations, which leads to higher prices of risky securities, $P = 1 - \gamma\omega(\Delta)$, and accordingly lower risk premia. By contrast, an increase in the costs of accessing risky markets leads to a lower Δ and a higher degree of concentration of risk. The

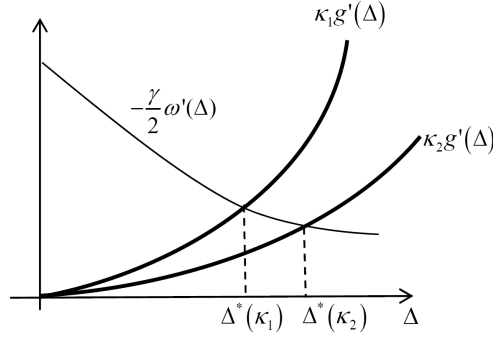


Figure 5: Illustration of the determination of Δ^* when a symmetric equilibrium exists. The figure illustrates that Δ^* increases when $\kappa_2 < \kappa_1$.

resulting decline in the extent of risk sharing leads to a drop in the prices of risky assets and an increase in risk premia.

These mechanisms of the model capture the stylized facts summarized in Section 2. We highlight one aspect of our analysis: the extent of market integration and cross-location portfolio flows, on one hand, and the magnitude of risk premia, on the other, are intimately linked. By contrast, representative-agent approaches to the determination of risk premia are — by their construction — limited in their ability to explain the empirically prevalent joint movements in risk premia and portfolio movements, since the representative agent always holds the market portfolio and prices adjust so as to keep the agent content with her holdings.

We conclude this section by noting that in Appendix A.1 we extend the results obtained so far to an intertemporal version of the model with recurrent shocks to participation costs. That extension strengthens the comparative statics conclusions established above by showing that, in a dynamic setting with participation-cost shocks, the model produces a) a negative correlation between capital flows and excess returns, b) time variation in excess returns that is unrelated to expected dividend growth, aggregate output etc., and c) return correlation across locations that exceeds the respective correlation of dividends. Furthermore, the additional risk caused by variation in the participation costs is priced, resulting in an increased expected excess return.

Next we turn our attention to a central issue of our analysis, pertaining to the existence of a symmetric equilibrium.

4.3 Asymmetric, location-invariant equilibria: Leverage and the diversity of financial strategies

Proposition 1 contains the premise that a symmetric equilibrium exists. Surprisingly, despite the symmetry of the model setup, a symmetric equilibrium may fail to exist. Instead, the market equilibrium may involve different choices (leverage ratios, portfolios of risky assets, wealth allocations, etc.) for agents in the same location, even though these agents have the same preferences and endowments and are allowed to make the same participation and portfolio choices.

These claims are explained by the observation that the necessary first-order conditions resulting in the prices $P_i = P = 1 - \gamma\omega(\Delta^*)$ are not generally sufficient. We now take a closer look at whether a symmetric equilibrium exists. Specifically, we fix the price $P = 1 - \gamma\omega(\Delta^*)$ of Proposition 1 and investigate whether (and under what conditions) the choices $w^f = 0$ and $\Delta = \Delta^*$ are indeed optimal for the investor.

To answer this question, we consider again the maximization problem (17). Taking $P = 1 - \gamma\omega(\Delta^*)$ as given, substituting into (18), and re-arranging implies that if investors allocate their wealth over a participation interval Δ (potentially different from Δ^*), then

$$1 - w^f = \frac{\omega(\Delta^*)}{\omega(\Delta)}. \quad (20)$$

Equation (20) contains an intuitive prediction. An investor allocating her wealth over a span $\Delta > \Delta^*$ is facing the same average returns, but a lower variance $\omega(\Delta)$, and hence a higher Sharpe ratio compared to an investor allocating her wealth over an interval of size Δ^* . Accordingly, the former investor finds it optimal to leverage her portfolio. This is reflected in equation (20), which states that $w^f < 0$ ($w^f > 0$) whenever $\Delta^* < \Delta$ ($\Delta^* > \Delta$).

An interesting implication of (19) is that the marginal benefit of an increased participation interval becomes larger with leverage. This is intuitive since increased leverage implies a more volatile wealth next period and hence a higher marginal benefit of reducing that variance by increasing Δ . In short, the choices of Δ and of leverage $1 - w^f$ are *complements*.

This complementarity can lead to an upward sloping marginal benefit of increased par-

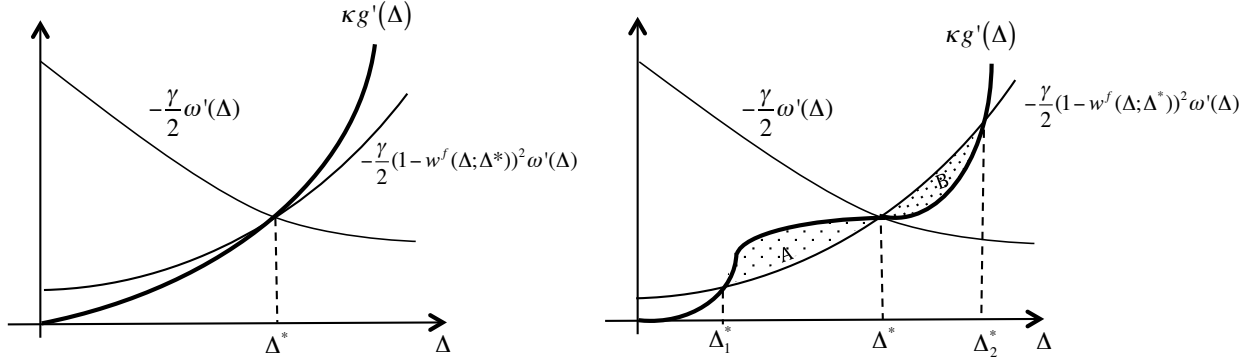


Figure 6: Illustration of the marginal benefit curve $-\frac{\gamma}{2}\left(\frac{\omega(\Delta^*)}{\omega(\Delta)}\right)^2\omega'(\Delta)$, the marginal cost curve $\kappa g'(\Delta)$ and the marginal benefit curve $-\frac{\gamma}{2}\omega'(\Delta)$ restricting $w^f = 0$. The left plot illustrates a case where a symmetric equilibrium exists whereas the right plot illustrates a case where a symmetric equilibrium fails to exist.

ticipation. Indeed, with our cash-flow specification, substituting (20) into (19) gives

$$\kappa g'(\Delta) = -\frac{\gamma}{2}\left(\frac{\omega(\Delta^*)}{\omega(\Delta)}\right)^2\omega'(\Delta). \quad (21)$$

Using the fact $\omega(\Delta) = \frac{\sigma^2}{12}(1-\Delta)^3$, the right hand side can be expressed as $-\frac{\gamma}{2}\left(\frac{\omega(\Delta^*)}{\omega(\Delta)}\right)^2\omega'(\Delta) = \frac{\gamma\sigma^2}{8}(1-\Delta^*)^6(1-\Delta)^{-4}$, which is increasing in Δ .

Figure 6 helps illustrate these notions. The figure depicts the marginal cost curve $\kappa g'(\Delta)$, the marginal benefit curve $-\frac{\gamma}{2}(1-w^f(\Delta; \Delta^*))^2\omega'(\Delta) = -\frac{\gamma}{2}\left(\frac{\omega(\Delta^*)}{\omega(\Delta)}\right)^2\omega'(\Delta)$, and also the curve $-\frac{\gamma}{2}\omega'(\Delta)$, i.e., the marginal benefit of participation fixing $w^f = 0$. The point where all three curves intersect corresponds to the point $\Delta = \Delta^*$. The left plot of Figure 6 illustrates a case where a symmetric equilibrium exists, whereas the right plot illustrates a case where a symmetric equilibrium fails to exist. The difference between the two plots is the shape of $g'(\Delta)$. In the left plot $g'(\Delta)$ intersects the marginal benefit curve only once, namely at Δ^* . For values smaller than Δ^* , the marginal benefit of participation is above the marginal cost and vice versa for values larger than Δ^* . Hence, in this case Δ^* is indeed the optimal choice for the investor.

This is no longer the case in the right plot. Here the marginal benefit curve intersects the marginal cost curve three times (at Δ_1^* , Δ^* , and Δ_2^*). Since the marginal benefit is below the

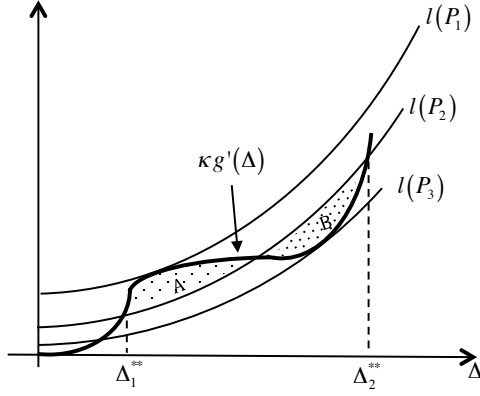


Figure 7: Illustration of an asymmetric equilibrium involving mixed strategies. The price adjusts so that area A is equal to area B . Accordingly, an investor is indifferent between Δ_1^{**} and Δ_2^{**} . Here, $P_3 < P_2 < P_1 < 1$.

marginal cost for values of Δ that are smaller and “close” to Δ^* , while the marginal benefit is larger than the marginal cost for values of Δ that are larger and “close” to Δ^* , Δ^* is a *local minimum*, and hence a suboptimal choice. By contrast the points Δ_1^* and Δ_2^* are local maxima. (In this particular example the point Δ_1^* is the global maximum, since the area A is larger than the area B .) The fact that Δ^* is not a maximum implies that there does not exist a symmetric equilibrium, since in a symmetric equilibrium it would have to be the case that $w^f = 0$.

The non-existence of a symmetric market equilibrium implies that one should look for equilibria where investors in the same location make different choices, even though they have the same preferences, endowments, and information. Figure 7 presents a simple graphical illustration of such an equilibrium in the context of the example depicted on the right plot of Figure 6.

Specifically we illustrate the construction of an equilibrium that features the same price $P_i = P$ for all markets, but where a fraction π of investors in every location i choose (w_1^f, Δ_1) , while the remaining fraction $(1 - \pi)$ of agents choose (w_2^f, Δ_2) . We refer to such an equilibrium as an asymmetric, location-invariant equilibrium. We introduce a function l

that captures the marginal benefit of participation as a function of Δ and P :

$$l(\Delta, P) \equiv -\frac{\gamma}{2} (1 - w^f(\Delta, P))^2 \omega'(\Delta) = -\frac{\gamma}{2} \left(\frac{1 - P}{\gamma} \right)^2 \frac{\omega'(\Delta)}{\omega^2(\Delta)}, \quad (22)$$

where the last equation follows from (18).

Figure 7 depicts the function $l(\Delta, P)$ for three values $P_1 < P_2 < P_3 < 1$. A first observation is that as P declines from P_3 to P_1 , the curve $l(\Delta, P)$ shifts up. Moreover there exists a level P_2 and associated values Δ_1^{**} and Δ_2^{**} for which the area “A” equals the area “B”, so that investors are indifferent between choosing Δ_1^{**} and Δ_2^{**} . Fixing these values of P, Δ_1^{**} , and Δ_2^{**} , we can determine the values of w_1^f and w_2^f from (18). As part of the proof of Proposition 2 (in the appendix), we show that these values of w_1^f and w_2^f satisfy $1 - w_1^f < 1 < 1 - w_2^f$. In order to clear the bond market, π has to be so that $\pi w_1^f + (1 - \pi)w_2^f = 0$, i.e., $\pi = \frac{w_2^f}{w_2^f - w_1^f} \in (0, 1)$.¹⁵ We also show in the appendix that for this value of π all markets for risky assets clear as well.

The next proposition generalizes the insights of the above illustrative example. It states the existence of an asymmetric, location-invariant equilibrium when a symmetric equilibrium fails to exist.

Proposition 2 *When the cost function g is such that a symmetric equilibrium fails to exist, there exists an asymmetric, location-invariant equilibrium. Specifically, there exist (at least) two tuples $\{\Delta_k, w_k^f\}$, $k \geq 2$, and $\pi_k > 0$ with $\sum_k \pi_k = 1$ and $\sum_k \pi_k (1 - w_k^f) = 1$, such that in every location i a fraction π_k of agents choose the interval, leverage, and portfolio combination $\{\Delta_k, w_k^f, dL^{(k)}\}$, where $dL^{(k)}$ is the measure given in (12) for $\Delta = \Delta_k$. The price of risky assets is uniquely determined in the class of location-invariant equilibria,¹⁶ and in particular a symmetric and an asymmetric location-invariant equilibrium involving different prices cannot co-exist.*

The model can therefore generate situations in which a symmetric equilibrium can fail to

¹⁵It holds that $\pi \in (0, 1)$ because $w_2^f < 0$ and $w_1^f > 0$.

¹⁶It is possible, for special parameter constellations, that in an asymmetric equilibrium an investor is indifferent between multiple (not just two) tuples of participation and leverage combinations. Accordingly, for the same price one could construct asymmetric equilibria with different combinations of asymmetric strategies. However — by construction — any two such equilibria would feature the same price for all assets. In this sense an asymmetric equilibrium is essentially unique.

exist. Instead, the market equilibrium features a diverse set of financial strategies, with some investors pursuing high-cost, high-Sharpe-ratio, high-leverage strategies and some investors pursuing low-cost, low-Sharpe-ratio, no-leverage strategies. The first type of strategies is akin to the sort of strategies offered by hedge funds, while the second type of strategies is akin to those offered by mutual funds.

A novel feature of our model is that it helps isolate a source of non-concavity of the participation decision that is distinct from non-convex participation-cost structures. Specifically, the continuity of the dividend structure in our framework along with the convexity of $g(\Delta)$ allow us to model the participation choice (i.e., the choice of Δ) as a concave decision problem *for fixed leverage*. Accordingly, we can illustrate the role of the complementarity between leverage and participation in rendering the participation problem non-concave. The distinct role of this complementarity can be easily overlooked in models with discrete markets where the participation choice itself (i.e., even with fixed leverage) is discrete and hence necessarily non-convex.

Clearly, if we were to introduce elements of non-convexity into the cost structure (for instance, because investors may choose non-contiguous participation locations, as in the next section or in Appendix A.2), there is an additional and conceptually distinct reason why the investor's objective is non-concave, leading to the adoption of diverse financial strategies.¹⁷

In summary, this section has established that the non-concavities in investors' objective functions introduce diversity in the optimal participation-leverage combinations chosen by different investors. Importantly, the equilibrium price plays a key role in determining the participation-leverage combinations that keep investors indifferent between different strategies. In the next section we identify a tension that arises when we introduce collateral constraints, so that the price simultaneously affects investors' indifference relations and their feasible leverage-participation combinations.

¹⁷A classic example, which also features different choices by ex-ante identical investors, due to fixed (hence non-convex) information costs, is Grossman and Stiglitz (1980). A difference between Grossman and Stiglitz (1980) and our setup, which becomes pertinent in the next section, is that the price adjusts to keep investors indifferent between different combinations of leverage and participation, while in Grossman and Stiglitz (1980) investors decide on paying information acquisition costs before observing the price.

5 Collateral Constraints and the Complementarity between Leverage and Participation

The previous section shows that equilibrium outcomes may feature heterogeneous investment strategies in every location despite the fact that investors are identical. An important aspect of these heterogeneous strategies is that they require different degrees of leverage. Even though in the model leverage can be chosen freely, in reality leverage is limited by collateral constraints. In this section we discuss the implications of such constraints. We present a simple example to show that leverage constraints open the possibility for multiple location-invariant equilibria. Accordingly, prices, capital flows, and leverage exhibit discontinuous dependence on changes to participation costs.

To model collateral constraints, we let $\chi \in [0, 1)$ denote a “haircut” parameter and require

$$\left[(1 - \chi) \int_0^1 P_j dX_j^{(i)} - F_i \right] + X_B^{(i)} \geq 0. \quad (23)$$

Equation (23) stipulates that the borrowing capacity of an investor (the first term — inside the square brackets — of (23)) do not exceed the amount the investor actually borrows (the second term of (23)). The investor’s borrowing capacity is computed as the market value of the collateral net of the haircut and of the obligation the investor has already assumed by committing to pay the participation costs in period 1.

The collateral constraint (23) is pervasive in reality, extensively used in the literature, and analytically convenient for our purposes. Therefore, we focus on the implications of this constraint in the body of the paper. In Appendix D we analyze an alternative, endogenous version of the constraint (23) motivated by the desire to prevent default in all states of nature. We show that the key findings of the present section remain the same.

To further streamline the exposition, we choose a particularly simple specification for the participation-cost structure that includes a discrete element of participation choice. This choice introduces an additional and direct reason for the existence of asymmetric equilibria featuring heterogeneous leverage. More importantly, it allows us to provide an analytically solvable example of participation and leverage choice that helps illustrate the role of leverage

constraints when these two choices interact.

Specifically, we postulate the cost function depicted in the middle graph of Figure 4. In that case, the optimal participation choice amounts to choosing the location of the points a_i and the number of the discrete points N .¹⁸ To keep computations as simple as possible, we assume furthermore that $b_N = \infty$ for $N > 2$, so that only choices involving $N \leq 2$ are feasible.¹⁹ Accordingly, an investor's problem comes down to choosing $N = 1$ (and paying no participation costs) or $N = 2$ and the distance d from her home location, resulting in participation costs equal to $\kappa f(d)$. (See Appendix A.2 for an extension to multiple locations.) In an effort to further streamline the analysis, we assume that $f'(d) = 0$. In that case the investor faces only a fixed cost of κf_0 when choosing $N = 2$, but can choose d freely (i.e., without incurring additional cost).

We start by analyzing equilibrium prices in this economy when we don't impose the constraint (23) on investors' decisions.

Proposition 3 *Assume that the borrowing constraint is not imposed on investors' decisions, and also assume that $\kappa \in (\kappa_1, \kappa_2)$, where $\kappa_1 \equiv \frac{1}{128} \frac{\gamma \sigma^2}{f_0}$, $\kappa_2 \equiv \frac{1}{8} \frac{\gamma \sigma^2}{f_0}$. Then there exists a unique asymmetric equilibrium in which a fraction $\pi > 0$ of the investors choose $N = 1$, while the rest choose $N = 2$ and $d = \frac{1}{2}$. The equilibrium price in all locations is*

$$P(\kappa) = 1 - \sigma \sqrt{\frac{\gamma \kappa f_0}{18}}, \quad (24)$$

and the choices w_i^f satisfy $w_1^f > 0$ and $w_2^f < 0$.

For values of κ smaller than κ_1 or larger than κ_2 the equilibrium is symmetric (and unique in the class of location-invariant equilibria) with everyone choosing $N = 2$ and $d = \frac{1}{2}$ or everyone choosing $N = 1$, respectively. Hence, without imposing the constraint (23), the left plot of Figure 8 gives a visual depiction of $P(\kappa)$, which is a continuous function of κ .

¹⁸Recall that in this case there is a minimum "fixed" cost that one needs to pay for every new location that she chooses given by $\kappa b_N \lim_{x \rightarrow 0} f(x) > 0$.

¹⁹The condition $b_N = \infty$ can be substantially weakened. Indeed all that is needed for $N = 1$ or $N = 2$ to be the only feasible choices is that $\kappa b_N \lim_{x \rightarrow 0} f(x) > 1$ for $N > 2$. To see this, use the normalization $P_B = 1$ along with the budget constraint to obtain $P_i = W_{0,i} = \int_0^1 P_j dX_j^{(i)} + X_B^{(i)} = (1 - \chi) \int_0^1 P_j dX_j^{(i)} - F_i + X_B^{(i)} + F_i + \chi \int_0^1 P_j dX_j^{(i)} \geq F_i + \chi \int_0^1 P_j dX_j^{(i)}$, where the inequality follows from (23). Noting that $P_j \leq 1$ and f is non-decreasing, the only feasible choices are $N = 1$ or $N = 2$ if $\kappa b_N \lim_{x \rightarrow 0} f(x) > 1$ for $N > 2$.

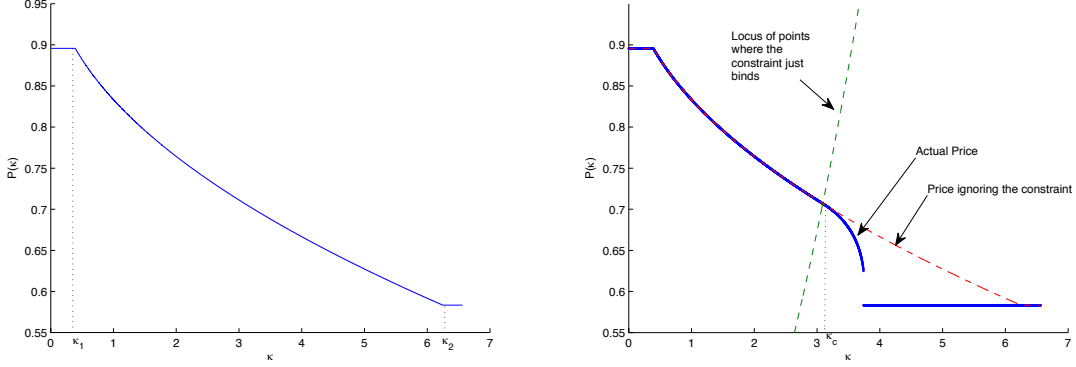


Figure 8: The left plot depicts the price $P(\kappa)$ assuming the leverage constraint is not binding. The downward-sloping dotted line on the right plot plots the same quantity, along with the actual price (solid line) and the locus of points such that the leverage constraint is just binding (upward sloping line). Hence for points above and to the left of the upward sloping line the constraint is not binding, whereas it is binding for points below and to the right. Parameters used in this example: $\gamma = 5$, $f_0 = 0.1$, $\chi = 0.2$, $\sigma = 1$.

When imposing the constraint (23), however, a different situation arises: For any given value of κ there may be multiple equilibria. A direct implication of equilibrium multiplicity is that it may become impossible to find a continuous mapping $\kappa \mapsto P(\kappa)$. Such a situation is depicted in the right plot of Figure 8, which graphs the equilibrium price in the absence of constraints, the locus of points where the constraint is binding, and the maximum price associated with an equilibrium when the constraint is binding.

To understand the source of multiple equilibria and why they imply the non-existence of a continuous mapping $\kappa \rightarrow P(\kappa)$, it is useful to refer to Figure 9. For each subplot we fix a value of κ . Given that value of κ , the line denoted “Indifference” in each subplot depicts combinations of P and possible choices of $(1 - w_2^f)$ such that the indifference relation $V_1 = V_2$ holds.²⁰ Similarly, the line denoted “Constraint” depicts combinations of P and $1 - w_2^f$ such that the constraint (23) binds with equality, i.e., the set of points such that²¹

$$\kappa f_0 = P \left(1 - \chi(1 - w_2^f) \right). \quad (25)$$

²⁰See equation (47) in the appendix for a formal statement of the indifference relation.

²¹Combining the investor’s budget constraint with (23) and repeating the same steps as in footnote 19 gives $W_{0,i} \geq F_i + \chi \int_0^1 P_j dX_j^{(i)}$. Evaluating this inequality with $P_i = P_j = P = W_{0,i}$, $P_B = 1$, $F_i = \kappa f_0$, and $\int_0^1 dX_j^{(i)} = 1 - w_i^f$ gives (25).

Accordingly, all the points that lie above that line are admissible combinations of P and $1 - w_2^f$. The line denoted “FOC” (in the first plot) depicts the combination of points that satisfy the (unconstrained) first-order condition for leverage

$$1 - P = \gamma \left(1 - w_2^f\right) \omega^{(N=2)},$$

where $\omega^{(N=2)} = \frac{\sigma^2}{48}$ is the minimal variance attainable for an investor choosing to participate in $N = 2$ locations. Finally, the line denoted “Lowest Price” is a reference line denoting the price that would obtain in the symmetric equilibrium where all investors participate exclusively in their own location.

The left plot depicts the case $\kappa = \kappa_c$. In that case the three lines “FOC”, “Indifference”, and “Constraint” intersect at the same point (C). This reflects that when $\kappa = \kappa_c$, the equilibrium combination of price and the (unconstrained) optimal leverage for an investor choosing $N = 2$ are such that the constraint just binds. Moreover, point C is the unique equilibrium.

For (moderately) higher values of κ , illustrated in the middle panel, the constraint binds actively, so that the first order condition for the optimal choice of leverage no longer holds with equality. Instead, candidate asymmetric equilibria are given by the intersection of “Indifference” and “Constraint”. Clearly, there are two such equilibria (points C' and D'). The symmetric equilibrium whereby everyone chooses $N = 1$ is yet another equilibrium. (To see this, note that point A' is to the right of point B'. This implies that if all investors choose $N = 1$ and zero leverage, then an individual investor would not benefit by deviating to $N = 2$; if she did deviate, her leverage would be given by the point B', which is above and to the left of the line labeled “Indifference” — hence inferior to choosing $N = 1$).

The right-most plot shows a case in which κ is large enough that there is no point of intersection between the lines “Indifference” and “Constraint”. At that point there can be no asymmetric equilibria. The equilibrium becomes symmetric, i.e., the price drops to the level that obtains when everyone chooses $N = 1$ and $w^f = 0$.

The multiplicity of equilibria implies that the mapping $\kappa \mapsto P(\kappa)$ is discontinuous at

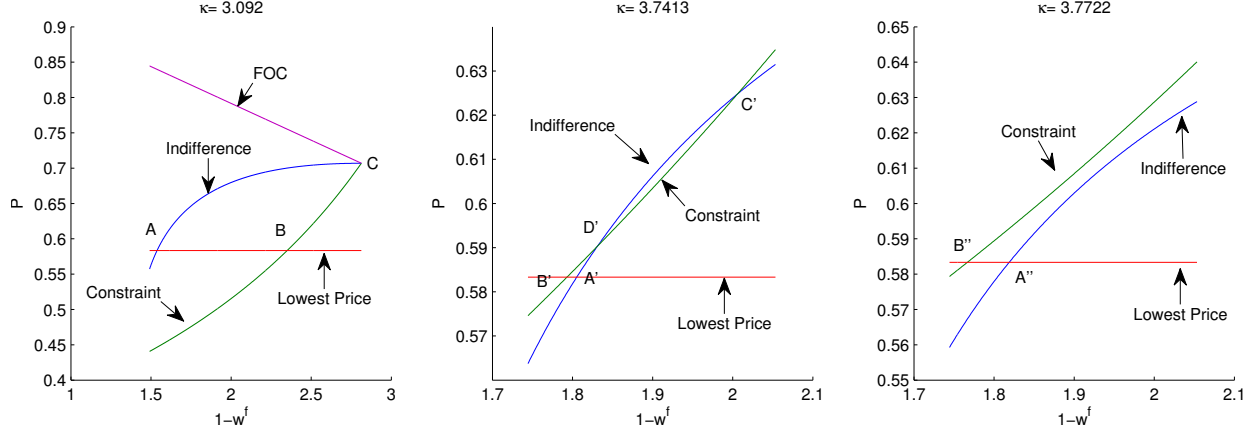


Figure 9: For three different values of the cost parameter κ , the three plots depict combinations of price (P) and leverage ($1 - w_2^f$) so that (i) investors are indifferent between adopting no- and high-leverage strategies (line labeled “Indifference”); (ii) investors’ choices of leverage ($1 - w_2^f$) are (unconstrained) optimal given P (line labeled “FOC”); and (iii) the leverage constraint just binds (line labeled “Constraint”). For reference, we also plot the equilibrium price associated with all investors participating exclusively in their own location (line labeled “Lowest price”). Parameters are identical to Figure 8.

some value $\kappa > \kappa_c$, no matter how one selects the equilibrium.²² This outcome, albeit in a different context, is reminiscent of Gennotte and Leland (1990).

To understand intuitively why multiple equilibria arise when the leverage constraint is binding, it is useful to start with the case when the constraint is not binding. In that case the incentives to pursue broad-participation strategies are substitutes across investors: The less other investors choose to participate in such strategies, the lower the prices of risky assets, and thus the higher an individual investor’s incentive to pursue such strategies. As is commonly the case when economic decisions are substitutes, the equilibrium is unique and features a price that keeps investors indifferent between broad- and narrow-participation strategies.

A different situation arises when the leverage constraint binds. Reductions in prices increase the Sharpe ratio of risky strategies, but also reduce the ability to leverage. Since

²²For instance if one chooses the equilibrium associated with the maximal price, then the discontinuity will occur for some value of κ between those depicted in the middle and the right-most panel of Figure 9. Similarly, if one chooses the equilibrium with the minimal price, then the jump will occur at some value of κ between those depicted in the left-most and middle graph. More generally, it is impossible to find a choice of equilibrium such that $P(\kappa)$ is continuous for the values of κ between the left-most and the right-most plots.

leverage and participation choices are complements, a decline in price may end up *lowering*, rather than increasing, the incentive to participate, rendering the choices to pursue broad-participation strategies complements across investors, rather than substitutes. As a result, there can be multiple equilibria: If prices are low, then leverage is constrained to be low and there isn't enough incentive to participate broadly, justifying low prices. If prices are high, then agents can leverage sufficiently to justify paying for broad participation, which supports high prices. Note that the cost κ can't be either too high or too low for these different equilibria to obtain. Fragility is an immediate consequence of the existence of multiple equilibria in a range of the cost parameter κ .

5.1 Discussion

We conclude the section with two remarks. First, while leverage constraints have already been used in the literature, our analysis goes further by pointing out that the complementarity between leverage and participation may result in broad-participation decisions being complements rather than substitutes across investors. In turn, this outcome leads to a discontinuous mapping from the cost of participation to prices, capital flows, and leverage. (Appendix D shows that this discontinuity survives in an extension of the model with leverage constrained endogenously in order to prevent default).

Second, we note that the complementarity between leverage and participation implies a tight connection between risky asset markets and bond markets. The previous section illustrates such a connection in the context of cost-parameter variations. An even more direct way of illustrating the tightness of that connection is to consider the following alternative thought experiment. Suppose that bond trading becomes practically infeasible for investors.²³ Then, a reasoning similar to the one in Section 4.2 shows that an asymmetric equilibrium will cease to exist; the equilibrium becomes one of the symmetric type, with a lower price for risky assets.²⁴

²³For example, a simple way of modeling such a situation — while staying within the confines of our framework — is to consider a substantial increase in the haircut parameter, with the effect of rendering leverage so unattractive that it is effectively impossible.

²⁴To see this graphically, note that an increase in the haircut parameter (χ) has the effect of rotating the line labeled “Constraint” in Figure 8 counter-clockwise around the point $(0, \kappa f_0)$.

6 Reduced Participation in a Subset of Locations, Entanglement, and Contagion

In the previous sections we assumed the same cost structure for all investors, regardless of location. This allowed us to focus on (potentially asymmetric but) location-invariant equilibria, which entail the same price in all locations. In this section we relax this assumption in order to study the effect of an adverse shock to financial participation in a subset of locations. The challenge in analyzing the model with location-specific costs is that an entire price function on $[0, 1]$ has to be computed, rather than a single value. To present the key insights of the model with location-specific costs, we restrict ourselves to a specific setup that is sufficiently simple to analyze, yet rich enough to illustrate how a reduction in market participation in a subset of risky markets can propagate across all markets.

Specifically, we consider the following stylized setup: for some positive $k < 1$ market access in locations $[-\frac{k}{2}, \frac{k}{2}]$ “breaks down”. Specifically, we assume that in these “affected” locations the cost parameter κ becomes infinite, so that investors choose to participate only in the bond market and the market for the local risky claim, but not in other markets. In the rest of the locations investors choose to participate in a single interval of length $\bar{\Delta}$, centered at their “home location” — this would be the outcome of a cost structure in these locations involving, e.g., $\kappa < \infty$, $b_N = \infty$ for $N > 1$, $g'(\bar{\Delta}) = 0$ for $\Delta < \bar{\Delta}$, and $g'(\Delta)$ sufficiently large for $\Delta > \bar{\Delta}$.

We solve this heterogeneous-participation-costs version of the model in the appendix. First, we calculate analytically the optimal demand of every investor given a price function (Lemma 3), and then aggregate the demands to solve for prices. We are unable to find a closed-form solution for the prices, but we can characterize the solution in terms of a linear integro-differential equation with delay, which can be solved numerically as easily as a matrix inversion problem.

To obtain a visual impression of the solution, Figure 10 depicts the price P_j for $j \in [0, 1]$ and compares it with the (symmetric) equilibrium price $P^*(\bar{\Delta})$ that would prevail if all agents in all locations chose a participation interval with length equal to $\Delta = \bar{\Delta}$. There are several noteworthy facts about Figure 10. As one might expect, prices in the set $A \equiv [-\frac{k}{2}, \frac{k}{2}]$

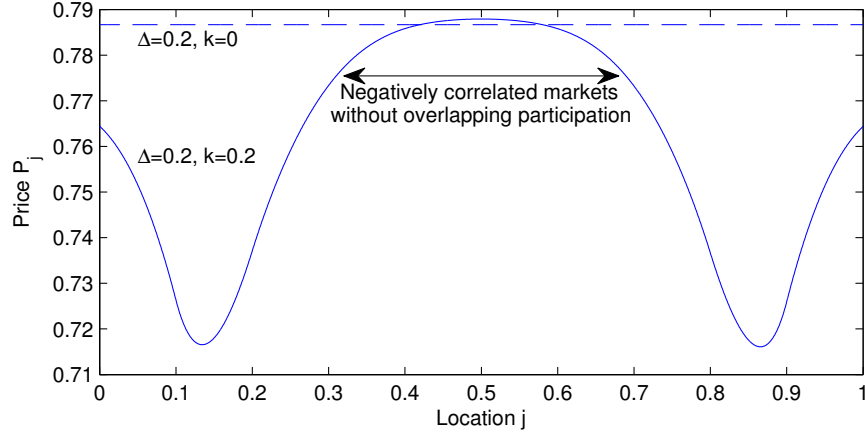


Figure 10: Contagion effects. The price function P_j depicts the price in different locations when $k = 0$ and $k = 0.2$. We set the length of the participation arc of “unaffected” investors to $\Delta = \bar{\Delta} = 0.2$. The locations between the arrows feature both negative correlation of dividends and no overlap between the participants in these markets and the set $A = [-\frac{k}{2}, \frac{k}{2}]$.

are lower than $P^*(\bar{\Delta})$. The more important fact is that prices in all other markets are affected, as well. This holds true even in markets in the interval $B \equiv [\frac{k}{2} + \bar{\Delta}, 1 - \frac{k}{2} - \bar{\Delta}]$, i.e., in locations that are at a larger distance than $\bar{\Delta}$ from the set A , so that agents in the set A would not be directly trading risky assets with agents in the set B , even if they were allowed to choose the same participation interval $\bar{\Delta}$ as everyone else. A third observation is that the prices in almost all markets are lower than $P^*(\bar{\Delta})$. Indeed, this reduction in price can happen even in markets with dividends that have a zero or negative correlation with the dividends of any risky security in A , as the figure illustrates.

The intuition behind Figure 10 is the following. By assumption, investors in the set A concentrate all their demand for risky assets in their home location. Hence, compared to the case in which everyone participates in an interval of length $\bar{\Delta}$ centered at their location, there is now lower demand for risky securities in locations neighboring the set A . The lower demand for risky securities in these locations leads to lower prices, which attracts demand from locations adjacent to the neighborhood of A . By tilting their portfolio towards these locations, investors remove demand for risky securities in locations even farther from A . Accordingly, prices in these farther locations need to drop in order to attract investors from locations adjacent to the neighbourhood of the neighbourhood of A , who also tilt their portfolios and so on.

This chain reaction implies that all locations are affected in equilibrium. In fact, even some prices in locations close to point $\frac{1}{2}$, the farthest from A , drop, as the following result states.²⁵

Proposition 4 *Assume $\bar{\Delta} + k < 1$ and that $\frac{1}{2} = \arg \max_{j \in [\frac{1}{2} - \bar{\Delta}, \frac{1}{2} + \bar{\Delta}]} P_j$. Then there exists a positive-measure subset $D \subset [\frac{1}{2} - \bar{\Delta}; \frac{1}{2} + \bar{\Delta}]$ such that $P(x) < P^*(\bar{\Delta})$ for all $x \in D$.*

Proposition 4 helps formalize the notion that the drop in prices caused by restricting market access for investors in the set A can permeate even a neighborhood of radius $\bar{\Delta}$ around the maximal-price location.

The extent of the change in risk premia in the various locations depends on the distance from the arc A , the length of the arc A , and most interestingly, on the length of the participation arc $\bar{\Delta}$. Figure 11 illustrates these statements. If $\bar{\Delta}$ is small, so that investors' portfolios are heavily exposed to risks in their vicinity, then even a small portfolio tilt towards a nearby location requires a relatively high risk compensation. By contrast, if investors' portfolios are invested across a broad range of locations, then they are more willing to absorb risks in their vicinity.

In model simulations not reported here due to space limitations, we have also considered the effect of local participation shocks when investors participate in multiple disjoint intervals, obtaining similar conclusions.²⁶

In summary, a surprising result of our analysis is that contagion occurs due to *limited*, rather than excessive, integration of risky markets. Indeed, contagion may become stronger the weaker the integration of risky markets.

An interesting implication of our model, if one adopts the interpretation of locations as asset classes, is the presence of spillover effects between asset classes that may share few or no common traders and have no obvious fundamental linkages. For instance, the model may

²⁵It is easy to show that, in the general case when a subset of investors are limited to a smaller investment set than the original arc of length $\bar{\Delta}$, and therefore in the setting of this section, the average risky-asset price in the economy declines.

²⁶In such situations, the direct impact of a local shock to participation is felt not only in the neighboring locations of the affected locations, but also in the other subintervals where the affected investors used to participate. This sets off multiple "waves" of contagion across the circle (i.e., Figure 10 features more than two local minima).

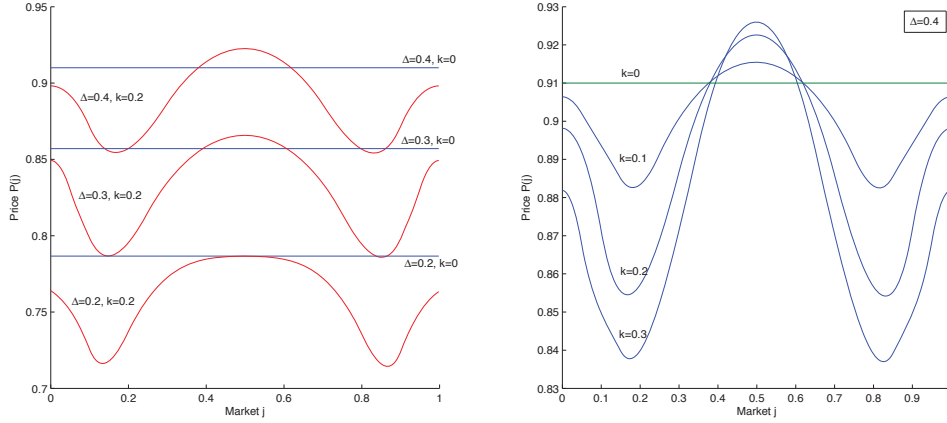


Figure 11: Contagion effects for various choices of $\bar{\Delta}$ and k . In the figures j denotes a specific market, k controls the length of the region with restricted access to markets, while Δ refers to $\bar{\Delta}$, that is the length of the participation arc for investors who can participate in “distant” markets.

help explain why shocks to Russian bonds affected Latin American debt during the LTCM crisis, or why downward (upward) jumps in the level (volatility) of the US stock market index routinely impact credit spreads in emerging markets.

We conclude this section by noting that the location-dependent version of the model has implications beyond the contagion ones we highlighted here. For instance, the complementarity between leverage and participation means that large arcs of participation and high degrees of leverage tend to be observed in the same locations.²⁷ Thus, countries with better participation technologies (e.g., countries with a higher degree of financial development) have higher leverage. This model prediction appears consistent with the evidence that a country such as the US has a riskier portfolio of overseas assets than a country such as China, while the US is a borrower in bond markets, as opposed to China, who is a lender.

7 Conclusion

We introduce a novel, location-invariant, and tractable structure of shocks on a circle that allows us to model the tradeoffs between integration and risk diversification on the one hand and participation costs on the other. The focus of the analysis is on the private

²⁷For instance, in the example of this section the locations in the set $A = [-\frac{k}{2}, \frac{k}{2}]$, where participation in risky markets is impaired, exhibit positive holdings of bonds.

incentives to incur costs to participate in various asset markets, and on the interdependence (“entanglement”) of markets in the presence of limited integration.

Our main findings can be grouped in three broad categories. First, participation decisions and leverage decisions are complements. Investors who choose to participate in a large number of markets also choose high degrees of leverage. Even when the participation- and leverage-choice problems are concave in isolation, the joint optimization problem may be non-concave. This non-concavity can lead to a heterogeneous market structure. Despite the fact that all investors are identical, some investors may opt for non-levered, low-participation investment strategies with others opting for levered, high-participation strategies.

Second, the non-concavity of an investor’s participation-and-leverage-choice problem can render participation choices complements rather than substitutes across investors when leverage is constrained. An implication is that small variations in participation costs can cause a change in the nature of equilibrium, which in turn implies discontinuous changes in prices, portfolio flows, and leverage.

Third, asset markets exhibit increased interdependence despite — in fact, because of — limited market integration. Limitations to market access in one subset of locations propagates across markets. Surprisingly, the impact of these local shocks on other markets may be strengthened when markets are less integrated. In an intertemporal version of the model, the joint determination of market integration and equilibrium prices implies price and return correlations that exceed those of the underlying dividends.

In conclusion, this paper provides a tractable framework to study several issues (limited market integration, endogenous leverage, contagion, etc.) through a common lens. The specific issues we analyzed are illustrative, but not exhaustive. We believe that the model can provide a simple framework to study the numerous situations where tradeoffs between participation frictions and the desire to diversify lead to endogenous and incomplete overlap in financial markets.

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Appendix — For Online Publication

A Extensions

A.1 An infinite horizon version of the baseline model

In this section we develop an intertemporal version of the model. The intertemporal version allows us to extend the intuitions of our comparative statics exercises to a framework where the shocks to the participation technology are recurrent.

Specifically, we keep the key assumptions of the baseline model (Section 4.2), and in particular the assumption that investors only participate in an arc surrounding their home location. However, we assume that investors maximize expected discounted utility from consumption

$$-\sum_{t=0}^{\infty} \beta^t E_0 [e^{-\gamma c_{t,i}}],$$

where t denotes (discrete) calendar time. We assume that dividends at location $i \in [0, 1)$ are given by

$$D_{t,i} = (1 - \rho) \sum_{k=-\infty}^t \rho^{t-k} \varepsilon_{k,i}, \quad (26)$$

where $\rho < 1$,

$$\varepsilon_{t,i} \equiv 1 + \sigma \left(B_i^{(t)} - \int_0^1 B_j^{(t)} dj \right), \quad (27)$$

and $B_i^{(t)}$ denotes a family of Brownian Bridges on $[0, 1]$ drawn independently across times $t \in \{-\infty, \dots, 1, 2, \dots, \infty\}$.

We make a few observations about this dividend structure. First, we note that equation (27) coincides with equation (3). Accordingly, $\int_0^1 \varepsilon_{t,i} di = 1$, and therefore, equation (26) implies that $\int_0^1 D_{t,i} = 1$ for all t , so that the aggregate dividend is always equal to one. Second, dividends at individual locations follow AR(1) processes, since equation (26) implies

$$D_{t,i} = (1 - \rho) \varepsilon_{t,i} + \rho D_{t-1,i}. \quad (28)$$

Moreover, since (27) coincides with (3), the increments of two dividend processes at two locations i and j have the covariance structure of equation (4).

In terms of participation decisions, we keep the same cost assumption as in the baseline model and further assume that investors participate in a single interval of length $\Delta_t^{(i)}$ centered at their “home” location. Participation costs are paid period by period in advance of trading. Specifically, an investor’s intertemporal budget constraint is given by

$$c_{t,i} + F_t \left(\Delta_t^{(i)} \right) + \int_0^1 P_{t,j} dX_{t,j}^{(i)} + P_{B,t} X_{B,t}^{(i)} = \int_0^1 (P_{t,j} + D_{t,j}) dX_{t-1,j}^{(i)} + X_{B,t-1}^{(i)}. \quad (29)$$

We note that in equation (29) we allow the entire cost function $F_t(\Delta) = \kappa_t g_t(\Delta)$ to be differ-

ent across different periods in order to capture the effect of repeated shocks to the participation technology.

This intertemporal version of the model presents a challenge that is absent in a static framework: If the interest rate varies over time, then the value function of an agent is not exponential in wealth. In fact, a closed-form expression for the value function most likely does not exist. Furthermore, once the value function is no longer exponential, portfolios are no longer independent of wealth, and hence the entire wealth distribution matters — an infinite-dimensional state variable.

In order to maintain the simple structure of the solution, therefore, we make necessary assumptions to achieve a constant interest rate in the presence of random participation costs, while safeguarding market clearing for bond markets, risky-asset markets, and consumption markets.²⁸ The following proposition states that it is possible to achieve this outcome by judiciously specifying the distribution of the participation costs. The main thrust of the proposition, however, concerns the expression for the risky-asset prices.

Proposition 5 *There exist an interval $[\Delta_l, \Delta_u]$, a (non-trivial) distribution function $\Psi(\cdot)$ on $[\Delta_l, \Delta_u]$, and a cost function $F(\cdot; \Delta_t) : [\Delta_l, \Delta_u] \rightarrow \mathbb{R}^+$ such that, if Δ_t is drawn in an i.i.d. fashion from Ψ , then*

- (i) *investors optimally choose $\Delta = \Delta_t$, thus incurring cost $F(\Delta_t; \Delta_t)$;*
- (ii) *the risk-free rate is constant over time and given as the unique positive solution to*

$$1 = \beta(1+r) E \left[e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta)} \right], \quad (30)$$

where the expectation is taken over the distribution of Δ ;
(iii) *the risky-asset prices equal*

$$P_{t,j}(\Delta_t, D_{t,j}) = \phi(D_{t,j} - 1) + \frac{1}{r} - \Phi_1 \omega(\Delta_t) - \Phi_0 \quad (31)$$

with $\phi \equiv \frac{\rho}{1+r-\rho}$ and

$$\Phi_1 = \gamma \frac{r(1-\rho)^2}{(1+r-\rho)^2} \quad (32)$$

$$\Phi_0 = \frac{\Phi_1}{r} \frac{E \left[e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta)} \omega(\Delta) \right]}{E \left[e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta)} \right]} > 0. \quad (33)$$

Furthermore, investors' optimal portfolios of risky assets are given by (12).

Equation (31) decomposes the price of a security into three components. As in all CARA models, one of these components equals the expected discounted value of future dividends, $\phi(D_{t,j} - 1) + r^{-1}$. The other two capture the risk premium. The term $\Phi_1 \omega(\Delta_t)$ is the risk premium associated with the realization of time- $t + 1$ dividend uncertainty, to which each investor is exposed according to

²⁸Alternatively, in the interest of simplicity, we could fix the interest exogenously to the model and let aggregate lending adjust accordingly.

the breadth Δ_t of her time- t portfolio. Finally, Φ_0 equals the sum of the expected discounted value of risk premia due to future realizations of dividend innovations and Δ_t .

We emphasize that the risk premium decreases with Δ_t and is common for all securities. Alternatively phrased, increases in capital movements across locations are correlated with higher prices for all risky securities (and hence lower expected excess returns). Importantly, these movements in the prices of risky securities are uncorrelated with movements in aggregate output or the interest rate, which are both constant by construction.

A further immediate implication of equation (31) is that the presence of repeated shocks to participation costs introduces correlation in security prices that exceeds that of their dividends. Indeed, taking two securities j and k , and noting that $Var(D_{t,j}) = Var(D_{t,k})$, we can use equation (31) to compute

$$corr(P_{t,j}, P_{t,i}) = \frac{Var(\Phi_1 \omega(\Delta_t)) + \phi^2 cov(D_{t,j}, D_{t,k})}{Var(\Phi_1 \omega(\Delta_t)) + \phi^2 Var(D_{t,j})} > \frac{cov(D_{t,j}, D_{t,k})}{Var(D_{t,j})} = corr(D_{t,j}, D_{t,i}).$$

The intuition is that movements in market integration cause common movements in the pricing of risk which make prices more correlated (and volatile) than the underlying dividends.

We collect some basic properties of the price due to the randomness in Δ_t in the following proposition.

- Proposition 6** (i) $P_{t,i}$ increases with Δ_t , and therefore $corr(P_{t,i}, \Delta_t) > 0$;
(ii) $E_t[P_{i,t+1} - (1+r)P_{i,t}]$ decreases with Δ_t ;
(iii) $corr(P_{t,i}, P_{t,j}) > corr(D_{t,i}, D_{t,j})$;
(iv) Φ_0 is higher (and hence the unconditional expected price is lower) than the one obtaining for Δ_t constant and equal to $E[\Delta]$.

A.2 Multiple arcs on the circle

The baseline model assumes that investors participate in markets spanning a single arc of length Δ around their “home” location. Extending the results to the general case where investors can choose to participate on multiple, disconnected arcs (as illustrated on the right-most graph of Figure 4) is straightforward and involves essentially no new insights. In this section we briefly sketch how to extend the results of the baseline model to this case and we show that allowing for this extra generality introduces an additional source of non-concavity into an investor’s optimization problem.

To start, we introduce the function

$$v(I) = \min_{N_i, \vec{a}_i, \vec{\Delta}_i, G_j^{(i)}} Var \left(\int_0^1 D_j dG_j^{(i)} \right)$$

$$s.t. \quad I \geq F \left(\vec{a}_i, \sum_{n=1}^{N_i} \Delta_{i,n} \right).$$

In words, the function $v(I)$ is the minimal variance, per share purchased, of the portfolio payoff that can be obtained by an investor who is willing to spend an amount I on participation costs. Proceeding similarly to Section 4.2 under the assumption that $P_j = P$ for all j , the facts that U is exponential and all D_j are normally distributed imply that maximizing utility over the choice of

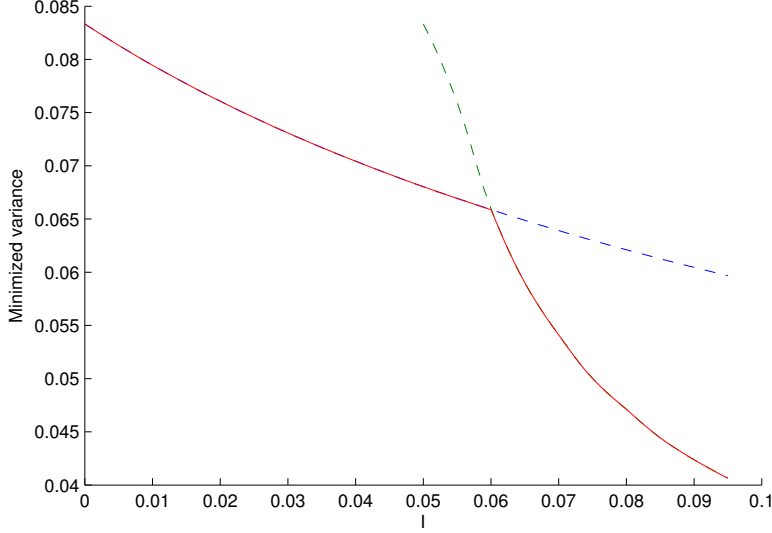


Figure 12: Numerical example to illustrate that $v(I)$ is non-convex. The figure depicts two (dotted) lines and the minimum of the two lines (solid line). The first dotted line starts at $I = 0$ and depicts the minimal variance that can be attained when participation costs are equal to I and the investor chooses to participate only on a single arc centered at her home location. The second dotted line starts at $I = 0.05$, i.e. at the minimum expenditure required to invest in two distinct arcs. This second dotted line depicts the minimal variance that can be attained when participation costs are equal to I and the investor can participate on two separate arcs with locations and lengths chosen so as to minimize variance. The function $v(I)$ (the minimum of the two dotted lines) is given by the solid line. For this example we chose $\sigma = 1$, $g(x) = 0.1 \times ((1-x)^{-6} - 1)$, $f(y) = 0.05 + 0.005 \times ((\frac{1}{2} - y)^{-2} - \frac{1}{0.25})$. (For $I > 0.1$ the function $v(I)$ would in general exhibit further kinks at the critical values I_N , $N \geq 2$, where the investor is indifferent between N and $N + 1$ distinct arcs.)

N_i , $\{a_{i,1}; \dots; a_{i,N_i}\}$, $\{\Delta_{i,1}; \dots; \Delta_{i,N_i}\}$, $G_j^{(i)}$, and w_i^f is equivalent to solving²⁹

$$\max_{w_i^f, I} Pw_i^f + (1 - w_i^f) \int_0^1 E[D_j] dG_j^{(i)} - \frac{\gamma}{2} (1 - w_i^f)^2 v(I) - I. \quad (34)$$

²⁹To ensure that the optimization problem (34) has a solution it is convenient either to impose a collateral constraint such as (23) or to introduce some (potentially small but positive) aggregate risk in dividends. Either of these assumptions coupled with the additional assumption $\lim_{\Delta \rightarrow 1} g(\Delta) = \infty$ suffices to ensure the existence of a solution to (34). Alternatively, one can ensure that (34) has an interior solution by requiring that, upon plugging in the optimal value of w_i^f , the maximand in (35) tends to negative infinity as I goes to infinity. Given the lower bound on P provided by the autarky equilibrium, it suffices that $\lim_{I \rightarrow \infty} (\frac{\sigma^2}{12})^2 \frac{\gamma}{v(I)} - I = -\infty$. This condition may be harder to verify than Assumption 1, since it is not readily expressed in terms of primitive parameters.

Given that $E[D_j] = 1$, equation (34) can be rewritten as

$$V = \max_{I, w_i^f} P w_i^f + \left(1 - w_i^f\right) - \frac{\gamma}{2} \left(1 - w_i^f\right)^2 v(I) - I. \quad (35)$$

In the baseline version of the model (Section 4.2), where the investor chooses to invest in a single arc around her home location, v is a convex function of the total cost I .³⁰ In the general case where investors' portfolios are invested on disconnected arcs, the function $v(I)$ is in general non-convex with kinks at the expenditure levels I_n where it becomes optimal to invest in $n + 1$ rather than n distinct arcs. Figure 12 provides an illustration. This non-convexity of $v(I)$, which may arise when (and only when) investors participate in markets located on multiple distinct arcs, constitutes an additional reason for the maximization problem (35) to be non-concave. This reason is distinct from the non-concavity arising from the interaction between leverage and participation decisions that we identify in Section 4.3, and strengthens the conclusion that a symmetric equilibrium may not exist.

B Proofs

Proof of Lemma 1. Property 2 follows immediately from integrating (3). To show property 3, note that, for any $i \in (0, 1)$, $\lim_{d(i,j) \rightarrow 0} D_j = \lim_{j \rightarrow i} D_j = D_i$ a.s. by the continuity of the Brownian motion. Continuity at 0 follows from the fact that $B_0 = B_1$.

We turn now to property 1. Since $E(B_i) = 0$ for all $i \in [0, 1]$, $E(D_j) = 1$. To compute $cov(D_i, D_j)$ we start by noting that $cov(B_s, B_t) = E(B_s B_t) = s(1 - t)$ for $s \leq t$. Therefore, for any $t \in [0, 1]$,

$$\begin{aligned} \int_0^1 E(B_t B_u) du &= \int_0^t u(1 - t) du + \int_t^1 t(1 - u) du \\ &= \frac{1}{2}(1 - t)t^2 + \frac{1}{2}(1 - t)^2 t = \frac{t(1 - t)}{2}. \end{aligned} \quad (36)$$

Accordingly,

$$\begin{aligned} Var\left(\int_0^1 B_u du\right) &= E\left[\left(\int_0^1 B_u du\right)^2\right] = E\left[\left(\int_0^1 B_u du\right)\left(\int_0^1 B_t dt\right)\right] \\ &= \int_0^1 \left(\int_0^1 E(B_u B_t) du\right) dt = \int_0^1 \frac{t(1 - t)}{2} dt = \frac{1}{12}, \end{aligned} \quad (37)$$

where the second line of (37) follows from Fubini's Theorem and (36). Combining (37) and (36)

³⁰To see this, note that $v'(I) = \frac{\omega'(\Delta)}{\kappa g'(\Delta)}$, where $\Delta(I) = g^{-1}\left(\frac{I}{\kappa}\right)$. Differentiating again gives $v''(I) = \frac{1}{\kappa} \frac{\omega''(\Delta)g'(\Delta) - \omega'(\Delta)g''(\Delta)}{(g'(\Delta))^2} \Delta'(I) > 0$.

gives

$$\begin{aligned} \frac{1}{\sigma^2} \text{Var}(D_t) &= \text{Var}(B_t) + \text{Var}\left(\int_0^1 B_u du\right) - 2\text{cov}\left(B_t, \int_0^1 B_u du\right) \\ &= t(1-t) + \frac{1}{12} - 2 \int_0^1 E(B_t B_u) du = \frac{1}{12}. \end{aligned} \quad (38)$$

This calculation finishes the proof of property 1. For property 4, take any $s \leq t$ and use (36) and (37) to obtain

$$\begin{aligned} \frac{\text{cov}(D_s, D_t)}{\sigma^2} &= \text{cov}\left(B_s - \int_0^1 B_u du, B_t - \int_0^1 B_u du\right) \\ &= E(B_s B_t) - E\left(B_s \int_0^1 B_u du\right) - E\left(B_t \int_0^1 B_u du\right) + \frac{1}{12} \\ &= s(1-t) - \frac{s(1-s)}{2} - \frac{t(1-t)}{2} + \frac{1}{12} \\ &= \frac{(s-t)(1+s-t)}{2} + \frac{1}{12}. \end{aligned} \quad (39)$$

This establishes property 4. ■

Proof of Proposition 1. We start by establishing the following lemma.

Lemma 2 *The (bounded-variation) function L with $L_{-\frac{\Delta}{2}-} = 0$ and $L_{\frac{\Delta}{2}} = 1$ that minimizes $\text{Var}\left(\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} D_j dL_j\right)$ is given by (12). Moreover, the minimal variance is equal to $\omega(\Delta)$.*

Proof of Lemma 2. To simplify notation, we prove a “shifted” version of the lemma, namely finding the minimal-variance portfolio on $[0, \Delta]$ rather than $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$. The two versions are clearly equivalent, since covariances depend only on the distances between locations, rather than the locations themselves.

We start by defining $q(d) = \frac{1}{12} - \frac{d(1-d)}{2}$ and therefore $q'(d) = -\frac{1}{2} + d$. In light of (4), $q(d) = \frac{1}{\sigma^2} \text{cov}(D_i, D_j)$ whenever $d(i, j) = d$. If $L_u = \int_{0-}^u dL_u$ is a variance-minimizing portfolio of risky assets, it must be the case that the covariance between any gross return $R_s = \frac{D_s}{P}$ for $s \in [0, \Delta]$ and the portfolio $\int_{0-}^{\Delta} R_u dL_u = \int_{0-}^{\Delta} \frac{D_u}{P} dL_u$ is independent of s . Thus, the quantity

$$\begin{aligned} \frac{1}{\sigma^2} \text{cov}\left(D_s, \int_{0-}^{\Delta} D_u dL_u\right) &= \frac{1}{\sigma^2} \left[\int_{0-}^s \text{cov}(D_s, D_u) dL_u + \int_s^{\Delta} \text{cov}(D_s, D_u) dL_u \right] = \\ &= \int_{0-}^s q(s-u) dL_u + \int_s^{\Delta} q(u-s) dL_u \end{aligned} \quad (40)$$

is independent of s . Letting $\tilde{L}(s) = 1 - L(s)$ and integrating by parts we obtain

$$\int_{0-}^s q(s-u) dL_u = L(s) q(0) - L(0-) q(s) + \int_{0-}^s L_u q'(s-u) du \quad (41)$$

$$\int_s^{\Delta} q(u-s) dL_u = \tilde{L}(s) q(0) - \tilde{L}(\Delta) q(\Delta-s) + \int_s^{\Delta} \tilde{L}_u q'(u-s) du. \quad (42)$$

Using (41) and (42) inside (40) and recognizing that $q(0) = \frac{1}{12}$, $L(0^-) = 0$, and $\tilde{L}(\Delta) = 0$, we obtain that (40) equals

$$Q(s) \equiv \frac{1}{12} + \int_{0^-}^s L_u q'(s-u) du + \int_s^\Delta \tilde{L}_u q'(u-s) du. \quad (43)$$

This expression is independent of $s \in [0, \Delta]$ if and only if $Q'(s) = 0$. Differentiating (43) and setting the resulting expression to zero yields

$$\begin{aligned} Q'(s) &= \int_{0^-}^s L_u q''(s-u) du - \int_s^\Delta \tilde{L}_u q''(u-s) du + L_s q'(0) - \tilde{L}_s q'(0) \\ &= \int_{0^-}^\Delta L_u du - \Delta + s - L_s + \frac{1}{2} = 0, \end{aligned} \quad (44)$$

where we used $q'(0) = -\frac{1}{2}$, $q'' = 1$, and $\tilde{L}(s) = 1 - L(s)$. Since (44) needs to hold for all $s \in [0, \Delta]$, it must be the case that $L_s = A + s$ for an appropriate constant A . To determine A , we substitute $L_s = A + s$ into (44) and solve for A to obtain

$$A = \frac{1 - \Delta}{2}.$$

It is immediate that the standardized portfolio corresponding to the solution L we computed is L^* of (12).

Using the variance-minimizing portfolio inside (43), implies after several simplifications, that $Q = \frac{1}{12}(1 - \Delta)^3$ and hence $\text{cov}\left(D_s, \int_{0^-}^\Delta D_u dL_u\right) = Q\sigma^2 = \omega(\Delta)$. Accordingly,

$$\begin{aligned} \text{Var}\left(\int_{0^-}^\Delta D_u dL_u\right) &= \text{cov}\left(\int_{0^-}^\Delta D_s dL_s, \int_{0^-}^\Delta D_u dL_u\right) = \int_{0^-}^\Delta \text{cov}\left(D_s, \int_{0^-}^\Delta D_u dL_u\right) dL_s \\ &= \omega(\Delta) \int_{0^-}^\Delta dL_s = \omega(\Delta). \end{aligned}$$

■

With Lemma 2 in hand it is possible to confirm that the allocations and prices of Proposition 1 constitute a symmetric equilibrium — assuming that one exists. We already argued that all agents choose the same standardized portfolio (as agent $\frac{\Delta}{2}$). Furthermore, since in a symmetric equilibrium all agents must hold the same allocation of bonds, clearing of the bond market requires $w_i^f = 0$ for all i . By equation (18), $w_i^f = 0$ is supported as an optimal choice for an investor only if $P_i = P$ is given by (15). Similarly, in light of (19), equation (14) is a necessary optimality condition for the interval Δ^* . Since the values of P and Δ^* implied by (15) and (14) are unique, they are necessarily the equilibrium values of P and Δ^* that characterize a symmetric equilibrium. Hence, when a symmetric equilibrium exists, it is unique in the class of symmetric equilibria.

Existence of a symmetric equilibrium implies that $w_i^f = 0$ is optimal, and so are the choices Δ^* and $G_{i+j}^{(i)} = L_j^*$ given prices $P_i = P$. It remains to show that markets clear. We already addressed bond-market clearing. To see that the stock markets clear, we start by noting that, since $P_i = W_{0,i} = P$ for all i , the market clearing condition amounts to $\int_{i \in [0,1)} dG_j^{(i)} = 1$. We have $\int_{i \in [0,1)} dG_j^{(i)} = \int_{i \in [0,1)} dL_{j-i}^* = \int_{j \in [0,1)} dL_j^* = 1$. ■

Proof of Proposition 2. Let $w^*(P)$ denote the set of optimal w_i^f solving the maximization problem (17) when the price in all markets is P . We first note that the assumption that no symmetric equilibrium exists implies that there exists no P such that $0 \in w^*(P)$. (If such a P existed, then we could simply repeat the arguments of Proposition 1 to establish the existence of a symmetric equilibrium with price $P_i = P$, and interval choice $\Delta_i = \Delta^*(P)$).

We next show that since there exists no P such that $0 \in w^*(P)$, it follows that $w^*(P)$ cannot be single-valued for all P . We argue by contradiction. Suppose to the contrary that $w^*(P)$ is single-valued. Since the theorem of the maximum implies that $w^*(P)$ is an upper-hemicontinuous correspondence, it follows that $w^*(P)$ is actually a continuous function. Inspection of (17) shows that $w^*(1) = 1$. Moreover, as $P \rightarrow -\infty$, the optimal solution to (17) becomes negative: $w^*(-\infty) < 0$. Then an application of the intermediate value theorem gives the existence of P such that $w^*(P) = 0$, a contradiction.

Combining the facts that a) there exists no P such that $0 \in w^*(P)$, b) $w^*(P)$ is multi-valued for at least one value of P , and c) $w^*(P)$ is upper-hemicontinuous, implies that there exists at least one P such that $\{w_1, w_2\} \in w^*(P)$ with $w_1 > 0$ and $w_2 < 0$. An implication of the necessary first-order condition for the optimality of the interval choice $\Delta^*(P)$ is that $\Delta^*(P)$ is also multi-valued with $\Delta_1 < \Delta_2$. Furthermore, since prices in all locations are equal, the (standardized) optimal portfolio of an agent choosing Δ_k is the variance-minimizing portfolio of Proposition 1, denoted $L^{*,k}$.

From this point onwards, an equilibrium can be constructed as follows. By definition, the tuples $\{\Delta_1, w_1, dL^{*,1}\}$ and $\{\Delta_2, w_2, dL^{*,2}\}$ are optimal. Hence it only remains to confirm that asset markets clear. Define $\pi \equiv -\frac{w_2}{w_1 - w_2} \in (0, 1)$. By construction, $\pi w_1 + (1 - \pi) w_2 = 0$ and, therefore, if in every location π agents choose $\{\Delta_1, w_1, dL^{*,1}\}$ and the remaining fraction $(1 - \pi)$ choose $\{\Delta_2, w_2, dL^{*,2}\}$, then the bond market clears by construction. To see that the stock markets clear, we start by noting that, since $P_i = W_{0,i} = P$, the market clearing condition for stock i amounts to

$$\pi \int_{[0,1]} dL_{j-i}^{*,1} + (1 - \pi) \int_{[0,1]} dL_{j-i}^{*,2} = 1,$$

which holds because $L_j^{*,k}$ for $k \in \{1, 2\}$ is a measure on the circle.

We prove next that symmetric and asymmetric equilibria (with different prices) cannot co-exist. Inspection of (17) shows that the optimal w_i^f is increasing in P in the sense that if $P_1 < P_2$ then $w_1 < w_2$ for any $w_1 \in w^*(P_1)$ and $w_2 \in w^*(P_2)$.³¹ Accordingly, if a symmetric equilibrium exists, i.e., if there is a \hat{P} such that $0 \in w^*(\hat{P})$ then there cannot exist $P \neq \hat{P}$ with the property that $\{w_1, w_2\} \in w(P)$ and yet $w_1 > 0$ and $w_2 < 0$, which is a requirement for the existence of an asymmetric equilibrium. Hence symmetric and asymmetric equilibria cannot co-exist. The fact that $w^*(P)$ is an increasing correspondence also implies that asymmetric equilibria are essentially unique, in the sense that asymmetric equilibria associated with different equilibrium prices cannot co-exist. ■

Remark 1 *The existence proof of an asymmetric equilibrium (when a symmetric equilibrium fails to exist) obtains also in the presence of the leverage constraint (23) that we introduce in Section 5.*

Proof of Proposition 3. Conjecture first that in equilibrium $P_j = P$ for all j and let $\pi \in [0, 1]$ denote the fraction of funds invested in the local market. Assuming that a given investor

³¹To see that this statement is correct, consider the maximum $\bar{V}(P, w_i^f)$ of the maximand in (17) over Δ and note that its cross-partial derivative with respect to P and w_i^f is positive: $\partial_P \partial_{w_i^f} \bar{V}(P, w_i^f) > 0$.

chooses $N = 2$ (that is, chooses to invest in her own location and another location at distance d), equation (4) allows the computation of the minimal portfolio variance:

$$\begin{aligned}\widehat{\omega}(d) &= \sigma^2 \min_{\pi} \left\{ \left(\pi^2 + (1 - \pi)^2 \right) \frac{1}{12} + 2\pi(1 - \pi) \left(\frac{1}{12} - \frac{d(1 - d)}{2} \right) \right\} \\ &= \sigma^2 \left(\frac{1}{12} - \frac{1}{4}d(1 - d) \right).\end{aligned}\quad (45)$$

The optimal distance d for an investor choosing $N = 2$ satisfies a first-order condition similar to equation (19), namely

$$-\frac{\gamma}{2} (1 - w^f)^2 \frac{\widehat{\omega}'(d)}{\widehat{\omega}^2(d)} = \kappa f'(d). \quad (46)$$

Since $\widehat{\omega}'(d) = 0$ when and only when $d = \frac{1}{2}$, and $f'(d) = 0$, it follows that $d = \frac{1}{2}$ is optimal for an investor choosing $N = 2$. Hence the minimal portfolio variance of an investor choosing $N = 1$ is equal to $\widehat{\omega}(0) = \frac{\sigma^2}{12}$, while the minimal portfolio variance for an investor choosing $N = 2$ is $\widehat{\omega}(\frac{1}{2}) = \frac{\sigma^2}{48}$. Assuming that the equilibrium is of the asymmetric, location-invariant type, we can use equation (17) to express the indifference between the choices $N = 1$, respectively $N = 2$ and $d = \frac{1}{2}$, as

$$Pw_1^f + (1 - w_1^f) - \frac{\gamma}{2} (1 - w_1^f)^2 \widehat{\omega}(0) = Pw_2^f + (1 - w_2^f) - \frac{\gamma}{2} (1 - w_2^f)^2 \widehat{\omega}\left(\frac{1}{2}\right) - \kappa f_0. \quad (47)$$

Using the first-order conditions for leverage

$$1 - P = \gamma (1 - w_2^f) \widehat{\omega}\left(\frac{1}{2}\right) = \gamma (1 - w_1^f) \widehat{\omega}(0) \quad (48)$$

inside (47) yields — after some simplifications — the equilibrium price (24).

To verify that the postulated equilibrium is indeed an equilibrium, we proceed as in the proof of Proposition 2. For $P(\kappa)$ to be an equilibrium price in all locations, it must also be case that $1 - w_1^f \leq 1 \leq 1 - w_2^f$, so that setting $\pi = \frac{w_2^f}{w_2^f - w_1^f} > 0$ ensures market clearing (of bond markets and all risky asset markets). In light of (48), the requirement $1 - w_1^f \leq 1 \leq 1 - w_2^f$ is equivalent to $P \in [1 - \gamma \widehat{\omega}(0), 1 - \gamma \widehat{\omega}(\frac{1}{2})]$. This requirement is satisfied as long as $\kappa \in (\kappa_1, \kappa_2)$. ■

Lemma 3 *Consider an investor located at $i \notin [-\frac{k}{2}, \frac{k}{2}]$, and therefore investing in markets $[i - \frac{\bar{\Delta}}{2}, i + \frac{\bar{\Delta}}{2}]$. Suppose that $P(x)$ is continuously differentiable everywhere on $[i - \frac{\bar{\Delta}}{2}, i + \frac{\bar{\Delta}}{2}]$. With $dX_l^{(i)}$ the number of shares purchased on the account of an investor at i in market l and $j \equiv i - \frac{\bar{\Delta}}{2}$,*

$$X_{j+\bar{\Delta}}^{(i)} = \frac{1}{\gamma \omega(\bar{\Delta})} \left[1 - \frac{1 - \bar{\Delta}}{2} (P_j + P_{j+\bar{\Delta}}) - \int_j^{j+\bar{\Delta}} P_u du \right]. \quad (49)$$

Furthermore, the function X is given by

$$X_{j+l}^{(i)} = \frac{P'_{j+l}}{\gamma \sigma^2} + X_{j+\bar{\Delta}}^{(i)} \frac{1 - \bar{\Delta} + 2l}{2} + \frac{1}{1 - \bar{\Delta}} \frac{P_{j+\bar{\Delta}} - P_j}{\gamma \sigma^2}. \quad (50)$$

If an investor is located at $i \in [-\frac{k}{2}, \frac{k}{2}]$ and only invests in market i then the respective demand for risky asset i is given by

$$\widehat{X}_i^{(i)} = \frac{1}{\gamma\omega(0)} (1 - P_i). \quad (51)$$

Proof of Lemma 3. Notice that optimization problem of agent i is equivalent to

$$\max_X P_i + \int_{j^-}^{j+\bar{\Delta}} (1 - P_u) dX_u - \frac{\gamma}{2} \text{Var} \left(\int_{j^-}^{j+\bar{\Delta}} D_u dX_u \right) \quad (52)$$

Thus, the first-order condition requires that

$$\gamma \text{cov} \left(D_s, \int_{j^-}^{j+\bar{\Delta}} D_u dX_u \right) = 1 - P_s \quad (53)$$

for all $s \in [j, j + \bar{\Delta}]$. Letting $q(d)$ be defined as in Lemma 2 we can rewrite (53) as

$$\int_{j^-}^s q(s-u) dX_u + \int_s^{j+\bar{\Delta}} q(u-s) dX_u = \frac{1 - P_s}{\gamma\sigma^2}. \quad (54)$$

Let $\widetilde{X}(s) = X(j + \bar{\Delta}) - X(s)$ and integrating by parts we obtain

$$\int_{j^-}^s q(s-u) dX_u = X(s)q(0) - X(j^-)q(s) + \int_j^s X_u q'(s-u) du \quad (55)$$

$$\int_s^{j+\bar{\Delta}} q(u-s) dX_u = \widetilde{X}(s)q(0) - \widetilde{X}(\bar{\Delta})q(\bar{\Delta}-s) + \int_s^{j+\bar{\Delta}} \widetilde{X}_u q'(u-s) du \quad (56)$$

Substituting (55) and (56) into (54), recognizing that $q(0) = \frac{1}{12}$, $X(j^-) = 0$, and $\widetilde{X}(j + \bar{\Delta}) = 0$, we obtain

$$\frac{1}{12}X(j + \bar{\Delta}) + \int_j^s X_u q'(s-u) du + \int_s^{j+\bar{\Delta}} \widetilde{X}_u q'(u-s) du = \frac{1 - P_s}{\gamma\sigma^2}. \quad (57)$$

Since this relation must hold for all s , we may differentiate both sides of (57) to obtain

$$\int_j^s X_u q''(s-u) du - \int_s^{j+\bar{\Delta}} \widetilde{X}_u q''(u-s) du + X_s q'(0) - \widetilde{X}_s q'(0) = -\frac{P'_s}{\gamma\sigma^2}. \quad (58)$$

This equation holds for all $s \in (j, j + \bar{\Delta})$. Noting that $q'' = 1$, $q'(0) = -\frac{1}{2}$, $\widetilde{X}(s) = X(j + \bar{\Delta}) - X(s)$, and using (58) to solve for X_s yields

$$X_s = \int_j^{j+\bar{\Delta}} X_u du + \left(s - j + \frac{1}{2} - \bar{\Delta} \right) X(j + \bar{\Delta}) + \frac{P'_s}{\gamma\sigma^2}. \quad (59)$$

Integrating (59) from j to $j + \bar{\Delta}$ and solving for $\int_j^{j+\bar{\Delta}} X_u du$ leads to

$$\int_j^{j+\bar{\Delta}} X_u du = \frac{1}{1-\bar{\Delta}} \left[X(j+\bar{\Delta}) \bar{\Delta} \left(\frac{1-\bar{\Delta}}{2} \right) + \frac{P(j+\bar{\Delta}) - P(j)}{\gamma\sigma^2} \right], \quad (60)$$

so that

$$\begin{aligned} X_s = \frac{1}{1-\bar{\Delta}} & \left[X(j+\bar{\Delta}) \bar{\Delta} \left(\frac{1-\bar{\Delta}}{2} \right) + \frac{P(j+\bar{\Delta}) - P(j)}{\gamma\sigma^2} \right] + \\ & \left(s - j + \frac{1}{2} - \bar{\Delta} \right) X(j+\bar{\Delta}) + \frac{P'_s}{\gamma\sigma^2}. \end{aligned} \quad (61)$$

Evaluating (57) at $s = j + \bar{\Delta}$, and noting that $q'(s) = -\frac{1}{2} + s$ leads to

$$\frac{1}{12} X(j+\bar{\Delta}) + \int_j^{j+\bar{\Delta}} X_u \left[-\frac{1}{2} + (\bar{\Delta} - u) \right] du = \frac{1 - P_{j+\bar{\Delta}}}{\gamma\sigma^2}. \quad (62)$$

An implication of (59) is that $X_u = X_j + \frac{P'_u - P'_j}{\gamma\sigma^2} + X(j+\bar{\Delta})u$. Using this expression for X_u inside (62), carrying out the requisite integrations and using integration by parts to express $\int_j^{j+\bar{\Delta}} \left(\frac{P'_u}{\gamma\sigma^2} \right) u du = \frac{P_{j+\bar{\Delta}}}{\gamma\sigma^2} (j+\bar{\Delta}) - \frac{P_j}{\gamma\sigma^2} j - \int_j^{j+\bar{\Delta}} \frac{P_u}{\gamma\sigma^2} du$, leads (after some simplifications) to

$$\begin{aligned} X(j+\bar{\Delta}) & \left(\frac{1}{12} + \frac{\bar{\Delta}^3}{6} - \frac{\bar{\Delta}^2}{4} \right) - \frac{\bar{\Delta}(1-\bar{\Delta})}{2} \left(X_j - \frac{P'_j}{\gamma\sigma^2} \right) - \frac{P_{j+\bar{\Delta}} - P_j}{2\gamma\sigma^2} + \int_{j-}^{j+\bar{\Delta}} \frac{P_u - P_j}{\gamma\sigma^2} du \\ & = \frac{1 - P_{\bar{\Delta}}}{\gamma\sigma^2}. \end{aligned} \quad (63)$$

Finally, evaluating (59) at j gives

$$\left(X_j - \frac{P'_j}{\gamma\sigma^2} \right) = \int_j^{j+\bar{\Delta}} X_u du + \left(\frac{1}{2} - \bar{\Delta} \right) X(j+\bar{\Delta}). \quad (64)$$

Equations (60), (63), and (64) are three linear equations in three unknowns. Solving for $X(j+\bar{\Delta})$ and using the definition of $\omega(\bar{\Delta})$ leads to (49). Equation (61) simplifies to (50). Finally, (51) is a direct consequence of (53) when $\bar{\Delta} = 0$. ■

Proof of Proposition 4. For any $j \in (\frac{k}{2}, \frac{1}{2}]$ and $l \in (-\frac{\bar{\Delta}}{2}, \frac{\bar{\Delta}}{2})$, we have from Lemma 3:

$$\begin{aligned} X_{j-\frac{\bar{\Delta}}{2}}^{(j)} &= \frac{P'_{j-\frac{\bar{\Delta}}{2}}}{\gamma\sigma^2} + \frac{P_{j-\frac{\bar{\Delta}}{2}} - P_{j+\frac{\bar{\Delta}}{2}}}{\gamma\sigma^2(1-\Delta)} + \frac{1-\Delta}{2} X_{j+\frac{\bar{\Delta}}{2}}^{(j)} \\ dX_{j+l}^{(j)} &= \left(\frac{P''_{j+l}}{\gamma\sigma^2} + X_{j+\frac{\bar{\Delta}}{2}}^{(j)} \right) dl \\ X_{j+\frac{\bar{\Delta}}{2}}^{(j)} - X_{(j+\frac{\bar{\Delta}}{2})^-}^{(j)} &= -\frac{P'_{j+\frac{\bar{\Delta}}{2}}}{\gamma\sigma^2} - \frac{P_{j-\frac{\bar{\Delta}}{2}} - P_{j+\frac{\bar{\Delta}}{2}}}{\gamma\sigma^2(1-\Delta)} + \frac{1-\Delta}{2} X_{j+\frac{\bar{\Delta}}{2}}^{(j)}. \end{aligned}$$

Specialize the first equation to $j = \frac{1}{2} + \frac{\bar{\Delta}}{2}$, the second to $j = \frac{1}{2} - l$ for all $l \in \left(-\frac{\bar{\Delta}}{2}, \frac{\bar{\Delta}}{2}\right)$, and the third to $j = \frac{1}{2} - \frac{\bar{\Delta}}{2}$ and aggregate to obtain the total demand for asset $\frac{1}{2}$:

$$1 = \frac{1-\Delta}{2} X_{\frac{1}{2}+\bar{\Delta}}^{(\frac{1}{2}+\frac{\bar{\Delta}}{2})} + \frac{1-\Delta}{2} X_{\frac{1}{2}}^{(\frac{1}{2}-\frac{\bar{\Delta}}{2})} + \int_{-\frac{\bar{\Delta}}{2}}^{\frac{\bar{\Delta}}{2}} X_{\frac{1}{2}-l+\frac{\bar{\Delta}}{2}}^{(\frac{1}{2}-l)} dl + \frac{P_{\frac{1}{2}}''}{\gamma\sigma^2} \Delta + \frac{P_{\frac{1}{2}-\bar{\Delta}} + P_{\frac{1}{2}+\bar{\Delta}} - 2P_{\frac{1}{2}}}{2\gamma\sigma^2(1-\Delta)}.$$

Suppose now that $P_j \geq 1 - \gamma\omega(\bar{\Delta})$ on $[\frac{1}{2} - \Delta, \frac{1}{2} + \Delta]$, with strict inequality on a positive measure set. It then follows from equation (49) that $X_{j+\frac{\bar{\Delta}}{2}}^{(j)} \leq 1$, so that

$$0 < \frac{P_{\frac{1}{2}}''}{\gamma\sigma^2} \Delta + \frac{P_{\frac{1}{2}-\bar{\Delta}} + P_{\frac{1}{2}+\bar{\Delta}} - 2P_{\frac{1}{2}}}{2\gamma\sigma^2(1-\Delta)}.$$

This inequality contradicts the assumption that P is maximized at $\frac{1}{2}$. ■

Proof of Proposition 5. We adopt a guess-and-verify approach. We start by noting that the beginning-of-period wealth of investor i at time $t+1$ is $W_{t+1,i} \equiv \int_0^1 (P_{t+1,j} + D_{t+1,j}) dX_{t,j}^{(i)} + X_{B,t}^{(i)}$. We then conjecture that, as long as

$$F(\Delta) = \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_t) \quad (65)$$

for some $M > -\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_u)$, (ii) and (iii) obtain. We show at the end that the function F can be chosen to ensure (i).

We also conjecture and verify that investors' holdings of risky assets $X_{t,j}^{(i)}$ coincide with $G_{t,j}^{(i)}$ of Proposition 1, and that their bond holdings equal

$$X_{B,t}^{(i)} = W_{t,i} - (1+r) \bar{P}_{t,i} - r\Phi_t, \quad (66)$$

where $\bar{P}_{t,i} \equiv \int_0^1 P_{t,j} dX_{t,j}^{(i)}$ is the average price that investor i pays for her portfolio. Here, to simplify notation, we defined $\Phi_t \equiv \Phi_1 \omega(\Delta_t) + \Phi_0$.

We first ensure that with these postulates markets clear. Clearly, all risky markets clear, since the holdings of risky assets are the same as in Proposition 1. To show that bond markets clear, we proceed inductively. First we note that investors are endowed with no bonds at time zero. Hence $\int_0^1 X_{B,-1}^{(i)} di = 0$ and therefore $\int_0^1 W_{0,i} di = \int_0^1 (P_{0,i} + D_{0,i}) di$. Next we postulate that $\int_0^1 X_{B,t-1}^{(i)} di = 0$, so that $\int_0^1 W_{t,i} di = \int_0^1 P_{t,i} di + \int_0^1 D_{t,j} dj$. Integrating our postulate (66) for $X_{B,t}^{(i)}$ across all investors, we obtain

$$\int_0^1 X_{B,t}^{(i)} di = \int_0^1 W_{t,i} di - (1+r) \int_0^1 \bar{P}_{t,i} di - r\Phi_t. \quad (67)$$

We next note that (a) $\int_0^1 D_{t,j} dj = 1$, by construction of the dividend process; (b) $\int_0^1 W_{t,i} di = \int_0^1 P_{t,i} di + \int_0^1 D_{t,j} dj = r^{-1} - \Phi_t + 1$, using the induction hypothesis, (31), and (a); and (c) $\int_0^1 \bar{P}_{t,i} di = \int_0^1 P_{t,i} di = r^{-1} - \Phi_t$. Using these three facts, it follows immediately that the right-hand side of (67) is zero, so that the bond market clears.

If investors set their bond holdings according to (66), then their budget constraint implies a consumption of

$$c_{t,i} = W_{t,i} - \frac{1}{1+r} X_{B,t}^{(i)} - \bar{P}_{t,i} - F_t. \quad (68)$$

Using the definition of $W_{t,i}$ and market clearing condition for bond holdings inside (68), and integrating across i implies that the market for consumption goods clears: $\int_0^1 c_{t,i} di = 1 - F_t$.

Having established market clearing given the postulated policies and prices, we next turn to optimality. Equation (68) implies

$$\begin{aligned} c_{t+1,i} - c_{t,i} &= W_{t+1,i} - W_{t,i} - \frac{1}{1+r} \left(X_{B,t+1}^{(i)} - X_{B,t}^{(i)} \right) - (\bar{P}_{t+1,i} - \bar{P}_{t,i}) - (F_{t+1} - F_t) \\ &= \left(\frac{r}{1+r} \right) (W_{t+1,i} - W_{t,i}) + \frac{r}{1+r} (\Phi_{t+1} - \Phi_t) - (F_{t+1} - F_t), \end{aligned} \quad (69)$$

where the second line follows from (66). We next use the definition of $W_{t,i}$ and (66) to obtain

$$\begin{aligned} W_{t+1,i} - W_{t,i} &= \int_0^1 (P_{t+1,j} + D_{t+1,j}) dX_{t,j}^{(i)} + X_{B,t}^{(i)} - W_{t,i} \\ &= \int_0^1 (P_{t+1,j} + D_{t+1,j}) dX_{t,j}^{(i)} - (1+r) \bar{P}_{t,i} - r \Phi_t. \end{aligned} \quad (70)$$

Substituting (70) into (69) and using (31) and (66) leads to

$$\begin{aligned} c_{t+1,i} - c_{t,i} &= \left(\frac{r}{1+r} \right) \left[(1+\phi) \int_0^1 D_{t+1,j} dX_{t,j}^{(i)} - (1+r) \phi \int_0^1 D_{t,j} dX_{t,j}^{(i)} - (1-r\phi) \int_0^1 dX_{t,j}^{(i)} \right] \\ &\quad - (F_{t+1} - F_t). \end{aligned} \quad (71)$$

Next use the fact $\bar{D}_{t+1,j} = \rho \bar{D}_{t,j} + (1-\rho) \bar{\varepsilon}_{t+1,j}$ along with $\phi = \frac{\rho}{1+r-\rho}$, $(1+\phi)\rho = (1+r)\phi$, and $(1+\phi)(1-\rho) = (1-r\phi)$ inside (71) to arrive at

$$c_{t+1,i} - c_{t,i} = \left(\frac{r}{1+r-\rho} \right) (1-\rho) \int_0^1 (\varepsilon_{t+1,j} - 1) dX_{t,j}^{(i)} - (F_{t+1} - F_t). \quad (72)$$

Having established (72), the dynamics of agent i 's consumption under our postulate, we next turn attention to the Euler equations, starting with the bond Euler equation

$$1 = \beta (1+r) E_t e^{-\gamma(c_{t+1,i} - c_{t,i})}. \quad (73)$$

Substituting (72) into (73) and noting that $\int_0^1 (\varepsilon_{t+1,j} - 1) dX_{t,j}^{(i)}$ is normally distributed with mean zero and variance $\omega(\Delta_t)$ gives

$$1 = \beta (1+r) e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_t) - \gamma F_t} E_t (e^{\gamma F_{t+1}}). \quad (74)$$

Now suppose that for any r and a given desired distribution $\Psi(\Delta)$ we set

$$F_t(\Delta_t; r) = \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_t). \quad (75)$$

Then equation (74) can be written as (30). Since $(1+r) E e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta)}$ is equal to 1 when $r = 0$ and increases monotonically to infinity as r increases, it follows that there exists a unique positive r such that equation (30) holds. For that value of r , all investors' bond Euler equations are satisfied.

Finally, we need to determine Φ_t so as to ensure that the Euler equations for risky assets hold, i.e., that

$$P_{t,j} = \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} (P_{t+1,j} + D_{t+1,j}) \right]. \quad (76)$$

To that end, we use (31) and (28) to express (76) as

$$\frac{1}{r} - \Phi_t + \phi(D_{t,j} - 1) = \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \left(\frac{1}{r} - \Phi_{t+1} + (1+\phi)(\rho D_{t,j} + (1-\rho)\varepsilon_{t+1,j}) - \phi \right) \right]. \quad (77)$$

We next note that

$$\beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \right] (1+\phi) \rho D_{t,j} = \frac{(1+\phi)\rho}{1+r} D_{t,j} = \phi D_{t,j} \quad (78)$$

using (73). Equation (78) simplifies (77) to

$$\begin{aligned} \frac{1}{r} - \Phi_t - \phi &= \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \left(\frac{1}{r} - \Phi_{t+1} + (1+\phi)(1-\rho)\varepsilon_{t+1,j} - \phi \right) \right] \\ &= \frac{1}{r(1+r)} - \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \Phi_{t+1} \right] + \frac{1}{1+r} (-\phi + (1+\phi)(1-\rho)) \\ &\quad + (1+\phi)(1-\rho) \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} (\varepsilon_{t+1,j} - 1) \right]. \end{aligned} \quad (79)$$

Using (72), Stein's Lemma, the fact that $\text{cov} \left(\int_0^1 (\varepsilon_{t+1,j} - 1) dX_{t,j}^{(i)}, \varepsilon_{t+1,j} \right) = \omega(\Delta)$ (see Proposition 1), and (73) implies

$$\beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \varepsilon_{t+1,j} \right] = \frac{1 - \gamma \frac{r}{1+r-\rho} (1-\rho) \omega(\Delta_t)}{1+r}. \quad (80)$$

Substituting (80) into (79) gives linear equations in Φ_0 and Φ_1 , solved by (32), respectively (33).

To complete the proof of the claim that Δ_t is chosen optimally, we provide an explicit example of a family of functions for $F_t(\Delta)$ that has the desired properties. To start, we compute the value function of an investor adopting the policies of Proposition 5. Equation (73) along with (72) imply

that

$$\begin{aligned} V(W_{t,i}, \Delta_t) &= -\frac{1}{\gamma} \sum_{t \geq 0} \beta^t E_t [e^{-\gamma c_{t,i}}] = -\frac{1}{\gamma} e^{-\gamma c_{0,i}} \left(1 + \sum_{t \geq 1} \beta^t E_t [e^{-\gamma \sum_{m=0}^{t-1} (c_{m+1,i} - c_{m,i})}] \right) \\ &= -\frac{1}{\gamma} e^{-\gamma c_{0,i}} \sum_{t \geq 0} (1+r)^{-t} = -\frac{1+r}{\gamma r} e^{-\gamma c_{0,i}}. \end{aligned}$$

In turn, equations (66), (68), and (75) imply

$$V(W_{t,i}, \Delta_t) = -\frac{1+r}{\gamma r} e^{-\frac{\gamma r}{1+r} W_{t,i} + z(\Delta_t)}, \quad (81)$$

where $z_t(\Delta_t) \equiv -\frac{r}{1+r} \gamma \Phi_t + M + \frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_t)$.

Next we suppose that we no longer impose that the investor choose $\Delta = \Delta_t$, (where Δ_t is the time- t random draw of Δ that we imposed in Proposition 5). Instead Δ is chosen optimally. However, prices are still given by $P_{t,j}(\Delta_t, D_{t,j})$ from equation (31). We will construct a function $\kappa_t g_t(\Delta)$ that renders the choice $\Delta = \Delta_t$ optimal at the total cost specified in (75).

Throughout we let $\bar{X}_t^{(i)}(\Delta; \Delta_t)$ denote the optimal number of total risky assets chosen by investor i , and assuming that that investor chooses Δ and prices are given by $P_{t,j}(\Delta_t, D_{t,j})$. For future reference, we note that by construction of the price function $P_{t,j}(\Delta_t, D_{t,j})$ it follows that $\bar{X}_t^{(i)}(\Delta_t; \Delta_t) = 1$. Using (81) the first order condition characterizing an optimal Δ is

$$F'_t(\Delta) = h(\Delta; \Delta_t),$$

where

$$h(\Delta; \Delta_t) = -\frac{1}{1+r} \frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \left(\bar{X}_t^{(i)}(\Delta; \Delta_t) \right)^2 \omega'(\Delta). \quad (82)$$

Next we fix a value of Δ_t and we simplify notation by writing $h(\Delta)$ rather than $h(\Delta; \Delta_t)$. We also let $q(x)$ denote some continuous function with $q(0) = 1$ and $q(x) > 1$ for $x > 0$. Let $\eta \in [0, 1]$, take some positive (small) $\varepsilon < \frac{\Delta_t}{2}$, and consider the function

$$F'_t(\Delta) = \begin{cases} \frac{\Delta}{\varepsilon} \eta h(\varepsilon) & \text{for } \Delta \leq \varepsilon \\ \eta h(\Delta) & \text{for } \Delta \in (\varepsilon, \Delta_t - \varepsilon] \\ \eta h(\Delta_t - \varepsilon) \frac{\Delta_t - \Delta}{\varepsilon} + h(\Delta_t) \frac{\Delta - \Delta_t + \varepsilon}{\varepsilon} & \text{for } \Delta \in (\Delta_t - \varepsilon, \Delta_t] \\ h(\Delta) q(\Delta) & \text{for } \Delta > \Delta_t + \varepsilon. \end{cases} \quad (83)$$

By construction, $F'_t(0) = 0$ and $F'_t(\Delta)$ is continuous and increasing in Δ . More importantly, $F'_t(\Delta_t) = h(\Delta_t)$, and hence $\Delta = \Delta_t$ satisfies the necessary first order condition (82). Moreover, since $F'_t(\Delta) < (>) h(\Delta)$ for $\Delta < (>) \Delta_t$, it follows that $\Delta = \Delta_t$ is optimal for any $\varepsilon > 0$ and $\eta \in [0, 1]$. Finally,

$$\lim_{\varepsilon \rightarrow 0} \int_0^{\Delta_t} F'_t(\Delta) d\Delta = \eta \int_0^{\Delta_t} h(x) dx > 0. \quad (84)$$

Now suppose that nature draws $\Delta_t = \Delta_u > 0$. By choosing M that is sufficiently close to

$-\frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\bar{\Delta})$ it follows that

$$0 < \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_u) < \int_0^{\Delta_u} h(x) dx. \quad (85)$$

Combining equations (84) and (85) it follows that for sufficiently small $\varepsilon > 0$ there exists some $\eta \in [0, 1]$ so that

$$\int_0^{\Delta_u} F'_t(x) dx = \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_u) > 0. \quad (86)$$

Hence, when $\Delta_t = \Delta_u$ the cost function $\kappa_t g_t(\Delta)$ renders $\Delta = \Delta_u$, while also satisfying (75). The same argument implies that for any value of Δ_t that satisfies

$$0 < \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_t) < \int_0^{\Delta_t} h(x) dx, \quad (87)$$

there exists $\eta \in [0, 1]$ and sufficiently small $\varepsilon > 0$ such that the optimal Δ coincides with Δ_t , and (75) holds. Continuity of $\omega(\Delta_t)$ and of $\int_0^{\Delta_t} h(x) dx$ in Δ_t implies that as long as $\underline{\Delta}$ is sufficiently close to Δ_u , there always exists $\eta \in [0, 1]$ and $\varepsilon > 0$ (both depending on the random draw Δ_t) such that $\Delta = \Delta_t$ is optimal and (75) holds. ■

Proof of Proposition 6. Parts (i)–(iii) are proved in the main body of the text. Part (iv) comes down to noticing that

$$\text{cov}(e^z, z) > 0 \quad (88)$$

for any random variable z — in particular, for $z = \omega(\Delta)$. The second statement of (iv) follows from the first and Jensen's inequality applied to the convex function ω . ■

C An interpretation of participation costs

Throughout the paper we maintain the assumption that participation in “distant” markets incurs participation costs. In this appendix³² we discuss how these costs could arise as information-acquisition costs that permit an investor to avoid the lower net returns earned by an investor unfamiliar with the asset class. Indeed, we wish to re-emphasize that we construe the notion of distance broadly, as a stand-in for the level of familiarity of investors in one location with all aspects of the financial environment in another. We also wish to emphasize that our notion of locations is meant to be very broad, in particular encompassing asset classes that may be especially opaque to uninformed investors (e.g., mortgage pools and small stocks in distant countries).

³²This appendix borrows from Gârleanu et al. (2013). We present a self-contained version of the model to keep the effort required of the interested reader to a minimum. Another related paper that derives portfolio concentration as a result of endogenous information acquisition is Van Nieuwerburgh and Veldkamp (2010).

C.1 Regular firms and common investors

Timing and the set of locations are the same as in Section 3. In each location there are measure-one continua of investors and firms, but both are of two types: Investors are either “common investors” or “swindlers”, while firms are either “regular” or “fraudulent”.

Common investors in each location constitute a fraction $\nu \in (0, 1)$ of the population in that location. They are identically endowed with an equal-weighted portfolio of all regular firms in that location i . The total measure of regular firms in each location is also ν . All regular firms in location i produce the same random output: $\hat{D}_{ik} = \hat{D}_i$, where k identifies the firm. The dividend is given by equation (3), as in the paper:

$$\hat{D}_i \equiv 1 + \left(B_i - \int_0^1 B_j dj \right). \quad (89)$$

Swindlers are a fraction $1 - \nu$ of the population in each location. Each swindler is endowed with the entirety of shares (normalized to one) of one fraudulent firm. Fraudulent firms produce zero output ($\hat{D}_{ik} = 0$).

For every firm in every location, there is a market for shares where any investor can submit a demand. As before, there exists a market for the riskless bond, available in zero net supply. Since investors don’t consume at time zero, we use the bond as the numeraire, and normalize its price to one ($r = 0$).

C.2 Budget constraints

Letting \hat{B}^{ci} denote the amount that a common investor in location i invests in riskless bonds, and \hat{X}_{jk}^{ci} denote a bivariate signed measure giving the number shares of firm k in location j she invests in, the time-one wealth of a common investor located in i is given by

$$\hat{W}_1^{ci} \equiv \hat{B}^{ci} + \int_{j \in \mathcal{L}} \int_{k \in [0,1]} D_{jk} d\hat{X}_{jk}^{ci}. \quad (90)$$

The first term on the right hand side of (90) is the amount that the investor receives from her bond position in period 1, while the second term captures the portfolio-weighted dividends of all the firms that the investor holds. The time-zero budget constraint of a common investor in location i is given by

$$\hat{B}^{ci} + \int_{j \in \mathcal{L}} \int_{k \in [0,1]} \hat{P}_{jk} d\hat{X}_{jk}^{ci} = \frac{1}{\nu} \int_{k \in [0,1]} \hat{P}_{ik} \rho_{ik} dk, \quad (91)$$

where ρ_{ik} is an indicator function taking the value one if the firm k in location i is a regular firm and zero otherwise, and \hat{P}_{jk} refers to the price of security k in location j . The left-hand side of (91) corresponds to the sum of the investor’s bond and risky-security spending, while the right-hand side reflects the value of the (equal-weighted) portfolio of regular firms the investor is endowed with.

C.3 Signals

Each investor may obtain a signal of the type — regular or fraudulent — of every firm in every location. The precision of these signals depends on the locations of the investor and the firm.

Specifically, we assume that each fraudulent firm in every location is assigned in an i.i.d. fashion a uniformly distributed index $u \in [0, 1 - \nu]$, which reflects the difficulty with which it can be identified as fraudulent. This index is not observed by anyone.

After the index is drawn, investors in every location i may obtain a signal about each firm in location j . (All investors in i who choose to become informed obtain the same signal about any given firm.) This signal characterizes the firm as either regular or fraudulent. The signal is imperfect. It correctly identifies every regular firm as such. However, it fails to identify all fraudulent firms: it correctly identifies a fraudulent firm that has drawn an index u with probability π_u and misclassifies it as regular with probability $1 - \pi_u$. Note that we take π_u independent of i and j .

We introduce the index u to obtain a heterogeneous distribution of the demand for the shares of fraudulent firms in the same location by investors in other locations. We assume that for some (positive-measure) set of values u $\pi_u = 1$, i.e., there exist in every location a positive measure of fraudulent firms that are correctly classified as fraudulent by all informed agents. No one knows, however, the identity of these firms before trading takes place. (Even conditional on prices and each agent's own trades, only the swindler can infer the u of her own firm in equilibrium).

Given this setup, Bayes' rule implies that the probability that a firm in location j is regular given that investor i 's signal identifies it as regular is given by

$$p \equiv \frac{\nu}{\nu + \int_0^{1-\nu} (1 - \pi_u) du}. \quad (92)$$

The law of large numbers implies then that p can also be interpreted as the fraction of firms in a given location j that are regular, given that the signal of investor i has identified them as regular.

The signals are costly. Specifically, the investor in location i can acquire signals about all the firms in a given location j , by paying exactly the same "participation costs" that we assume in Section 3.2. We denote the cost function by \hat{F} .

C.4 Earnings manipulation and swindler's problem

We next introduce an assumption whose sole purpose is to ensure endogenously that agents do not short fraudulent shares. Before proceeding, we note that one can dispense with all the assumptions of this section, by simply imposing a no-shorting constraint.

Specifically, we assume that swindlers have the ability to manipulate the earnings of fraudulent firms. A swindler l in location i has the ability to borrow any amount $\hat{L}^{il} \geq 0$ of her choosing at time 0, divert these funds into the firm, and report earnings equal to \hat{L}^{il} in period 1. (Equivalently, we could assume that the swindler can take an action to produce earnings \hat{L}^{il} by incurring a personal non-pecuniary cost of effort, which would have a value \hat{L}^{il} in monetary terms.)

The budget constraint of a swindler is similar to (91) except that $\nu^{-1} \int_{k \in [0,1]} \hat{P}_{ik} \rho_{(i,k)} dk$ is replaced by \hat{P}_{il} :

$$\hat{B}^{sil} + \int_{j \in \mathcal{L}} \int_{k \in [0,1]} \hat{P}_{jk} d\hat{X}_{jk}^{sil} = \hat{P}_{il}. \quad (93)$$

Note that, as before, the notation allows investors' portfolios to have atoms, which is further useful here because, in equilibrium, swindlers optimally hold a non-infinitesimal quantity of shares of their own firms. We denote the post-trade number of shares held by the swindler who owns firm l in location i by $\hat{S}^{il} = d\hat{X}_{il}^{sil}$.

The time-1 wealth of a swindler is

$$\hat{W}_1^{sil} \equiv \hat{B}^{sil} + \int_{j \in \mathcal{L}} \int_{k \in [0,1]} \hat{D}_{jk} d\hat{X}_{jk}^{sil} + \hat{L}^{il} (\hat{S}^{il} - 1). \quad (94)$$

The difference with (90) is the term $\hat{L}^{il} (\hat{S}^{il} - 1)$, which represents the swindler's consumption gains when performing a diversion \hat{L}^{il} , while owning a post-trade number of shares in her company equal to \hat{S}^{il} . This increase is intuitive. If $\hat{S}^{il} - 1 < 0$, i.e., if the swindler reduces her ownership of shares by being a net seller, then she has no incentive to perform earnings diversion since she will recover only a fraction of her personal funds that she diverts into the company; thus $\hat{L}^{il} = 0$. If, however, the swindler is a net buyer of her own security ($\hat{S}^{il} - 1 > 0$), then the ability to manipulate earnings becomes infinitely valuable, since \hat{L}^{il} can be chosen arbitrarily large. Intuitively, the swindler can report arbitrarily large profits at the expense of outside investors who hold negative positions (short sellers) in the fraudulent firm. As a consequence, in equilibrium all other investors optimally refrain from shorting, even when they know for sure that the firm is fraudulent. The reasoning is as follows: With positive probability $\pi_u = 1$; accordingly all investors' signals identify the fraudulent firm as such, and investors do not find it optimal to submit a positive demand for that firm in equilibrium. Therefore, any prospective shorter understands that *any* short position that she establishes implies $\hat{S}^{il} > 1$, and is (unboundedly) loss-making. Since the quality of the signal (u) is not observed by anyone³³, a prospective shorter must assign positive probability to such an occurrence, and hence avoids short-selling.

Before proceeding, we reiterate that our earnings-manipulation assumption serves only as a deterrent to shorting, and is intentionally stylized so as to expedite the presentation of the results that follow. In reality there are many other reasons why investors are deterred from short selling firms with tightly controlled float, such as "short squeezes", which we do not model here.³⁴ The exact nature of the shorting deterrent is inconsequential for our results. (See, e.g., Lamont (2012) for an empirical study documenting various short-selling deterrence mechanisms employed by firms.)

C.5 Optimization problem

All investor are maximizing a CARA utility with parameter $\hat{\gamma}$ over time-1 wealth.

Common investors are price takers. Taking the interest rate, the prices for risky assets, and the actions of the swindlers as given for all firms in all locations, a common investor maximizes her expected utility subject to (91). The investor conditions on her own information set \mathcal{F}_i (i.e., on her signals about every security), as well as on the prices of all securities in all markets.

The problem of the swindler is similar to the one of the common investor with two exceptions: a) she takes into account the impact of her trading on the price of her stock, and b) she needs to decide whether to manipulate the earnings of her company. Similarly to a common investor, the swindler who owns firm l in location i maximizes her utility over \hat{B}^{sil} and $d\hat{X}_{jk}^{sil}$ subject to the budget constraint (93).

³³In equilibrium only the swindler can infer the u of her own firm.

³⁴A short squeeze refers to the possibility of cornering the shorting market by restricting the amount of securities that are available for lending and forcing short sellers to close out their positions. Short squeezes can be detrimental to short sellers. We do not model these deterrence mechanisms here since they would require the introduction of more trading periods in the model.

C.6 Equilibrium

An equilibrium is a collection of prices \hat{P}_i for all risky assets, asset demands, and bond holdings expressed by all investors in all locations, such that: 1) Markets for all securities clear: $\nu \int_{i \in \mathcal{L}} d\hat{X}_{jk}^{ci} + \int_{i \in \mathcal{L}} \int_{l=\nu}^1 d\hat{X}_{jk}^{sil} = 1$ for all j, k ; 2) Risky-asset and bond holdings, $\{\hat{X}_{jk}^{ci}, \hat{B}^{ci}\}$, are optimal for regular investors in all locations given prices and the investors' expectations; 3) Optimal bond holdings \hat{B}^{sil} , diversion amounts \hat{L}^{sil} , and asset holdings for all securities \hat{X}_{jk}^{sil} (including a swindler's own holdings of her own firm \hat{S}^{il}) are optimal for swindlers given their expectations; 4) All investors update their beliefs about the type of stock k in location j by using all available information to them — prices, interest rate, and private signals — and Bayes' rule, whenever possible.

Our equilibrium concept contains elements of both a rational expectations equilibrium and a Bayes-Nash equilibrium. All investors make rational inferences about the type of each security based on their signals, the equilibrium prices, and the interest rate, by using Bayes' rule and taking the optimal actions of all other investors (regular and swindlers) in all locations as given. The assumption of a continuum of regular investors implies that they are price takers in all markets. In forming dividend anticipations about a given security, they take prices and the optimal diversion strategies of swindlers as given.

Swindlers, on the other hand, are endowed with the shares of a fraudulent company and can manipulate its earnings. Thus they take into account the impact of their trades on the security they are trading. In formulating a demand for their security, swindlers have to consider how different prices might affect the perceptions of other investors about the type of their security. As is standard, Bayes' rule disciplines investors' beliefs only for demand realizations that are observed in equilibrium. As is usual in a Bayes-Nash equilibrium, there is freedom in specifying how out-of-equilibrium prices affect investor posterior distributions of security types.

We note that the distinction between regular investors, who are rational price-takers and swindlers who are strategic about the impact of their actions on the price of their firm is helpful for expediting the presentation of results, but not crucial. We can show³⁵ that our equilibrium concept is the limit (as the number of traders approaches infinity) of a sequence of economies with finite numbers of traders — both regular and swindlers — who are all strategic about their price impact and rational about their inferences, as in Kyle (1989).

C.7 Non-participation

We are finally ready to state the result of this appendix, which states that non-participation can arise as the result of a choice not to obtain signals about given markets. As in Section 3, we focus for simplicity on location invariance.

Proposition 7 *Consider an equilibrium to the economy in Section 3, defined by the price P and the share holdings $X_j^{(i)}$. Consider also the class of asymmetric-information economies, defined in this appendix, that are indexed by ν and obey the restrictions $\hat{\gamma}(\nu) = \gamma/\nu$, $\hat{F} = \nu F$, and $p(\nu)/\nu$ decreases in ν with $\lim_{\nu \rightarrow 0} p(\nu)/\nu = \infty$.*

Then there exists a value $\bar{\nu} > 0$ such that, as long as $\nu \leq \bar{\nu}$, an equilibrium exists in the asymmetric-information economy that exhibits the following properties.

- (i) *All prices in location i are equal, $\hat{P}_{ik} = \hat{P}_i = \hat{P} = pP$.*

³⁵See Gârleanu et al. (2013) for details.

- (ii) *There is neither shorting nor dividend manipulation in equilibrium— in particular, of fraudulent firms.*
- (iii) *If an investor i in the participation-cost economy participates in a set \mathcal{S}_i of locations, then the same investor acquires signals for locations \mathcal{S}_i and no others, and only trades in these locations; in fact, the share holdings are proportional (excluding swindler's holdings of own firm share):*

$$\int_{k \in \mathcal{L}} d\hat{X}_{jk}^{ci} = \int_{k \in \mathcal{L}} d\hat{X}_{jk}^{sil} = \frac{\nu}{p} dX_j^{(i)}. \quad (95)$$

Proposition 7 provides an information-theoretic interpretation of participation costs. The investors in the main body of the paper can be thought of as acting in an environment in which they have the following choice. They can either pay a “participation” cost — exactly the same as specified in Section 3.2 — to learn informative signals of the types of securities in a given location, or simply limit their investments in that location to uninformed (index) portfolios. According to Proposition 7, in case their information advantage in markets where they are informed is sufficiently large compared to markets where they are uninformed, then it is optimal to allocate their limited capital exclusively in locations where they are not subject to an informational disadvantage, thus foregoing some diversification benefits. Put differently, they only invest in a location when they have paid the cost to become informed about it.

C.8 Proof of Proposition 7

Proof. We structure the proof in two steps. We start by assuming that no shorting or dividend manipulation is possible, and verify that the prescribed prices and portfolios constitute an equilibrium. We then check that property (ii) obtains as a result of optimal behavior.

Step 1. Suppose first that investors in the asymmetric-information economy (AIE) obtain signals and invest in the same locations \mathcal{S}_i that the investors in the base-case, participation-cost economy (PCE) choose to participate in. We note that a bad signal identifies a firm as fraudulent for sure, and therefore the investor does not purchase any share in such a firm. Since the signals on any market j contain noise (so that every firm has the same probability of being misclassified), it is optimal to allocate the funds in location j in an equal way across the firms that are identified as regular. In particular, letting K_j^i be the set of firms in location j for which agent i 's signal is positive, the quantity

$$d\hat{Z}_j^i \equiv \int_{k \in K_j^i} \rho_{jk} d\hat{X}_{jk}^{ci} \quad (96)$$

is deterministic and equal to

$$d\hat{Z}_j^i = p \int_{k \in K_j^i} d\hat{X}_{jk}^{ci} \equiv p d\hat{X}_j^{ci}. \quad (97)$$

Given the prices, all competitive investors maximize

$$E \left[\int_{j \in \mathcal{S}_i} \int_{k \in K_j^i} \left(\hat{D}_{jk} - \hat{P} \right) d\hat{X}_{jk}^{ci} \right] - \frac{\hat{\gamma}}{2} Var \left(\int_{j \in \mathcal{S}_i} \int_{k \in K_j^i} \hat{D}_{jk} d\hat{X}_{jk}^{ci} \right) \quad (98)$$

$$\begin{aligned} &= E \left[\int_{j \in \mathcal{S}_i} \int_{k \in K_j^i} \left(\hat{D}_j - \hat{P} \frac{1}{p} \right) \rho_{jk} d\hat{X}_{jk}^{ci} \right] - \frac{\hat{\gamma}}{2} Var \left(\int_{j \in \mathcal{S}_i} \int_{k \in K_j^i} \hat{D}_j \rho_{jk} d\hat{X}_{jk}^{ci} \right) \\ &= \int_{j \in \mathcal{S}_i} \left(E[p\hat{D}_j] - \hat{P} \right) d\hat{X}_j^{ci} - \frac{\hat{\gamma}}{2} Var \left(\int_{j \in \mathcal{S}_i} p\hat{D}_j d\hat{X}_j^{ci} \right) \\ &= \int_{j \in \mathcal{S}_i} (E[D_j] - P) d\hat{Z}_j^i - \frac{\gamma}{2\nu} Var \left(\int_{j \in \mathcal{S}_i} D_j d\hat{Z}_j^i \right). \end{aligned} \quad (99)$$

It consequently follows that, if $dX_j^{(i)}$ satisfies the first-order conditions of the investor in the PCE, $d\hat{Z}_{jk}^i = \nu dX_j^{(i)}$ satisfies the first-order conditions of a common investor in the AIE. We therefore conclude that (95) holds on \mathcal{S}_i .

To verify that the markets for all regular firms clear, it suffices to note that common investors in i purchase $d\hat{Z}_j^{ci}$ shares of regular firms in location j , thus $\nu dX_j^{(i)}$. The swindlers in i solve exactly the same problem, in addition to that of trading in their own firm. Consequently the total demand for regular firms in location j is $\nu \int_i dX_j^{(i)} = \nu$, the same as the supply. We verify later that markets clear for the fraudulent firms, as well.

We next check that for ν sufficiently low no agent wishes to invest outside \mathcal{S}_i . The agent's concave objective implies that, if a better portfolio existed than the one optimal on \mathcal{S}_i , then trading towards that portfolio would be beneficial. Thus, if \hat{W}_1^{ci} is the optimal wealth achievable on \mathcal{S}_i , then, for $t \in (0, 1)$,

$$\begin{aligned} &E[\hat{W}_1^{ci}] - \frac{\hat{\gamma}}{2} Var(\hat{W}_1^{ci}) \\ &\leq E \left[\hat{W}_1^{ci} + t \int_{j,k} \left(\hat{D}_{jk} - \hat{P} \right) d\hat{X}_{jk} \right] - \frac{\hat{\gamma}}{2} Var \left(\hat{W}_1^{ci} + t \int_{j,k} \hat{D}_{jk} d\hat{X}_{jk} \right), \end{aligned}$$

which gives, by letting t go to zero,

$$E \left[\int_{j,k} \left(\hat{D}_{jk} - \hat{P} \right) d\hat{X}_{jk} \right] - \hat{\gamma} Cov \left(\hat{W}_1^{ci}, \int_{j,k} \hat{D}_{jk} d\hat{X}_{jk} \right) \geq 0. \quad (100)$$

Since both sides are linear in $d\hat{X}_{jk}$, and the inequality holds with equality for $j \in \mathcal{S}_i$, it must hold for at least one location $j \notin \mathcal{S}_i$. (Remember that the agent cannot distinguish between firms

in location j .) Thus,

$$\begin{aligned}
0 &\leq E \left[\int_k \left(\hat{D}_{jk} - \hat{P} \right) d\hat{X}_{jk} \right] - \hat{\gamma} Cov \left(\hat{W}_1^{ci}, \int_k \hat{D}_{jk} d\hat{X}_{jk} \right) \\
&= (\nu - \hat{P}) \int_k d\hat{X}_{jk} - \hat{\gamma} Cov \left(\hat{W}_1^{ci}, \nu \hat{D}_j \right) \int_k d\hat{X}_{jk} \\
&= (\nu - pP - \gamma Cov(W_1^i, \nu D_j)) \int_k d\hat{X}_{jk}, \\
&= \nu \left(1 - \gamma Cov(W_1^i, D_j) - \frac{p}{\nu} P \right) \int_k d\hat{X}_{jk},
\end{aligned} \tag{101}$$

where we used the fact that agent i in the AIE takes the same risky positions (when restricted to \mathcal{S}_i) as agent i in the PCE, multiplied by ν .

Expression (101), however, is clearly negative as long as $P > 0$ and p/ν large enough. This conclusion represents a contradiction, thus implying that the investor cannot achieve a higher utility by purchasing (a non-zero measure of) shares located outside \mathcal{S}_i .

We can write down the investor's certainty-equivalent as a function of the choice \mathcal{S}_i by combining (99) with (95) and adding the information costs:

$$\begin{aligned}
&\nu \left(\int_{j \in \mathcal{S}_i} (E[D_j] - P) dX_j^{(i)} - \frac{\gamma}{2} Var \left(\int_{j \in \mathcal{S}_i} D_j dX_j^{(i)} \right) \right) - \hat{F} \\
&= \nu \left(\int_{j \in \mathcal{S}_i} (E[D_j] - P) dX_j^{(i)} - \frac{\gamma}{2} Var \left(\int_{j \in \mathcal{S}_i} D_j dX_j^{(i)} \right) - F \right).
\end{aligned} \tag{102}$$

The tradeoff between diversification and costs is therefore the same in the AIE as in the PCE, implying that any choice of \mathcal{S}_i optimal in the AIE is optimal in the PCE.

Step 2. Consider now the problem of a swindler. Her investment in her own (fraudulent) firm is independent of the choices she makes with respect to information acquisition and investment in the other firms — in particular, in these respects she behaves just like a common investor.

In her own firm, the swindler submits a demand that may affect prices. We assume the following off-the-equilibrium-path beliefs: a firm whose price is not equal to P is inferred to be fraudulent with probability one. Since no one buys a firm believed fraudulent for sure, the swindler's only chance of making a profit off their fraudulent firm is to submit a demand that is perfectly elastic at price P . Indeed, it is immediate to see that, if the demand by investors other than the swindler at P is positive, then the swindler makes a profit, equal to P times this demand.

Furthermore, if no investor shorts this firm, then the aggregate demand (excluding the swindler) is positive, and therefore the swindler does not manipulate dividends. We discussed this decision in detail above, following equation (94).

Conversely, if the aggregate demand by investors other than the swindler were negative, then the swindler would manipulate in unlimited amounts — $\hat{L}^{il} = \infty$ — and the agents who shorted would make an infinite loss. The assumptions made on π (i.e., on the correlation structure of the firm-specific shocks u) imply that, with strictly positive probability, all investors' signals identify firm l as fraudulent. (However, investors do not know that other investors have also identified such a firm as fraudulent). This means that, with positive probability, no investor beside the swindler submits a positive demand for this firm. A shorting agent, therefore, would make an infinite loss

for any short position, since in that case market clearing would imply $\hat{S}^{il} > 1$.³⁶ ■

D An alternative formulation of the leverage constraint

In this section we elaborate further on the interaction between borrowing constraints and high price sensitivity to participation costs. Specifically, we introduce a “limited-liability” constraint that places an endogenous bound on borrowing, and show that it plays a similar role to constraint (23) in the text. In particular, we show a qualitatively similar amplification result to Section 5, as depicted in Figure 8: An increase in the participation cost parameter implies an amplified – and possibly discontinuous – reaction of the equilibrium price as compared to the case where the limited-liability constraint is not imposed.

Before formalizing and analyzing the constraint, we provide a new dividend structure. An important novel feature of this structure is that all dividends are non-negative, so that the notion of limited liability is economically meaningful. Specifically, let Γ_j be a Gamma process on $[0, 1)$, so that for $u > s$ we have

$$\Gamma_u - \Gamma_s \sim \Gamma(k(u - s); \nu).$$

Extending $d\Gamma$ to the entire real line as before — that is, via $d\Gamma_s = d\Gamma_{s \bmod 1}$ — we define

$$D_j = \mu + \int_{j-\frac{1}{2}}^{j+\frac{1}{2}} w_{s-j} d\Gamma_s \quad (103)$$

for some $\mu \geq 0$ and weights $w_i \geq 0$ periodic with period 1 and symmetric around 0. In the interest of concreteness, in our numerical illustration below we define $w_i = 1$ if $i \in [-\frac{1}{4}, \frac{1}{4}]$ and $w_i = 0$ otherwise. Conveniently, for this choice of w , D_j and $D_{j+\frac{1}{2}}$ are independent.

More important, specification (103) generally implies that dividends are positive and the joint distribution of the dividends in any n locations depends exclusively on the distances on the circle between the locations.

An agent located in location i maximizes utility over end-of-period wealth $W_{1,i}$ net of participation costs, that is, she maximizes

$$-\frac{1}{\gamma} E_0 e^{-\gamma(W_{1,i} - F_i)},$$

where F_i refers to the participation costs incurred by the agent, depending on her participation choices. For the participation costs we adopt the same structure as in Section 5. Specifically we assume that by paying a cost κ , an investor can participate not only in her location but also in the location diametrically “opposite” hers on the circle. Otherwise the investor can only invest in the risky asset in her own location. Proceeding as in Section 5, the indifference of agent i between

³⁶To make this argument precise for an economy with a continuum of agents, in Gârleanu et al. (2013) we consider a sequence of finite economies with increasing numbers of strategic traders and show that investors do not find it optimal to short stocks in any equilibrium along the sequence. However, their price impact declines monotonically as the number of traders increases. (Intuitively, the reason is that the danger of trading against a swindler receiving zero outside demand for her share is present no matter how small is the size of a short position.) Since we want to construe our economy with a continuum of agents as a limit of finite economies, we must assume that any short position against a swindler receiving zero outside demand would give rise to earnings manipulation.

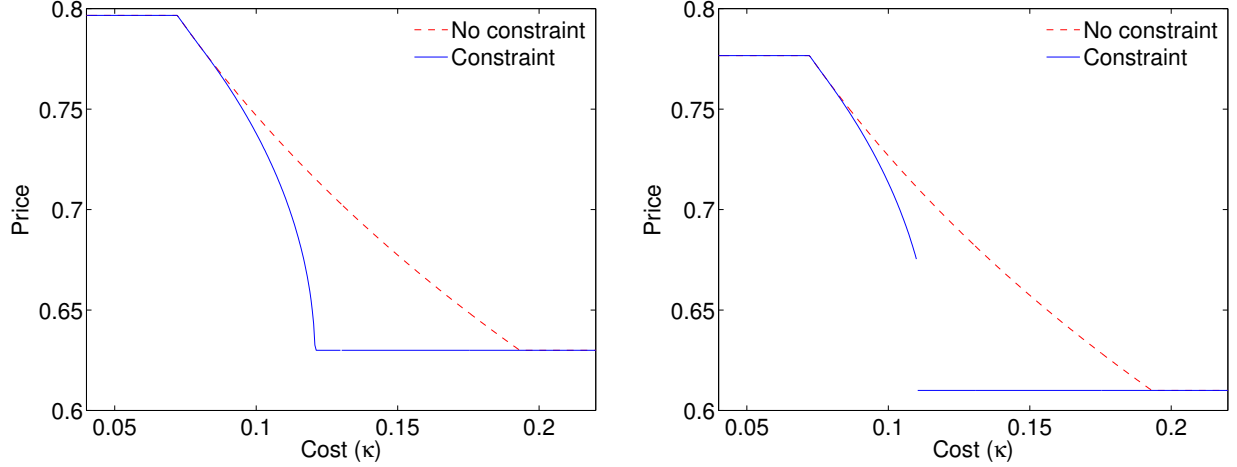


Figure 13: The figure illustrates the higher sensitivity of the price to the diversification cost in the presence of the constraint. In the left panel, the price decreases continuously to the value obtaining with no diversification. In the right panel, the price jumps to this value when $\kappa = \mu$. In either case, the slope of the solid line (the price in the presence of a leverage constraint) exhibits a steeper decline than the dotted line (the price in the absence of a leverage constraint). The common parameters used here are $k = 20$, $v = 10$, and $\gamma = 10$; in the left panel $\mu = 0.13$, while in the right one $\mu = 0.11$.

investing exclusively in location i and incurring the cost κ to participate also in location $i + \frac{1}{2}$ means

$$\max_{1-w_2^f} -Ee^{-\gamma\left(P+\frac{1}{2}(1-w_2^f)\sum_{j=i,i+\frac{1}{2}}(D_j-P)-\kappa\right)} = \max_{1-w_1^f} -Ee^{-\gamma\left(P+(1-w_1^f)(D_i-P)\right)}, \quad (104)$$

where we have used the definition of an agent's objective and her budget constraint. Note that $1 - w_2^f$, respectively $1 - w_1^f$, is the leverage choice of an agent who decides to invest across the two locations, respectively only in her own location.

For future reference, we provide an analytic expression for the dependence of P on κ in the absence of any constraint on leverage. The implicit function theorem applied to equation (104) yields³⁷

$$\frac{dP}{d\kappa} = -\frac{1}{w_1^f - w_2^f} < 0, \quad (105)$$

Now suppose that, due to the no-recourse nature of lending contracts, borrowing is restricted so as to ensure that there is no default in equilibrium.³⁸ Thus, borrowing is subject to the constraint

$$X_2^S D^{\min} + (X_2^B - \kappa) \geq 0, \quad (106)$$

³⁷It is possible to show that there exist values of κ for which only asymmetric equilibria exist.

³⁸Richer contracts, through which the borrower and lender share some risk, can be envisaged. Note, however, that such a contract would allow an agent (partial) diversification across locations at zero cost, and thus run counter to the central friction of the paper.

where D^{\min} is the smallest possible dividend in period 1, X_2^S is the number of shares chosen by investor a , and $-X_2^B$ is the amount borrowed by the investor. Using the time-zero budget constraint and noting that $D^{\min} = \mu$ and $X_2^S - 1 = -w_2^f$, equation (106) becomes

$$-w_2^f(P - \mu) \leq \mu - \kappa. \quad (107)$$

Attaching a Lagrange λ to the constraint (107) we obtain

$$\frac{dP}{d\kappa} = -\frac{1 + \lambda}{w_2^f - (1 + \lambda)w_1^f} < -\frac{1}{w_2^f - w_1^f} < 0. \quad (108)$$

Equation (108) shows an amplification effect: In the presence of the constraint small changes in κ translate into larger drops in the price than would obtain in its absence. This stronger reaction of the price in the presence of the constraint is illustrated in Figure 13, and is analogous to Figure 8, where the constraint is modeled as a collateral constraint. The intuition for the increased price sensitivity, captured by (108), is quite immediate: An increase in κ not only requires the price to decrease in order to avoid a decrease in V_2 relative to V_1 (the effect behind equation (105)); it also pushes the price down to counteract the direct effect of tightening of the constraint.

Depending on the parameters, the price may even drop discontinuously to the value obtaining in the no-diversification equilibrium. The point is made most starkly in the case $\kappa = 0$ and $\mu = 0$. At these values there is diversification, and no leverage. Any increase in κ , on the other hand, drives the price discontinuously down to the no-diversification value. More generally, it can happen that, as κ approaches μ , enough agents continue to diversify — even if their leverage is virtually zero — for the price to be above the no-diversification value that obtains when $\kappa > \mu$. However, once κ exceeds μ the price drops discontinuously, as the right panel of Figure 13 illustrates. Thus, as in Section 5, a small change in κ can cause the nature of the equilibrium to change, which induces a discontinuous change in the price.

To conclude, even if one modeled borrowing limitations as resulting from a no-default requirement, the price function is steeper in κ than when the constraint is absent, and can even be discontinuous, similar to section 5, where the constraint is modeled as a simple leverage constraint.