$$
\begin{aligned}
&A\cos(ct-\varphi)\\
&= A(\cos ct\cos(-\varphi)-\sin ct\sin(-\varphi)) && \text{(by 4(i))}\\
&= A(\cos ct\cos\varphi+\sin ct\sin\varphi) && \text{(cos is even and sin is odd)}\\
&= \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cos ct+\frac{b}{\sqrt{a^2+b^2}}\sin ct\right) && \text{(substitution)}\\
&= a\cos ct+b\sin ct.
\end{aligned}
$$

□