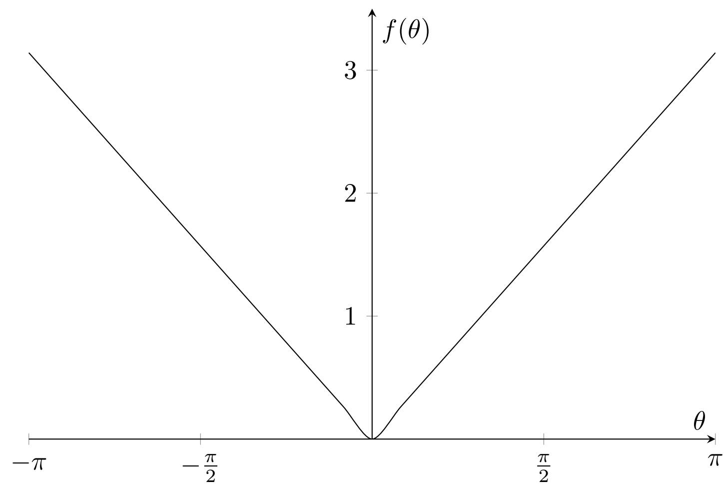
(a)



(b) The Fourier coefficients are

$$\begin{aligned}\hat{f}(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |\theta|\, e^{-in\theta} d\theta \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |\theta| \cos n\theta d\theta - \frac{1}{2\pi} i \int_{-\pi}^{\pi} |\theta| \sin n\theta d\theta.\end{aligned}$$

$|\theta| \cos n\theta$ is even and $|\theta| \sin n\theta$ is odd, so

$$\hat{f}(n) = \frac{1}{\pi}\int_0^{\pi} \theta \cos n\theta d\theta.$$

We have $\hat{f}(0) = \pi/2$. For $n \neq 0$, we have

$$\begin{aligned}\hat{f}(n) &= \frac{1}{\pi}\int_0^{\pi} \theta \cos n\theta d\theta \\ &= \frac{\cos \pi n + \pi n \sin \pi n - 1}{\pi n^2} \\ &= \frac{-1 + (-1)^n}{\pi n^2}\end{aligned}$$

□

(c) The series is cosine since f is even, so

$$f(\theta) \sim \frac{\pi}{2} + \sum_{n \neq 0} \frac{-1 + (-1)^n}{\pi n^2} \cos n\theta.$$

Eliminating zero terms,

$$f(\theta) \sim \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ odd} \geq 1} \frac{\cos n\theta}{n^2}. \tag{1}$$

(d) Observe that $\sum \left|n^{-2}\right|$ converges absolutely, so we have equality in (1). Taking $\theta = 0$, we get

$$
\begin{aligned}
0 &= \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ odd} \geq 1} \frac{1}{n^2} \\
\iff \sum_{n \text{ odd} \geq 1} \frac{1}{n^2} &= \frac{\pi^2}{8}.
\end{aligned}
$$

From example 2 in section 1.1, we have

$$
g(\theta) \sim \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos n\theta}{n^2}
$$

when $g(\theta) = (\pi - \theta)^2/4$ for $0 \leq \theta \leq 2\pi$. Since $\sum n^{-2}$ converges absolutely, we also have equality. Taking $\theta = 0$,

$$
\begin{aligned}
\frac{\pi^2}{4} &= \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2} \\
\iff \sum_{n=1}^{\infty} \frac{1}{n^2} &= \frac{\pi^2}{6}.
\end{aligned}
$$

$\square$