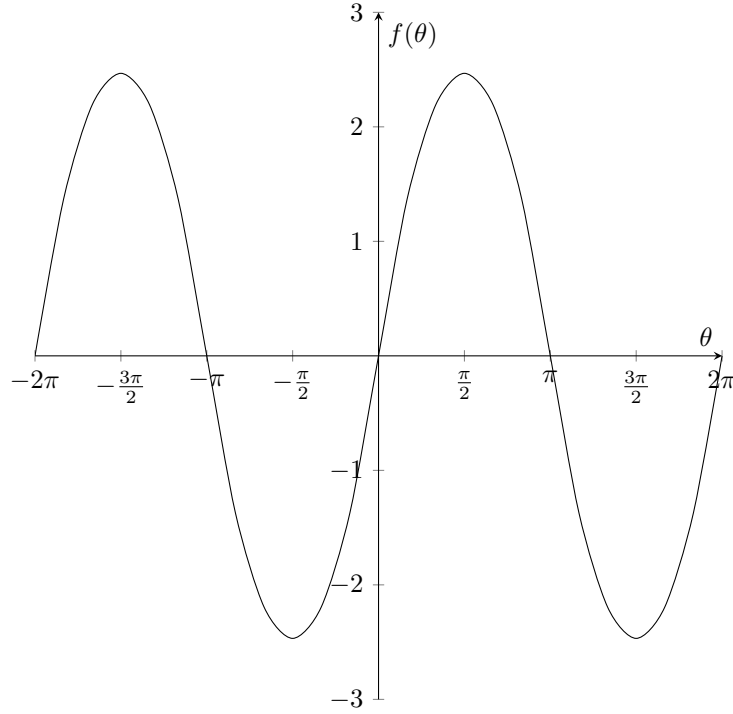


(a)



(b) Since  $f$  is odd, its Fourier series is sine with coefficients

$$\begin{aligned}
 A_k &= \frac{2}{\pi} \int_0^\pi f(\theta) \sin k\theta d\theta \\
 &= \frac{2}{\pi} \int_0^\pi \theta(\pi - \theta) \sin k\theta d\theta \\
 &= \frac{2 - \pi k \sin \pi k - 2 \cos \pi k}{k^3} \\
 &= \frac{2 - 2(-1)^k}{k^3}
 \end{aligned}$$

For  $k$  even,  $A_k = 0$ . For  $k$  odd,  $A_k = 4/k^3$ . Thus,

$$f(x) \sim \frac{2}{\pi} \sum_{k=1}^{\infty} A_k \sin k\theta = \frac{8}{\pi} \sum_{k \text{ odd} \geq 1} \frac{\sin k\theta}{k^3}.$$

Since  $\sum |k^{-3}|$  converges absolutely, we have equality by corollary 2.3:

$$f(x) = \frac{8}{\pi} \sum_{k \text{ odd} \geq 1} \frac{\sin k\theta}{k^3}.$$

□