For all $n = 1, 2, \dots$ and $m = 1, \dots, n$ define the interval $I_{nm} \subseteq [0, 2\pi]$ as the m^{th} slice when $[0, 2\pi]$ is divided into n, or

$$I_{nm} = \begin{cases} [0, 2\pi/n] & \text{for } m = 1 \\ (2\pi(m-1)/n, 2\pi m/n] & \text{for } m = 2, \dots, n. \end{cases}$$

Define a sequence of intervals $\{J_k\}_{k=1}^{\infty}$ to enumerate I_{nm} with

$$\begin{aligned} &J_1 = I_{11}, \\ &J_2 = I_{21}, J_3 = I_{22}, \\ &J_4 = I_{31}, J_5 = I_{32}, J_6 = I_{33}, \end{aligned}$$

and so on. Define a sequence of functions $\{f_k\}_{k=1}^{\infty}$ by $f_k = \chi_{J_k}$, where χ is the indicator function.

Then for every k, $f_k = \chi_{J_k} = \chi_{I_{nm}}$ for some n and m and

$$\frac{1}{2\pi} \int_0^{2\pi} |f_k(\theta)|^2 \, d\theta = \frac{1}{2n\pi}.$$

This tends to 0 as we limit $k \to \infty$ because $n \to \infty$ with k.

Observe that every point $x = [0, 2\pi]$ belongs to J_k for infinitely many k, so $f_k(x) = 1$ infinitely many times. However, there are also infinitely many k for which J_k does not contain x and $f_k(x) = 0$. Hence, the limit $\lim_{k\to\infty} f_k(x)$ does not exist for any x.