The n^{th} Dirichlet D_n is defined as

$$D_n(x) = \sum_{k=-n}^{n} e^{ikx}.$$

This is a geometric sequence, which we can simplify to

$$D_n(x) = \frac{\omega^{-n} - \omega^{n+1}}{1-\omega},$$

where we let $\omega = e^{ix}$

F_N is defined as

$$NF_N(x) = \sum_{n=0}^{N-1} D_n(x)$$

for $N = 1, 2, \ldots$. Then

$$\begin{aligned}
NF_N(x) &= \sum_{n=0}^{N-1} \frac{\omega^{-n} - \omega^{n+1}}{1-\omega} \\
&= \frac{1}{1-\omega} \sum_{n=0}^{N-1} \left(\omega^{-n} - \omega^{n+1}\right) \\
&= \frac{\omega^0 - \omega^1 + \omega^{-1} - \omega^2 + \omega^{-2} - \omega^3 + \cdots + \omega^{1-N} - \omega^N}{1-\omega} \\
&= \frac{\omega^0 + \omega^{-1} + \cdots + \omega^{1-N}}{1-\omega} - \frac{\omega^N + \omega^{N-1} + \cdots + \omega^1}{1-\omega} \\
&= \frac{1-\omega^{-N}}{(1-\omega)(1-\omega^{-1})} - \frac{\omega^N - 1}{(1-\omega)(1-\omega^{-1})} \\
&= \frac{2 - \omega^N - \omega^{-N}}{2 - \omega - \omega^{-1}} \\
&= \frac{1 - \cos Nx}{1 - \cos x} \\
&= \frac{\sin^2(Nx/2)}{\sin^2(x/2)},
\end{aligned}$$

where for the last line we use

$$\begin{aligned}
1 - \cos\alpha &= \cos(\alpha/2 - \alpha/2) - \cos(\alpha/2 + \alpha/2) \\
&= 2\sin(\alpha/2)\sin(\alpha/2) \\
&= 2\sin^2(\alpha/2).
\end{aligned}$$

Then

$$F_N(x) = \frac{1}{N} \frac{\sin^2(Nx/2)}{\sin^2(x/2)}.$$

$\square$