

We are given that

$$f(x) = \begin{cases} \frac{xh}{p} & \text{for } 0 \leq x \leq p \\ \frac{h(\pi-x)}{\pi-p} & \text{for } p \leq x \leq \pi \end{cases}$$

From the formula for the Fourier sine coefficients, we have

$$\begin{aligned} A_m &= \frac{2}{\pi} \int_0^\pi f(x) \sin mx \, dx \\ &= \frac{2}{\pi} \int_0^p \frac{xh}{p} \sin mx \, dx + \frac{2}{\pi} \int_p^\pi \frac{h(\pi-x)}{\pi-p} \sin mx \, dx, \end{aligned}$$

where we use the fact that $f(x)$ is piecewise. By algebra,

$$A_m = \frac{2h}{\pi p} \int_0^p x \sin mx \, dx + \frac{2h}{(\pi-p)} \int_p^\pi \sin mx \, dx - \frac{2h}{\pi(\pi-p)} \int_p^\pi x \sin mx \, dx \quad (1)$$

Integrating,

$$\begin{aligned} \int_p^\pi \sin mx \, dx &= -\frac{1}{m} [\cos mx]_p^\pi \\ &= \frac{1}{m} \cos mp - \frac{1}{m} \cos \pi m. \end{aligned}$$

Letting $a, b \in \mathbb{R}$ and integrating by parts,

$$\begin{aligned} \int_a^b x \sin mx \, dx &= \left[x \int \sin mx \, dx \right]_a^b - \int_a^b \frac{dx}{dx} \int \sin mx \, dx \, dx \\ &= -\frac{1}{m} [x \cos mx]_a^b + \frac{1}{m} \int_a^b \cos mx \, dx \\ &= -\frac{1}{m} [x \cos mx]_a^b + \frac{1}{m^2} [x \sin mx]_a^b \\ &= \frac{1}{m} (a \cos ma - b \cos mb) + \frac{1}{m^2} (\sin mb - \sin ma). \end{aligned}$$

Substituting for a and b ,

$$\begin{aligned} \int_0^p x \sin mx \, dx &= -\frac{p}{m} \cos mp + \frac{1}{m^2} \sin mp \\ \int_p^\pi x \sin mx \, dx &= \frac{p}{m} \cos mp - \frac{\pi}{m} \cos \pi m - \frac{1}{m^2} \sin mp. \end{aligned}$$

Substituting into (1),

$$\frac{m}{2h} A_m = -\frac{1}{\pi} \cos mp + \frac{1}{\pi mp} \sin mp$$

$$\begin{aligned}
& + \frac{1}{(\pi - p)} \cos mp - \frac{1}{(\pi - p)} \cos \pi m \\
& - \frac{p}{\pi(\pi - p)} \cos mp + \frac{1}{(\pi - p)} \cos m\pi + \frac{1}{\pi m(\pi - p)} \sin mp \\
& = -\frac{1}{\pi} \cos mp + \frac{1}{(\pi - p)} \cos mp - \frac{p}{\pi(\pi - p)} \cos mp \\
& \quad + \frac{1}{\pi m p} \sin mp + \frac{1}{\pi m(\pi - p)} \sin mp \\
& = \left(\frac{1}{\pi - p} - \frac{1}{\pi} - \frac{p}{\pi(\pi - p)} \right) \cos mp + \left(\frac{1}{\pi m p} + \frac{1}{\pi m(\pi - p)} \right) \sin mp.
\end{aligned}$$

Observe that

$$\frac{1}{\pi - p} - \frac{1}{\pi} - \frac{p}{\pi(\pi - p)} = \frac{\pi - \pi + p - p}{\pi(\pi - p)} = 0$$

and

$$\frac{1}{\pi m p} + \frac{1}{\pi m(\pi - p)} = \frac{\pi - p + p}{\pi m p(\pi - p)} = \frac{1}{m p(\pi - p)},$$

so

$$A_m = \frac{2h}{m^2} \frac{\sin mp}{p(\pi - p)}$$

as desired.

For $0 < h$, $0 < p < \pi$, we have $A_m = 0$ iff $\sin mp = 0$. When $p = \pi/2$, $\sin m\pi/2 = 0$ for $m = 2, 4, \dots$, so the second, fourth, and so on, harmonics are missing. Similarly, when $p = \pi/3$, $\sin m\pi/3 = 0$ for $m = 3, 6, \dots$, so the third, sixth, and so on, harmonics are missing.