

Let

$$A_m = \frac{2h}{m^2} \frac{\sin mp}{p(\pi - p)} \quad \text{and} \quad f(x) = \begin{cases} \frac{xh}{p} & \text{for } 0 \leq x \leq p, \\ \frac{h(\pi - x)}{\pi - p} & \text{for } p \leq x \leq \pi. \end{cases}$$

Let $C = 2h/p(\pi - p) \geq 0$. Then

$$|A_m| = C \frac{|\sin mp|}{m^2} \leq \frac{C}{m^2},$$

so

$$|A_m \sin mp| = |A_m| |\sin mx| \leq \frac{C}{m^2}.$$

Since $\sum_{m=1}^{\infty} 1/m^2$ converges,

$$\sum_{m=1}^{\infty} A_m \sin mp$$

converges absolutely.

We also know that f is continuous and by chapter 1, exercise 9, we have that $\sum_{m=1}^{\infty} A_m \sin mp$ is the Fourier series of f . Then by corollary 2.3,

$$f(x) = \sum_{m=1}^{\infty} A_m \sin mp$$

for all $x \in [0, \pi]$. □