The Weiner process is a Gaussian process with mean zero and a non-stationary covariance function $k_w(x, x') = \min(x, x')$, which is defined for $x \geq 0$. Notice that for $x_* = 0$, $\bar{f}_* = 0$ by our prior and $\mathbb{V}[f_*] = k_w(0, 0) = 0$, so the Weiner process always has $f(0) = 0$.

The Brownian bridge (or *tied-down Weiner process*) is a Weiner process that passes through $f(1) = 0$. We obtain its equations by conditioning the Weiner process on this.

We have

$$\bar{f}_* = k_w(x_*, x)((k_w(x, x))^{-1}y = 0$$

by equation (2.25) in the book, since $x = 1$ and $y = 0$.

We also have

$$\begin{aligned}
\operatorname{cov}(\mathbf{f}_*) &= k_w(\mathbf{x}_*^\top, \mathbf{x}_*^\top) - k_w(\mathbf{x}_*^\top, x)(k_w(x, x))^{-1}k_w(x, \mathbf{x}_*^\top) \\
&= k_w(\mathbf{x}_*^\top, \mathbf{x}_*^\top) - k_w(\mathbf{x}_*^\top, 1)k_w(1, \mathbf{x}_*^\top)
\end{aligned}$$

by equation (2.24). Note that $\mathbf{x}_*$ is a vector of n_* 1-dimensional test points and $\mathbf{k}$ is a vector of n_* evaluations of the test points against the one training point.

We can use this covariance to define a kernel

$$\begin{aligned}
k_b(x, x') &= \operatorname{cov}(f(x), f(x')) \\
&= k_w(x, x') - k_w(x, 1)k_w(1, x') \\
&= \min(x, x') - \min(x, 1)\min(x', 1).
\end{aligned}$$

For $0 \leq x, x' \leq 1$,

$$k_b(x, x') = \min(x, x') - xx'.$$

The Brownian bridge is thus a Gaussian process with mean 0 and covariance $k_b(x, x') = \min(x, x') - xx'$.

A computer program that draws samples from this process at a finite grid of points in $[0, 1]$ is included in the repository.