Let X be a matrix of $n-1$ observations. Let $\mathbf{x}_\circ$ be an additional observation. Let $\mathbf{x}_*$ be an arbitrary test point.

Define

$$\begin{aligned}
K &= k(X, X), \\
\mathbf{k}_\circ &= k(X, \mathbf{x}_\circ), \\
k_{\circ\circ} &= k(\mathbf{x}_\circ, \mathbf{x}_\circ), \\
\mathbf{k}_* &= k(X, \mathbf{x}_*), \\
k_{\circ *} &= k(\mathbf{x}_\circ, \mathbf{x}_*), \\
k_{**} &= k(\mathbf{x}_*, \mathbf{x}_*).
\end{aligned}$$

Then setting $M = K + \sigma_n^2 I$ and $c = k_{\circ\circ} + \sigma_n^2 - \mathbf{k}_\circ^\top M^{-1}\mathbf{k}_\circ$,

$$\begin{aligned}
&\mathrm{var}_{n-1}(f(\mathbf{x}_*)) \\
&= k_{**} - \mathbf{k}_*^\top \left(K + \sigma_n^2 I\right)^{-1} \mathbf{k}_*, \\
&= k_{**} - \mathbf{k}_*^\top M^{-1} \mathbf{k}_*,
\end{aligned}$$

$$\begin{aligned}
&\mathrm{var}_n(f(\mathbf{x}_*)) \\
&= k_{**} - \begin{bmatrix} \mathbf{k}_*^\top & k_{\circ *} \end{bmatrix} \left(\begin{bmatrix} K & \mathbf{k}_\circ \\ \mathbf{k}_\circ^\top & k_{\circ\circ} \end{bmatrix} + \sigma_n^2 I \right)^{-1} \begin{bmatrix} \mathbf{k}_* \\ k_{\circ *} \end{bmatrix} \\
&= k_{**} - \begin{bmatrix} \mathbf{k}_*^\top & k_{\circ *} \end{bmatrix} \begin{bmatrix} K + \sigma_n^2 I & \mathbf{k}_\circ \\ \mathbf{k}_\circ^\top & k_{\circ\circ} + \sigma_n^2 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{k}_* \\ k_{\circ *} \end{bmatrix} \\
&= k_{**} - \begin{bmatrix} \mathbf{k}_*^\top & k_{\circ *} \end{bmatrix} \begin{bmatrix} M^{-1} + c^{-1} M^{-1} \mathbf{k}_\circ \mathbf{k}_\circ^\top M^{-1} & -c^{-1} M^{-1} \mathbf{k}_\circ \\ -c^{-1} \mathbf{k}_\circ^\top M^{-1} & c^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{k}_* \\ k_{\circ *} \end{bmatrix},
\end{aligned}$$

where we perform inversion using an identity from section A.3. Then

$$\begin{aligned}
&\mathrm{var}_{n-1}(f(\mathbf{x}_*)) - \mathrm{var}_n(f(\mathbf{x}_*)) \\
&= c^{-1} \mathbf{k}_*^\top M^{-1} \mathbf{k}_\circ \mathbf{k}_\circ^\top M^{-1} \mathbf{k}_* - c^{-1} \mathbf{k}_*^\top M^{-1} \mathbf{k}_\circ - c^{-1} \mathbf{k}_\circ^\top M^{-1} \mathbf{k}_* + c^{-1} \\
&= \frac{\left(\mathbf{k}_\circ^\top M^{-1} \mathbf{k}_*\right)^2 - 2\mathbf{k}_\circ^\top M^{-1} \mathbf{k}_* + 1}{c} \\
&= \frac{\left(1 - \mathbf{k}_\circ^\top M^{-1} \mathbf{k}_*\right)^2}{c} \\
&= \frac{\left(1 - \mathbf{k}_\circ^\top \left(K + \sigma_n^2 I\right)^{-1} \mathbf{k}_*\right)^2}{k_{\circ\circ} + \sigma_n^2 - \mathbf{k}_\circ^\top \left(K + \sigma_n^2 I\right)^{-1} \mathbf{k}_\circ}.
\end{aligned}$$

The numerator is the square of a real number, so it is non-negative. The denominator is the Schur complement of

$$k\left(\begin{bmatrix} X & \mathbf{x}_\circ \end{bmatrix}, \begin{bmatrix} X & \mathbf{x}_\circ \end{bmatrix}\right) + \sigma_n^2 I = \begin{bmatrix} K + \sigma_n^2 I & \mathbf{k}_\circ \\ \mathbf{k}_\circ^\top & k_{\circ\circ} + \sigma_n^2 \end{bmatrix}.$$

By assumption k is positive-definite and $\sigma_n^2 > 0$, so this matrix is also positive definite as the sum of positive-definite matrices. Then its Schur complement is positive by Boyd and Vandenberghe, section A.5.5.

Hence, $\mathrm{var}_{n-1}(f(\mathbf{x}_*)) - \mathrm{var}_n(f(\mathbf{x}_*)) \geq 0$ and

$$\mathrm{var}_{n-1}(f(\mathbf{x}_*)) \geq \mathrm{var}_n(f(\mathbf{x}_*)).$$

$\square$