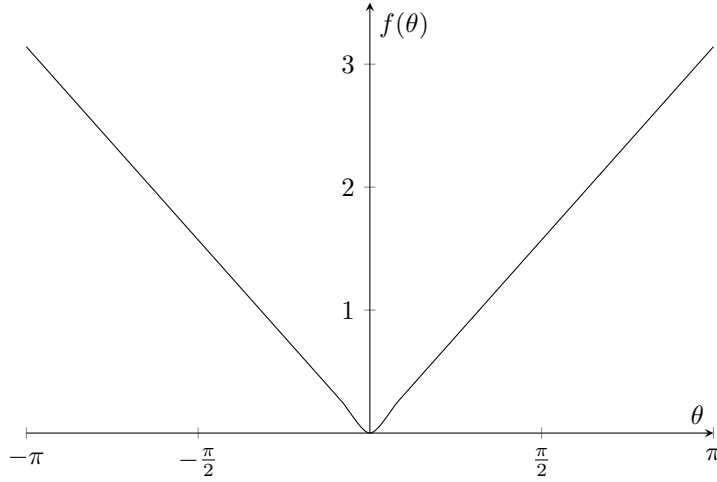


(a)



(b) The Fourier coefficients are

$$\begin{aligned}\hat{f}(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |\theta| e^{-in\theta} d\theta \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |\theta| \cos n\theta d\theta - \frac{1}{2\pi} i \int_{-\pi}^{\pi} |\theta| \sin n\theta d\theta.\end{aligned}$$

$|\theta| \cos n\theta$  is even and  $|\theta| \sin n\theta$  is odd, so

$$\hat{f}(n) = \frac{1}{\pi} \int_0^{\pi} \theta \cos n\theta d\theta.$$

We have  $\hat{f}(0) = \pi/2$ . For  $n \neq 0$ , we have

$$\begin{aligned}\hat{f}(n) &= \frac{1}{\pi} \int_0^{\pi} \theta \cos n\theta d\theta \\ &= \frac{\cos \pi n + \pi n \sin \pi n - 1}{\pi n^2} \\ &= \frac{-1 + (-1)^n}{\pi n^2}\end{aligned}$$

□

(c) The series is cosine since  $f$  is even, so

$$f(\theta) \sim \frac{\pi}{2} + \sum_{n \neq 0} \frac{-1 + (-1)^n}{\pi n^2} \cos n\theta.$$

Eliminating zero terms,

$$f(\theta) \sim \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ odd} \geq 1} \frac{\cos n\theta}{n^2}. \quad (1)$$

- (d) Observe that  $\sum |n^{-2}|$  converges absolutely, so we have equality in (1). Taking  $\theta = 0$ , we get

$$\begin{aligned} 0 &= \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ odd} \geq 1} \frac{1}{n^2} \\ \iff \sum_{n \text{ odd} \geq 1} \frac{1}{n^2} &= \frac{\pi^2}{8}. \end{aligned}$$

From example 2 in section x.x, we have

$$g(\theta) \sim \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos n\theta}{n^2}$$

when  $g(\theta) = (\pi - \theta)^2/4$  for  $0 \leq \theta \leq 2\pi$ . Since  $\sum n^{-2}$  converges absolutely, we also have equality. Taking  $\theta = 0$ ,

$$\begin{aligned} \frac{\pi^2}{4} &= \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2} \\ \iff \sum_{n=1}^{\infty} \frac{1}{n^2} &= \frac{\pi^2}{6}. \end{aligned}$$

□