Define

$$s_k = \sum_{j=0}^{k} c_j \quad \text{and} \quad \sigma_n = \frac{1}{n+1} \sum_{k=0}^{n} s_k.$$

We are given that $s_k \to s$ as $k \to \infty$.

Pick an arbitrary $\epsilon > 0$.

Since $s_k \to s$, there exists B such that $|s_k - s| < B$ for all k. There also exists $N \geq 0$ such that for all $k \geq N$, $|s_k - s| \leq \varepsilon/2$.

Set $M = \max\{0, 2NB/\varepsilon - N - 1\}$. Then for for all $n \geq M$,

$$\begin{aligned}
&2N\frac{B}{\varepsilon} - N - 1 \leq M \leq n \\
&\iff BN + (n + 1 - N)\frac{\varepsilon}{2} \leq (n+1)\varepsilon \\
&\implies \sum_{k=0}^{N-1} |s_k - s| + \sum_{k=N}^{n} |s_k - s| \leq (n+1)\varepsilon \\
&\implies \left| \sum_{k=0}^{n} s_k - (n+1)s \right| \leq (n+1)\varepsilon \\
&\iff \left| \frac{1}{n+1} \sum_{k=0}^{n} s_k - s \right| = |\sigma_n - s| \leq \varepsilon.
\end{aligned}$$

$\square$