We have

$$\begin{aligned}
\left|\hat{f}_k(n) - \hat{f}_k(n)\right| &= \left|\int_0^1 f_k(x)e^{-2\pi inx}\,dx - \int_0^1 f(x)e^{-2\pi inx}\,dx\right| \\
&= \left|\int_0^1 \left(f_k(x) - f(x)\right)e^{-2\pi inx}\,dx\right| \\
&\leq \int_0^1 \left|\left(f_k(x) - f(x)\right)e^{-2\pi inx}\right|\,dx \\
&= \int_0^1 |f_k(x) - f(x)|\,dx.
\end{aligned}$$

The last integral is independent of n. It tends to 0 as $k \to \infty$ by assumption. □