(a) Define
$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}.$$
We first show that this series converges for every $z \in \mathbb{C}$.

Define
$$a_n = \frac{|z^n|}{n!}.$$
Then
$$\frac{a_{n+1}}{a_n} = \frac{\left|z^{n+1}\right| n!}{|z^n|\,(n+1)!} = \frac{|z|^{n+1}\,n!}{|z|^n\,(n+1)!} = \frac{|z|}{n+1}.$$
Applying the ratio test,
$$\lim_{n\to\infty} \frac{|z|}{n+1} = 0,$$
so the series $\sum_{n=0}^{\infty} a_n$ converges.

Recalling that $|z^n/n!| = |z^n|/n! = a_n$, we have that
$$e^z = \sum_{n=0}^{z^n}$$
converges by 3(c).

We now show that the convergence is uniform on every bounded subset of $\mathbb{C}$. Pick an arbitrary bounded $S \subset \mathbb{C}$ and an arbitrary $\epsilon > 0$. We will show that there exists an integer M such that for all $N > M$ and $s \in S$,
$$\left| \sum_{n=0}^{N} \frac{z^n}{n!} - e^x \right| < \epsilon. \tag{1}$$
Note that (1) is equivalent to
$$\left| \sum_{n=N+1}^{\infty} \frac{z^n}{n!} \right| < \epsilon \tag{2}$$
after cancelling the first N terms of the series.

Choose c such that $c > |s|$ for all $s \in S$. We know from above that
$$e^c = \sum_{n=0}^{\infty} \frac{c^n}{n!}$$
converges. Then there exists an integer M such that for all $N > M$,
$$\left| \sum_{n=0}^{N} \frac{c^n}{n!} - e^c \right| < \epsilon,$$

or after cancelling the first N terms of the series,

$$\sum_{n=N+1}^{\infty} \frac{c^n}{n!} < \epsilon.$$

Observe that for all n,

$$\frac{c^n}{n!} > \frac{|z|^n}{n!} = \left|\frac{z^n}{n!}\right|,$$

so

$$\sum_{n=N+1}^{\infty} \left|\frac{z^n}{n!}\right| < \epsilon.$$

For every partial sum from $N+1$ to some N' we have

$$\left|\sum_{n=N+1}^{N'} \frac{z^n}{n!}\right| \leq \sum_{n=N+1}^{N'} \left|\frac{z^n}{n!}\right|$$

by the triangle inequality. Taking the limit,

$$\begin{aligned}\left|\sum_{n=N+1}^{\infty} \frac{z^n}{n!}\right| &\leq \sum_{n=N+1}^{\infty} \left|\frac{z^n}{n!}\right| \\ &< \epsilon,\end{aligned}$$

which matches (2), concluding the proof. $\square$