Let $f : [-\pi, \pi] \to \mathbb{R}$ such that

$$f(\theta) = \begin{cases} 0 & \text{if } |\theta| > \delta, \\ 1 - |\theta|/\delta & \text{if } |\theta| \le \delta. \end{cases}$$

f is even, so its Fourier series is cosine with coefficients

$$A_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) \cos n\theta d\theta.$$

For $n = 0$, we have

$$\begin{aligned} A_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) d\theta \\ &= \frac{1}{2\pi} \int_{-\delta}^{\delta} \left(1 - \frac{|\theta|}{\delta}\right) d\theta \\ &= \frac{1}{\pi} \int_{0}^{\delta} \left(1 - \frac{\theta}{\delta}\right) d\theta \\ &= \frac{\delta}{2\pi}. \end{aligned}$$

For $n = 1, 2, \ldots$, we have

$$\begin{aligned} A_n = A_{-n} &= \frac{1}{\pi} \int_{0}^{\delta} \left(1 - \frac{\theta}{\delta}\right) \cos n\theta d\theta \\ &= \frac{1}{\pi} \int_{0}^{\delta} \cos n\theta d\theta - \frac{1}{\pi\delta} \int_{0}^{\delta} \theta \cos n\theta d\theta \\ &= \frac{\sin n\delta}{\pi n} - \frac{n\delta \sin n\delta + \cos n\delta - 1}{n^2 \pi \delta} \\ &= \frac{1 - \cos n\delta}{n^2 \pi \delta} \end{aligned}$$

Hence,

$$f(\theta) \sim \frac{\delta}{2\pi} + 2 \sum_{n=1}^{\infty} \frac{1 - \cos n\delta}{n^2 \pi \delta} \cos n\theta.$$

We conclude by proving equality. It is easy to see that f is a continuous function on the circle. We can bound

$$\left| \frac{1 - \cos n\delta}{n^2 \pi \delta} \right| \le \frac{2}{n^2 \pi \delta},$$

so $\sum_{n=1}^{\infty} A_n$ is absolutely convergent. Hence, by corollary 2.3,

$$f(\theta) = \frac{\delta}{2\pi} + 2 \sum_{n=1}^{\infty} \frac{1 - \cos n\delta}{n^2 \pi \delta} \cos n\theta.$$

□