Technical Report: Modified DAC Method for Moving Rigid Body Localization Using TOAs Only

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***Abstract*—This report presents the modified divide-and-conquer (DAC) method for moving rigid body localization (MRBL) using time-of-arrival (TOA) measurements only.**

I. Modified DAC Method for MRBL Using TOAs Only

Since the modified DAC method for MRBL is very similar to the original DAC method in [1], we only give the key steps in the following.

1) Estimation of the Sensor Positions: The transformed model (25) is related to the global sensor positions and it can be used to estimate them. More specifically, by letting $\boldsymbol{\theta}_1 = [\boldsymbol{s}_{i,k}^{oT}, \|\boldsymbol{s}_{i,k}^{o}\|^2]^T$ for any $i \in \{1,\ldots,N\}$ and $k \in \{1,\ldots,K\}$, (25) can be written in a matrix form as

$$\boldsymbol{h}_1 - \boldsymbol{G}_1\boldsymbol{\theta}_1 = \boldsymbol{B}_1\boldsymbol{\nu}_1, \tag{1}$$

where

$$\begin{aligned}
&\boldsymbol{G}_1 = \begin{bmatrix} -2\boldsymbol{a}_1^T & 1 \\ \vdots & \vdots \\ -2\boldsymbol{a}_M^T & 1 \end{bmatrix}, \ \boldsymbol{h}_1 = \begin{bmatrix} d_{i1,k}^2 - \|\boldsymbol{a}_1\|^2 \\ \vdots \\ d_{iM,k}^2 - \|\boldsymbol{a}_M\|^2 \end{bmatrix}, \\
&\boldsymbol{B}_1 = 2\mathrm{diag}\left\{d_{i1,k}^o, \ldots, d_{iM,k}^o\right\}, \\
&\boldsymbol{\nu}_1 = [n_{i1,k}, \ldots, n_{iM,k}]^T.
\end{aligned} \tag{2}$$

The LWLS solution to (1) is

$$\hat{\boldsymbol{\theta}}_1 = (\boldsymbol{G}_1^T\boldsymbol{W}_1\boldsymbol{G}_1)^{-1}\boldsymbol{G}_1^T\boldsymbol{W}_1\boldsymbol{h}_1, \tag{3}$$

where $\boldsymbol{W}_1 = (\boldsymbol{B}_1\boldsymbol{R}_1\boldsymbol{B}_1^T)^{-1}$ with $\boldsymbol{R}_1$ being the covariance matrix of $\boldsymbol{\nu}_1$. Note that $\boldsymbol{B}_1$ also contains the true range values, and we handle this issue similarly to the approach in this work, i.e., we first set $\boldsymbol{W}_1$ to identity to obtain an initial estimate by (3), and then update it by using the initial estimate.

The relations among the elements of $\hat{\boldsymbol{\theta}}_1$ are not considered in (3). From their relations, we can form the following model:

$$\boldsymbol{h}_2 - \boldsymbol{G}_2\boldsymbol{\theta}_2 = \boldsymbol{\nu}_2, \tag{4}$$

where

$$\boldsymbol{h}_2 = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{1,(1:3)} \odot \hat{\boldsymbol{\theta}}_{1,(1:3)} \\ \hat{\boldsymbol{\theta}}_{1,(4)} \end{bmatrix}, \ \boldsymbol{G}_2 = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{1}^T \end{bmatrix}, \ \boldsymbol{\theta}_2 = [\boldsymbol{s}_{i,k}^o \odot \boldsymbol{s}_{i,k}^o]. \tag{5}$$

Its LWLS solution is

$$\hat{\boldsymbol{\theta}}_2 = (\boldsymbol{G}_2^T\boldsymbol{W}_2\boldsymbol{G}_2)^{-1}\boldsymbol{G}_2^T\boldsymbol{W}_2\boldsymbol{h}_2, \tag{6}$$

where $\boldsymbol{W}_2 = [\boldsymbol{S}(\boldsymbol{G}_1^T\boldsymbol{W}_1\boldsymbol{G}_1)^{-1}\boldsymbol{S}]^{-1}$ with $\boldsymbol{S} = \mathrm{diag}\{[2\hat{\boldsymbol{\theta}}_{1,(1:3)}^T, 1]^T\}$.

The sensor position estimate at time k is

$$\hat{\boldsymbol{s}}_{i,k} = \mathrm{diag}(\mathrm{sgn}(\hat{\boldsymbol{\theta}}_{1,(1:3)}))[\sqrt{\hat{\theta}_{2,(1)}}, \sqrt{\hat{\theta}_{2,(2)}}, \sqrt{\hat{\theta}_{2,(3)}}]^T, \tag{7}$$

and the estimation covariance matrix is

$$\mathrm{cov}(\hat{\boldsymbol{s}}_{i,k}) = \boldsymbol{B}_2^{-1}(\boldsymbol{G}_2^T\boldsymbol{W}_2\boldsymbol{G}_2)^{-1}\boldsymbol{B}_2^{-T}, \tag{8}$$

where $\boldsymbol{B}_2 = 2\mathrm{diag}\{\hat{\boldsymbol{s}}_{i,k}\}$.

2) Estimation of the Rigid Body Parameters: According to (30), the following CWLS problem similar to (35) using $\hat{\boldsymbol{s}}_{i,k}$ can be formulated:

$$\begin{aligned}
&\min_{\boldsymbol{y}} \ (\hat{\boldsymbol{A}}\boldsymbol{z} - \hat{\boldsymbol{s}})^T\hat{\boldsymbol{W}}(\hat{\boldsymbol{A}}\boldsymbol{z} - \hat{\boldsymbol{s}}) \\
&\text{s.t. } \boldsymbol{Q}^T\boldsymbol{Q} = \boldsymbol{I}, \det(\boldsymbol{Q}) = 1,
\end{aligned} \tag{9}$$

where the weighting matrix

$$\begin{aligned}
\hat{\boldsymbol{W}} =& \mathrm{Diag}\{\mathrm{cov}(\hat{\boldsymbol{s}}_{1,1}), \ldots, \mathrm{cov}(\hat{\boldsymbol{s}}_{N,1}), \ldots, \mathrm{cov}(\hat{\boldsymbol{s}}_{1,K}), \ldots, \\
&\mathrm{cov}(\hat{\boldsymbol{s}}_{N,K})\}^{-1},
\end{aligned}$$

and

$$\begin{aligned}
&\boldsymbol{z} = [\mathrm{vec}(\boldsymbol{Q})^T, \mathrm{vec}(\boldsymbol{P})^T, \boldsymbol{t}^T, \boldsymbol{v}^T]^T, \\
&\hat{\boldsymbol{s}} = [\hat{\boldsymbol{s}}_{1,1}^T, \ldots, \hat{\boldsymbol{s}}_{N,1}^T, \ldots, \hat{\boldsymbol{s}}_{1,K}^T, \ldots, \hat{\boldsymbol{s}}_{N,K}^T]^T, \\
&\hat{\boldsymbol{A}} = [\hat{\boldsymbol{A}}_{1,1}^T, \ldots, \hat{\boldsymbol{A}}_{N,1}^T, \ldots, \hat{\boldsymbol{A}}_{1,K}^T, \ldots, \hat{\boldsymbol{A}}_{N,K}^T]^T,
\end{aligned} \tag{10}$$

with $\hat{\boldsymbol{A}}_{i,k} = [(\boldsymbol{c}_i^T \otimes \boldsymbol{I}), -(\boldsymbol{c}_i^T \otimes kT\boldsymbol{I}), \boldsymbol{I}, kT\boldsymbol{I}]$. Similarly, by introducing an auxiliary matrix $\boldsymbol{Z} = \boldsymbol{z}\boldsymbol{z}^T$ and performing relaxation, problem (9) can be relaxed as

$$\begin{aligned}
&\min_{\boldsymbol{Z},\boldsymbol{z}} \ \mathrm{tr}\{\hat{\boldsymbol{A}}^T\hat{\boldsymbol{W}}\hat{\boldsymbol{A}}\boldsymbol{Z} - 2\hat{\boldsymbol{s}}^T\hat{\boldsymbol{W}}\hat{\boldsymbol{A}}\boldsymbol{z} + \hat{\boldsymbol{s}}^T\hat{\boldsymbol{W}}\hat{\boldsymbol{s}}\} \\
&\text{s.t. } (36), \begin{bmatrix} \boldsymbol{Z} & \boldsymbol{z} \\ \boldsymbol{z}^T & 1 \end{bmatrix} \succeq \boldsymbol{0}.
\end{aligned} \tag{11}$$

After solving (11), the same orthogonalization and refinement steps described in Section III of the main manuscript are followed to obtain the DAC solution.

References

[1] S. Chen and K. C. Ho, "Accurate localization of a rigid body using multiple sensors and landmarks," *IEEE Trans. Signal Process.*, vol. 63, no. 24, pp. 6459-6472, Dec. 2015.