CSC384 Assignment 1 - Advanced Heuristic Function

1. Describe how one can calculate the advanced heuristic value for any state of the puzzle.

The advanced heuristic function we will be implementing is a modification of the Manhattan distance heauristic. For any state n, the advanced heuristic value, $h(n)$, will be the distance between the 2x2 piece's current location and the bottom opening. Let the bottom opening be defined by the following squares, {(1, 3), (1, 4), (2, 3), (2, 4)}, as mentioned above. However, we have the following exception. When a piece other than the 2x2 piece that is blocking the bottom opening, we will charge 1 + the Manhattan Distance between the 2x2 piece's current location and the bottom opening). The rationale behind this is that we add one extra unit of cost to move the blocking piece out of the way and then apply the Manhattan Distance heuristic.

2. Why is your advanced heuristic admissible?

This advanced heuristic can be shown to be admissible in the following manner. We want to show that for every state n, the advanced heuristic value, $h(n)$, is greater than or equal to 0 and less than or equal to the cost of the cheapest path from state n to a goal state. We will prove this by looking at two different cases.

For the first case, we will look at the states where no piece, other than potentially the 2x2 piece, is blocking the bottom opening. In this case, $h(n)$ will be equal to the value given by the Manhattan distance heuristic. Since we already know the Manhattan distance heuristic is an admissible function, this shows that the advanced heuristic is admissible for these states.

For the second case, we will look at the states where there is a piece other than the 2x2 piece that is blocking the bottom opening. For such states, we know from the first case that if the bottom opening was not blocked in such a manner, then its advanced heuristic value, $h(n)$, is less than or equal to the cost of the cheapest path from state n to a goal state. We also know that it will take at least 1 move, which is equivalent to a cost charge of at least 1, to remove the piece that is blocking the bottom opening. By definition, the advanced heuristic value for states from this case is $h(n) + 1$. Hence, we have that $h(n) + 1 <=$ the cost of the cheapest path from state n to a goal state where state n has a piece other than the 2x2 piece that is blocking the bottom opening.

3. Why does your advanced heuristic dominate the Manhattan distance heuristic?

This advanced heuristic can be shown to dominate the Manhattan distance heuristic in the following manner. For a state n, let the advanced heuristic value be given by $h(n)$ and let the Manhattan distance heuristic value be given by $h_1(n)$.

For the first condition, we want to show that for every state n, $h(n) \geq h_1(n)$. This is true because the only possible values that $h(n)$ evaluates to are either $h_1(n)$ or $h_1(n) + 1$. Since $h_1(n) \geq h_1(n)$ and $h_1(n) + 1 \geq h_1(n)$, we have shown that the first condition is satisfied.

For the second condition, we want to show that for at least one state m, $h(m) > h_1(m)$. Take any one of the states where there is a piece other than the 2x2 piece that is blocking the bottom opening. Let s be that state. We know by the definition of the advanced heuristic that $h(s) = 1 + h_1(s)$. Since $1 + h_1(s) > h_1(s)$, we have shown that the second condition is satisfied as well.

Hence, we have shown that both conditions required to show that the advanced heuristic dominates the Manhattan distance heuristic are satisfied. Therefore, we can conclude that the advanced heuristic dominates the Manhattan distance heuristic, as needed.