Modeling Wildlife Populations: Insights from a Predator-Prey Simulation Study

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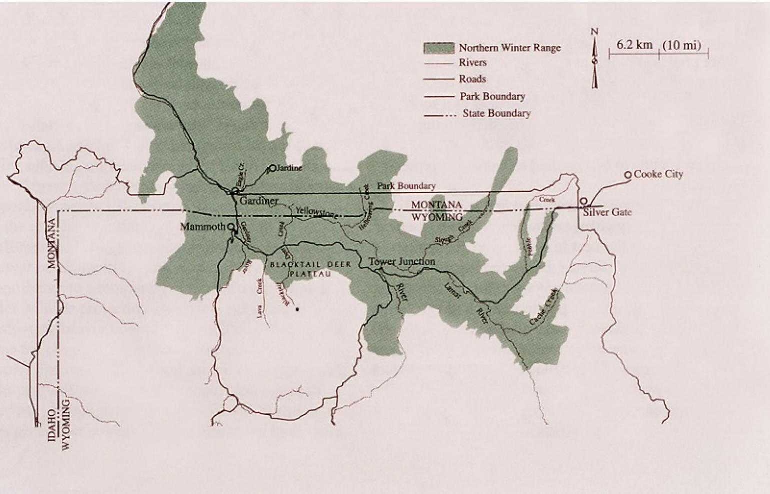
Motivation

- Reintroduction of wolves has shifted ecosystem dynamics.
- Studying the population dynamics between elk and wolves in the Yellowstone ecosystem can provide valuable insights.
- Conclusions made can help inform conservation and management strategies for other ecosystems.
- Forecasts can help ensure populations are behaving as expected.

Dynamics of the Yellowstone Ecosystem

- Species of interest: Elk and Wolves
- Specifically northern range herds



Data

- Obtained from National Park Service Website
  - Elk data: https://www.nps.gov/yell/learn/nature/elk.htm
  - Wolf data: https://www.nps.gov/yell/learn/nature/upload/FINAL-FOR-APPROVAL-WOLF-REPORT-2020_508R.pdf

Lotka-Volterra Equations

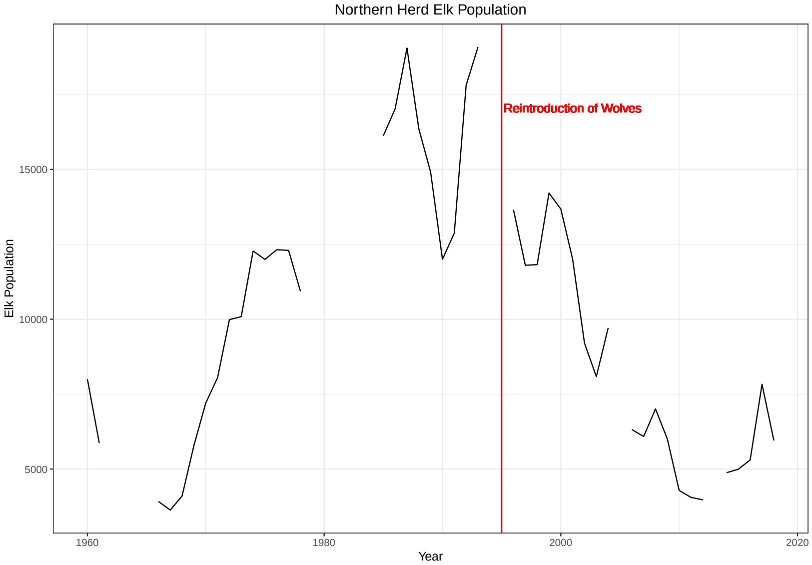
Lotka-Volterra Equations: Introduction

- Nonlinear differential equations which describe the fluctuating population dynamics of predator and prey
- Extension of Logistic Growth Model
- $x(t)$ represents the population size of prey at time t
- $y(t)$ represents the population size of predators at time t

Lotka-Volterra Equations: Prey equation

- Prey equation: $\frac{dx}{dt} = \alpha x - \beta xy$
  - Represents the instantaneous growth rate
  - x is the population of prey (elk)
  - y is the population of predators (wolves)
  - Where α is the exponential growth rate of the prey
  - Where β is the predation rate.
- Assumptions
  - The prey population has an unlimited supply of resources.
  - Exponential growth in absence of predators. No carrying capacity.

Elk Populations in the Absense of predators



Lotka-Volterra Equations: Predator equation

- Predator equation: $\frac{dy}{dt} = -\gamma y + \delta xy$
  - Represents the instantaneous growth rate
  - x is the population of prey (elk).
  - y is the population of predators (wolves).
  - Where γ is the shrinkage rate of the predator population.
  - Where δ is the predator growth rate as a factor of the product of populations.
- Assumptions
  - The food supply of the predator population depends entirely on the size of the prey population.
  - The rate of predation on the prey is assumed to be proportional to the rate at which the predators and the prey meet.

Parameter Estimation

Potential Approaches

- $\frac{dx}{dt} = \alpha x - \beta xy$
- $\frac{dx}{dt} = -\gamma y + \delta xy$
- Need to estimate: $\alpha, \beta, \gamma, \delta$

1. Nonlinear Least Squares
2. Bayesian Approximation

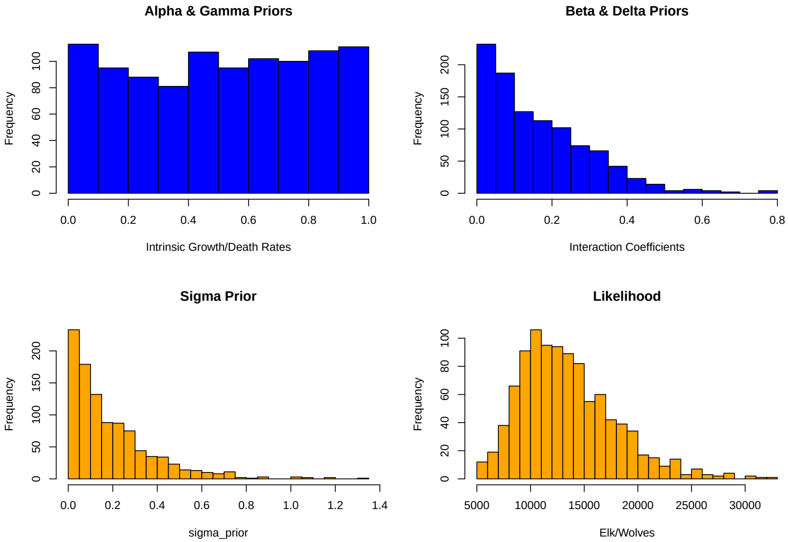
Bayesian Approximation: Quick Example

- Example
  - Likelihood: $p(y_1, ..., y_n|\lambda) \sim \text{Poisson}(\lambda)$
  - Prior: $p(\lambda) \sim \text{Gamma}(\alpha_0, \beta_0)$
  - Posterior: $p(\lambda|y_1, .., y_n) = \frac{p(y_1, \ldots, y_n|\lambda) * p(\lambda)}{p(y_1, \ldots, y_n)}$
- Our case
  - $p(\alpha, \beta, \gamma, \delta|y_{[1,k]}, ..., y_{[n,k]})$

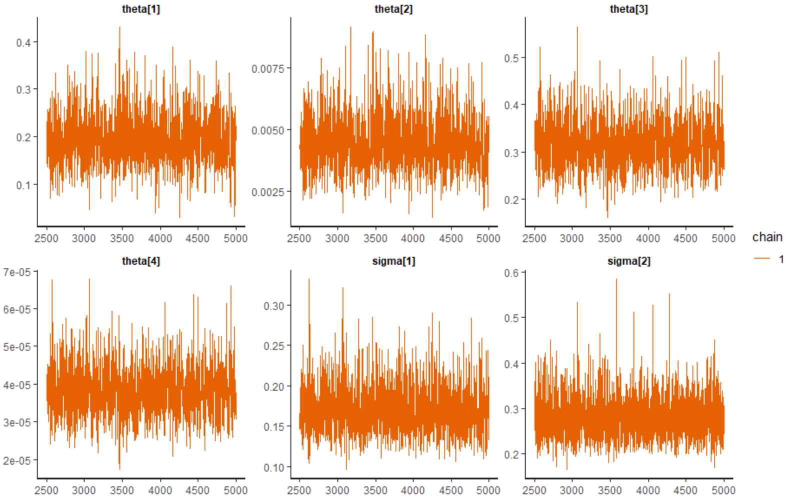
Bayesian Approximation: Likelihood & Priors

- Likelihood:
  - $y_{n,k} \sim \text{lognormal}(log(z_{n,k}), \sigma_k)$
- Priors
  - $\alpha, \gamma \sim \text{Beta(1,1)}$
  - $\beta, \delta \sim \text{Beta(1,5)}$

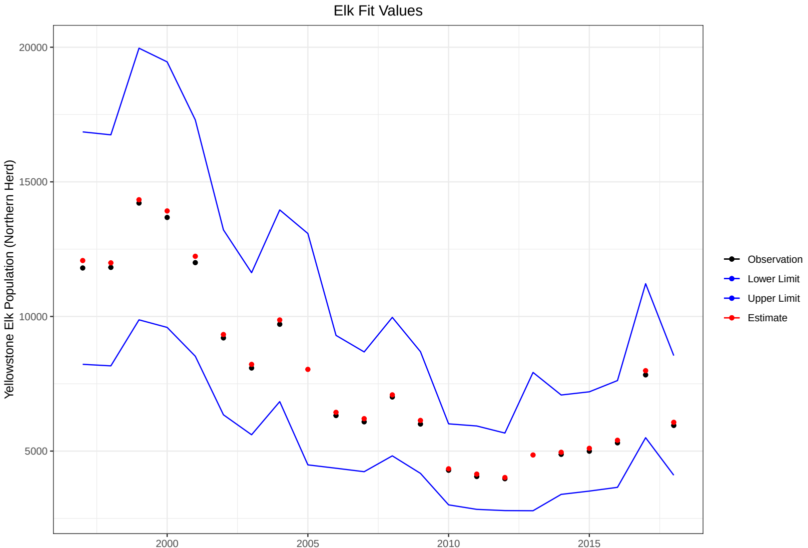
Prior Predictive Checks



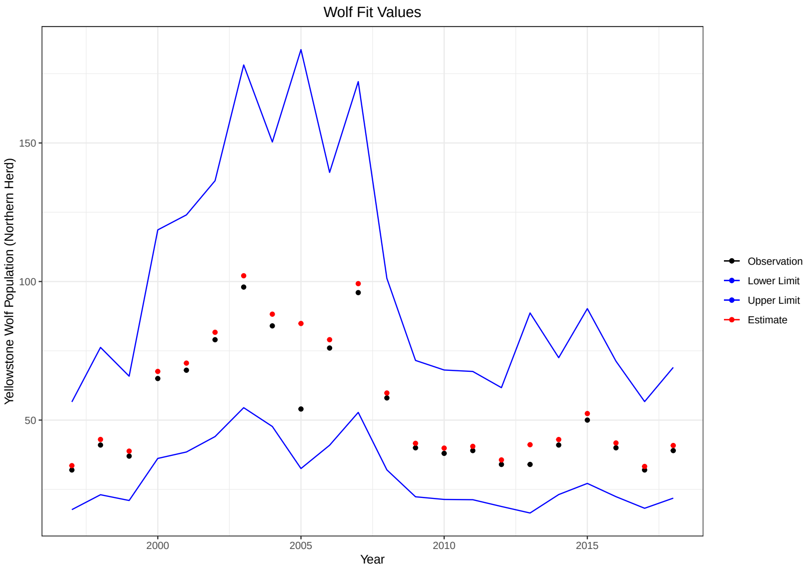
Posterior Convergence Check



Checking Model Fit: Elk



Checking Model Fit: Wolves

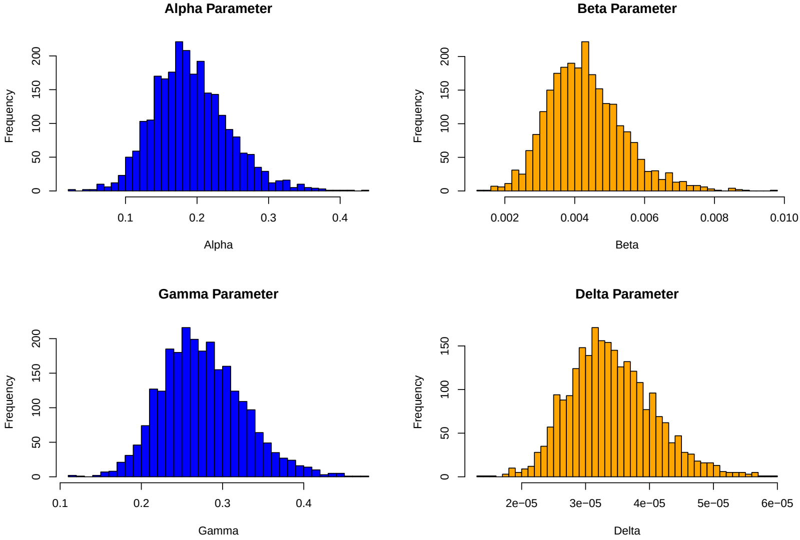


Forecasts & Inference

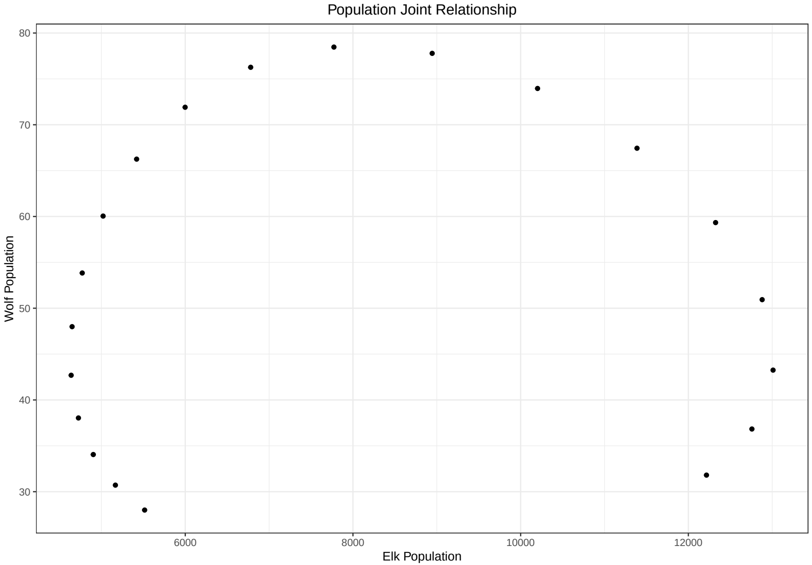
Estimated Parameters

	Estimate	S.E.	Lower 95% CI	Upper 95% CI
α	0.192	0.053	0.101	0.31
β	0.004	0.001	0.002	0.007
γ	0.276	0.051	0.188	0.387
δ	0	0	0	0
σ_{elk}	0.18	0.032	0.128	0.253
σ_{wolves}	0.29	0.051	0.212	0.405

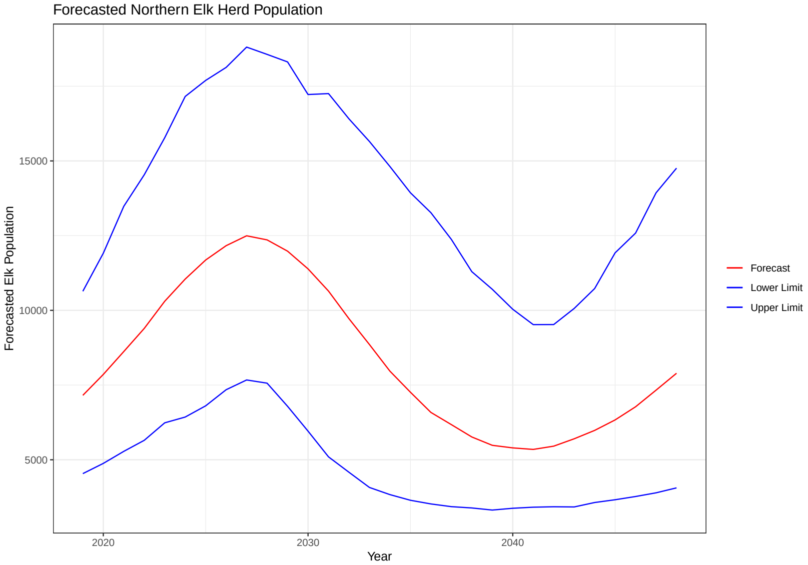
Parameter Estimation: Visualization



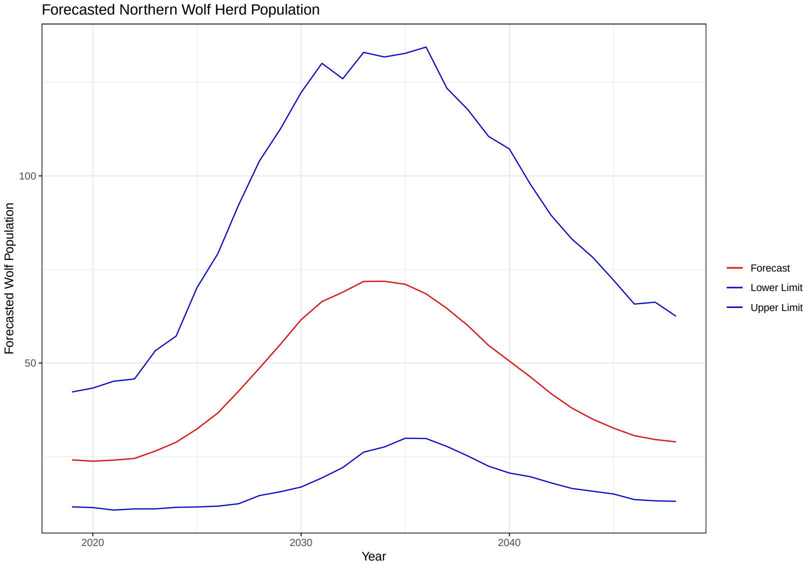
Joint Relationship



Forecasts: Elk



Forecasts: Wolves



Conclusions

- Expect the elk population to peak in 2027 - 2029.
- Expect the wolf population to peak in 2031 - 2033.
- Expect the intrinsic growth rate of elk to be between 0.10 to 0.32
  - Estimated to be 0.20.
- Positive β coefficient.
- Expected intrinsic shrinkage rate of wolves to be between 0.18 to 0.38
  - Estimated to be 0.27.
- $\delta < \beta$

Limitations & Considerations

- Limited to only elk & wolf populations.
- How would our estimates vary if we used uninformative priors?
- Instead of considering elk_missing as a random variable we could condition it on the number of wolves.
- Instead of the continuous lognormal likelihood we could try the discrete possion likelihood.