23. Dyadisk 7 [Dec. 24]

23.1. An (ignorant's) question about dendric subshifts.

(I apologize for the poor quality of my writing - this has been written very fast only today ...)

Let us think of a subshift X which for some level $n \geqslant 1$ has Rauzy graph $R_n(X)$ that has many long edges and only a few "switches" (= bifurcation points), all of them simple (i.e. the local blow-up graph ["extension graph"] consist of two vertices of valence 1 and one vertex of valence 2, so it is in particular a tree). Now I pass to the next level Rauzy graph $R_{n+1}(X)$, which differs from the previous one in that every switch moves one step further into its "cusp direction" (i.e. the direction of the "2-valence vertex"). I pass to the next level and so on, until two of the switches collide, in which case I may or may not have a problem, as the resulting local blow-up graph could disconnect, or inherit a non-trivial cycle, or (in the good case) stay a tree.

In the case where a non-trivial cycle is generated, however, it may well be (indeed this happens very often) that a "short time later" the cycle splits, by another switch-collision which cancels the previous one, and all is fine again.

My guess is (but I may be wrong) that if we pass to a lager class of subshifts which admits such "cancelling pairs" of switch collisions in their Rauzy development, then the resulting "almost dendric" subshifts share most or even all of the intrinsic properties of dendric subshifts. Hence my personal suggestion for a less combinatorial (and hence admittedly less comfortable) but perhaps more conceptual definition for such "almost dendric" subshifts would be to allow, in the definition of the extension graph, that a vertex is split into several "subvertices" if for some sufficiently large $m \geqslant 1$ there don't exist common extensions of length m from such subvertices (for $m = 1$ this would give the classical extension graph). The defining property for an *almost dendric* subshift should then be that all such "split extension graphs" must be trees.

If my morning reading in France G.'s papers during her unfortunately for me not visible talk is correct, for a general morphisms σ and a minimal dendric subshift X the image subshift $\sigma(X) = \sigma \cdot X$ (= the subshift generated by the image language $\sigma(\mathcal{L}(X))$ will in general not be dendric, unless some rather strong combinatorial conditions on σ are imposed. My (optimistic) hope is that, in contrast, the image subshift $\sigma(X)$ of an almost dendric subshift X is always almost dendric (or at least eventually dendric, meaning all sufficiently long words in the language are almost dendric), as long as σ is recognizable in X.

I wonder whether anything in that direction has been considered by the dendric subcommunity, and if so, whether it seems to work, or where the problems are (which I certainly have overlooked ...). Any suggestion for reading is well come !