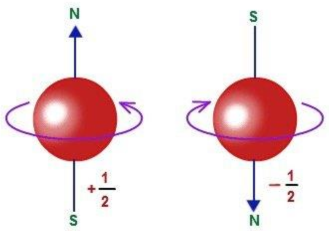
Lectures 21-25:EPR Paradox and CHSH Game

CS 401: Quantum Computing
Dr. Kell, Spring 2023

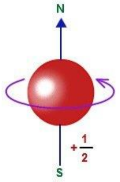
Measurement in Alternate Bases

***Disclaimer:** physics part of explanation is very bastardized/incorrect!

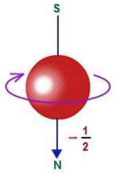
Physical Property used for Qubit*



“Electron Spin”

$|0\rangle \longleftrightarrow$ 

“Spin up”

$|1\rangle \longleftrightarrow$ 

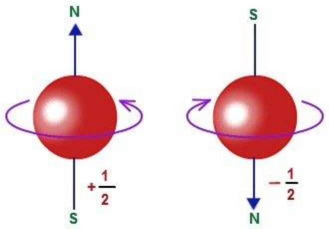
“Spin down”

Measurement in Alternate Bases

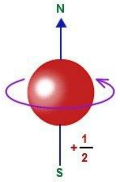
***Disclaimer:** physics part of explanation is very bastardized/incorrect!

Physical Property used for Qubit*

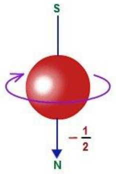
$\longrightarrow$



"Electron Spin"

$|0\rangle \longleftrightarrow$ 

"Spin up"

$|1\rangle \longleftrightarrow$ 

"Spin down"

Denote:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Plus"

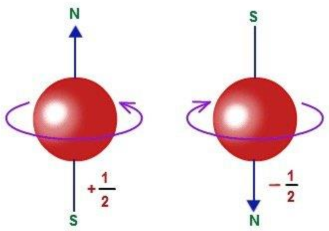
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Minus"

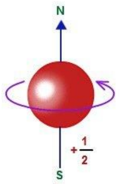
Measurement in Alternate Bases

*Disclaimer: physics part of explanation is very bastardized/incorrect!

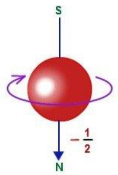
Physical Property used for Qubit*



"Electron Spin"

$|0\rangle$ 

"Spin up"

$|1\rangle$ 

"Spin down"

Denote:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Plus"

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Minus"

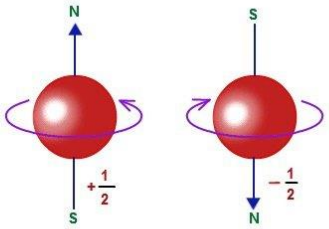
Observe: we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \quad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

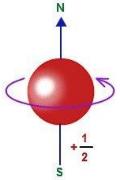
Measurement in Alternate Bases

***Disclaimer:** physics part of explanation is very bastardized/incorrect!

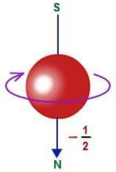
Physical Property used for Qubit*



"Electron Spin"

$|0\rangle \longleftrightarrow$ 

"Spin up"

$|1\rangle \longleftrightarrow$ 

"Spin down"

Denote:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Plus"

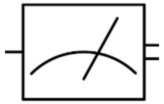
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Minus"

Observe: we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \quad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

Measuring in "Computational Basis":



$|0\rangle$ vs $|1\rangle$

State $|0\rangle$ = 100% chance spin up, 0% spin down.

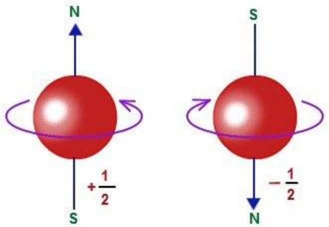
State $|1\rangle$ = 0% chance spin up, 100% spin down.

Both states $|+\rangle$ and $|-\rangle$ = 50% chance spin up, 50% spin down.

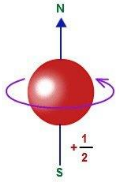
Measurement in Alternate Bases

*Disclaimer: physics part of explanation is very bastardized/incorrect!

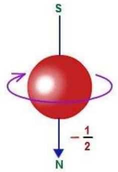
Physical Property used for Qubit*



"Electron Spin"

$|0\rangle \longleftrightarrow$ 

"Spin up"

$|1\rangle \longleftrightarrow$ 

"Spin down"

Denote:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Plus"

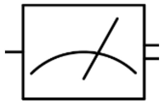
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Minus"

Observe: we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \quad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

Measuring in "Computational Basis":



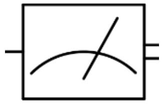
$|0\rangle$ vs $|1\rangle$

State $|0\rangle$ = 100% chance spin up, 0% spin down.

State $|1\rangle$ = 0% chance spin up, 100% spin down.

Both states $|+\rangle$ and $|-\rangle$ = 50% chance spin up, 50% spin down.

Measuring in "Plus-minus Basis"



$|+\rangle$ vs $|-\rangle$

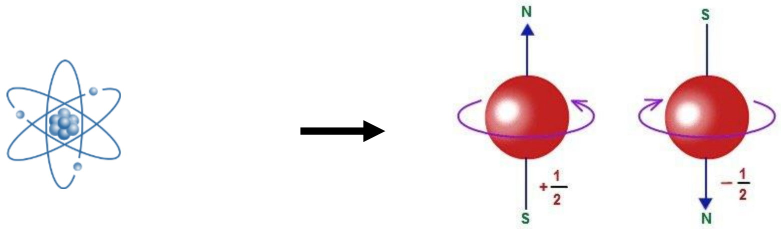
State $|+\rangle$ = 100% chance spin plus, 0% spin minus.

State $|-\rangle$ = 0% chance spin plus, 100% spin minus.

Both states $|0\rangle$ and $|1\rangle$ = 50% chance spin plus, 50% spin minus.

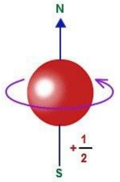
Measurement in Alternate Bases

*__Disclaimer:__ physics part of explanation is very bastardized/incorrect!

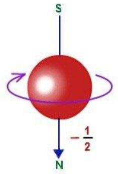


Physical Property used for Qubit*

"Electron Spin"

$|0\rangle \longleftrightarrow$ 

"Spin up"

$|1\rangle \longleftrightarrow$ 

"Spin down"

Denote:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Plus"

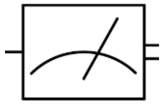
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

"Spin Minus"

Observe: we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \quad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

Measuring in "Computational Basis":



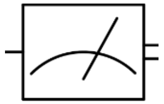
$|0\rangle$ vs $|1\rangle$

State $|0\rangle$ = 100% chance spin up, 0% spin down.

State $|1\rangle$ = 0% chance spin up, 100% spin down.

Both states $|+\rangle$ and $|-\rangle$ = 50% chance spin up, 50% spin down.

Measuring in "Plus-minus Basis"



$|+\rangle$ vs $|-\rangle$

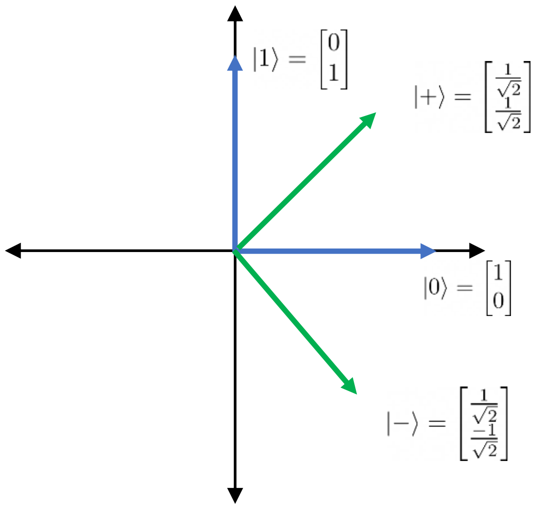
State $|+\rangle$ = 100% chance spin plus, 0% spin minus.

State $|-\rangle$ = 0% chance spin plus, 100% spin minus.

Both states $|0\rangle$ and $|1\rangle$ = 50% chance spin plus, 50% spin minus.

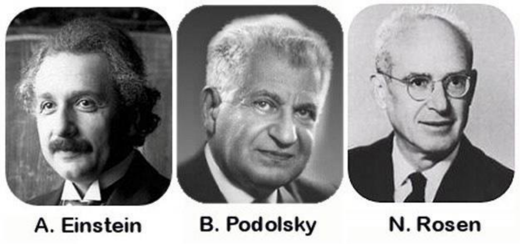
Geometric Intuition

- We can choose any axis for what will determine the binary outcome of measurement.
- Probabilities are determined by coordinates of state in the axis of measurement

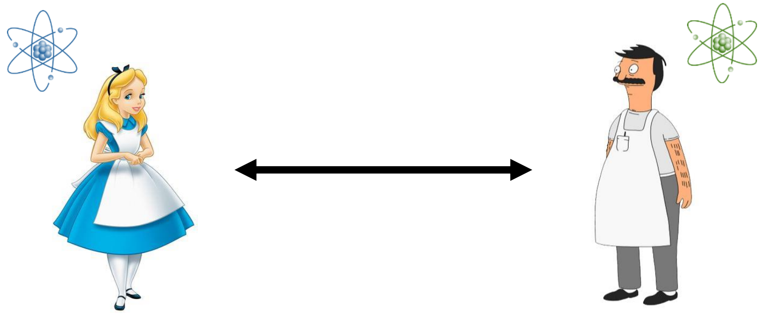


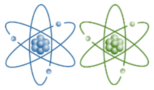
EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)



Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.

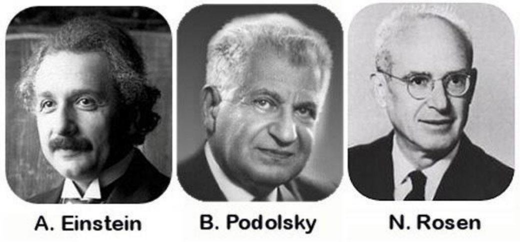




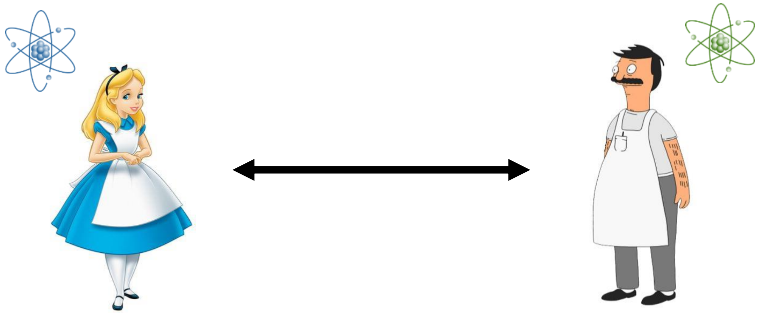
$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

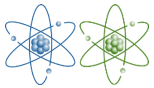
EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)



Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



 $= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

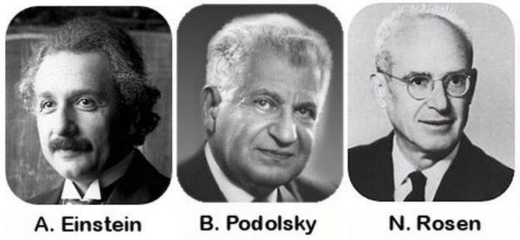
First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

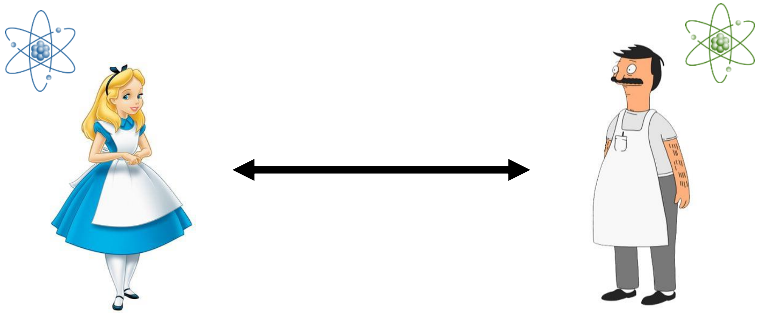
…we can express the EPR pair as:

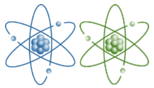
EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)



Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



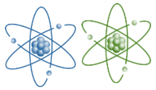


$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

…we can express the EPR pair as:



$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

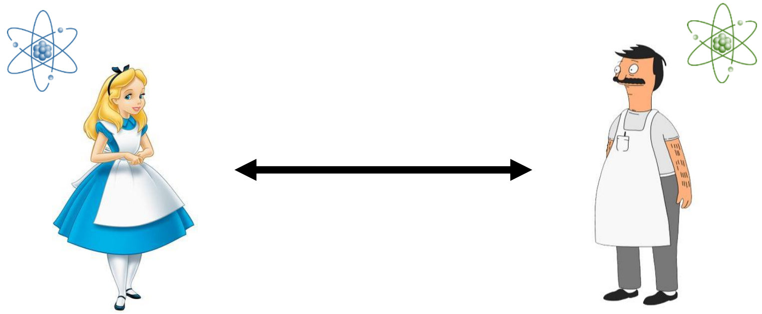
$$= \frac{1}{2}\left[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\right]$$

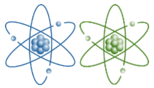
EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)

A. Einstein B. Podolsky N. Rosen

Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.

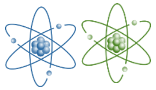


 $$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

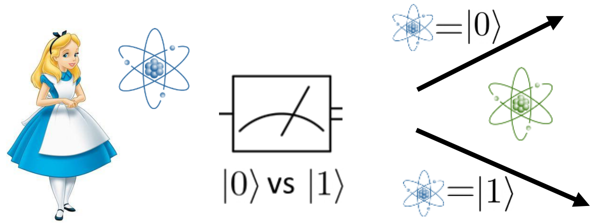
$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

…we can express the EPR pair as:

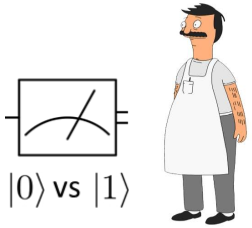
 $$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

$$= \frac{1}{2}\left[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\right]$$

Case 1: Alice measures in 0-1 basis…

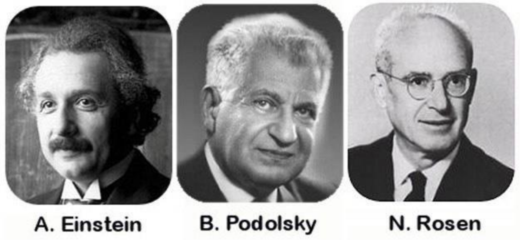


…Bob measures in 0-1 basis.

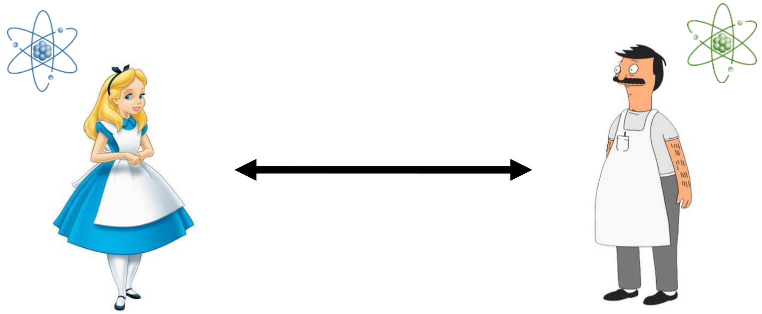


EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)



Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

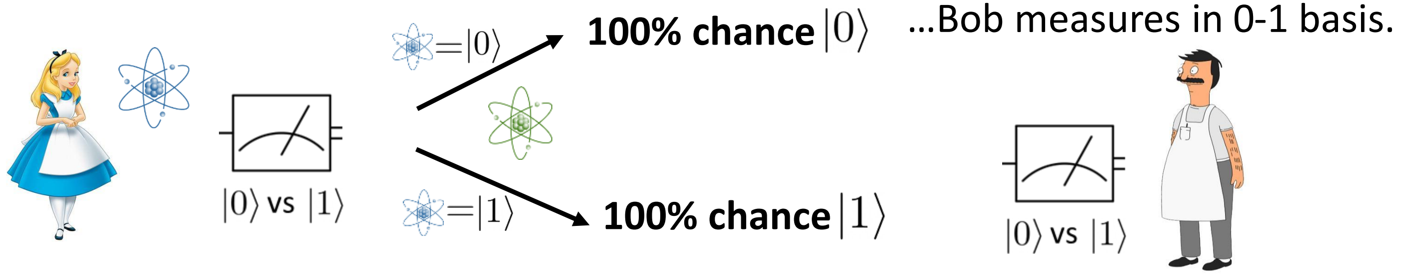
$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

...we can express the EPR pair as:

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

$$= \frac{1}{2}\left[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\right]$$

Case 1: Alice measures in 0-1 basis…

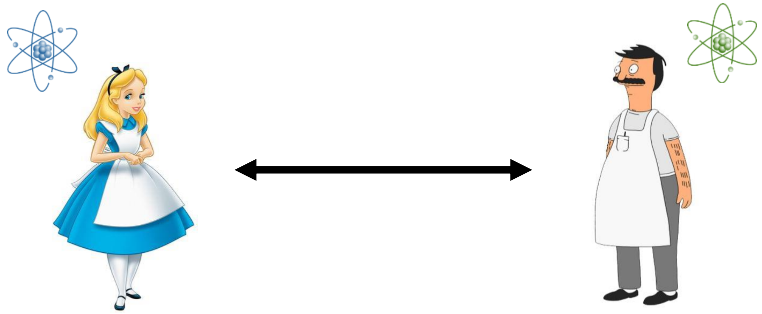


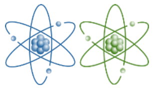
EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)

A. Einstein B. Podolsky N. Rosen

Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



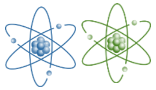


$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

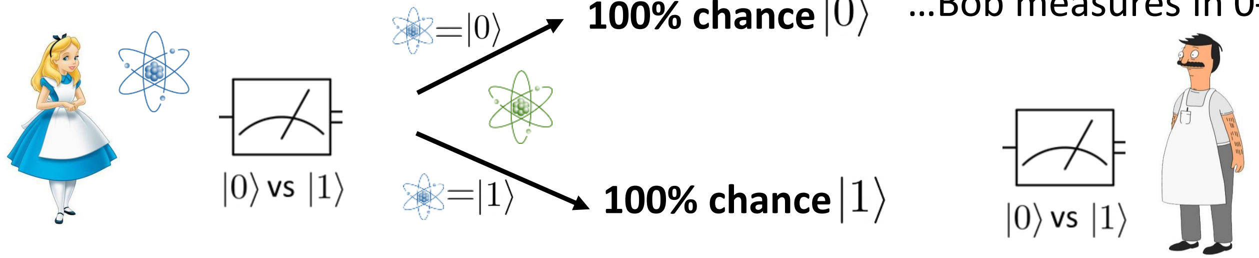
…we can express the EPR pair as:



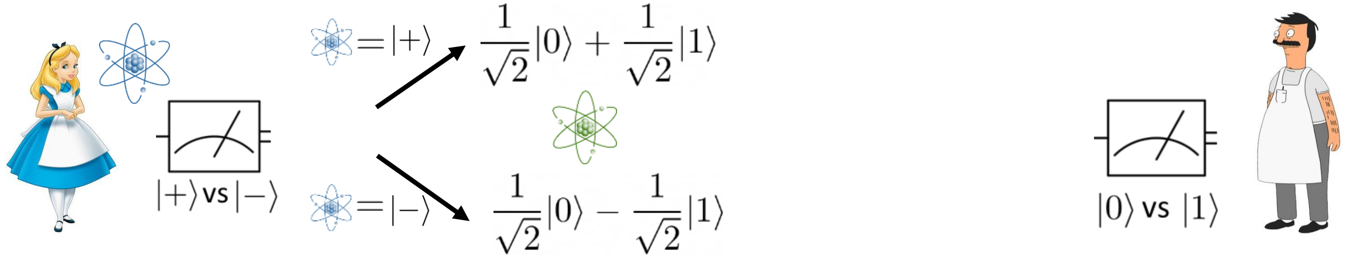
$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

$$= \frac{1}{2}\left[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\right]$$

Case 1: Alice measures in 0-1 basis… …Bob measures in 0-1 basis.

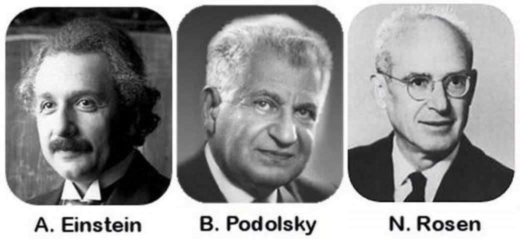


Case 2: Alice measures in plus-minus basis… …Again Bob measures in 0-1 basis.

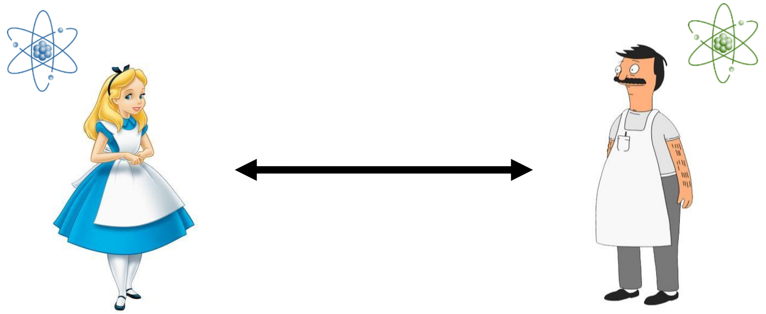


EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)



Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

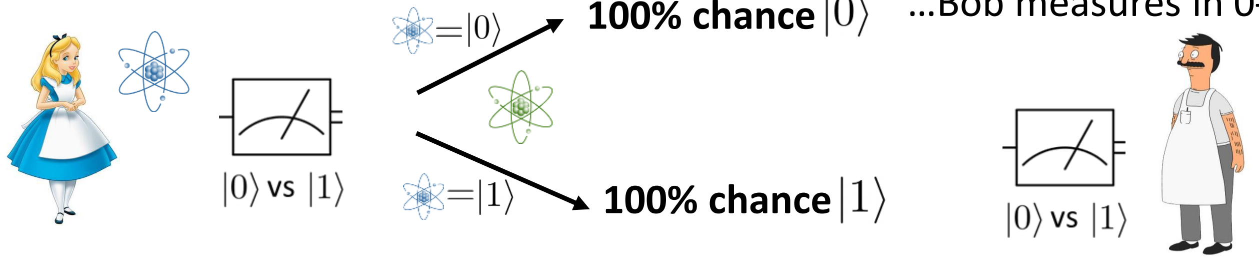
$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

…we can express the EPR pair as:

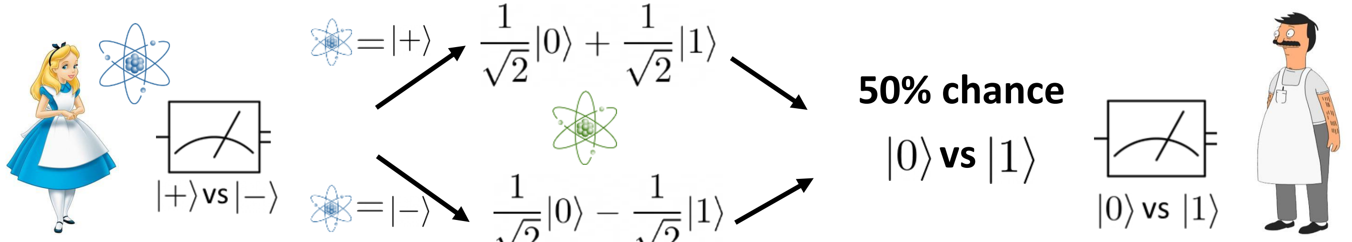
$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

$$= \frac{1}{2}\Big[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\Big]$$

Case 1: Alice measures in 0-1 basis… …Bob measures in 0-1 basis.



Case 2: Alice measures in plus-minus basis… …Again Bob measures in 0-1 basis.

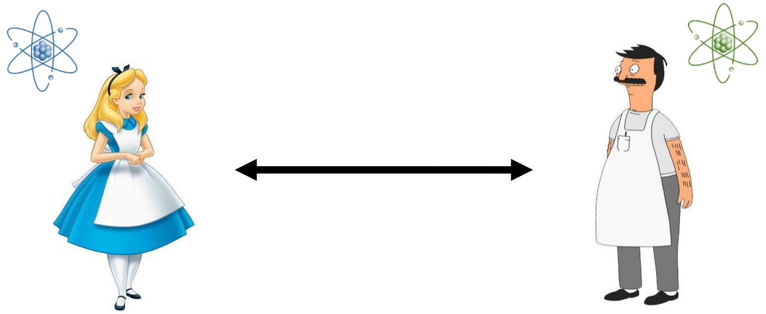


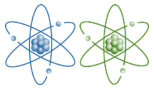
EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)

A. Einstein B. Podolsky N. Rosen

Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



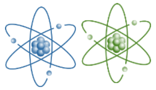


$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

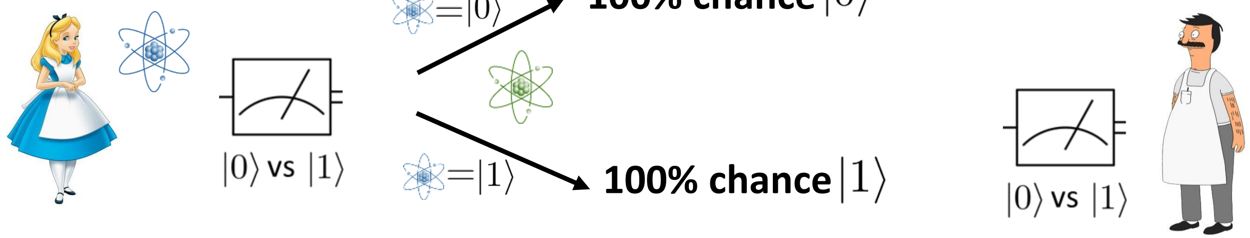
…we can express the EPR pair as:



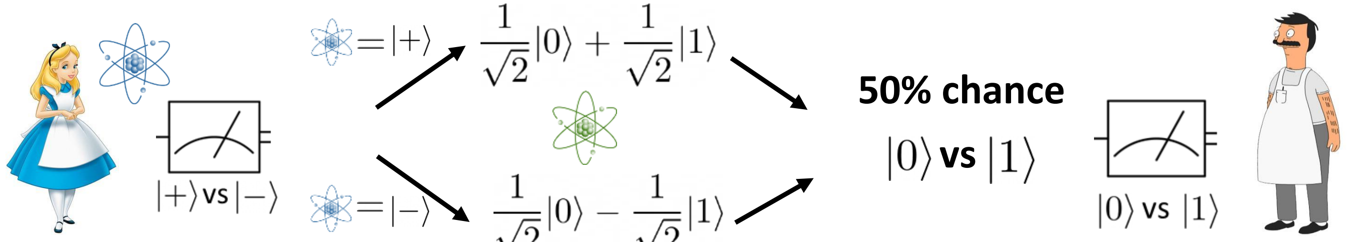
$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

$$= \frac{1}{2}\Big[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\Big]$$

Case 1: Alice measures in 0-1 basis… …Bob measures in 0-1 basis.



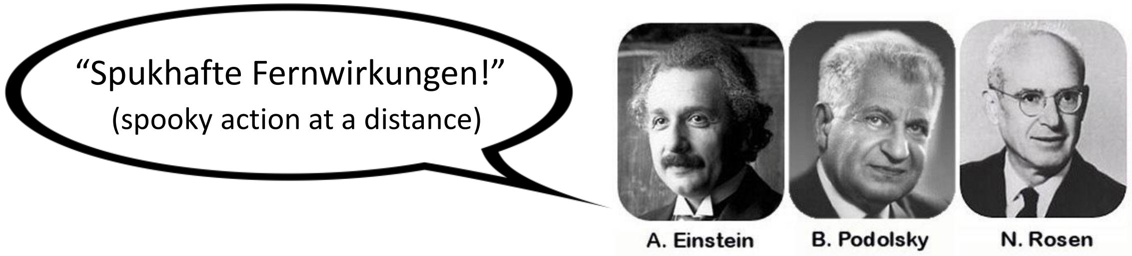
Case 2: Alice measures in plus-minus basis… …Again Bob measures in 0-1 basis.



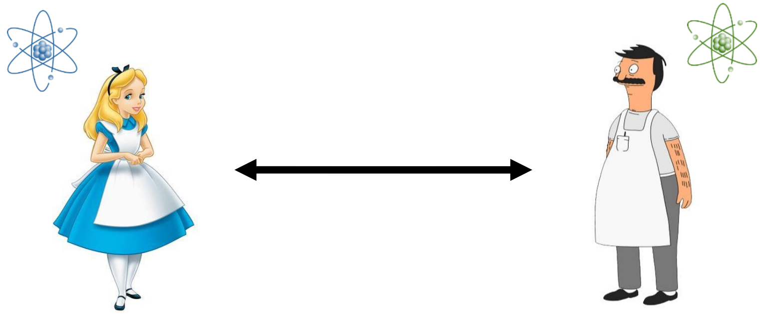
Punchline: If QM describes properties of the atoms that are tangibly real, why should Alice's choice of measurement affect Bob's measurement outcome? (Non-locality)

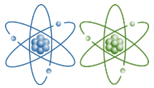
EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)



Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.

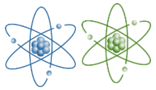


 $= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

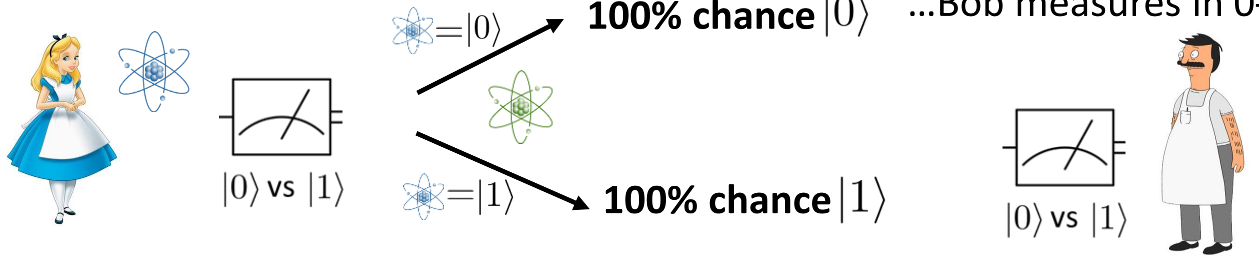
$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

…we can express the EPR pair as:

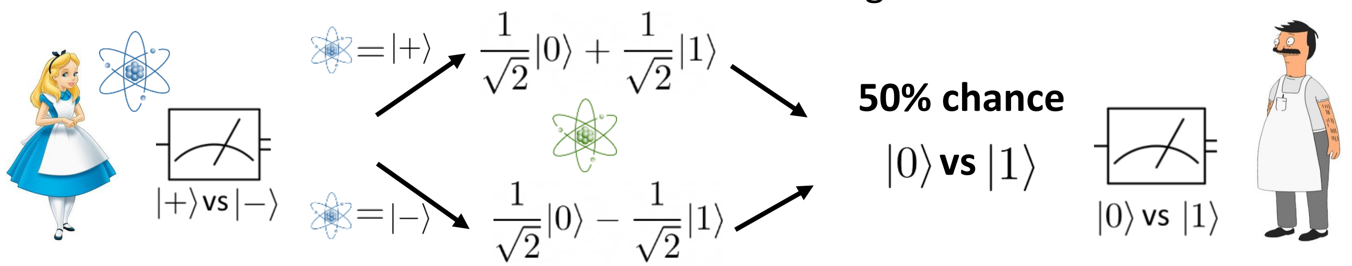
 $= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$

$$= \frac{1}{2}\left[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\right]$$

Case 1: Alice measures in 0-1 basis… …Bob measures in 0-1 basis.



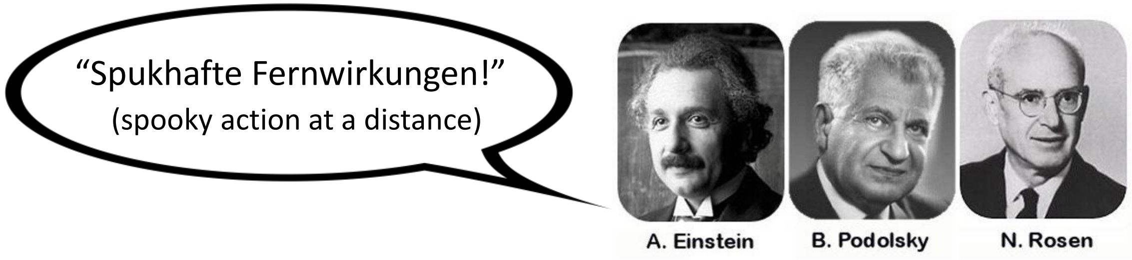
Case 2: Alice measures in plus-minus basis… …Again Bob measures in 0-1 basis.



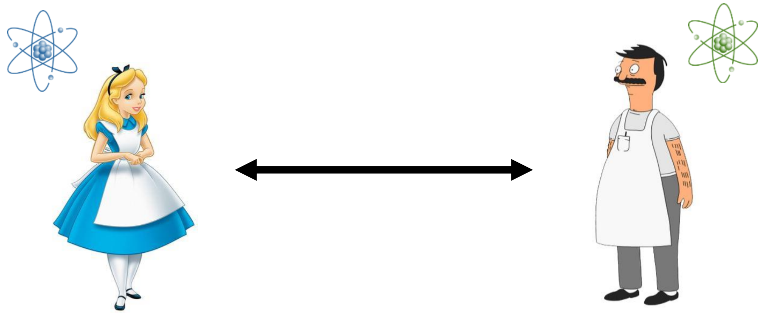
Punchline: If QM describes properties of the atoms that are tangibly real, why should Alice's choice of measurement affect Bob's measurement outcome? (Non-locality)

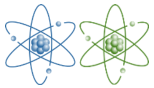
EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)



Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.

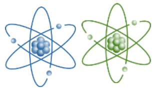


 $= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

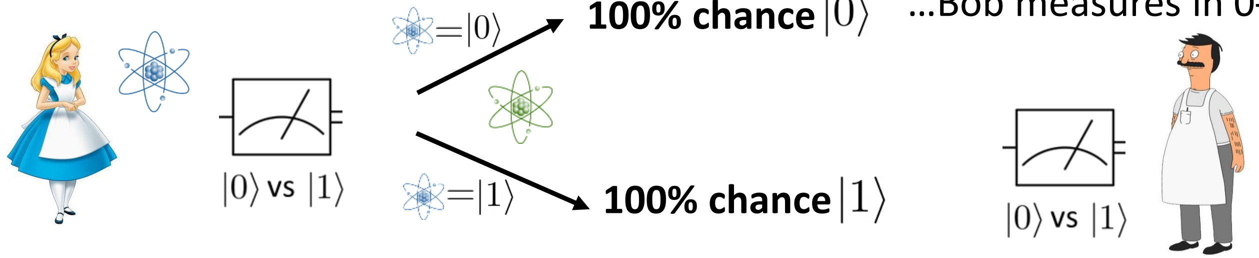
$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

…we can express the EPR pair as:

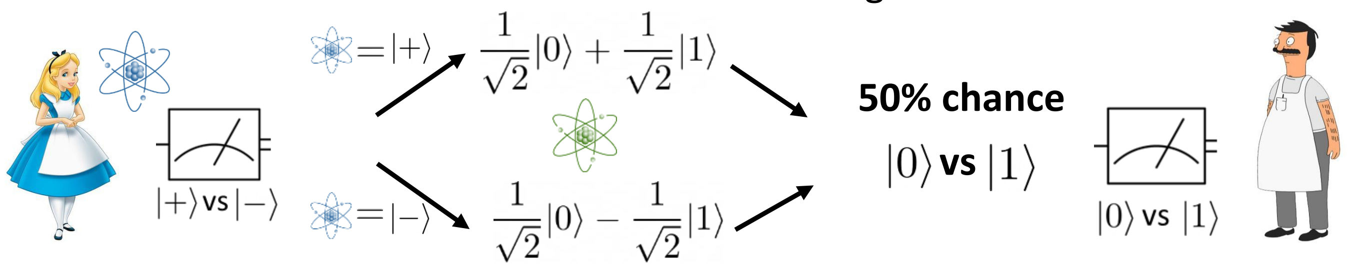
 $= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$

$$= \frac{1}{2}\left[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\right]$$

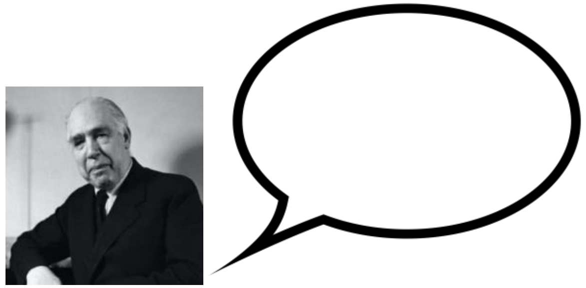
Case 1: Alice measures in 0-1 basis… …Bob measures in 0-1 basis.



Case 2: Alice measures in plus-minus basis… …Again Bob measures in 0-1 basis.



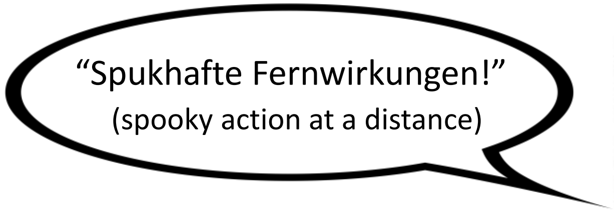
Punchline: If QM describes properties of the atoms that are tangibly real, why should Alice's choice of measurement affect Bob's measurement outcome? (Non-locality)



Niels Bohr

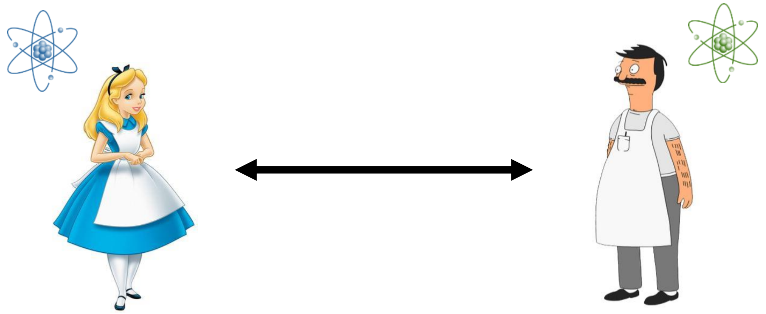
EPR Paradox

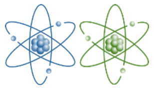
(Thought experiment by Einstein, Podolsky, Rosen, 1935)



A. Einstein B. Podolsky N. Rosen

Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



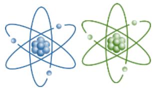


$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

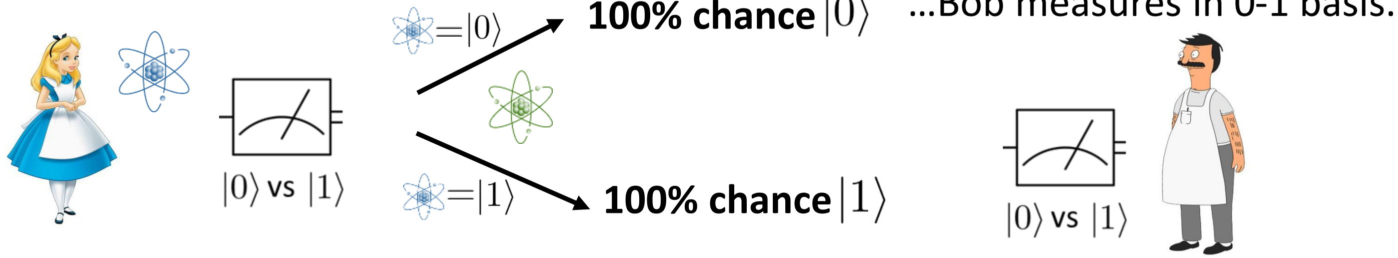
...we can express the EPR pair as:



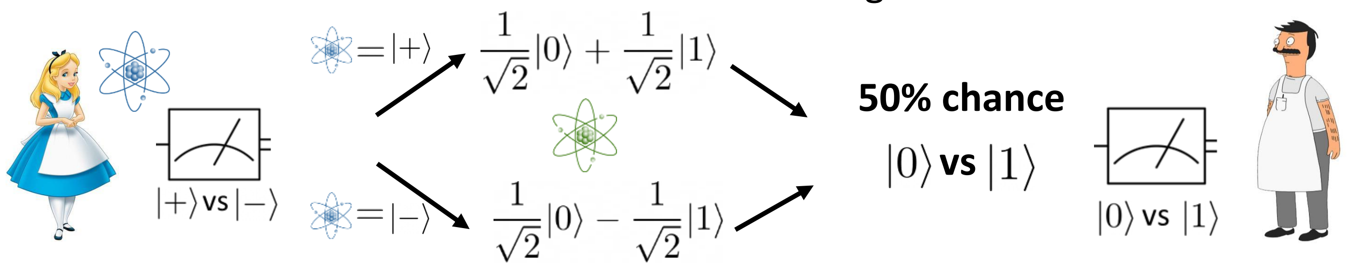
$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

$$= \frac{1}{2}\Big[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\Big]$$

Case 1: Alice measures in 0-1 basis...



Case 2: Alice measures in plus-minus basis... ...Again Bob measures in 0-1 basis.



Punchline: If QM describes properties of the atoms that are tangibly real, why should Alice’s choice of measurement affect Bob’s measurement outcome? (Non-locality)

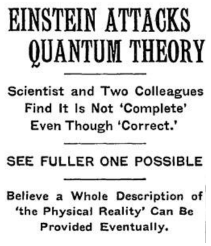
Niels Bohr

EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)

"Spukhafte Fernwirkungen!"
(spooky action at a distance)


A. Einstein B. Podolsky N. Rosen

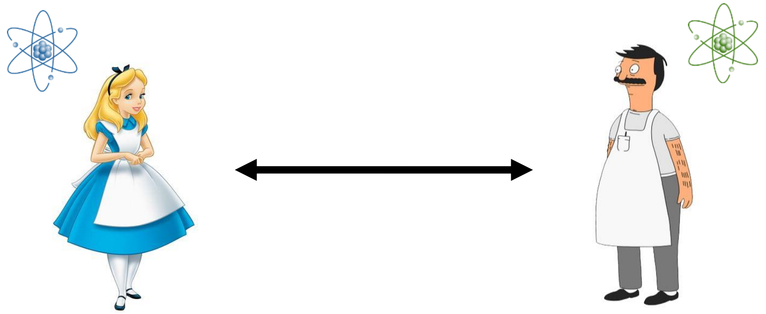

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

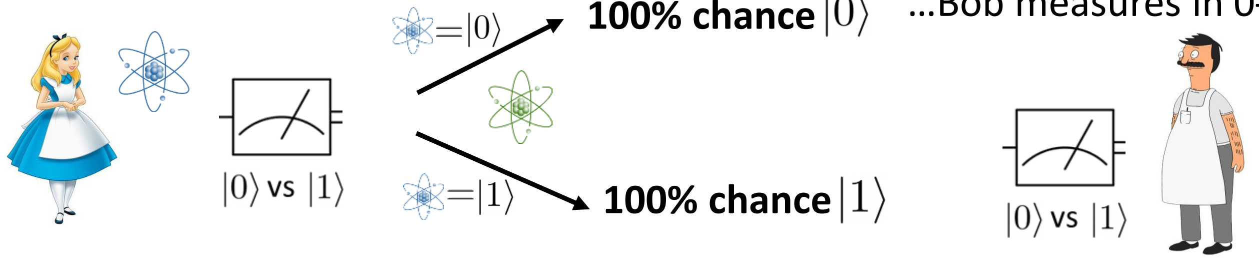
$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

...we can express the EPR pair as:

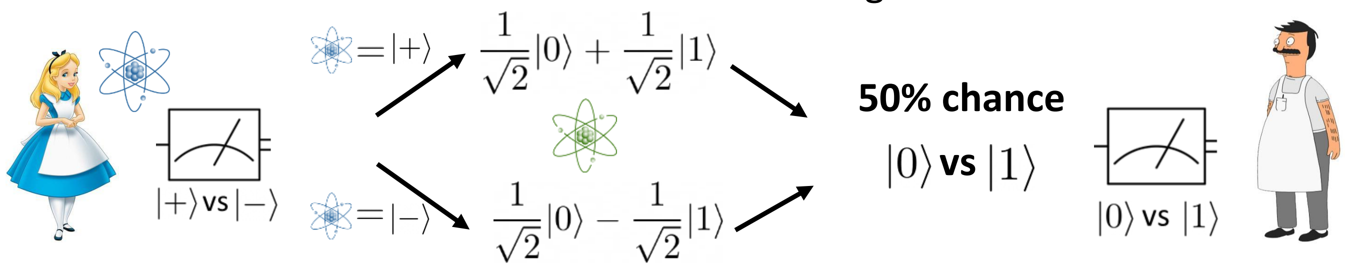
$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

$$= \frac{1}{2}\left[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\right]$$

Case 1: Alice measures in 0-1 basis... ...Bob measures in 0-1 basis.



Case 2: Alice measures in plus-minus basis... ...Again Bob measures in 0-1 basis.



Punchline: If QM describes properties of the atoms that are tangibly real, why should Alice's choice of measurement affect Bob's measurement outcome? (Non-locality)

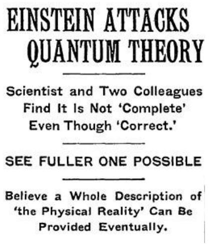

Niels Bohr

EPR Paradox

(Thought experiment by Einstein, Podolsky, Rosen, 1935)

“Spukhafte Fernwirkungen!”
(spooky action at a distance)

A. Einstein B. Podolsky N. Rosen

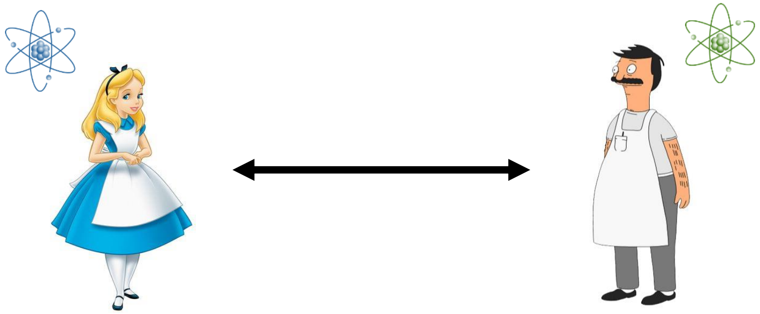

EINSTEIN ATTACKS QUANTUM THEORY

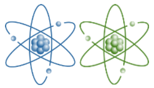
Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Alice and Bob put two qubits in entangled EPR pair, and then separate themselves by a large distance.



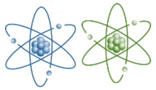


$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

First Observe: since we can express states $|0\rangle$ and $|1\rangle$ as:

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \qquad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

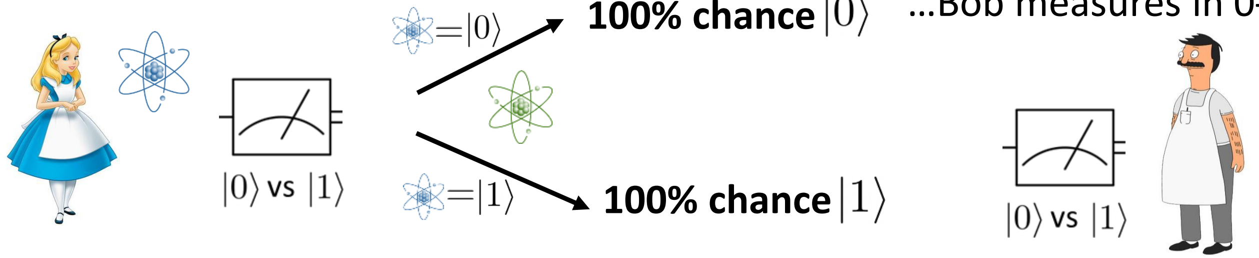
…we can express the EPR pair as:



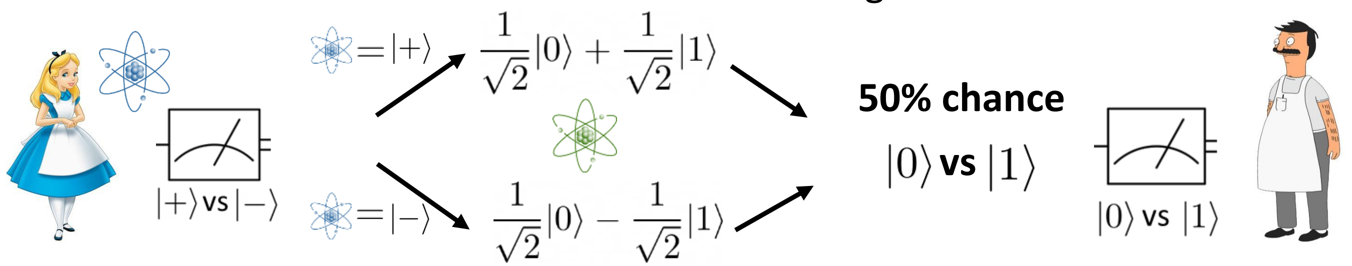
$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|-0\rangle\right] + \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle\right]$$

$$= \frac{1}{2}\left[|+0\rangle + |-0\rangle + |+1\rangle - |-1\rangle\right]$$

Case 1: Alice measures in 0-1 basis… …Bob measures in 0-1 basis.



Case 2: Alice measures in plus-minus basis… …Again Bob measures in 0-1 basis.



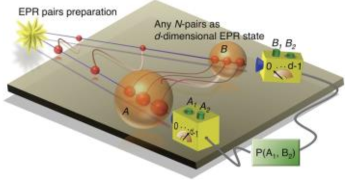
Punchline: If QM describes properties of the atoms that are tangibly real, why should Alice’s choice of measurement affect Bob’s measurement outcome? (Non-locality)

Niels Bohr

Next lecture: Can non-locality be verified experimentally?

John Bell (1964)

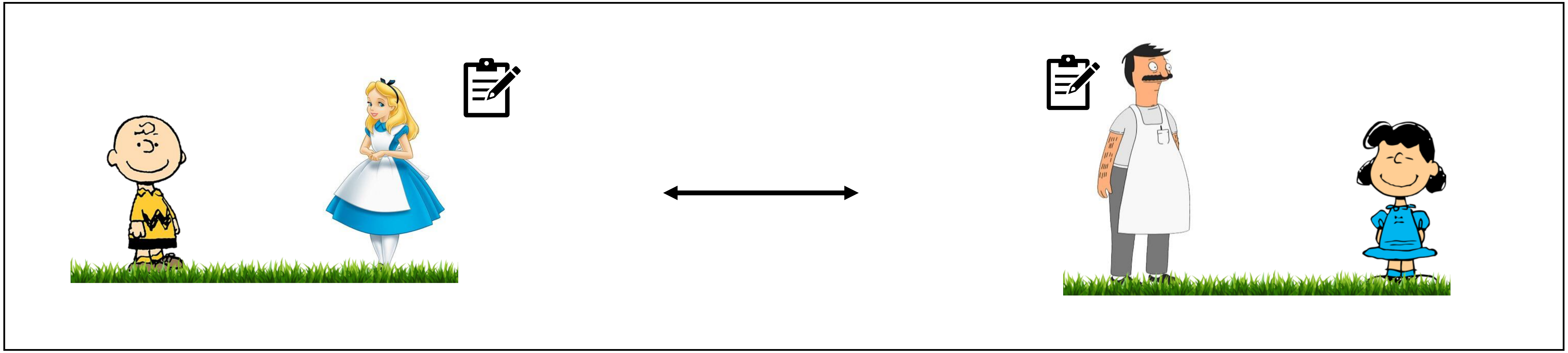
Answer: Amazing yes!
“Bell Experiments” highly suggest that nature is non-local.



CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

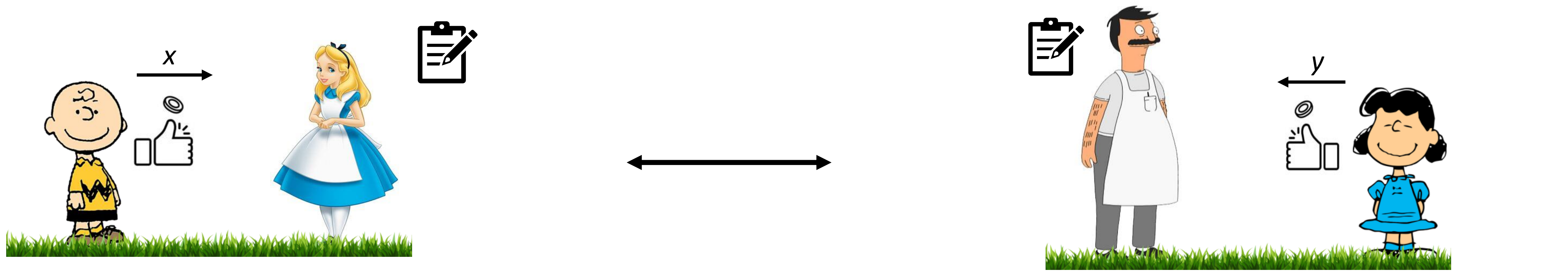
1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.



CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.



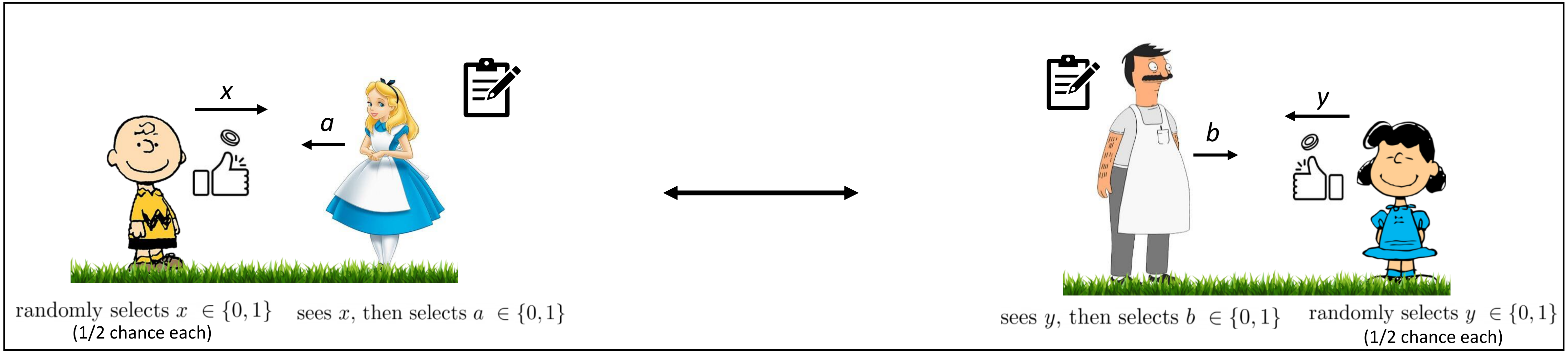
randomly selects $x \in \{0,1\}$
(1/2 chance each)

randomly selects $y \in \{0,1\}$
(1/2 chance each)

CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.

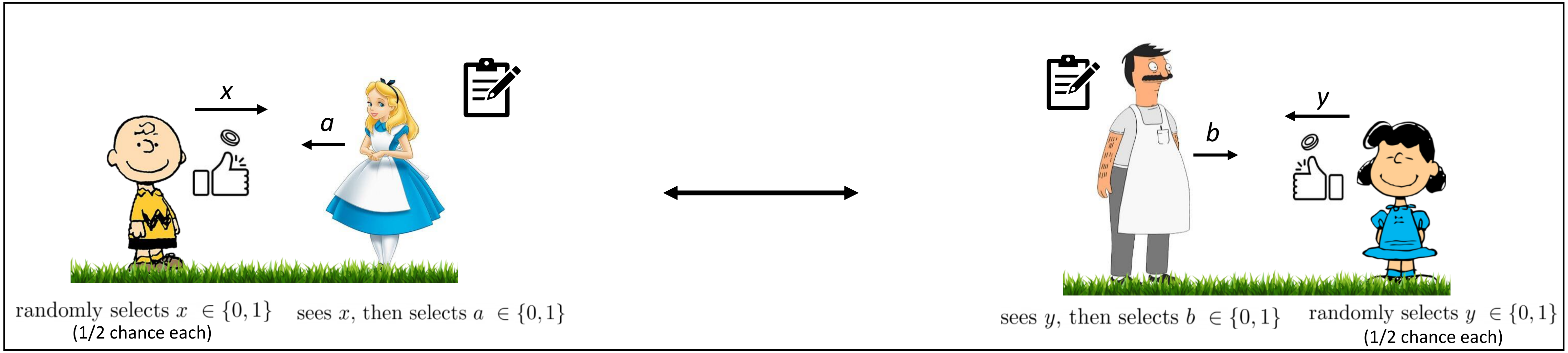


CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.
5. Alice and Bob win if $a \oplus b = x \wedge y$. Objective is to maximize chance of winning.

("a+b mod 2" = "x and y")



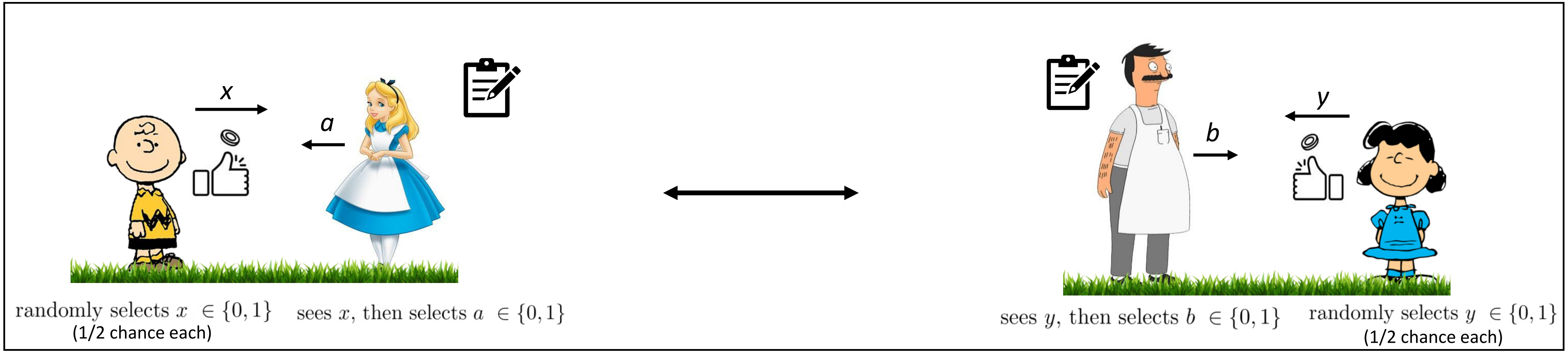
CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.
5. Alice and Bob win if $a \oplus b = x \wedge y$. Objective is to maximize chance of winning.

("a+b mod 2" = "x and y")

Initial attempt at a classical strategy?



CHSH Game

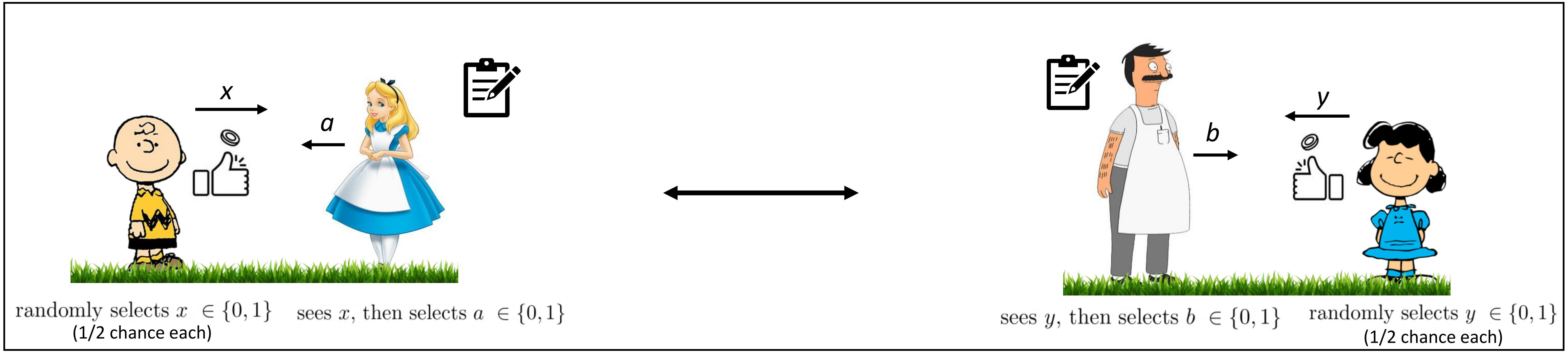
(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.
5. Alice and Bob win if $a \oplus b = x \wedge y$. Objective is to maximize chance of winning.

("a+b mod 2" = "x and y")

Initial attempt at a classical strategy?

Always pick a = 0 b = 0. Chance of winning: ?



CHSH Game

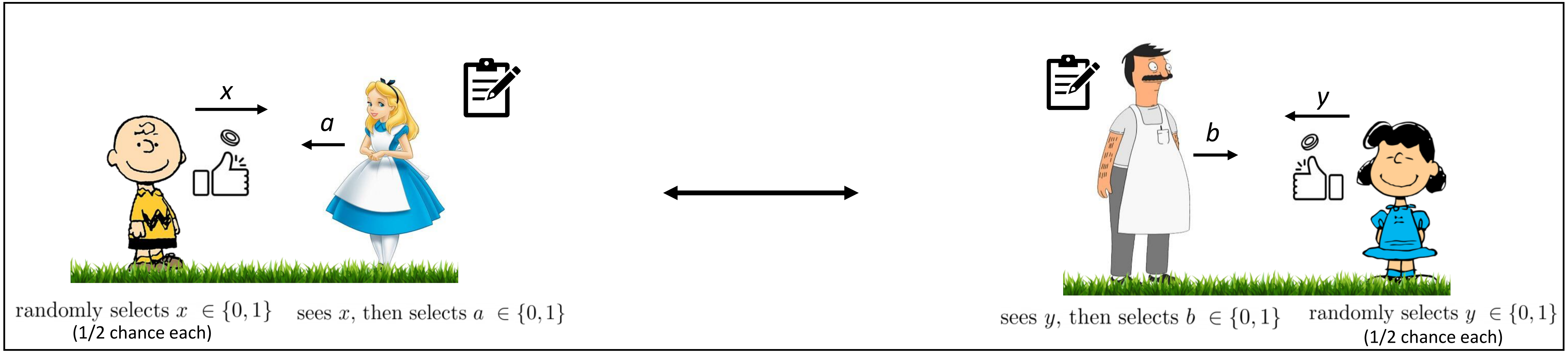
(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.
5. Alice and Bob win if $a \oplus b = x \wedge y$. Objective is to maximize chance of winning.

("a+b mod 2" = "x and y")

Initial attempt at a classical strategy?

Always pick a = 0 b = 0. Chance of winning: 3/4



CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

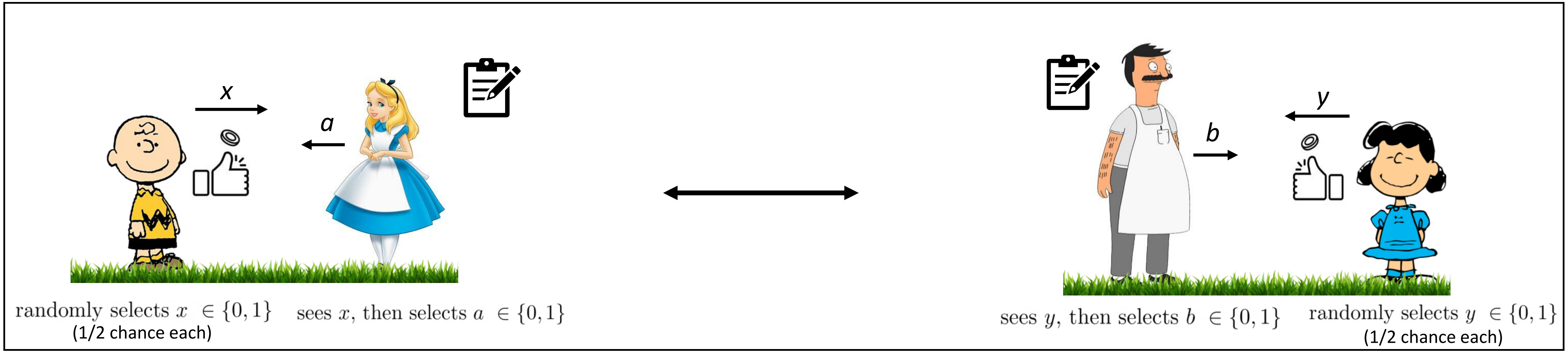
1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.
5. Alice and Bob win if $a \oplus b = x \wedge y$. Objective is to maximize chance of winning.

("a+b mod 2" = "x and y")

Initial attempt at a classical strategy?

Always pick a = 0 b = 0. Chance of winning: 3/4

Theorem: 3/4 is best possible win probability for classical strategy.



CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.
5. Alice and Bob win if $a \oplus b = x \wedge y$. Objective is to maximize chance of winning.

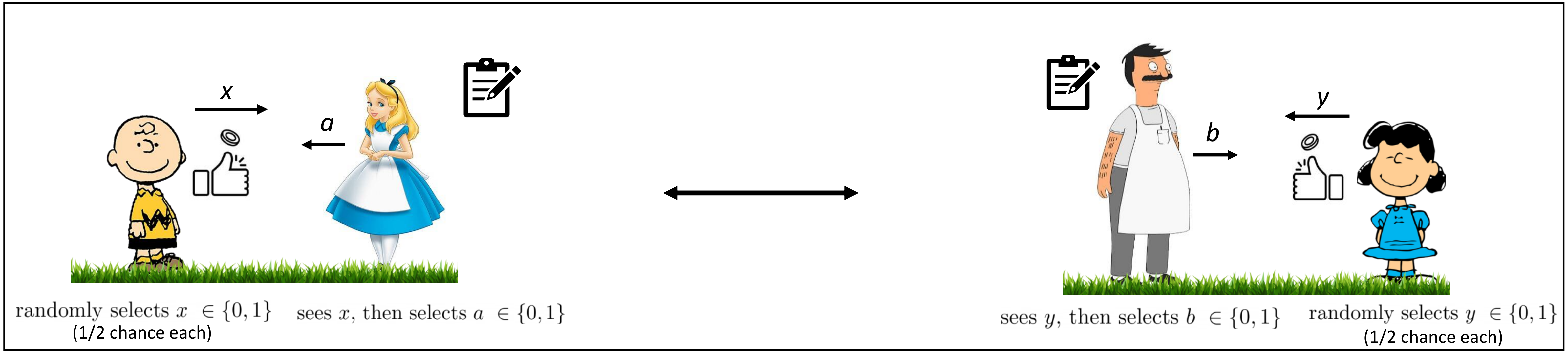
("a+b mod 2" = "x and y")

Initial attempt at a classical strategy?

Always pick a = 0 b = 0. Chance of winning: 3/4

Theorem: 3/4 is best possible win probability for classical strategy.

Better quantum strategy?



CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.
5. Alice and Bob win if $a \oplus b = x \wedge y$. Objective is to maximize chance of winning.

("a+b mod 2" = "x and y")

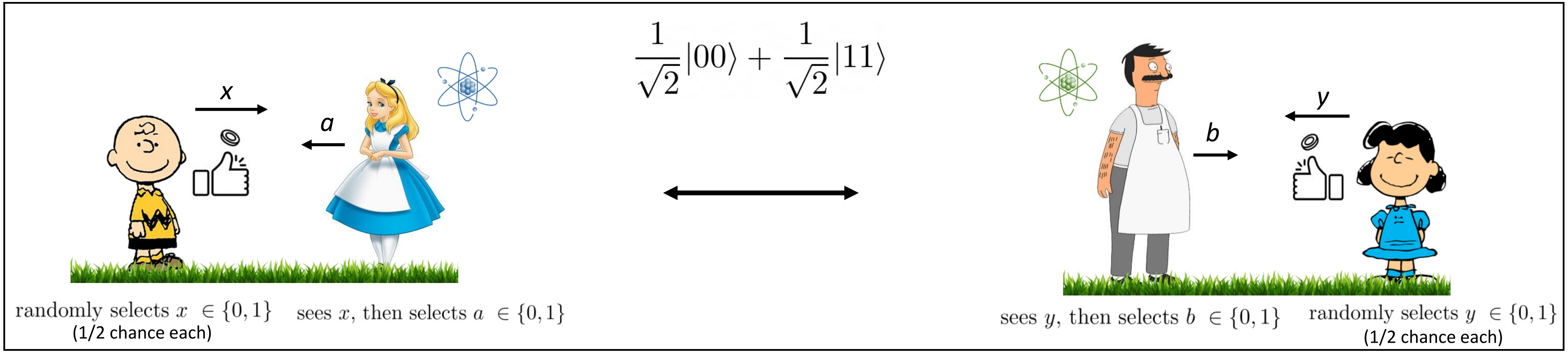
Initial attempt at a classical strategy?

Always pick a = 0 b = 0. Chance of winning: 3/4

Theorem: 3/4 is best possible win probability for classical strategy.

Better quantum strategy?

Yes! Using EPR pair Alice/Bob can win with prob ≈ 0.85



CHSH Game

(Alternative formulation/proof of Bell's Theorem by Clauser, Horne, Shimony, and Holt, 1969)

1. Alice and Bob initially meet in-person. Can share information and trade notes.
2. Alice and Bob separate by a far distance to meet Charlie and Lucy, respectively.
3. Charlie picks a random value x = 0 or 1. Lucy picks a random value y = 0 or 1.
4. After seeing x, Alice picks value a = 0 or 1. After seeing y, Bob picks value b = 0 or 1.
5. Alice and Bob win if $a \oplus b = x \wedge y$. Objective is to maximize chance of winning.

("a+b mod 2" = "x and y")

Initial attempt at a classical strategy?

Always pick a = 0 b = 0. Chance of winning: 3/4

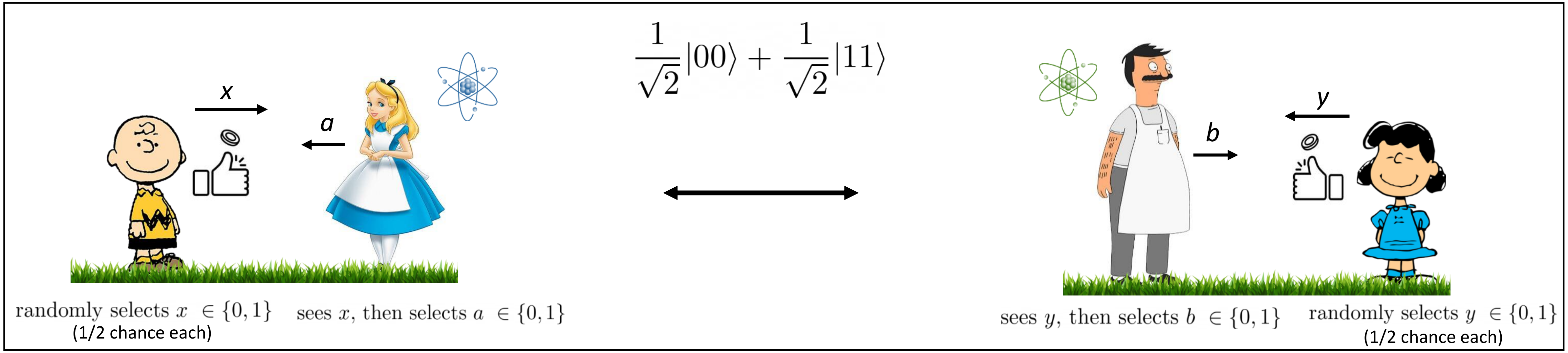
Theorem: 3/4 is best possible win probability for classical strategy.

Better quantum strategy?

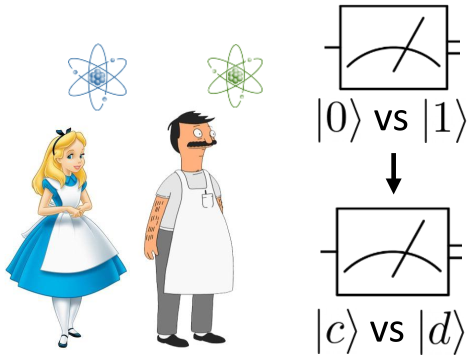
Yes! Using EPR pair Alice/Bob can win with prob ≈ 0.85

Punchline:

- Experiments simulate this game and win with probability close to 0.85.
- Suggests that physics is in fact non-local.

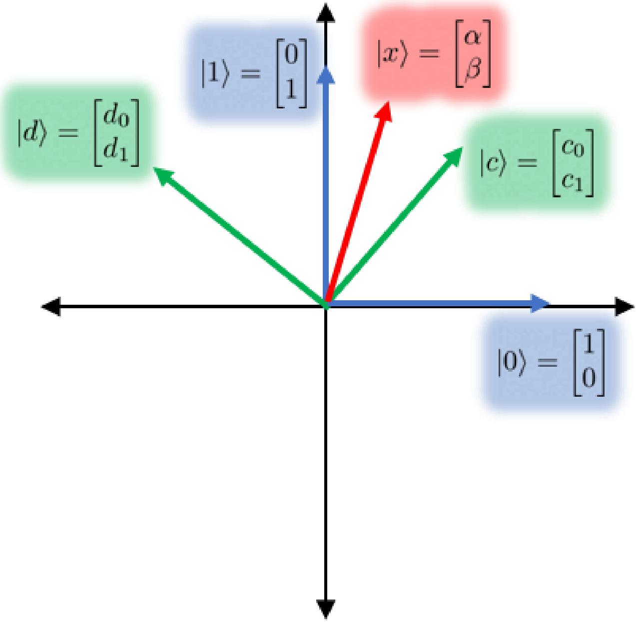


First-step: Understanding "Change of Basis"

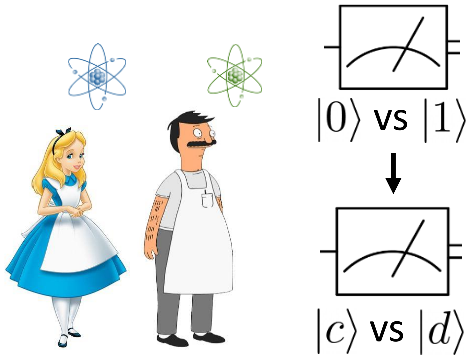


Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$



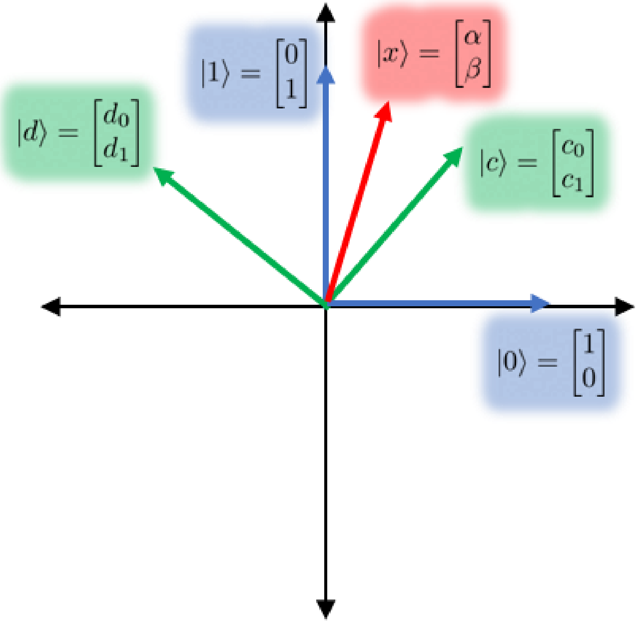
First-step: Understanding "Change of Basis"



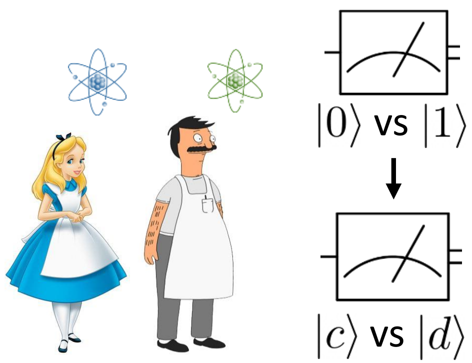
Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$

High-level Approach: find matrix that translates |x> between coordinate systems



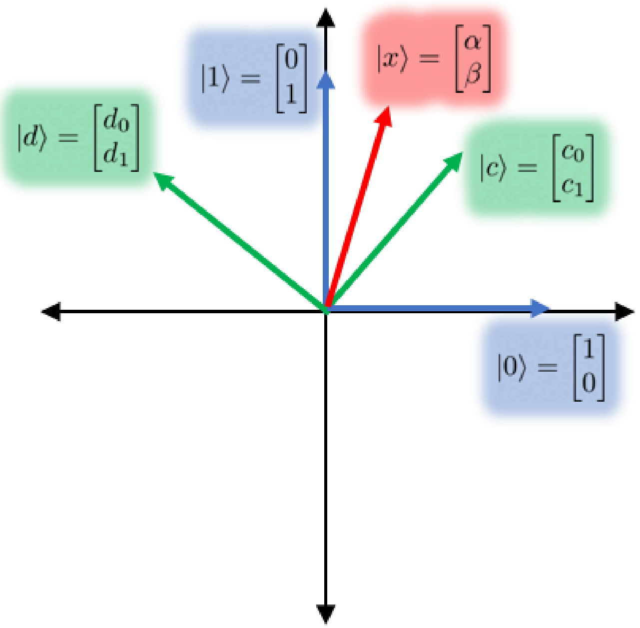
First-step: Understanding “Change of Basis”



Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$

High-level Approach: find matrix that translates |x> between coordinate systems

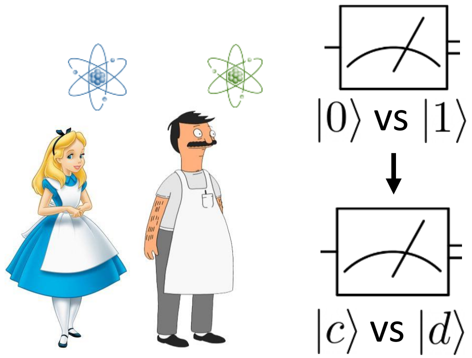


Easier first step: translate coordinates using green axis to coordinates using blue axis.

$$\begin{bmatrix} & ? & \end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}c_0\\c_1\end{bmatrix}$$

$$\begin{bmatrix} & ? & \end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}d_0\\d_1\end{bmatrix}$$

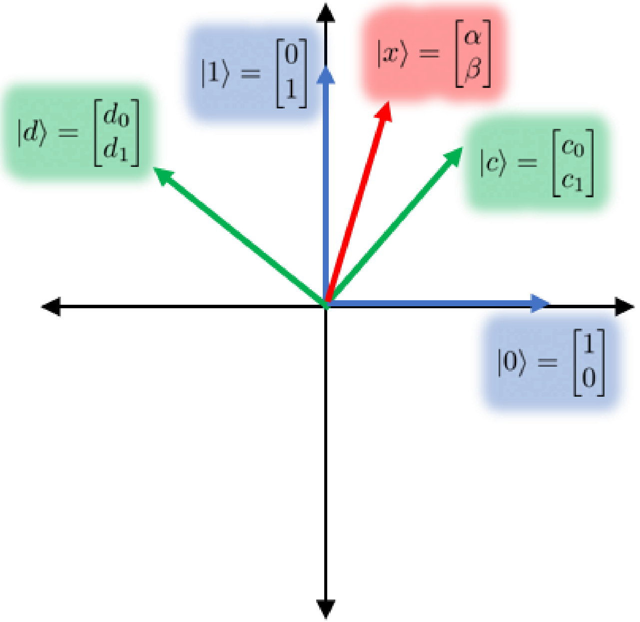
First-step: Understanding "Change of Basis"



Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$

High-level Approach: find matrix that translates |x> between coordinate systems

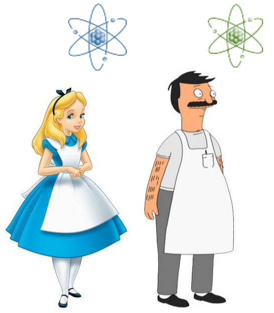


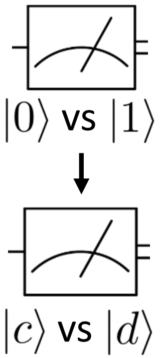
Easier first step: translate coordinates using green axis to coordinates using blue axis.

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \end{bmatrix}$$

First-step: Understanding "Change of Basis"

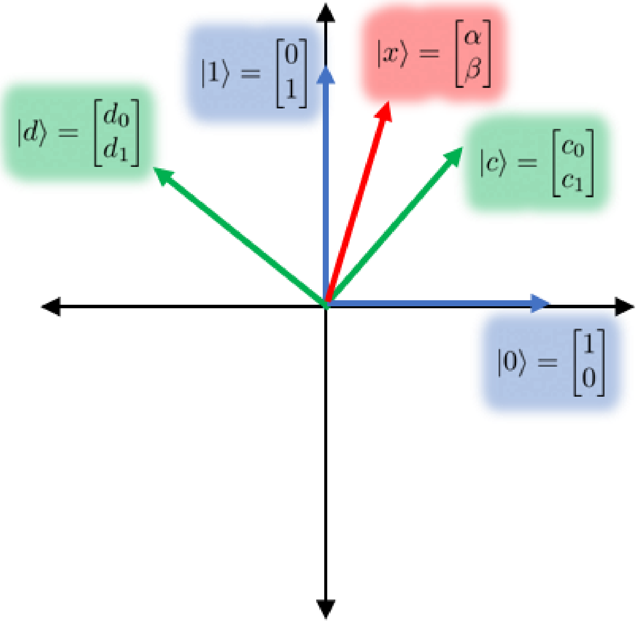




Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$

High-level Approach: find matrix that translates |x> between coordinate systems

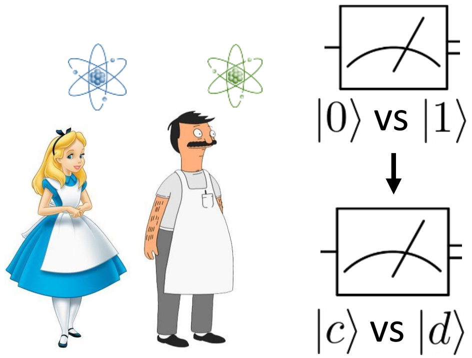


Easier first step: translate coordinates using green axis to coordinates using blue axis.

$$\begin{bmatrix}c_0 & d_0\\c_1 & d_1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}c_0\\c_1\end{bmatrix}$$

$$\underbrace{\begin{bmatrix}c_0 & d_0\\c_1 & d_1\end{bmatrix}}_{M}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}d_0\\d_1\end{bmatrix}$$

First-step: Understanding “Change of Basis”



Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

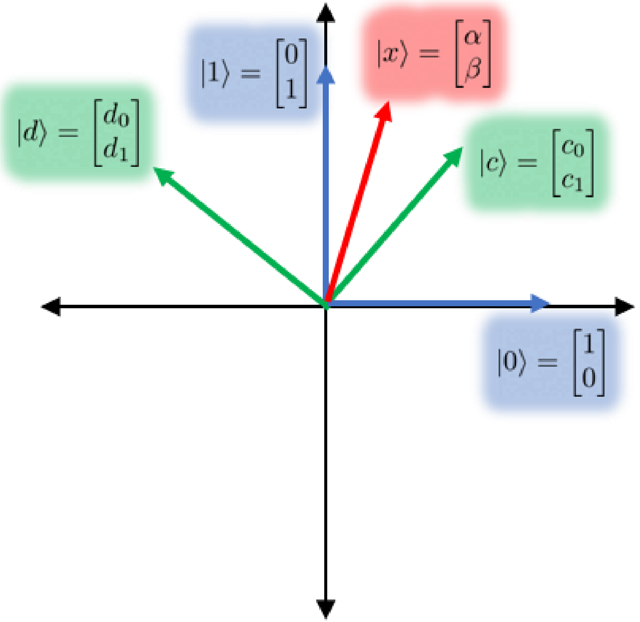
Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$

High-level Approach: find matrix that translates |x> between coordinate systems

Our goal: translate coordinates using blue axis to coordinates using green axis .

$$\begin{bmatrix} & ? & \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} & ? & \end{bmatrix}\begin{bmatrix} d_0 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

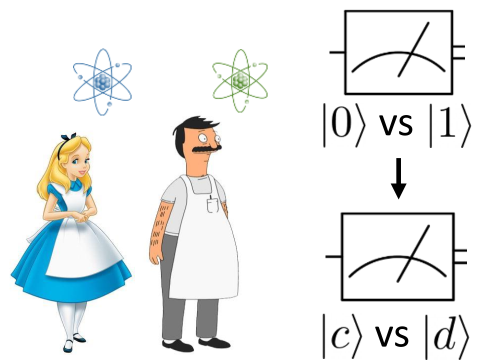


Easier first step: translate coordinates using green axis to coordinates using blue axis.

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}}_{M}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \end{bmatrix}$$

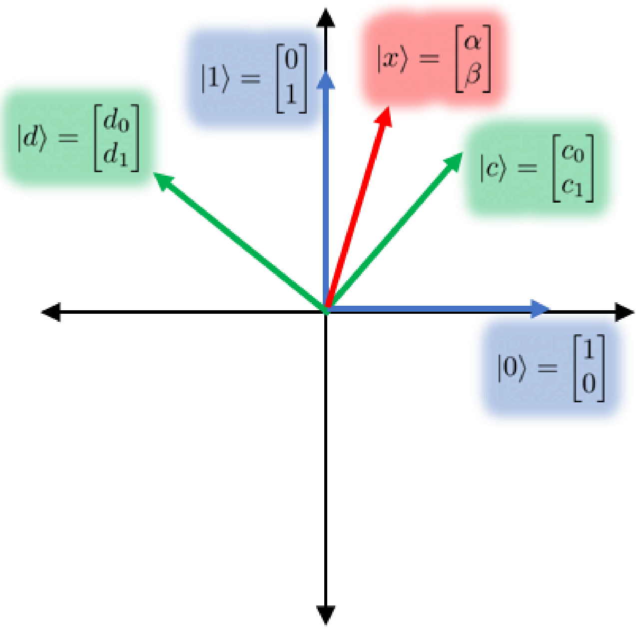
First-step: Understanding “Change of Basis”



Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$

High-level Approach: find matrix that translates |x> between coordinate systems



Easier first step: translate coordinates using green axis to coordinates using blue axis.

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}}_{M} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \end{bmatrix}$$

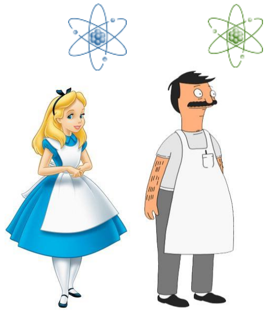
Our goal: translate coordinates using blue axis to coordinates using green axis .

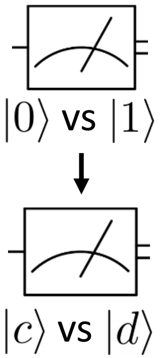
$$\begin{bmatrix} & M^{-1} & \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} & M^{-1} & \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Multiply by inverse matrix

First-step: Understanding “Change of Basis”

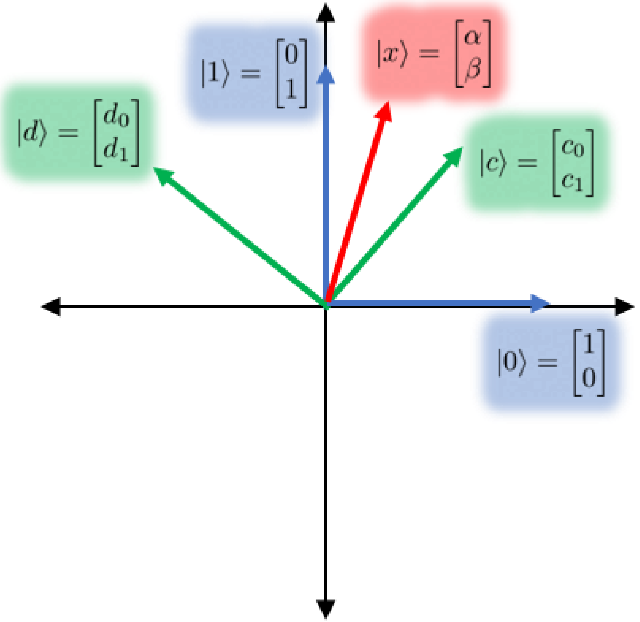




Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$

High-level Approach: find matrix that translates |x> between coordinate systems



Easier first step: translate coordinates using green axis to coordinates using blue axis.

$$\underbrace{\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}}_{M} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \end{bmatrix}$$

M

Our goal: translate coordinates using blue axis to coordinates using green axis .

$$\begin{bmatrix} & M^{-1} & \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} & M^{-1} & \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Multiply by inverse matrix

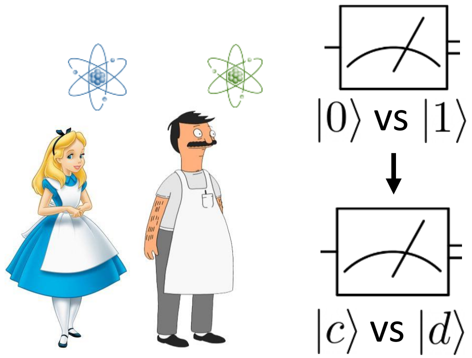
General formula for Inverse of 2x2 Matrix

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}^{-1} = \frac{1}{c_0 d_1 - c_1 d_0} \begin{bmatrix} d_1 & -d_0 \\ -c_1 & c_0 \end{bmatrix}$$

Thus: we can find $|x\rangle = ?|c\rangle + ?|d\rangle$ with:

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}^{-1} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

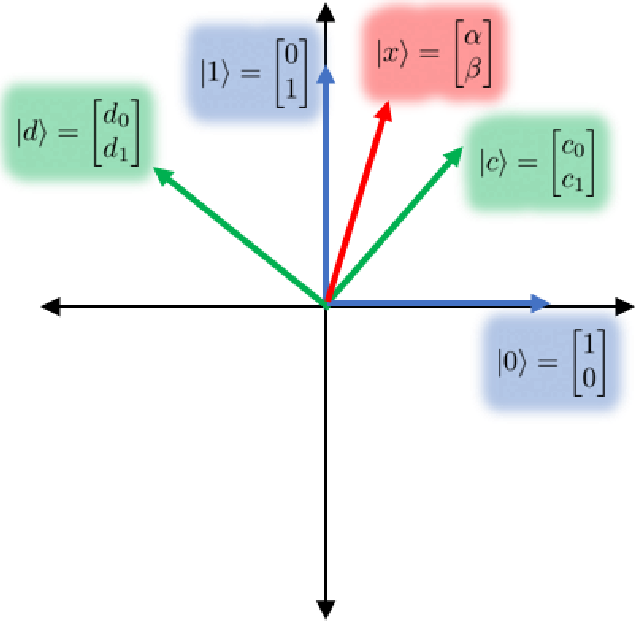
First-step: Understanding "Change of Basis"



Given: $|x\rangle = \alpha|0\rangle + \beta|1\rangle$

Want to express: $|x\rangle = ?|c\rangle + ?|d\rangle$

High-level Approach: find matrix that translates |x> between coordinate systems



Easier first step: translate coordinates using green axis to coordinates using blue axis.

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix} \boxed{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \boxed{\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}}$$

$$\underbrace{\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}}_{M} \boxed{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = \boxed{\begin{bmatrix} d_0 \\ d_1 \end{bmatrix}}$$

Our goal: translate coordinates using blue axis to coordinates using green axis .

$$\begin{bmatrix} & M^{-1} & \end{bmatrix} \boxed{\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}} = \boxed{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$\begin{bmatrix} & M^{-1} & \end{bmatrix} \boxed{\begin{bmatrix} d_0 \\ d_1 \end{bmatrix}} = \boxed{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

Multiply by inverse matrix

General formula for Inverse of 2x2 Matrix

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}^{-1} = \cancel{\frac{1}{c_0 d_1 - c_1 d_0} \begin{bmatrix} d_1 & -d_0 \\ -c_1 & c_0 \end{bmatrix}}$$

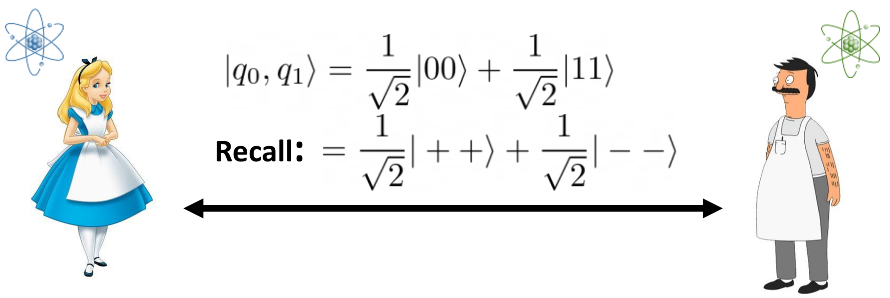
Thus: we can find $|x\rangle = ?|c\rangle + ?|d\rangle$ with:

$$\begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}^{-1} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \rightarrow \begin{bmatrix} c_0 & d_0 \\ c_1 & d_1 \end{bmatrix}^{T} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

when M is unitary

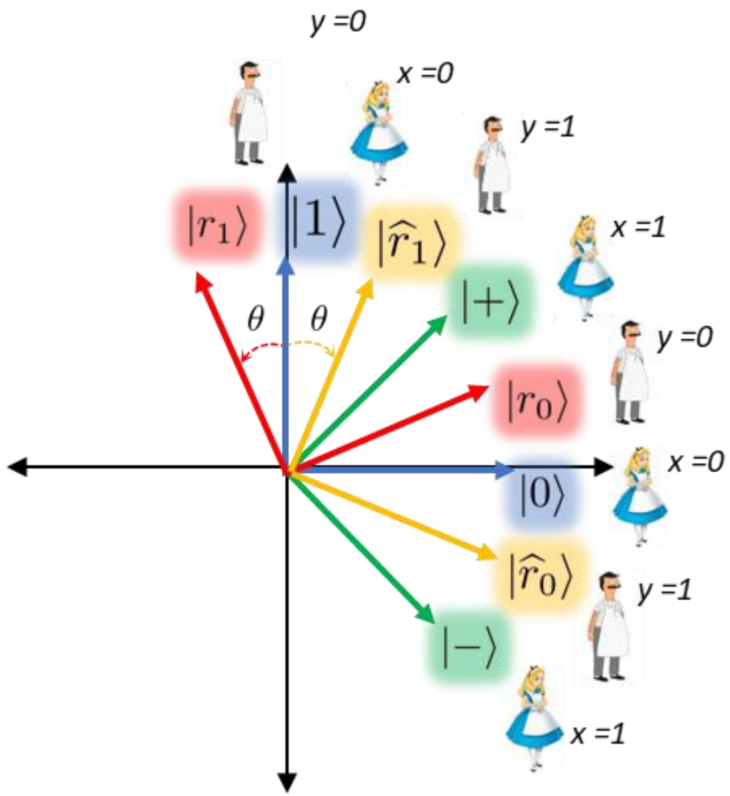
Quantum Strategy

Step 1: create EPR pair before separating



$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$



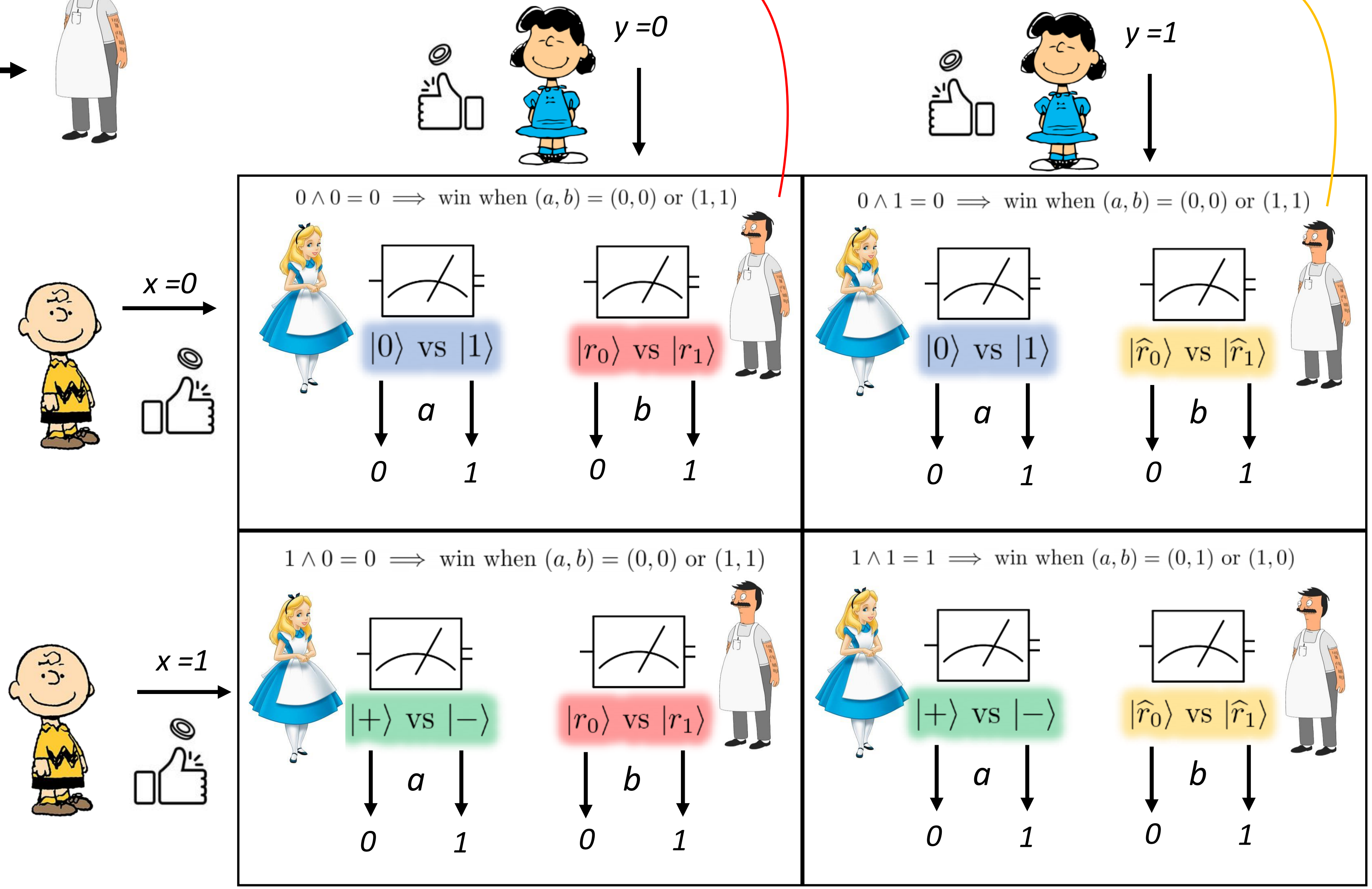
Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$$
$$|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

Clockwise Rotation

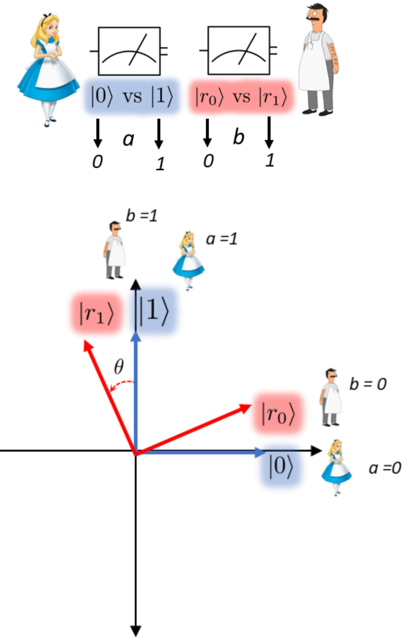
$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$$
$$|\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

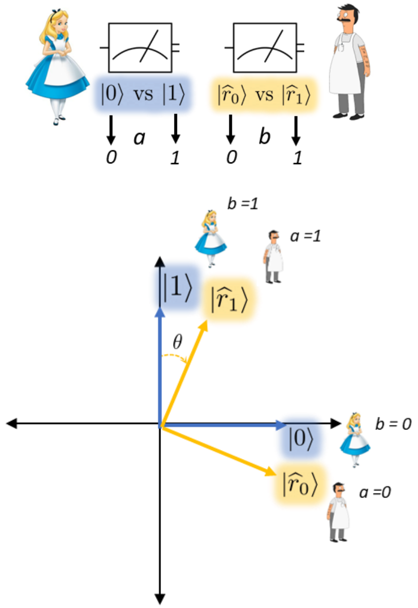
$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



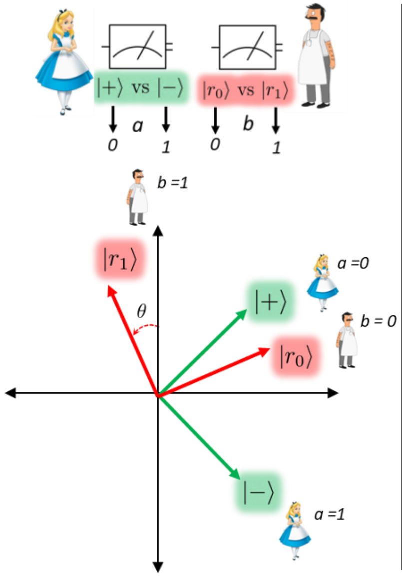
Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

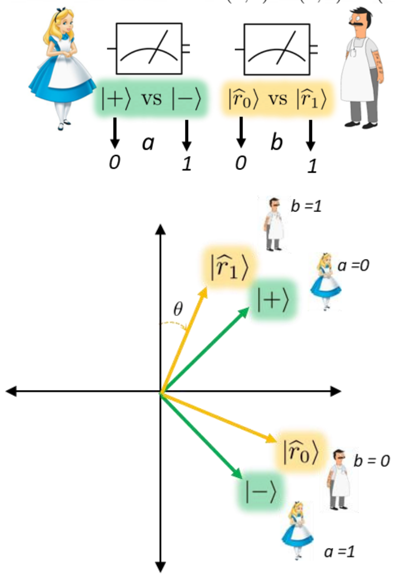
$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$

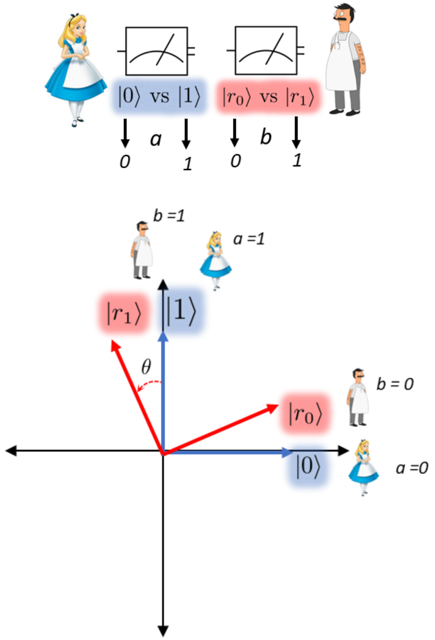


$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



Counterclockwise Rotation

$0 \wedge 0 = 0 \implies$ win when $(a, b) = (0, 0)$ or $(1, 1)$



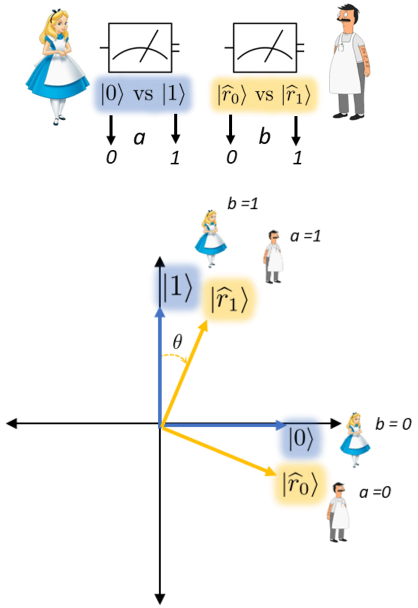
$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix}$$

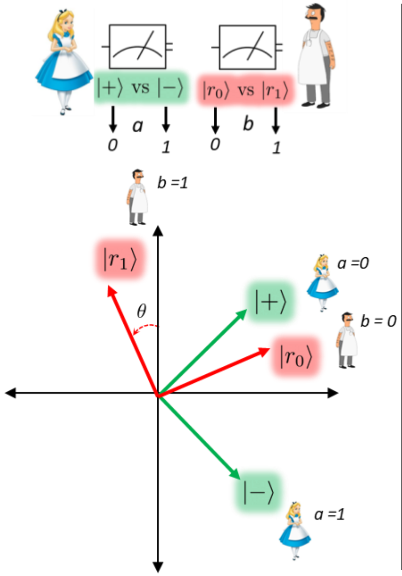
Clockwise Rotation

$0 \wedge 1 = 0 \implies$ win when $(a, b) = (0, 0)$ or $(1, 1)$

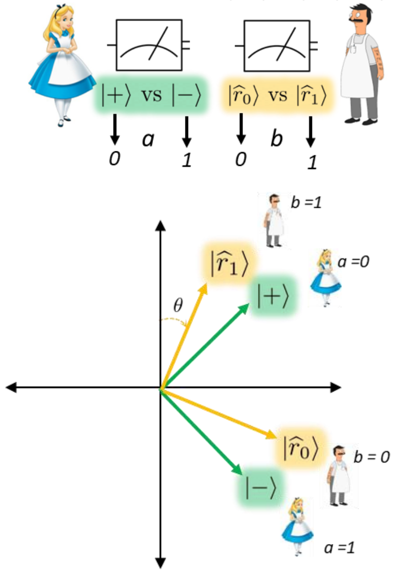


$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

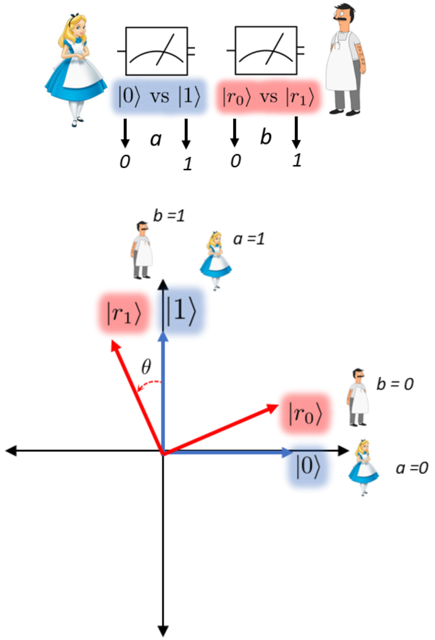
$1 \wedge 0 = 0 \implies$ win when $(a, b) = (0, 0)$ or $(1, 1)$



$1 \wedge 1 = 1 \implies$ win when $(a, b) = (0, 1)$ or $(1, 0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



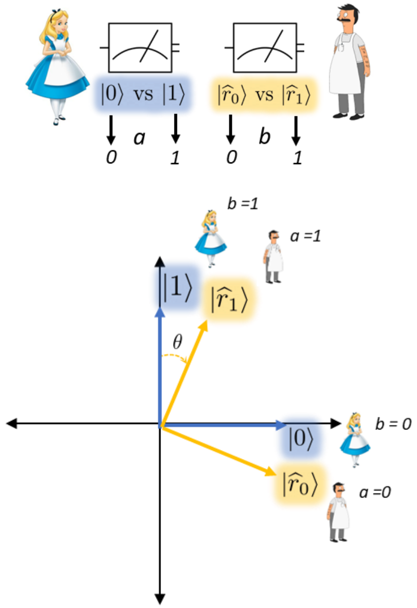
Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

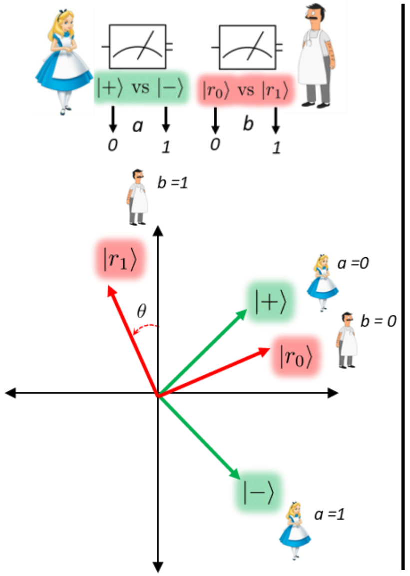
$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



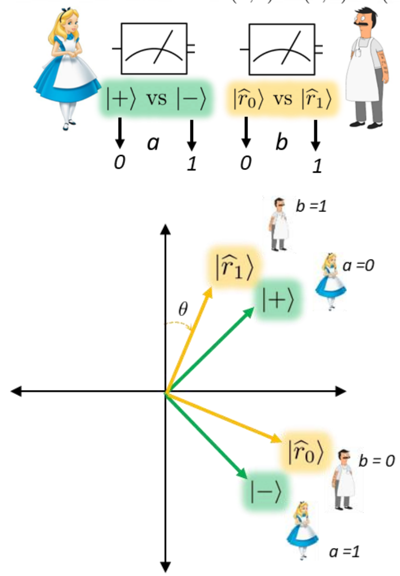
Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

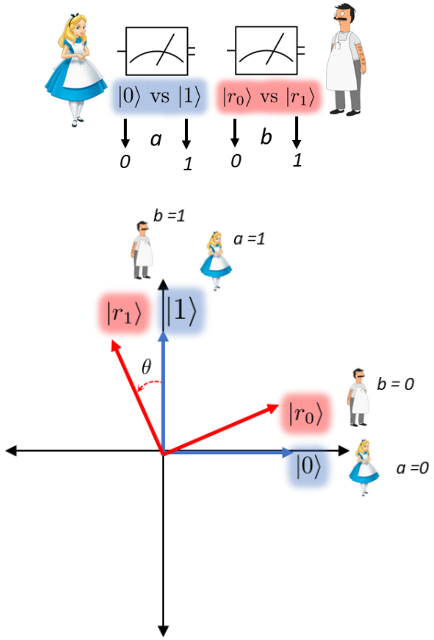
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

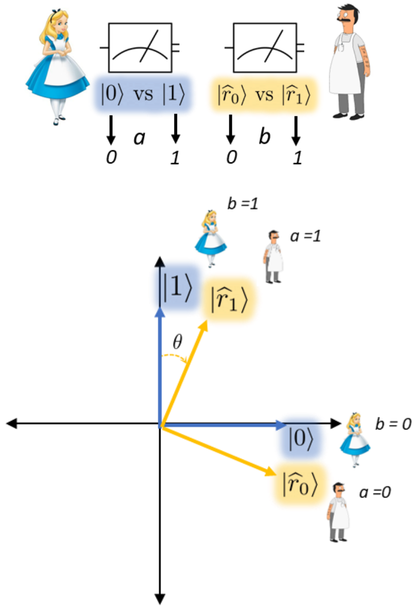
$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

$M^{-1} =$ clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$$

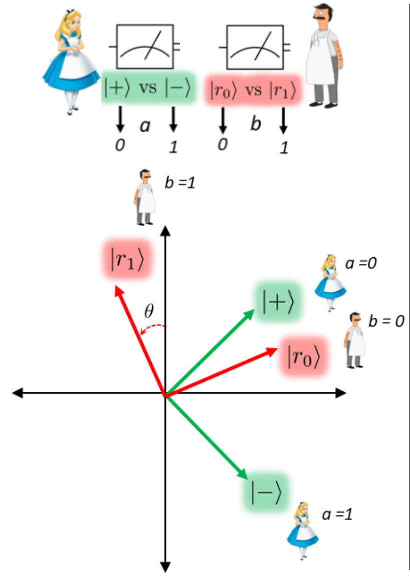
$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



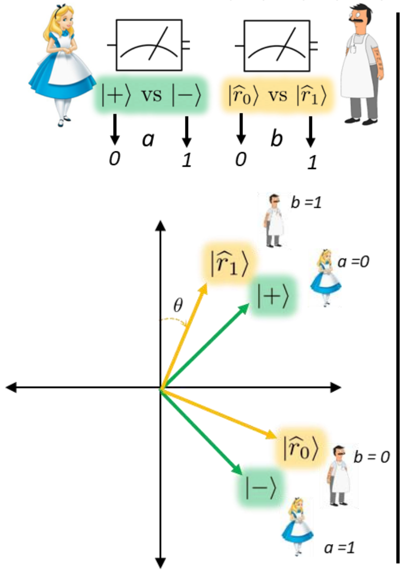
Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

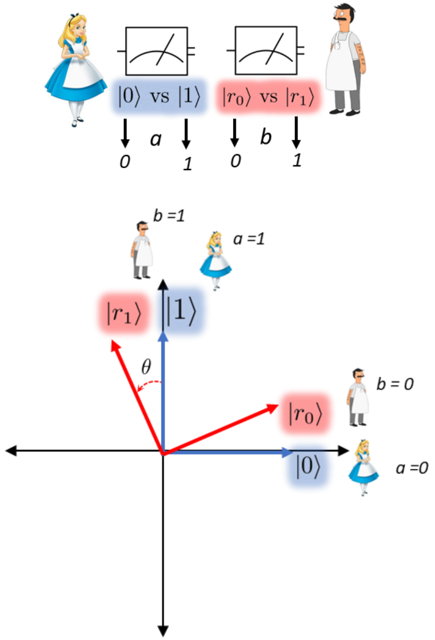
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

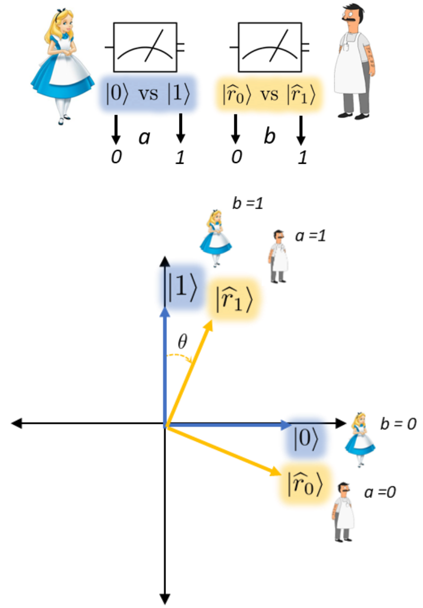
$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$ $|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

$M^{-1} =$ clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

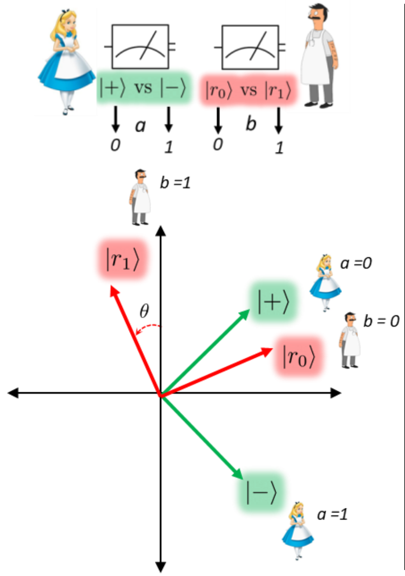
$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



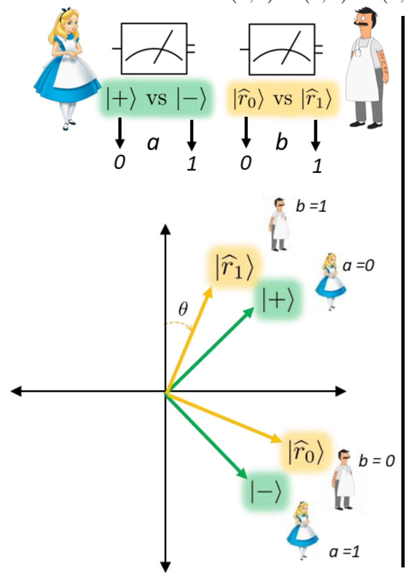
Clockwise Rotation

$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$ $|\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$

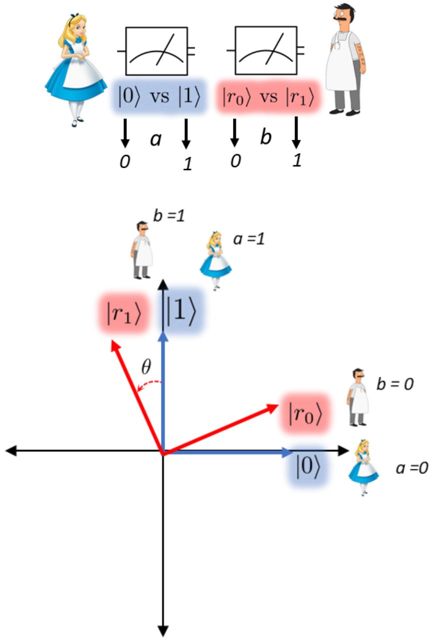
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a, b) = (0, 0)$ or $(1, 1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

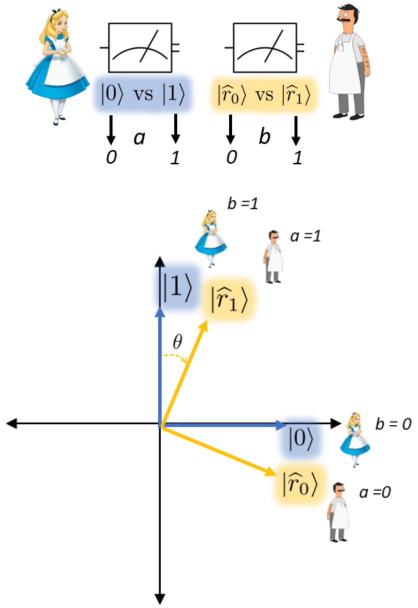
$$\boxed{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}\boxed{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\boxed{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}\boxed{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

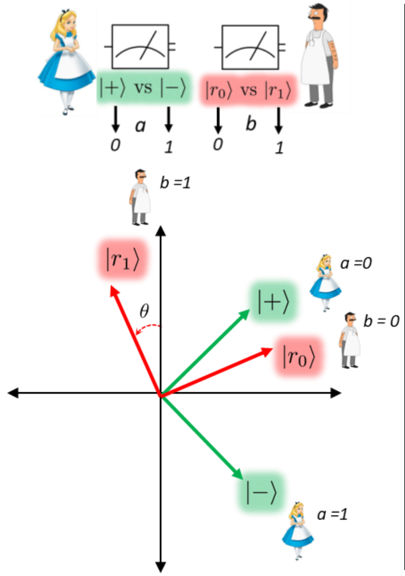
$0 \wedge 1 = 0 \implies$ win when $(a, b) = (0, 0)$ or $(1, 1)$



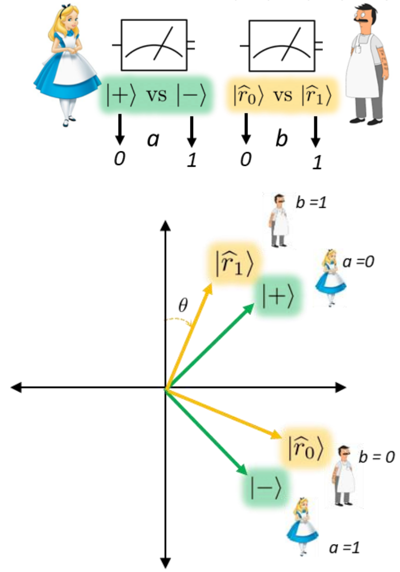
Clockwise Rotation

$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

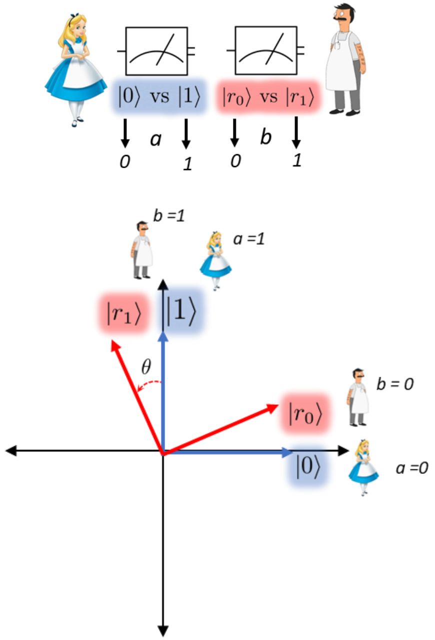
$1 \wedge 0 = 0 \implies$ win when $(a, b) = (0, 0)$ or $(1, 1)$



$1 \wedge 1 = 1 \implies$ win when $(a, b) = (0, 1)$ or $(1, 0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

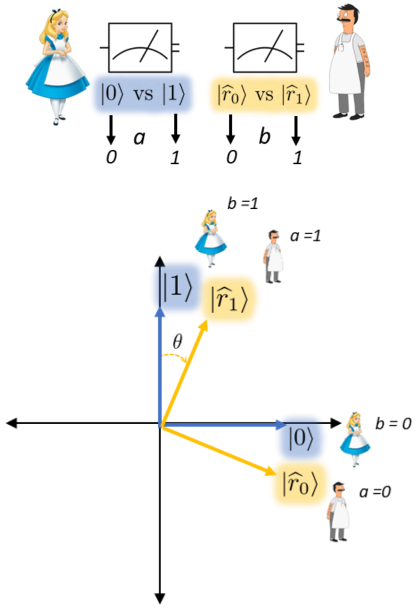
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\cos\theta|0r_0\rangle - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \cos\theta|1r_1\rangle\right]$$

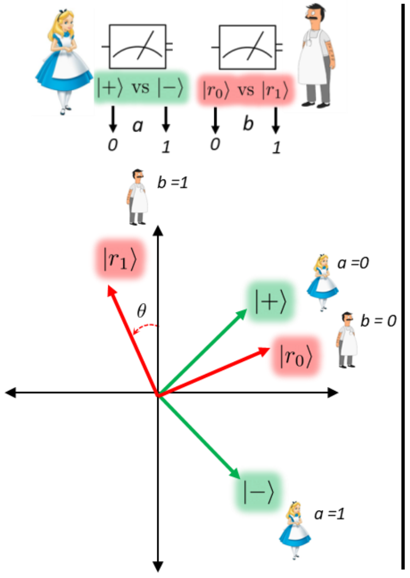
$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



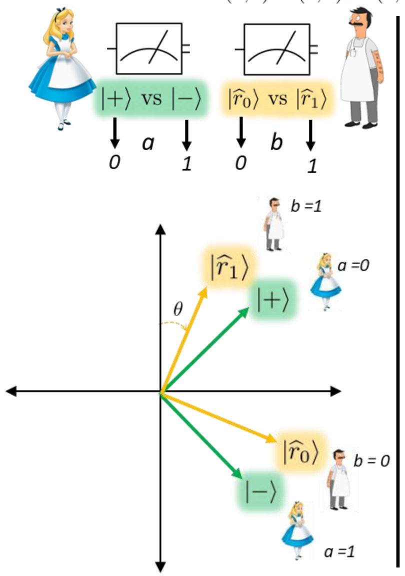
Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$

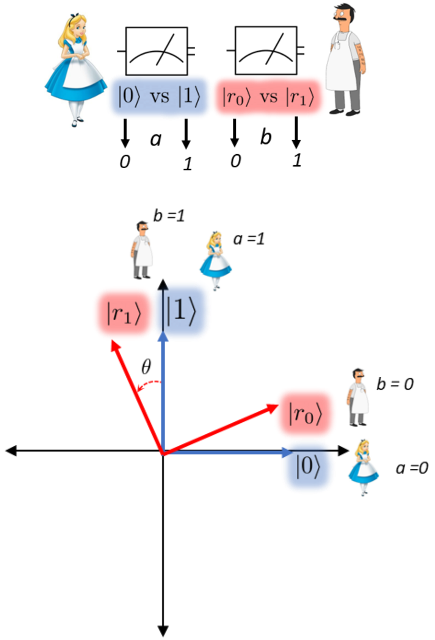


$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



Counterclockwise Rotation

$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

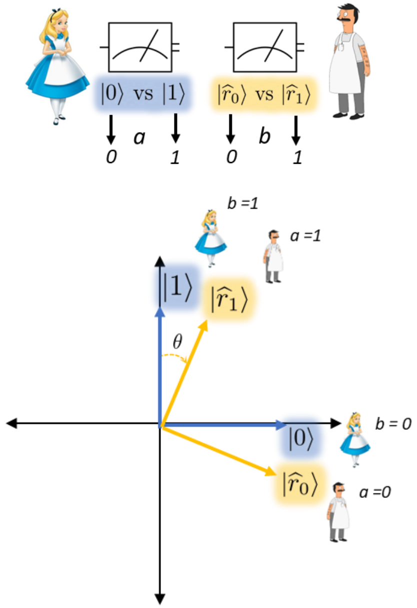
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\underbrace{\boxed{\cos\theta|0r_0\rangle}}_{\text{win}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \underbrace{\boxed{\cos\theta|1r_1\rangle}}_{\text{win}}\right]$$

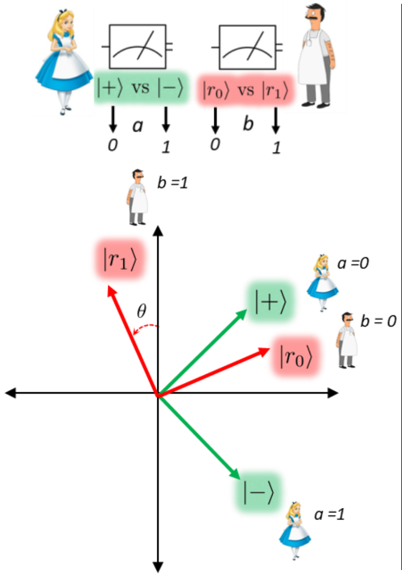
Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

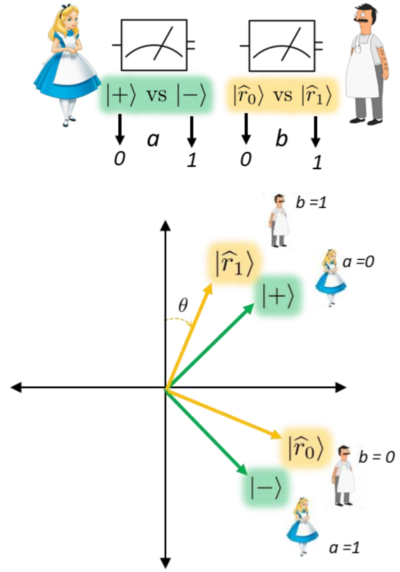
$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



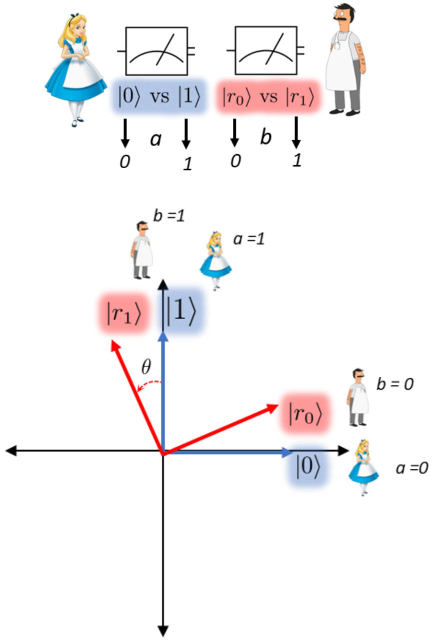
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

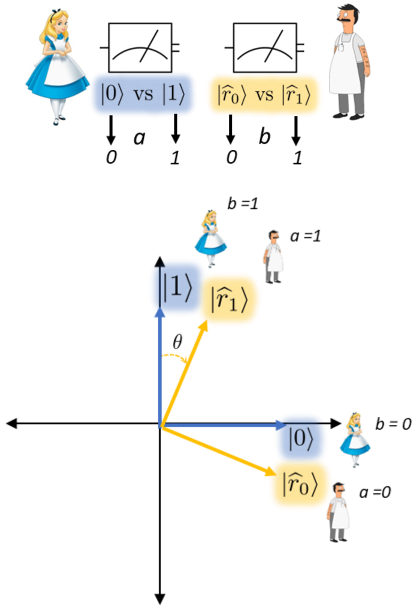
$$\frac{1}{\sqrt{2}}\left[\underbrace{\boxed{\cos\theta|0r_0\rangle}}_{\text{win}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \underbrace{\boxed{\cos\theta|1r_1\rangle}}_{\text{win}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

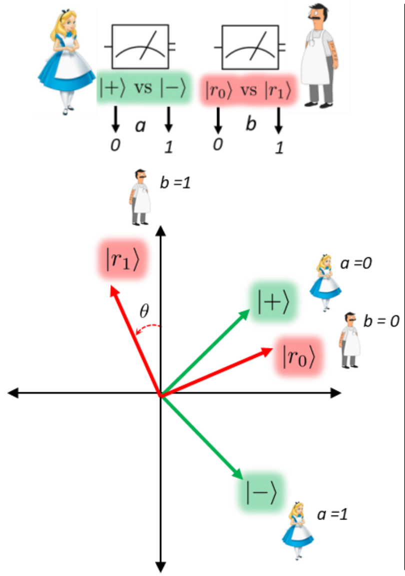
Clockwise Rotation

$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$

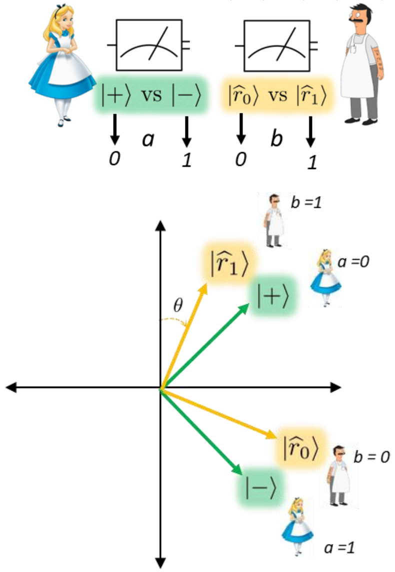
$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



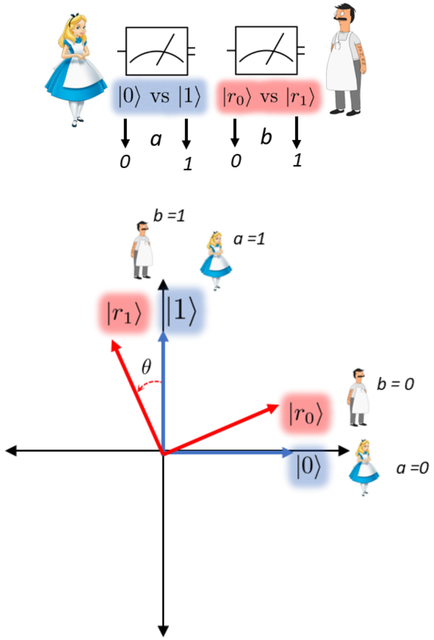
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

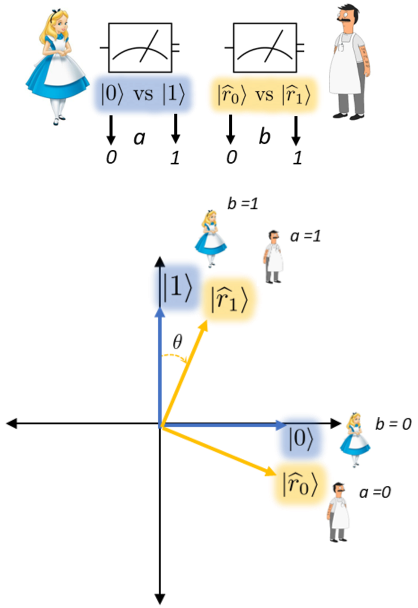
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\text{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\text{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



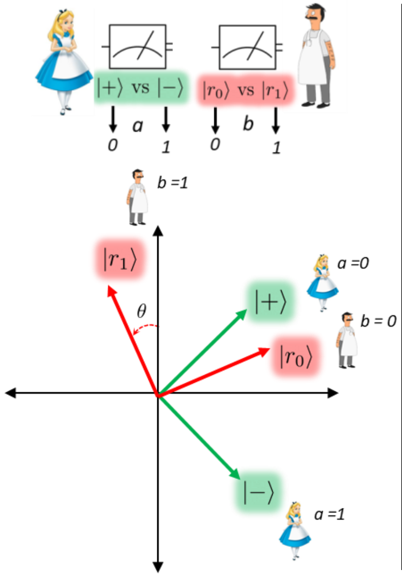
Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

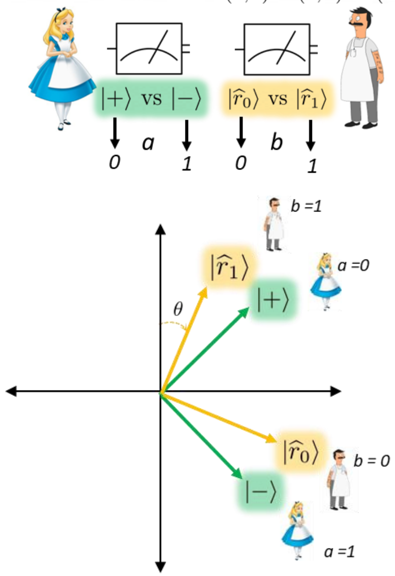
M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

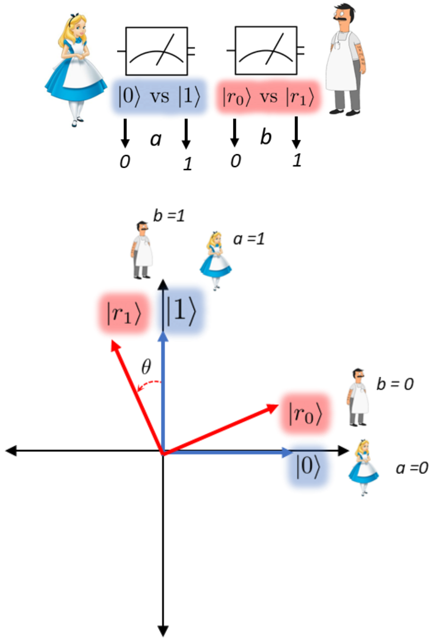
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

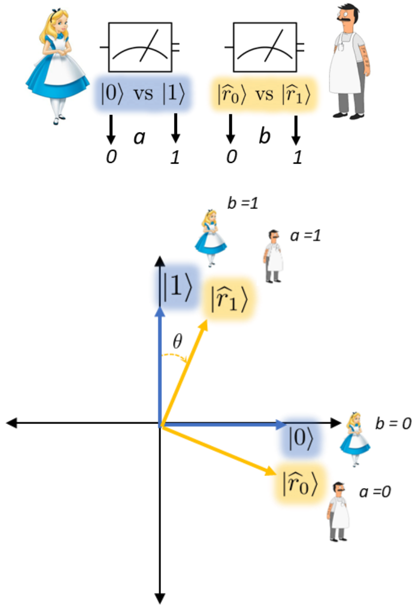
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

win **win**

$$\frac{1}{\sqrt{2}}\left[\boxed{\cos\theta|0r_0\rangle} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \boxed{\cos\theta|1r_1\rangle}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



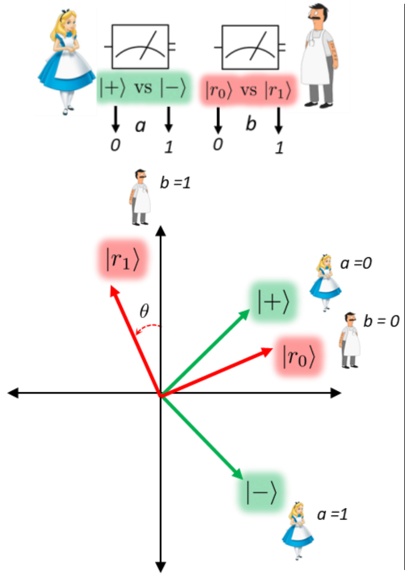
Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

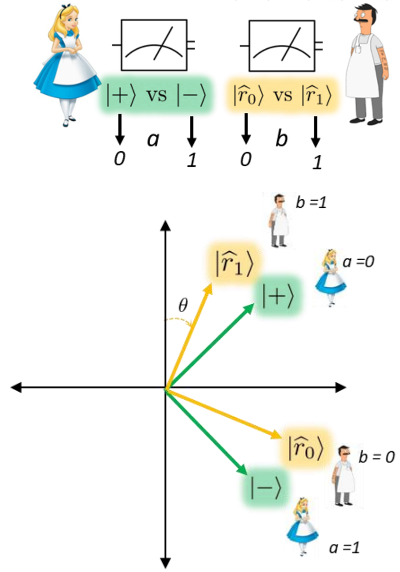
M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

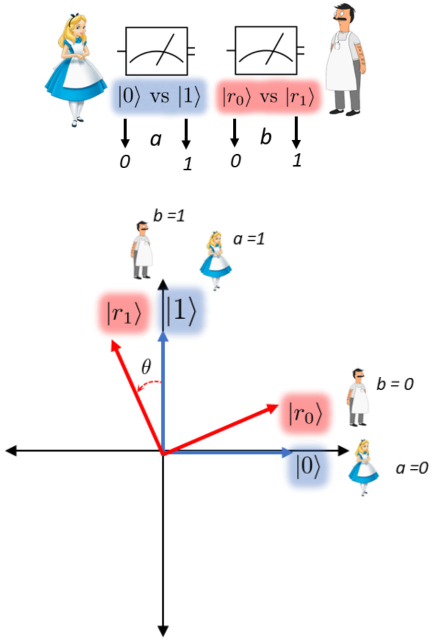
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta \\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta \\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

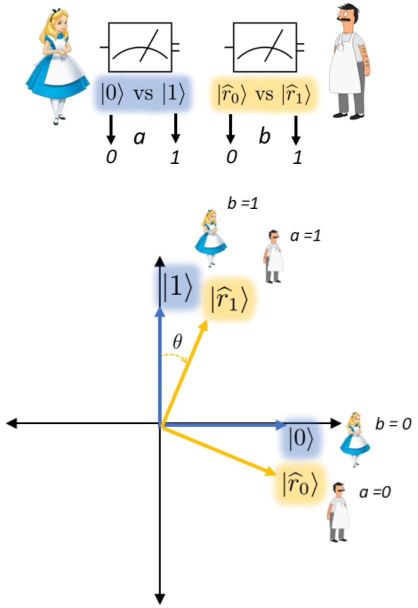
$$\frac{1}{\sqrt{2}}\left[\overbrace{\boxed{\cos\theta|0r_0\rangle}}^{\text{win}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overbrace{\boxed{\cos\theta|1r_1\rangle}}^{\text{win}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$

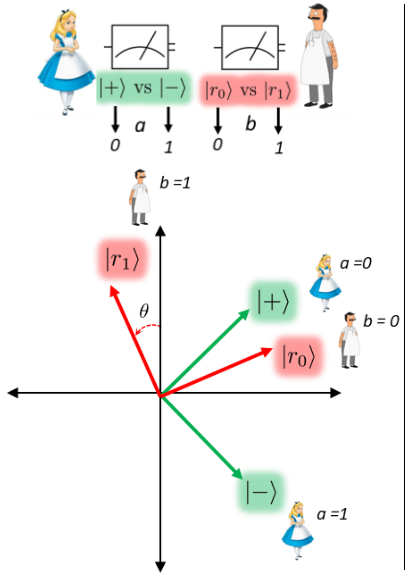


M^{-1} = counterclockwise

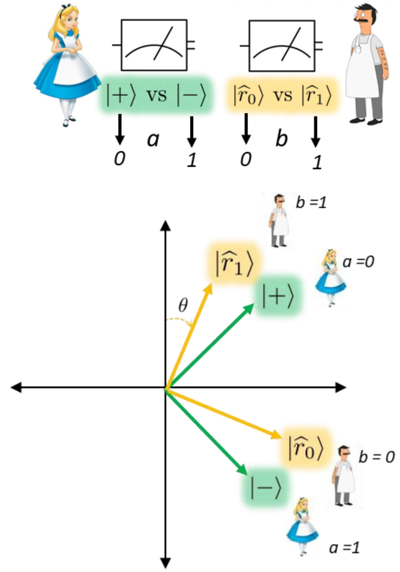
$$\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

$$\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta \\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\hat{r}_0\rangle + \cos\theta|\hat{r}_1\rangle$$

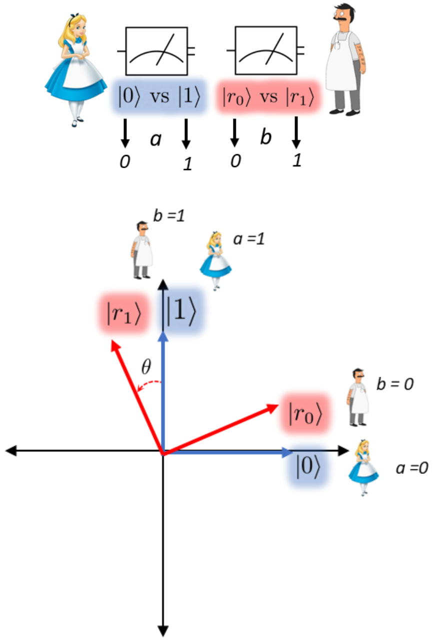
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

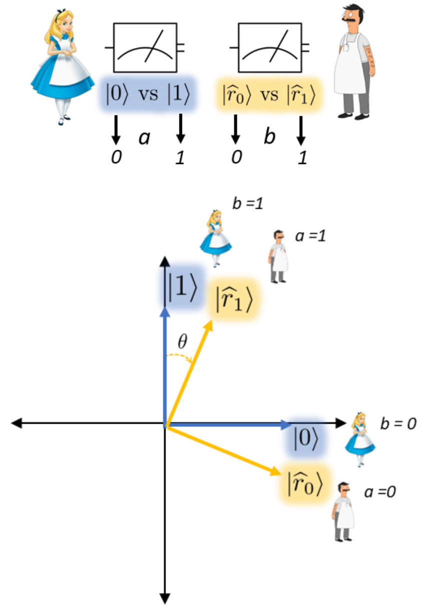
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\text{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\text{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Clockwise Rotation

$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

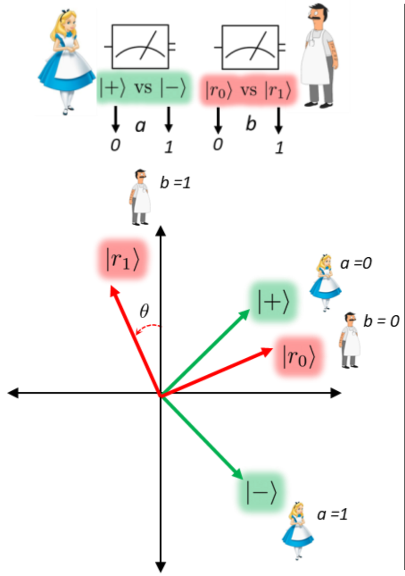
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|\widehat{r}_0\rangle + \sin\theta|\widehat{r}_1\rangle$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\widehat{r}_0\rangle + \cos\theta|\widehat{r}_1\rangle$$

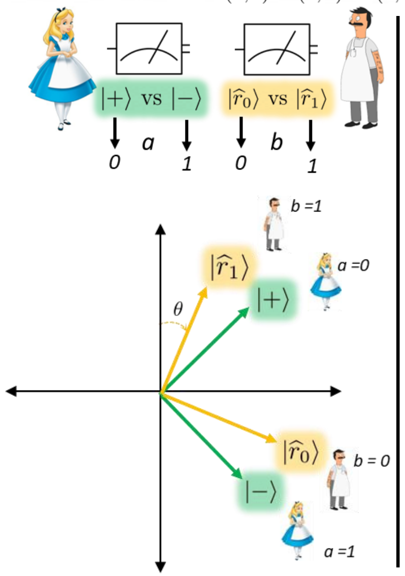
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\cos\theta|0\widehat{r}_0\rangle + \sin\theta|0\widehat{r}_1\rangle - \sin\theta|1\widehat{r}_0\rangle + \cos\theta|1\widehat{r}_1\rangle\right]$$

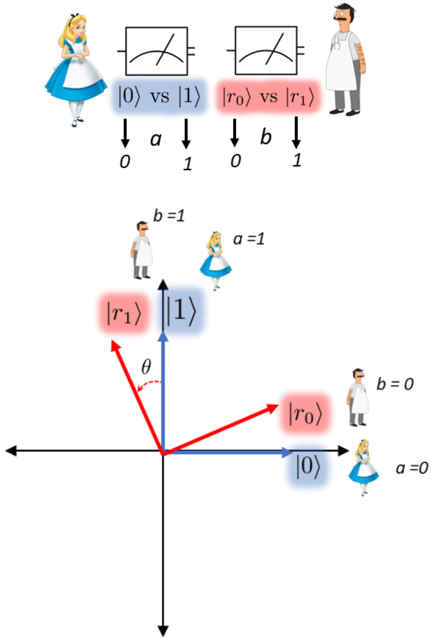
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

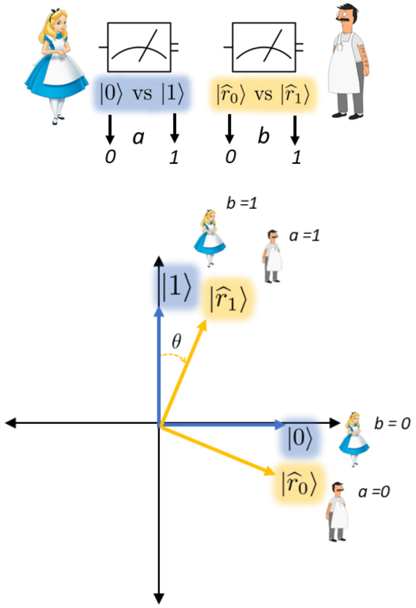
$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

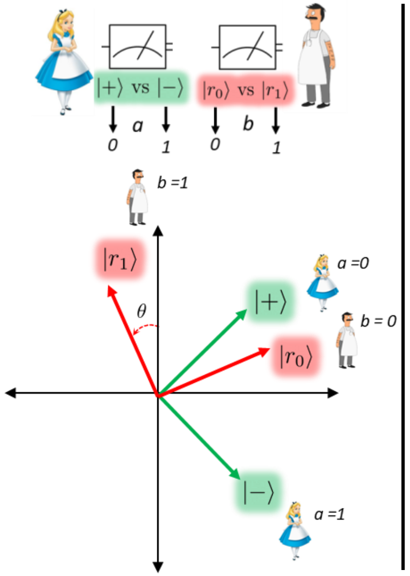
$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\hat{r}_0\rangle + \cos\theta|\hat{r}_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

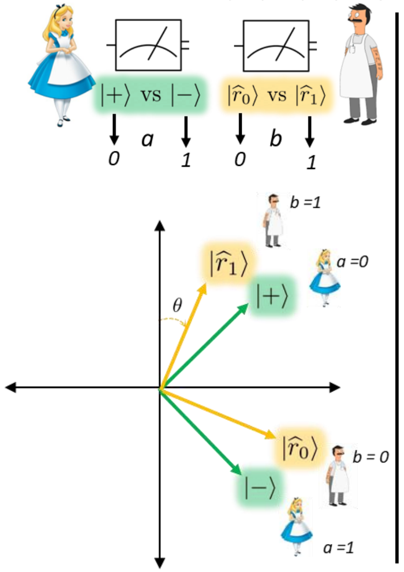
$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0\hat{r}_0\rangle}} + \sin\theta|0\hat{r}_1\rangle - \sin\theta|1\hat{r}_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1\hat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

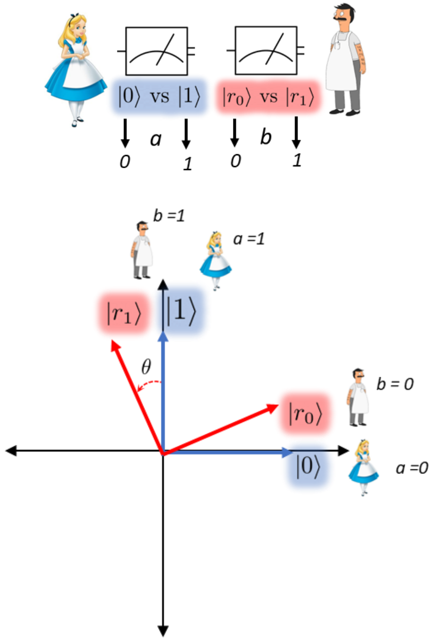
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

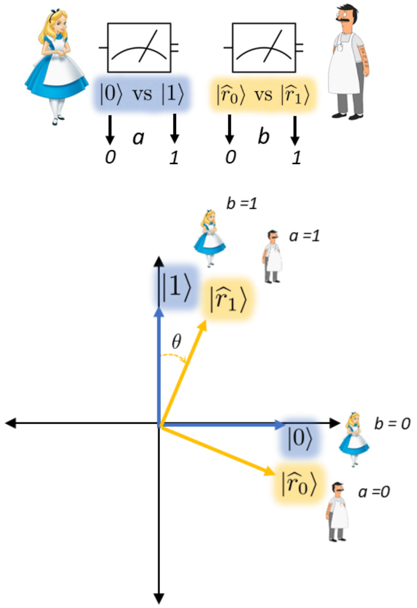
$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\text{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\text{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

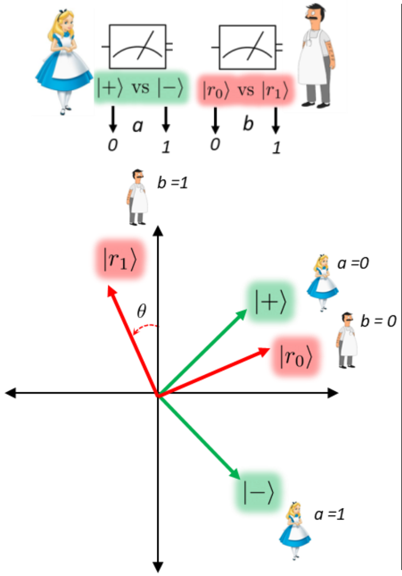
$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\hat{r}_0\rangle + \cos\theta|\hat{r}_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\text{win}}{\boxed{\cos\theta|0\hat{r}_0\rangle}} + \sin\theta|0\hat{r}_1\rangle - \sin\theta|1\hat{r}_0\rangle + \overset{\text{win}}{\boxed{\cos\theta|1\hat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

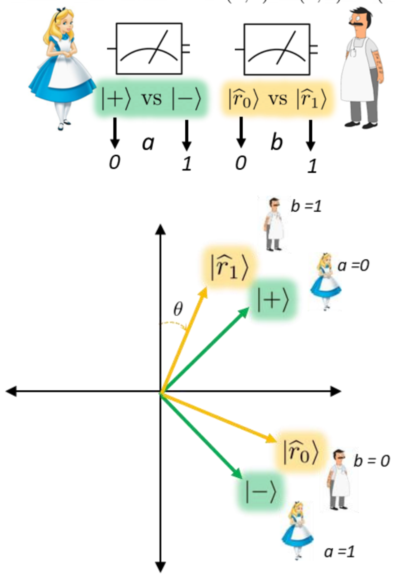
$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

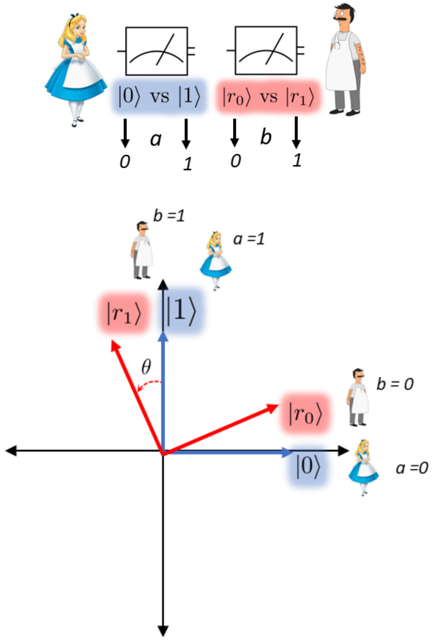
$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta + \sin\theta\\ -\sin\theta + \cos\theta\end{bmatrix}$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



Counterclockwise Rotation

$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

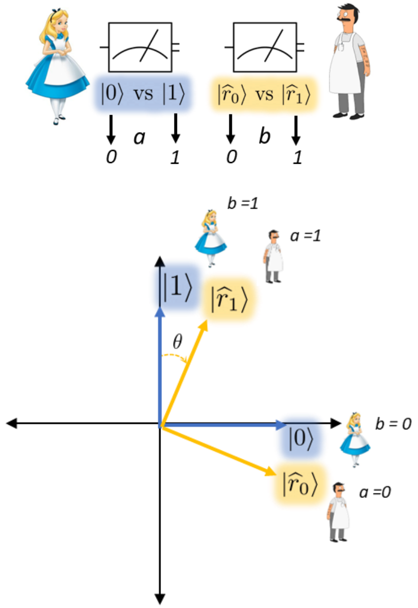
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\text{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\text{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

Clockwise Rotation

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\widehat{r}_0\rangle + \sin\theta|\widehat{r}_1\rangle$$

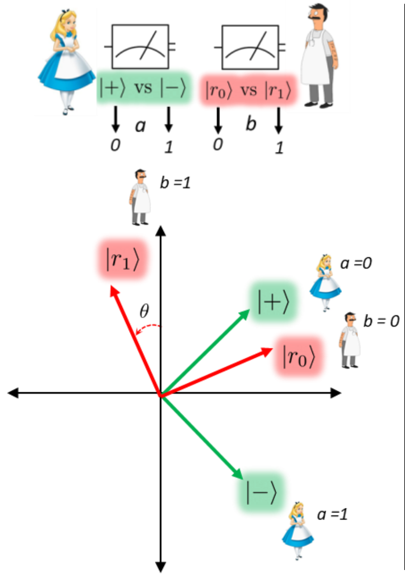
$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\widehat{r}_0\rangle + \cos\theta|\widehat{r}_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\text{win}}{\boxed{\cos\theta|0\widehat{r}_0\rangle}} + \sin\theta|0\widehat{r}_1\rangle - \sin\theta|1\widehat{r}_0\rangle + \overset{\text{win}}{\boxed{\cos\theta|1\widehat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$

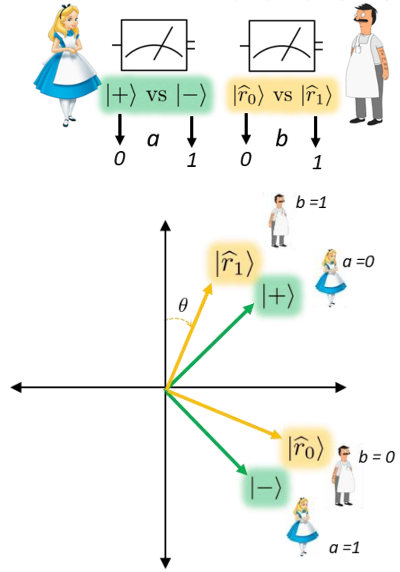


M^{-1} = clockwise

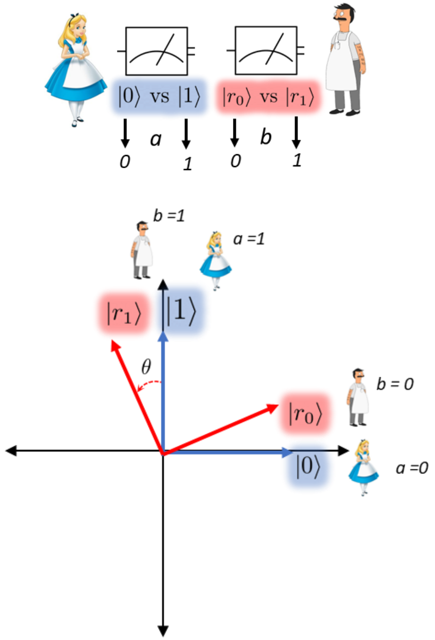
$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta + \sin\theta\\ -\sin\theta + \cos\theta\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

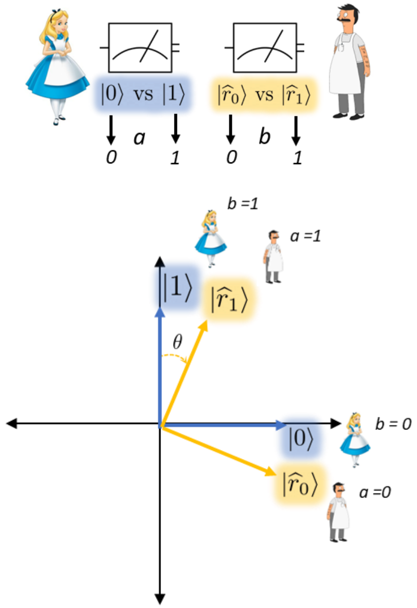
$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Clockwise Rotation

$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\widehat{r}_0\rangle + \sin\theta|\widehat{r}_1\rangle$$

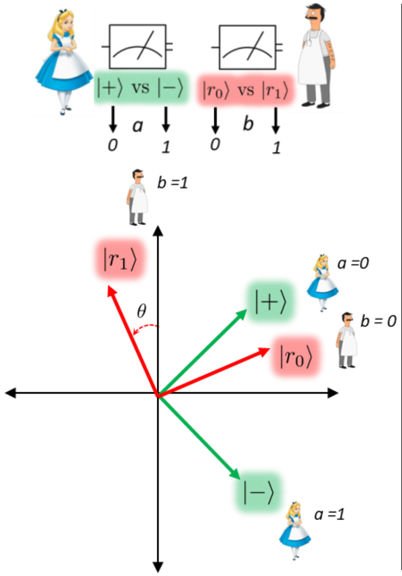
$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\widehat{r}_0\rangle + \cos\theta|\widehat{r}_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0\widehat{r}_0\rangle}} + \sin\theta|0\widehat{r}_1\rangle - \sin\theta|1\widehat{r}_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1\widehat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



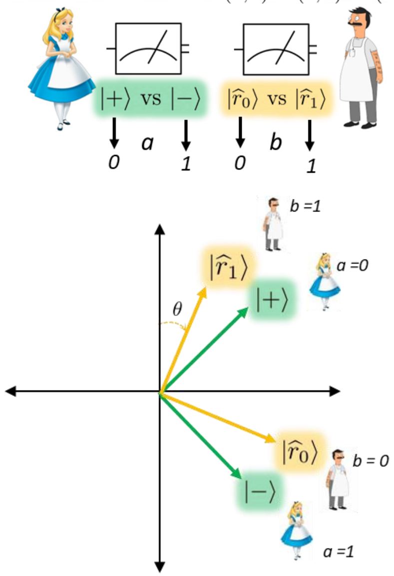
M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta + \sin\theta\\ -\sin\theta + \cos\theta\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

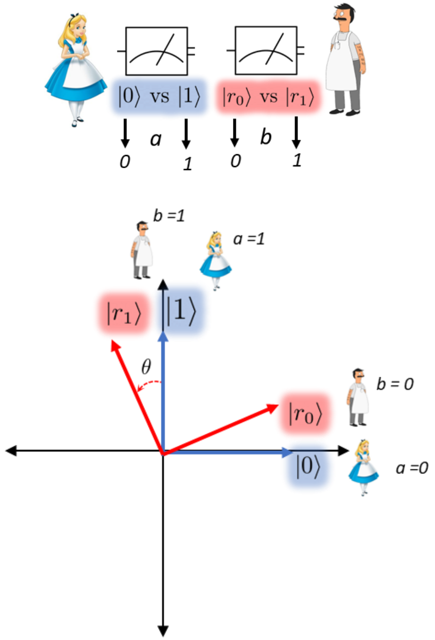
$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta - \sin\theta\\ -\sin\theta - \cos\theta\end{bmatrix}$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



Counterclockwise Rotation

$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

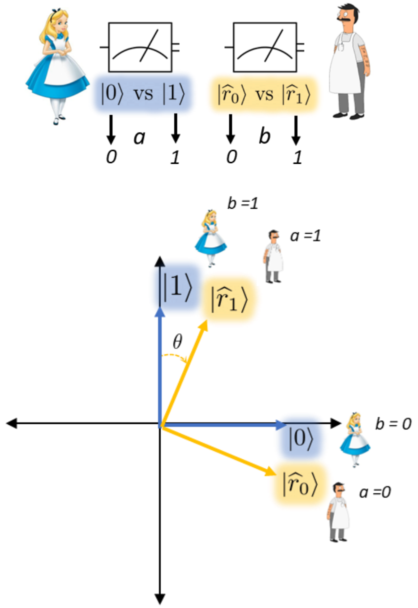
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

Clockwise Rotation

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

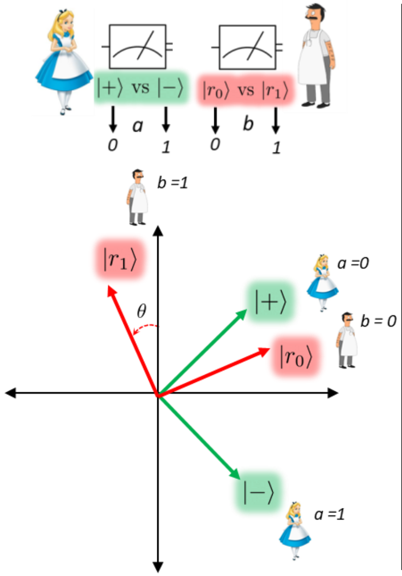
$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\hat{r}_0\rangle + \cos\theta|\hat{r}_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0\hat{r}_0\rangle}} + \sin\theta|0\hat{r}_1\rangle - \sin\theta|1\hat{r}_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1\hat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

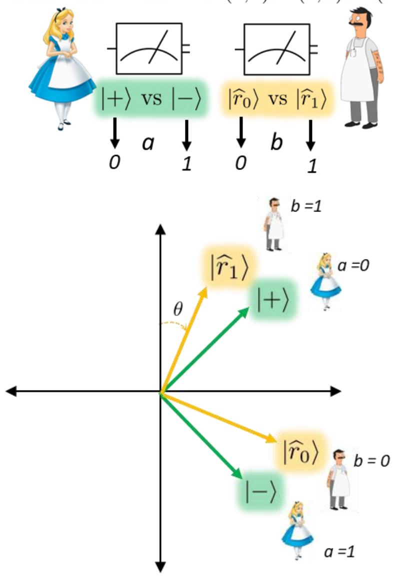
$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta + \sin\theta\\ -\sin\theta + \cos\theta\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta - \sin\theta\\ -\sin\theta - \cos\theta\end{bmatrix}$$

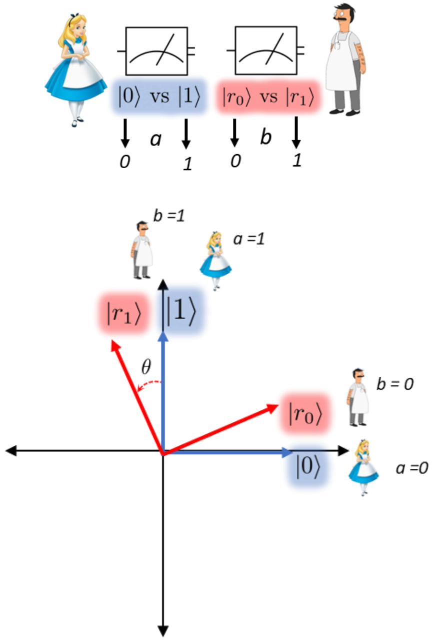
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\right]$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



Counterclockwise Rotation

$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\downarrow$$

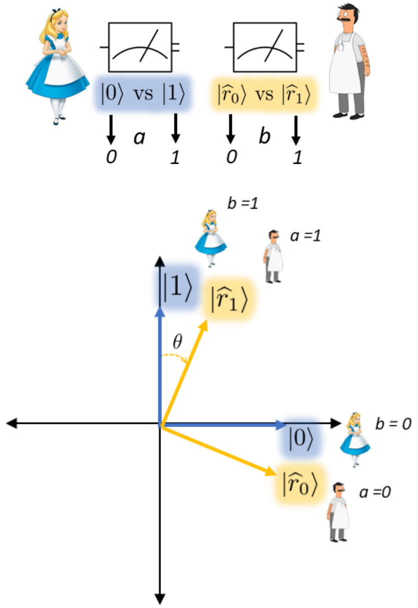
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\Big[\underbrace{\boxed{\cos\theta|0r_0\rangle}}_{\text{win}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \underbrace{\boxed{\cos\theta|1r_1\rangle}}_{\text{win}}\Big]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

Clockwise Rotation

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|\widehat{r}_0\rangle + \sin\theta|\widehat{r}_1\rangle$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\widehat{r}_0\rangle + \cos\theta|\widehat{r}_1\rangle$$

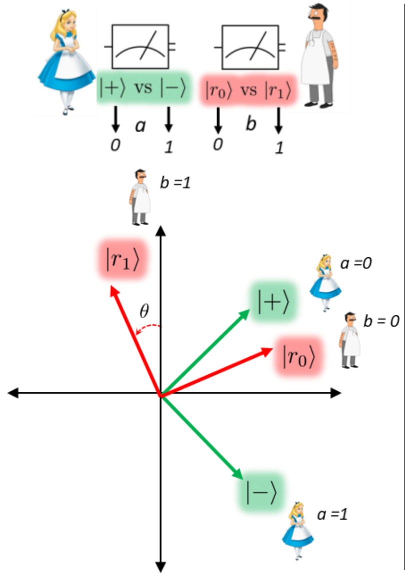
$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\Big[\underbrace{\boxed{\cos\theta|0\widehat{r}_0\rangle}}_{\text{win}} + \sin\theta|0\widehat{r}_1\rangle - \sin\theta|1\widehat{r}_0\rangle + \underbrace{\boxed{\cos\theta|1\widehat{r}_1\rangle}}_{\text{win}}\Big]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta + \sin\theta \\ -\sin\theta + \cos\theta \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\big[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\big]$$

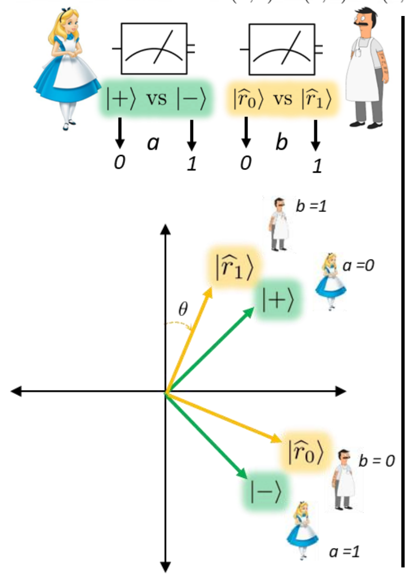
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta - \sin\theta \\ -\sin\theta - \cos\theta \end{bmatrix}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\big[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\big]$$

$$\downarrow$$

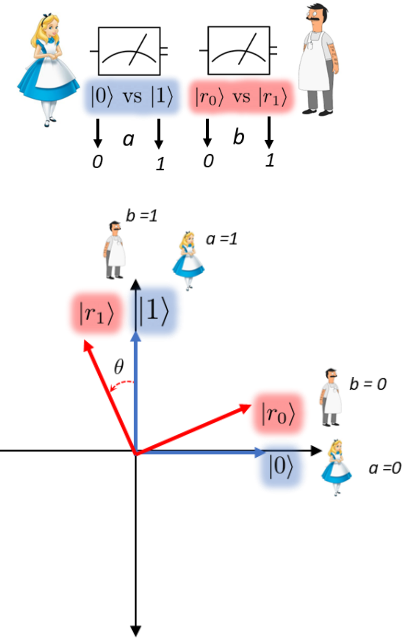
$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



Counterclockwise Rotation

$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\downarrow$$

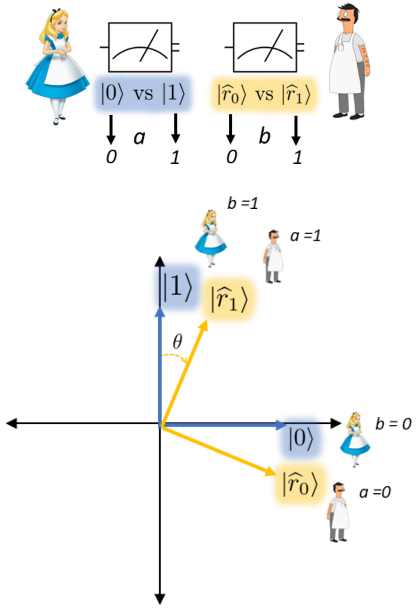
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

Clockwise Rotation

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|\widehat{r}_0\rangle + \sin\theta|\widehat{r}_1\rangle$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\widehat{r}_0\rangle + \cos\theta|\widehat{r}_1\rangle$$

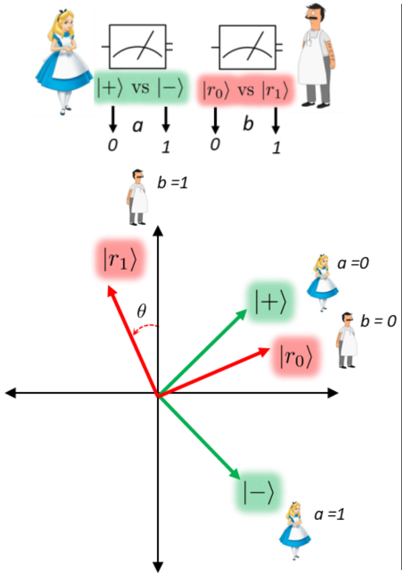
$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0\widehat{r}_0\rangle}} + \sin\theta|0\widehat{r}_1\rangle - \sin\theta|1\widehat{r}_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1\widehat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta + \sin\theta \\ -\sin\theta + \cos\theta \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta - \sin\theta \\ -\sin\theta - \cos\theta \end{bmatrix}$$

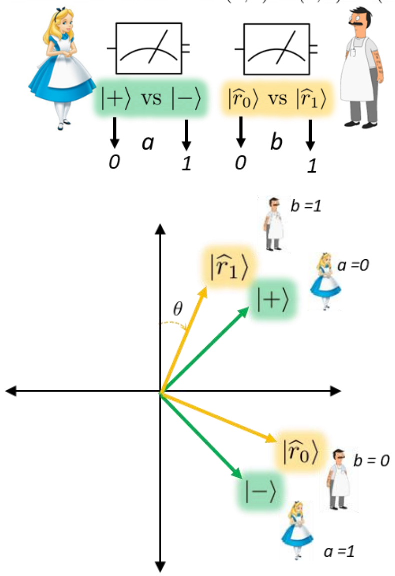
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\right]$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

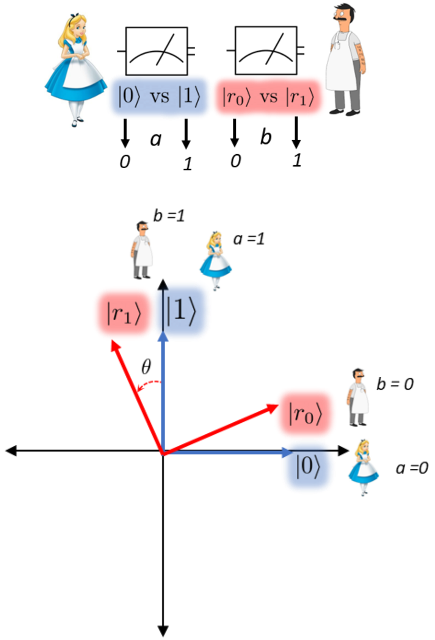
$$\frac{1}{2}\left[(\cos\theta + \sin\theta)|+r_0\rangle + (-\sin\theta + \cos\theta)|+r_1\rangle + (\cos\theta - \sin\theta)\sin\theta|-r_0\rangle + (-\sin\theta - \cos\theta)|-r_1\rangle\right]$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



Counterclockwise Rotation

$0 \wedge 0 = 0 \implies$ win when $(a, b) = (0,0)$ or $(1,1)$



$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

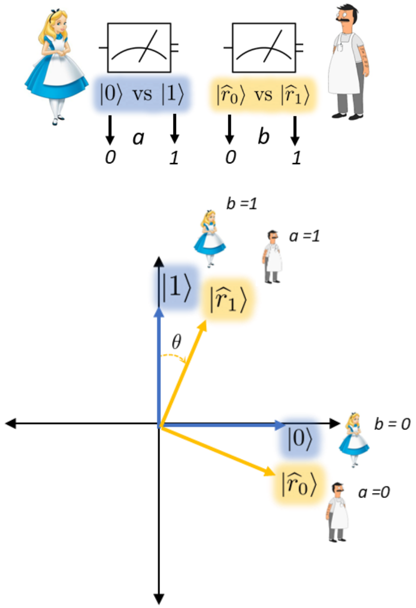
$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

Clockwise Rotation

$0 \wedge 1 = 0 \implies$ win when $(a, b) = (0,0)$ or $(1,1)$



$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\widehat{r}_0\rangle + \sin\theta|\widehat{r}_1\rangle$$

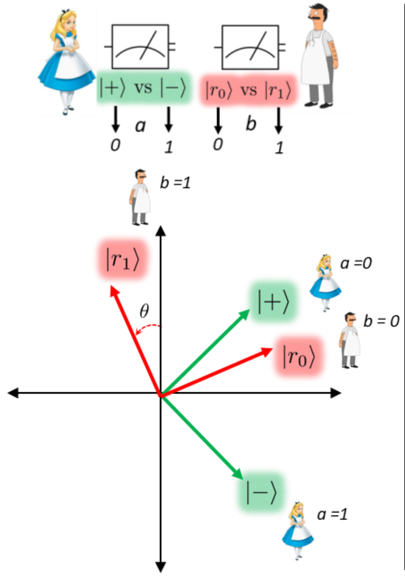
$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\widehat{r}_0\rangle + \cos\theta|\widehat{r}_1\rangle$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0\widehat{r}_0\rangle}} + \sin\theta|0\widehat{r}_1\rangle - \sin\theta|1\widehat{r}_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1\widehat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a, b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta + \sin\theta\\ -\sin\theta + \cos\theta\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta - \sin\theta\\ -\sin\theta - \cos\theta\end{bmatrix}$$

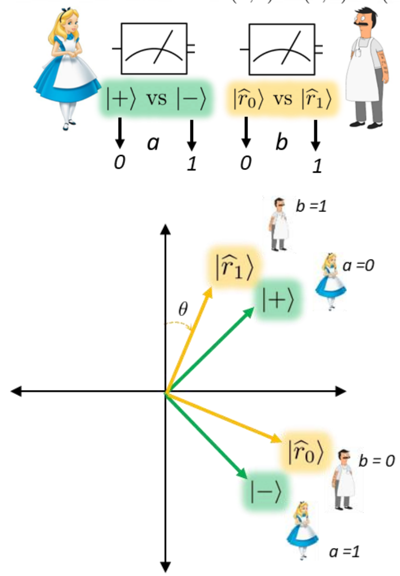
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\right]$$

$$\downarrow$$

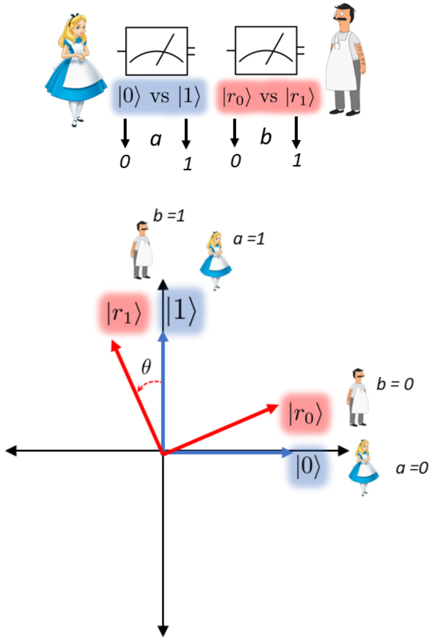
$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

$$\frac{1}{2}\left[\overset{\textbf{win}}{\boxed{(\cos\theta + \sin\theta)|{+}r_0\rangle}} + (-\sin\theta + \cos\theta)|{+}r_1\rangle + (\cos\theta - \sin\theta)\sin\theta|{-}r_0\rangle + \overset{\textbf{win}}{\boxed{(-\sin\theta - \cos\theta)|{-}r_1\rangle}}\right]$$

$1 \wedge 1 = 1 \implies$ win when $(a, b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

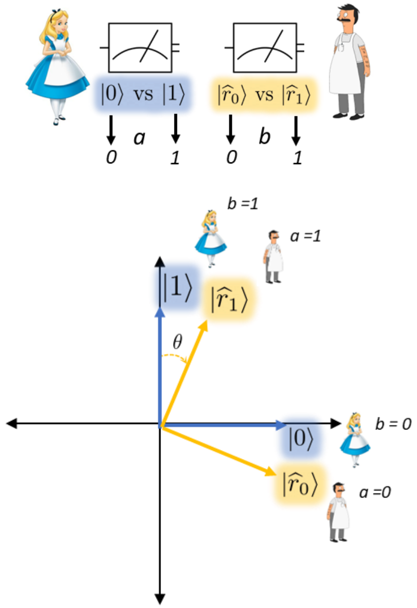
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Clockwise Rotation

$$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|\widehat{r}_0\rangle + \sin\theta|\widehat{r}_1\rangle$$

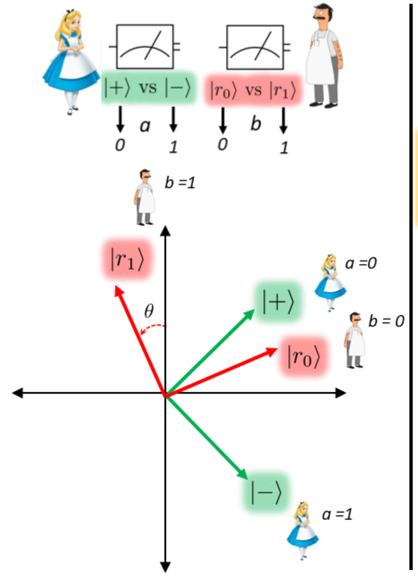
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\widehat{r}_0\rangle + \cos\theta|\widehat{r}_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0\widehat{r}_0\rangle}} + \sin\theta|0\widehat{r}_1\rangle - \sin\theta|1\widehat{r}_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1\widehat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta + \sin\theta \\ -\sin\theta + \cos\theta \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta - \sin\theta \\ -\sin\theta - \cos\theta \end{bmatrix}$$

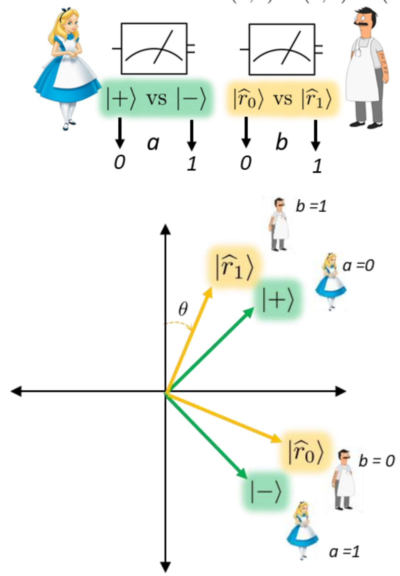
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\right]$$

$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

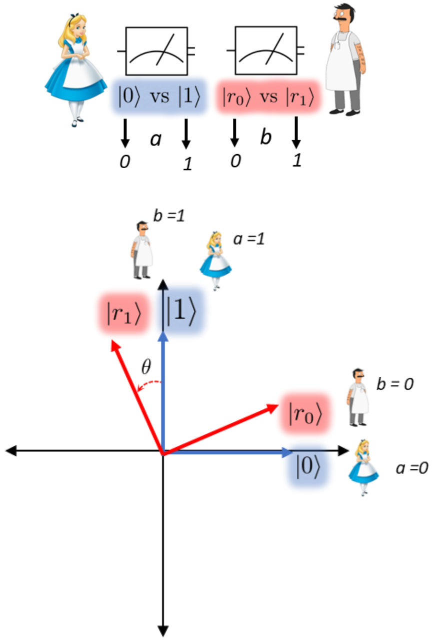
$$\frac{1}{2}\left[\overset{\textbf{win}}{\boxed{(\cos\theta + \sin\theta)|+r_0\rangle}} + (-\sin\theta + \cos\theta)|+r_1\rangle + (\cos\theta - \sin\theta)\sin\theta|-r_0\rangle + \overset{\textbf{win}}{\boxed{(-\sin\theta - \cos\theta)|-r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta + \sin\theta}{2}\right)^2 + \left(\frac{-\cos\theta - \sin\theta}{2}\right)^2 =$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



$0 \wedge 0 = 0 \implies$ win when $(a, b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

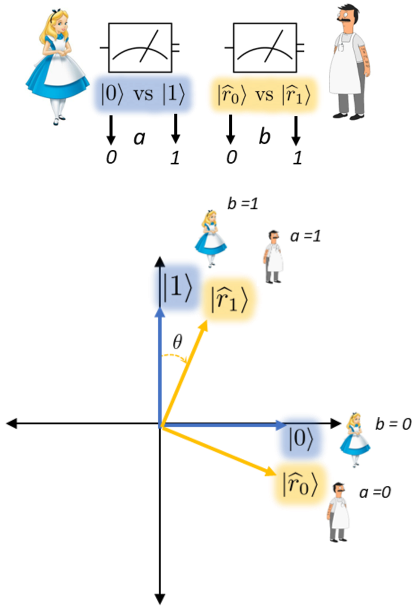
$$\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a, b) = (0,0)$ or $(1,1)$



Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

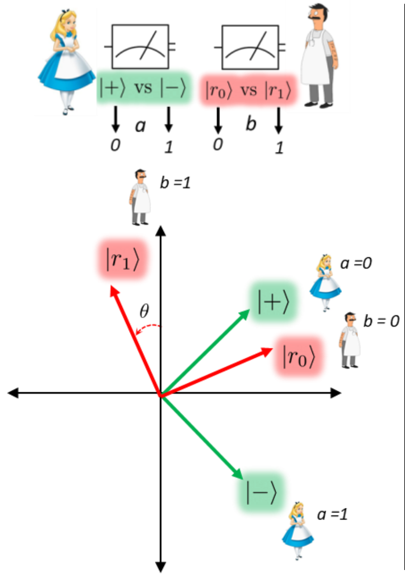
$$\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\hat{r}_0\rangle + \cos\theta|\hat{r}_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0\hat{r}_0\rangle}} + \sin\theta|0\hat{r}_1\rangle - \sin\theta|1\hat{r}_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1\hat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a, b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta + \sin\theta\\ -\sin\theta + \cos\theta\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$$\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta - \sin\theta\\ -\sin\theta - \cos\theta\end{bmatrix}$$

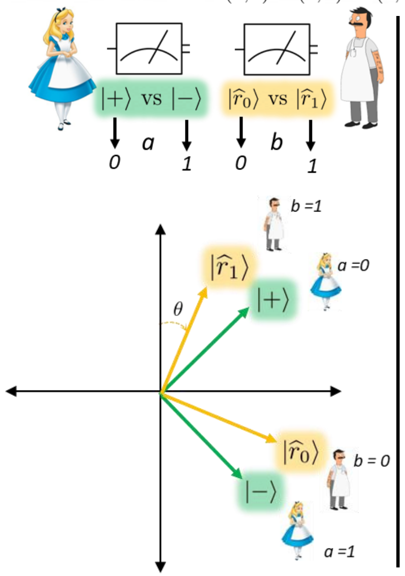
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\right]$$

$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

$$\frac{1}{2}\left[\overset{\textbf{win}}{\boxed{(\cos\theta + \sin\theta)|+r_0\rangle}} + (-\sin\theta + \cos\theta)|+r_1\rangle + (\cos\theta - \sin\theta)\sin\theta|-r_0\rangle + \overset{\textbf{win}}{\boxed{(-\sin\theta - \cos\theta)|-r_1\rangle}}\right]$$

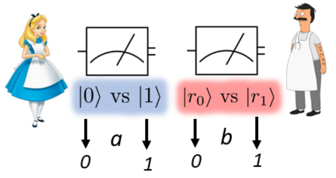
$$\Pr[\text{win}] = \left(\frac{\cos\theta + \sin\theta}{2}\right)^2 + \left(\frac{-\cos\theta - \sin\theta}{2}\right)^2 = \frac{1}{2} + \cos\theta\sin\theta$$

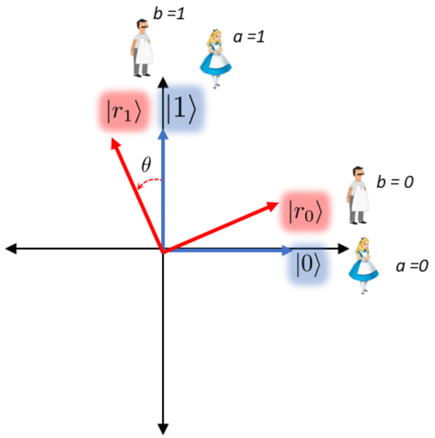
$1 \wedge 1 = 1 \implies$ win when $(a, b) = (0,1)$ or $(1,0)$



Counterclockwise Rotation

$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$





$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

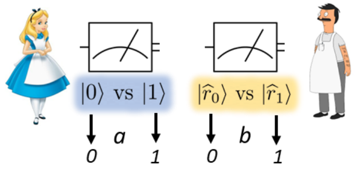
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

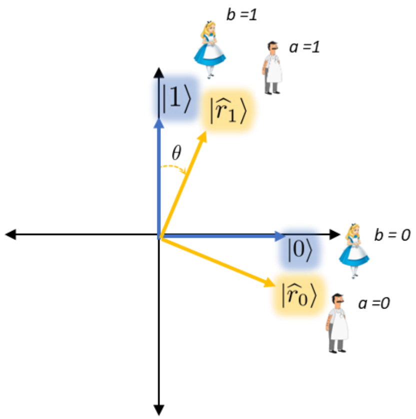
$$\frac{1}{\sqrt{2}}\Big[\underbrace{\boxed{\cos\theta|0r_0\rangle}}_{\text{win}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \underbrace{\boxed{\cos\theta|1r_1\rangle}}_{\text{win}}\Big]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

Clockwise Rotation

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$





$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

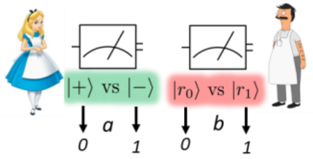
$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\hat{r}_0\rangle + \cos\theta|\hat{r}_1\rangle$$

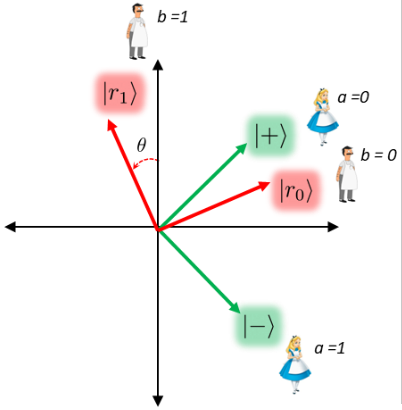
$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\Big[\underbrace{\boxed{\cos\theta|0\hat{r}_0\rangle}}_{\text{win}} + \sin\theta|0\hat{r}_1\rangle - \sin\theta|1\hat{r}_0\rangle + \underbrace{\boxed{\cos\theta|1\hat{r}_1\rangle}}_{\text{win}}\Big]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$





M^{-1} = clockwise

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta + \sin\theta\\ -\sin\theta + \cos\theta\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta - \sin\theta\\ -\sin\theta - \cos\theta\end{bmatrix}$$

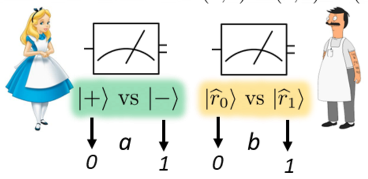
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\right]$$

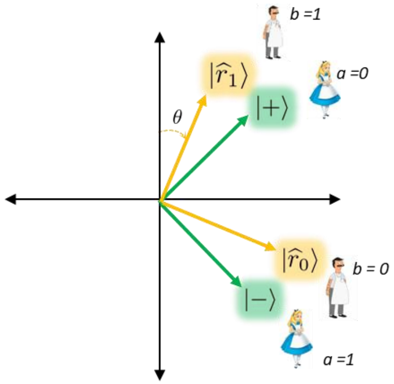
$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

$$\frac{1}{2}\Big[\underbrace{\boxed{(\cos\theta + \sin\theta)|+r_0\rangle}}_{\text{win}} + (-\sin\theta + \cos\theta)|+r_1\rangle + (\cos\theta - \sin\theta)\sin\theta|-r_0\rangle + \underbrace{\boxed{(-\sin\theta - \cos\theta)|-r_1\rangle}}_{\text{win}}\Big]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta + \sin\theta}{2}\right)^2 + \left(\frac{-\cos\theta - \sin\theta}{2}\right)^2 = \frac{1}{2} + \cos\theta\sin\theta$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$





M^{-1} = counterclockwise

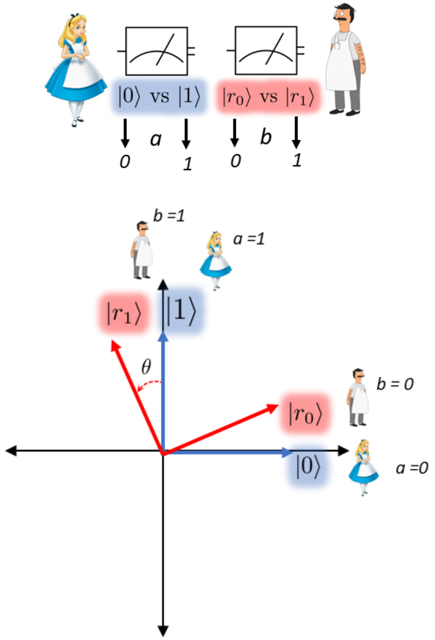
$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta - \sin\theta\\ \sin\theta + \cos\theta\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|\hat{r}_0\rangle + (\sin\theta + \cos\theta)|\hat{r}_1\rangle\right]$$

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta + \sin\theta\\ \sin\theta - \cos\theta\end{bmatrix}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|\hat{r}_0\rangle + (\sin\theta - \cos\theta)|\hat{r}_1\rangle\right]$$

$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

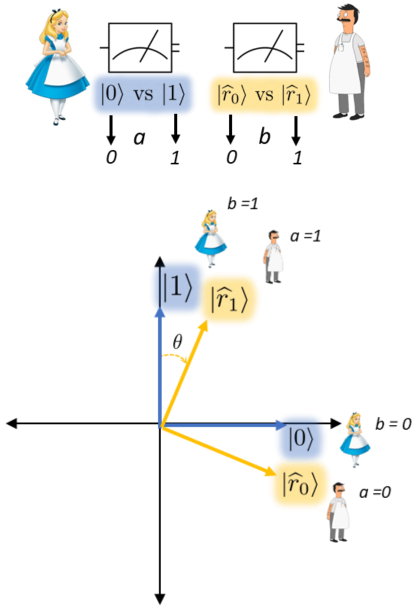
$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\hat{r}_0\rangle + \cos\theta|\hat{r}_1\rangle$$

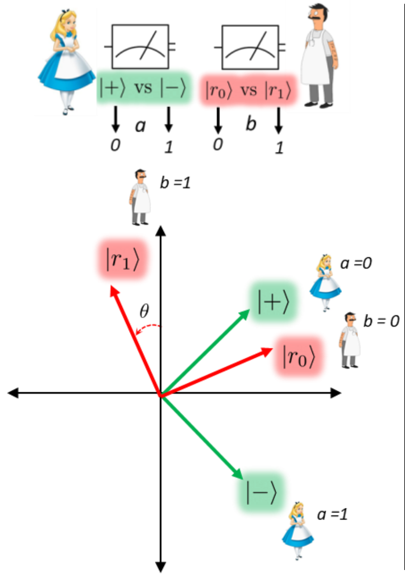
$$\downarrow$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\textbf{win}}{\boxed{\cos\theta|0\hat{r}_0\rangle}} + \sin\theta|0\hat{r}_1\rangle - \sin\theta|1\hat{r}_0\rangle + \overset{\textbf{win}}{\boxed{\cos\theta|1\hat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta + \sin\theta \\ -\sin\theta + \cos\theta \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta - \sin\theta \\ -\sin\theta - \cos\theta \end{bmatrix}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\right]$$

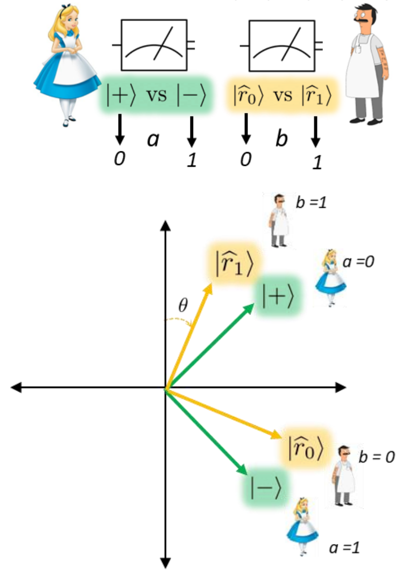
$$\downarrow$$

$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

$$\frac{1}{2}\left[\overset{\textbf{win}}{\boxed{(\cos\theta + \sin\theta)|+r_0\rangle}} + (-\sin\theta + \cos\theta)|+r_1\rangle + (\cos\theta - \sin\theta)\sin\theta|-r_0\rangle + \overset{\textbf{win}}{\boxed{(-\sin\theta - \cos\theta)|-r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta + \sin\theta}{2}\right)^2 + \left(\frac{-\cos\theta - \sin\theta}{2}\right)^2 = \frac{1}{2} + \cos\theta\sin\theta$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta - \sin\theta \\ \sin\theta + \cos\theta \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|\hat{r}_0\rangle + (\sin\theta + \cos\theta)|\hat{r}_1\rangle\right]$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta + \sin\theta \\ \sin\theta - \cos\theta \end{bmatrix}$$

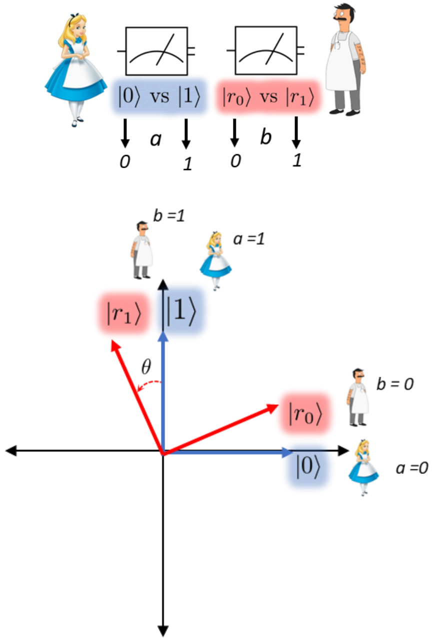
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|\hat{r}_0\rangle + (\sin\theta - \cos\theta)|\hat{r}_1\rangle\right]$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

$$\frac{1}{2}\left[(\cos\theta - \sin\theta)|+\hat{r}_0\rangle + (\sin\theta + \cos\theta)|+\hat{r}_1\rangle + (\cos\theta + \sin\theta)|-\hat{r}_0\rangle + (\sin\theta - \cos\theta)|-\hat{r}_1\rangle\right]$$

$0 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Counterclockwise Rotation

$$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \quad |r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|r_0\rangle - \sin\theta|r_1\rangle$$

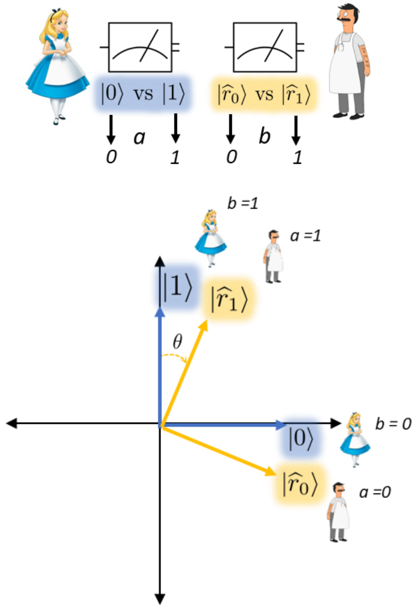
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = \sin\theta|r_0\rangle + \cos\theta|r_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\text{win}}{\boxed{\cos\theta|0r_0\rangle}} - \sin\theta|0r_1\rangle + \sin\theta|1r_0\rangle + \overset{\text{win}}{\boxed{\cos\theta|1r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$0 \wedge 1 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



Clockwise Rotation

$$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle \quad |\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \rightarrow |0\rangle = \cos\theta|\hat{r}_0\rangle + \sin\theta|\hat{r}_1\rangle$$

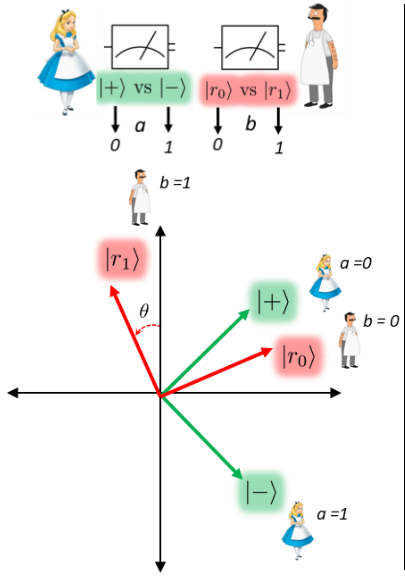
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \rightarrow |1\rangle = -\sin\theta|\hat{r}_0\rangle + \cos\theta|\hat{r}_1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle =$$

$$\frac{1}{\sqrt{2}}\left[\overset{\text{win}}{\boxed{\cos\theta|0\hat{r}_0\rangle}} + \sin\theta|0\hat{r}_1\rangle - \sin\theta|1\hat{r}_0\rangle + \overset{\text{win}}{\boxed{\cos\theta|1\hat{r}_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}}\right)^2 = \cos^2\theta$$

$1 \wedge 0 = 0 \implies$ win when $(a,b) = (0,0)$ or $(1,1)$



M^{-1} = clockwise

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta + \sin\theta \\ -\sin\theta + \cos\theta \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|r_0\rangle + (-\sin\theta + \cos\theta)|r_1\rangle\right]$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta - \sin\theta \\ -\sin\theta - \cos\theta \end{bmatrix}$$

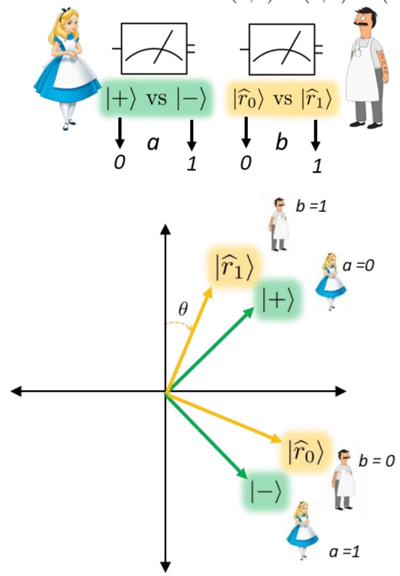
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|r_0\rangle + (-\sin\theta - \cos\theta)|r_1\rangle\right]$$

$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

$$\frac{1}{2}\left[\overset{\text{win}}{\boxed{(\cos\theta + \sin\theta)|{+}r_0\rangle}} + (-\sin\theta + \cos\theta)|{+}r_1\rangle + (\cos\theta - \sin\theta)\sin\theta|{-}r_0\rangle + \overset{\text{win}}{\boxed{(-\sin\theta - \cos\theta)|{-}r_1\rangle}}\right]$$

$$\Pr[\text{win}] = \left(\frac{\cos\theta + \sin\theta}{2}\right)^2 + \left(\frac{-\cos\theta - \sin\theta}{2}\right)^2 = \frac{1}{2} + \cos\theta\sin\theta$$

$1 \wedge 1 = 1 \implies$ win when $(a,b) = (0,1)$ or $(1,0)$



M^{-1} = counterclockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta - \sin\theta \\ \sin\theta + \cos\theta \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta - \sin\theta)|\hat{r}_0\rangle + (\sin\theta + \cos\theta)|\hat{r}_1\rangle\right]$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\theta + \sin\theta \\ \sin\theta - \cos\theta \end{bmatrix}$$

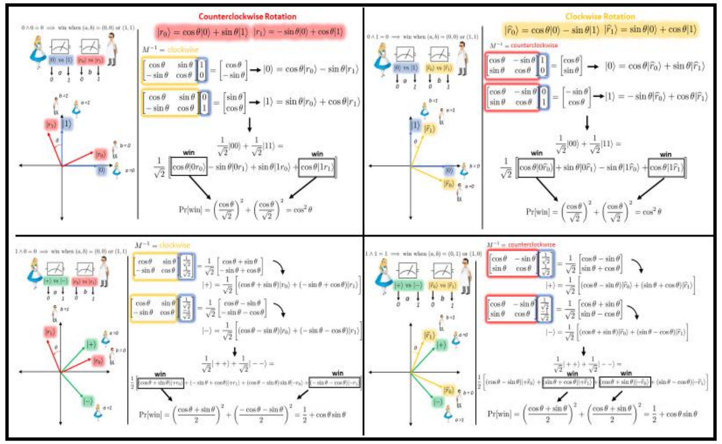
$$|-\rangle = \frac{1}{\sqrt{2}}\left[(\cos\theta + \sin\theta)|\hat{r}_0\rangle + (\sin\theta - \cos\theta)|\hat{r}_1\rangle\right]$$

$$\frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle =$$

$$\frac{1}{2}\left[(\cos\theta - \sin\theta)|{+}\hat{r}_0\rangle + \overset{\text{win}}{\boxed{(\sin\theta + \cos\theta)|{+}\hat{r}_1\rangle}} + \overset{\text{win}}{\boxed{(\cos\theta + \sin\theta)|{-}\hat{r}_0\rangle}} + (\sin\theta - \cos\theta)|{-}\hat{r}_1\rangle\right]$$

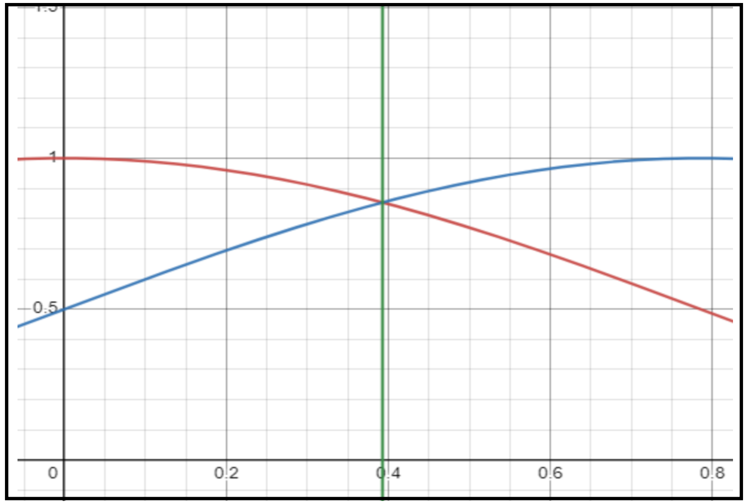
$$\Pr[\text{win}] = \left(\frac{\cos\theta + \sin\theta}{2}\right)^2 + \left(\frac{\cos\theta + \sin\theta}{2}\right)^2 = \frac{1}{2} + \cos\theta\sin\theta$$

Calculating Optimal Theta



Analysis implies optimal theta is given by:

$$\cos^2\theta = \frac{1}{2} + \cos\theta\sin\theta$$



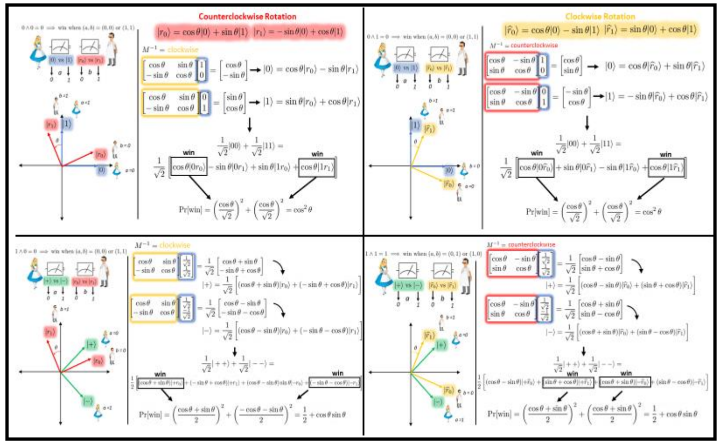
— = $\cos^2\theta$ — = $\frac{1}{2} + \cos\theta + \sin\theta$ | = $\pi/8$

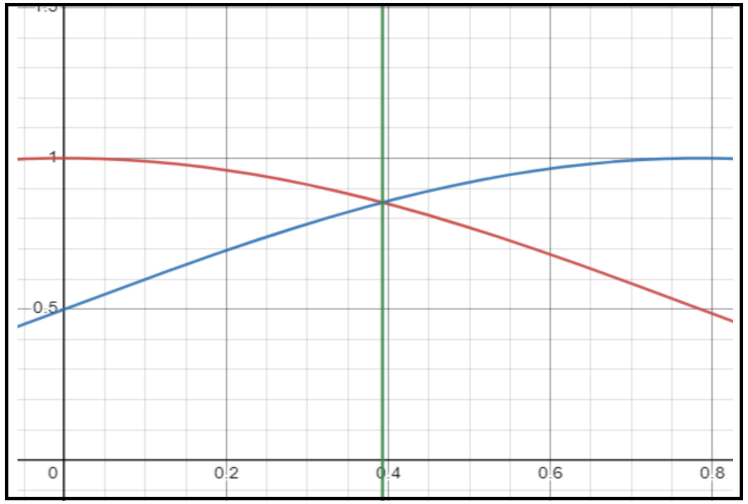
$$\theta = \frac{\pi}{8} \longrightarrow \cos^2\left(\frac{\pi}{8}\right) \approx 0.85$$

optimal theta

chance of winning

Calculating Optimal Theta





= $\cos^2\theta$ = $\frac{1}{2} + \cos\theta + \sin\theta$ = $\pi/8$

Analysis implies optimal theta is given by:

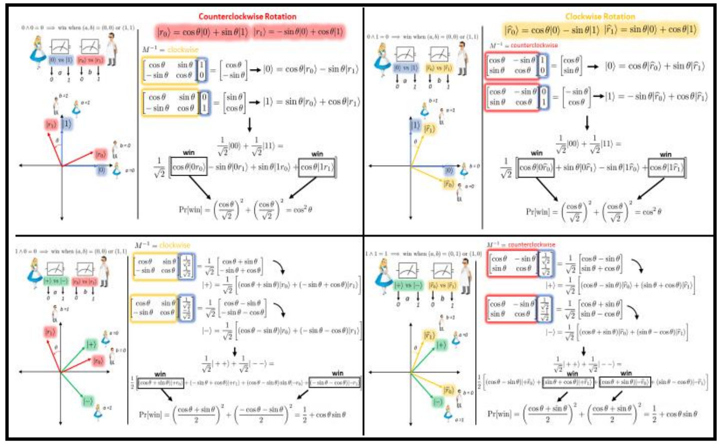
$$\cos^2\theta = \frac{1}{2} + \cos\theta\sin\theta$$

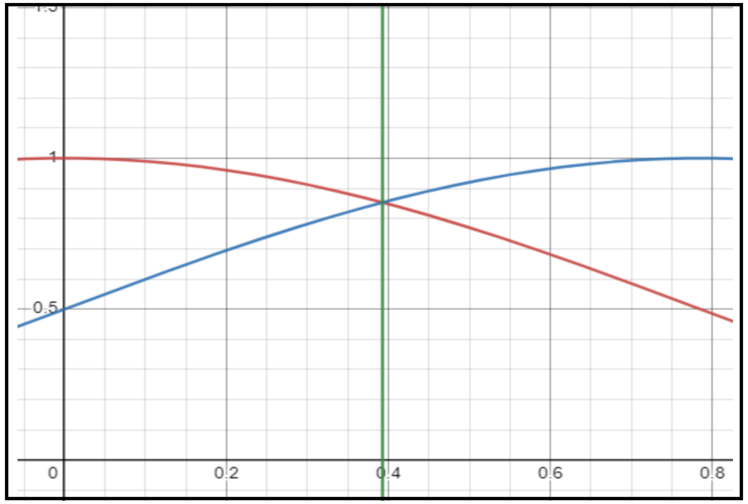
$$\theta = \frac{\pi}{8} \longrightarrow \cos^2\left(\frac{\pi}{8}\right) \approx 0.85$$

optimal theta   chance of winning

Question: Can we do better using a different strategy?

Calculating Optimal Theta





— $= \cos^2\theta$ — $= \frac{1}{2} + \cos\theta + \sin\theta$ | $= \pi/8$

Analysis implies optimal theta is given by:

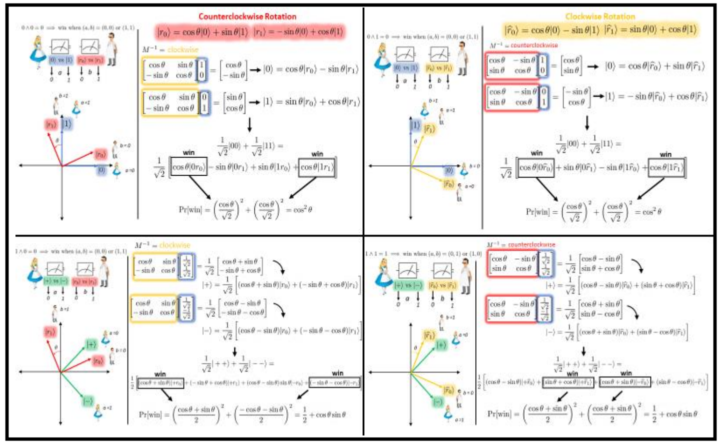
$$\cos^2\theta = \frac{1}{2} + \cos\theta\sin\theta$$

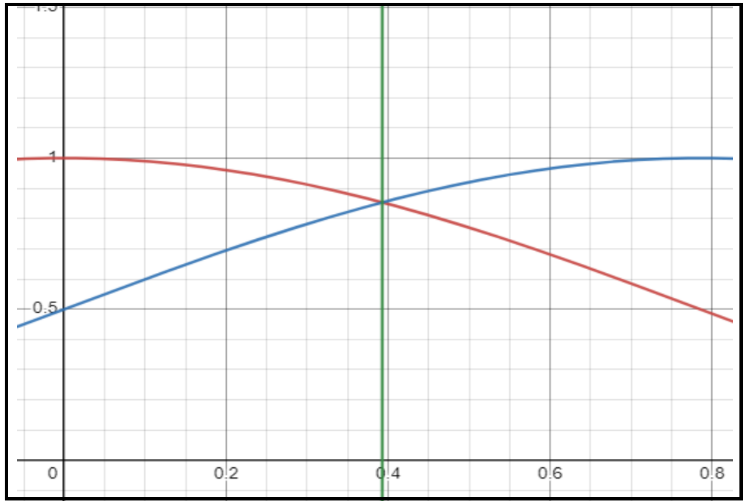
$$\theta = \frac{\pi}{8} \longrightarrow \cos^2\left(\frac{\pi}{8}\right) \approx 0.85$$

optimal theta chance of winning

Question: Can we do better using a different strategy? **Answer:** no.

Calculating Optimal Theta





— $= \cos^2 \theta$ — $= \frac{1}{2} + \cos \theta + \sin \theta$ | $= \pi/8$

Analysis implies optimal theta is given by:

$$\cos^2 \theta = \frac{1}{2} + \cos \theta \sin \theta$$

$$\theta = \frac{\pi}{8} \longrightarrow \cos^2\left(\frac{\pi}{8}\right) \approx 0.85$$

optimal theta — chance of winning

Question: Can we do better using a different strategy? **Answer:** no.

Tsirelson's Bound (1980): no quantum strategy can do better than $\cos^2 \pi/8$ chance of winning.

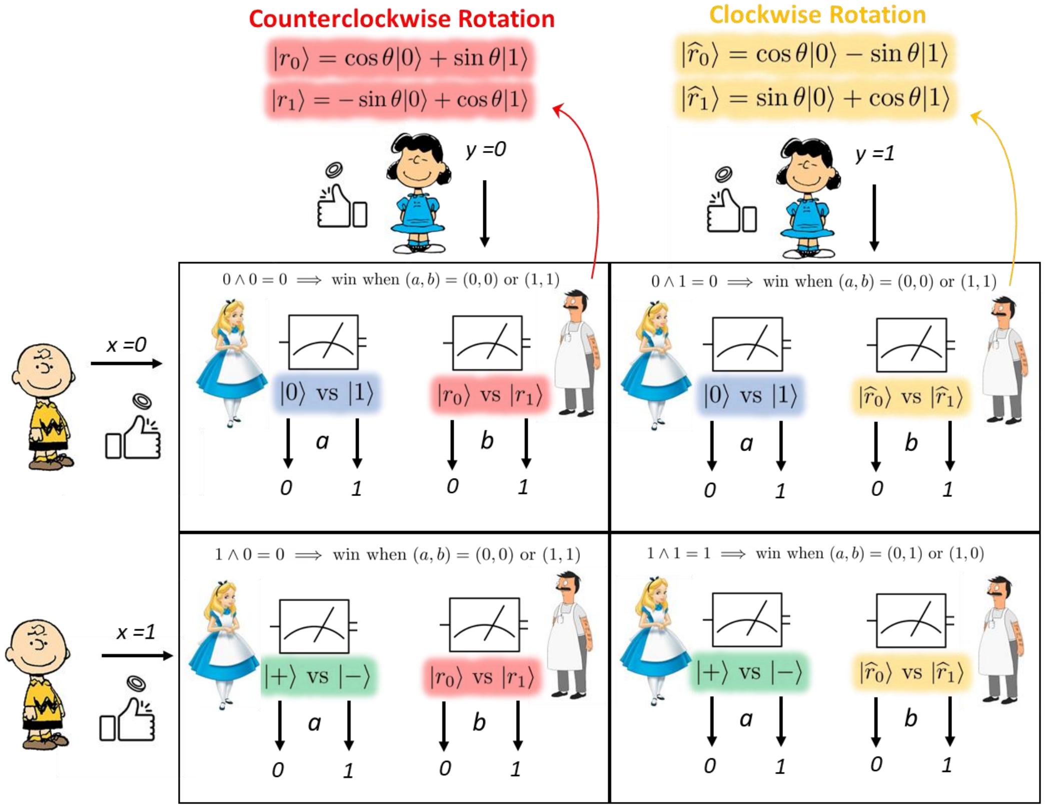
Circuit Implementation?

Step 1: create EPR pair before separating

$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$

Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

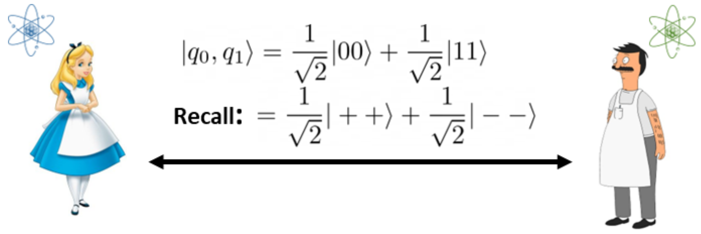


Circuit Implementation?

Step 1: create EPR pair before separating

$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$



Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

Counterclockwise Rotation

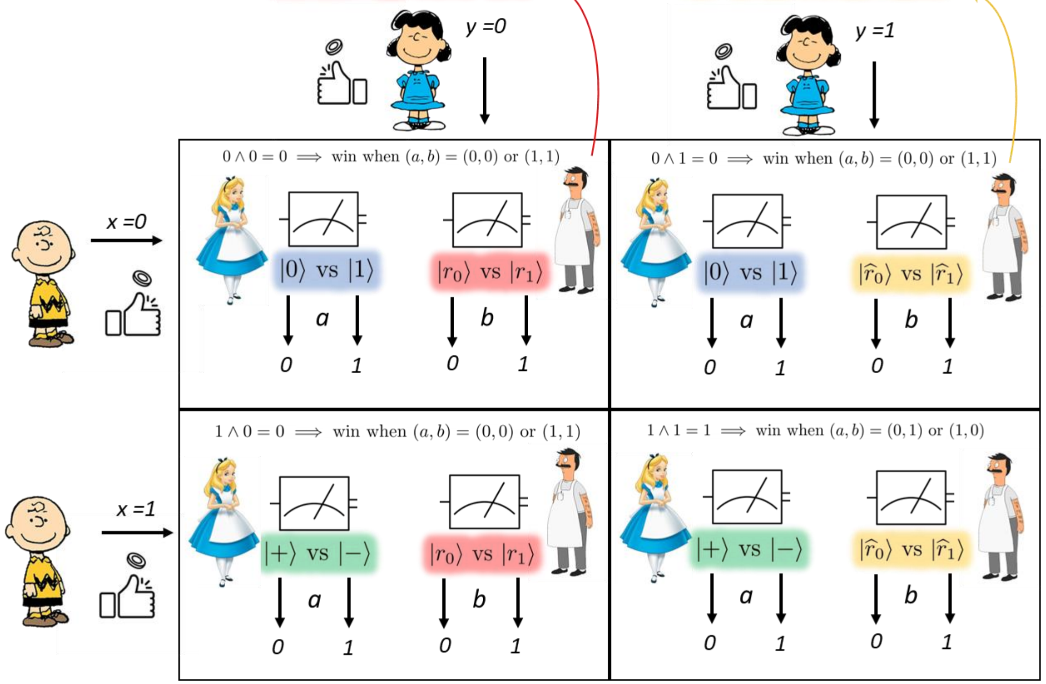
$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

Clockwise Rotation

$|\widehat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$

$|\widehat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$



Question 1: How do we apply a rotation to quantum state?

Circuit Implementation?

Step 1: create EPR pair before separating

$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$

Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

Counterclockwise Rotation

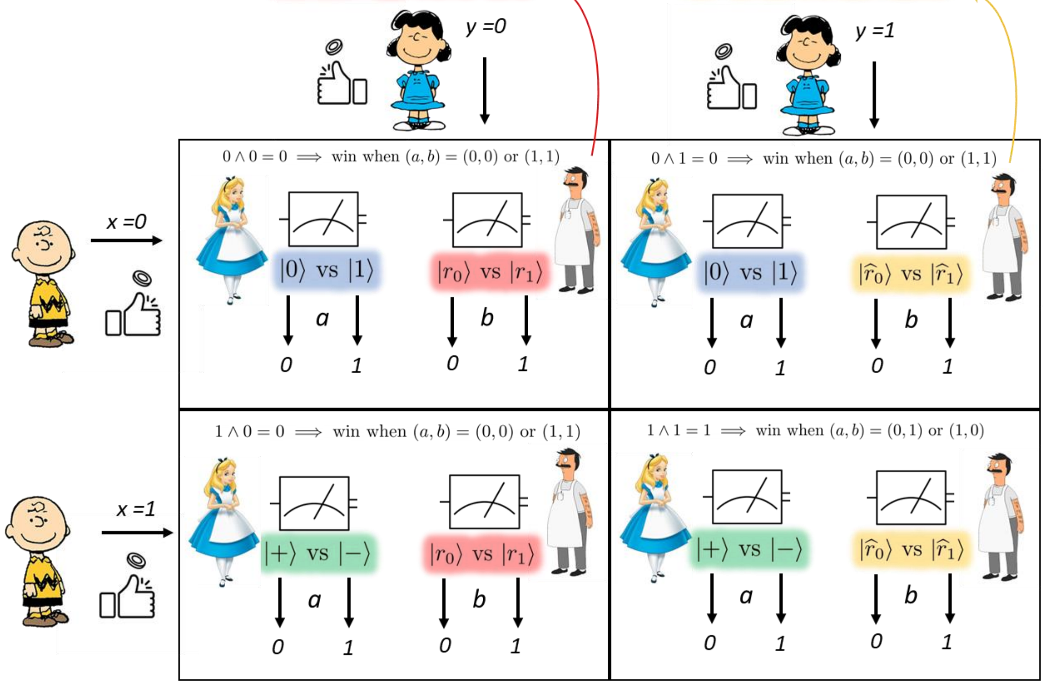
$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

Clockwise Rotation

$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$

$|\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$



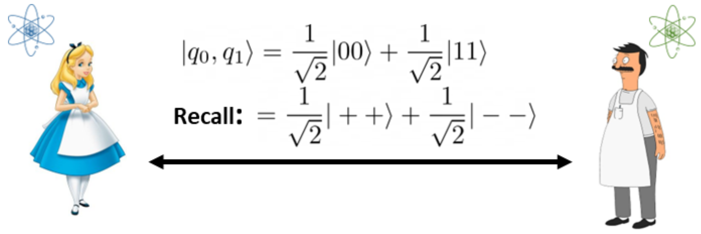
Question 1: How do we apply a rotation to quantum state?

$$R_y(\theta) \longleftrightarrow \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Ry-gate (rotate state by theta/2 radians)

Circuit Implementation?

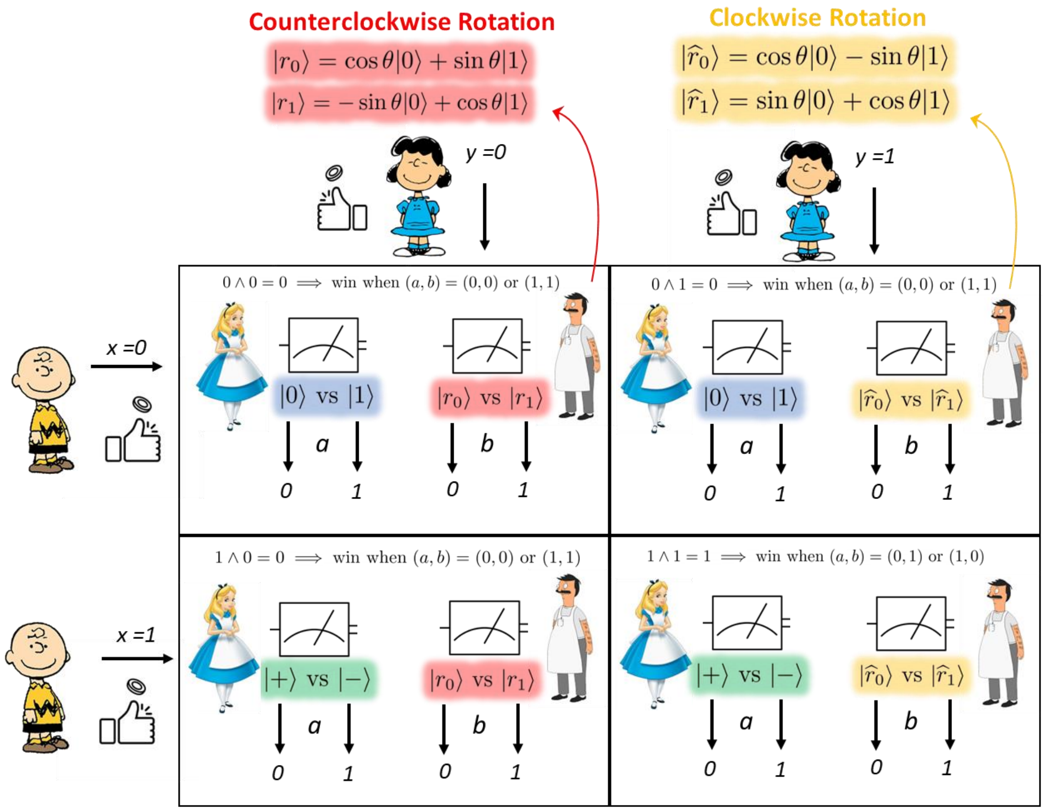
Step 1: create EPR pair before separating



$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$

Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)



Question 1: How do we apply a rotation to quantum state?

$$-\boxed{R_y(\theta)}- \longleftrightarrow \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Ry-gate (rotate state by theta/2 radians)

Question 2: How do we measure in alternate basis in Qiskit?

Circuit Implementation?

Step 1: create EPR pair before separating

$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$

Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

Counterclockwise Rotation

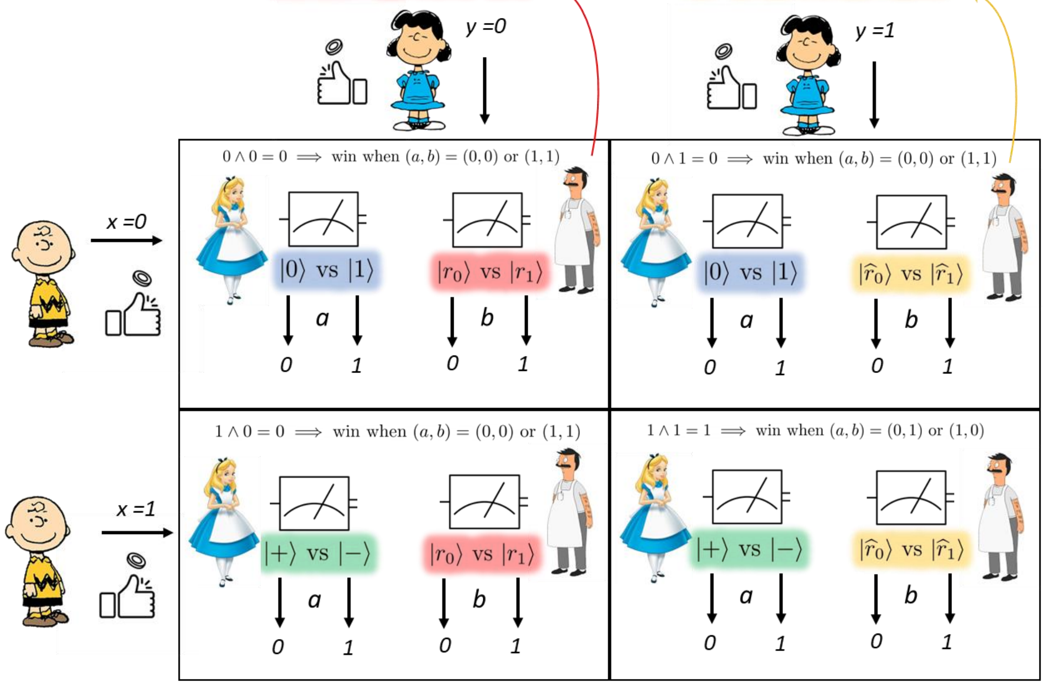
$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

Clockwise Rotation

$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$

$|\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$



Question 1: How do we apply a rotation to quantum state?

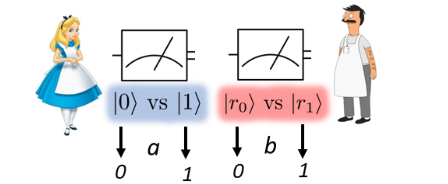
$$R_y(\theta) \longleftrightarrow \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Ry-gate (rotate state by theta/2 radians)

Question 2: How do we measure in alternate basis in Qiskit?

Answer: We don't need to.

State when x =0 and y = 0



$$\frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|0r_0\rangle - \sin\left(\frac{\pi}{8}\right)|0r_1\rangle + \sin\left(\frac{\pi}{8}\right)|1r_0\rangle + \cos\left(\frac{\pi}{8}\right)|1r_1\rangle\right]$$

Circuit Implementation?

Step 1: create EPR pair before separating

$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$

Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

Counterclockwise Rotation

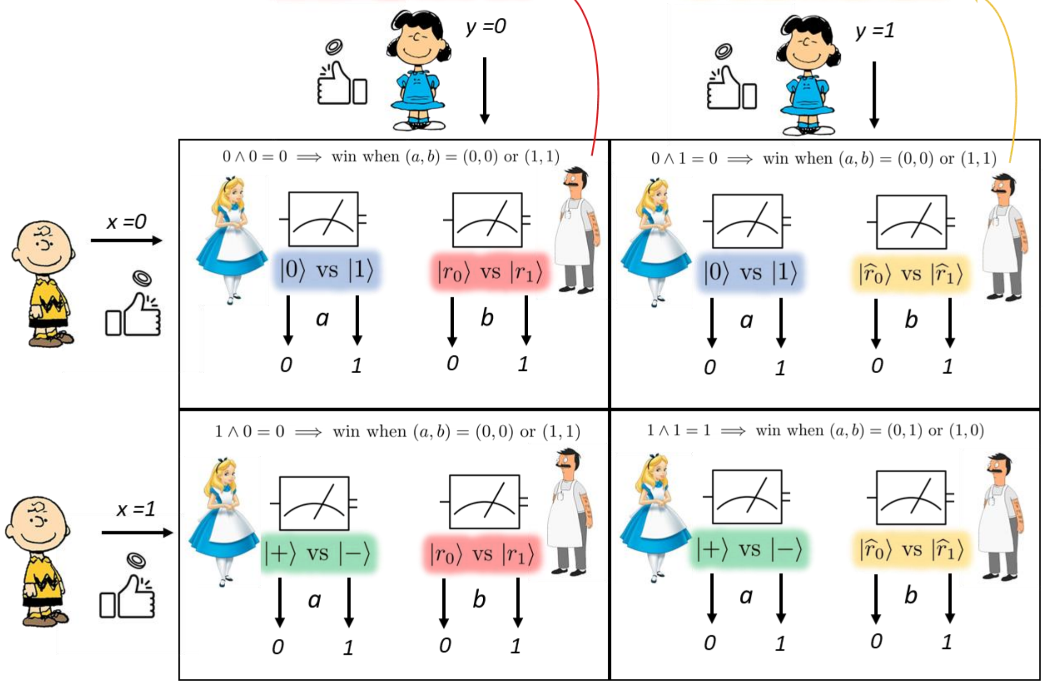
$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

Clockwise Rotation

$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$

$|\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$



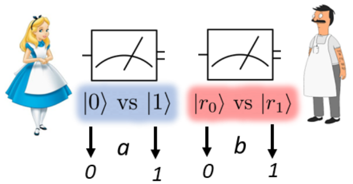
Question 1: How do we apply a rotation to quantum state?

$$R_y(\theta) \longleftrightarrow \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Ry-gate (rotate state by theta/2 radians)

Question 2: How do we measure in alternate basis in Qiskit?

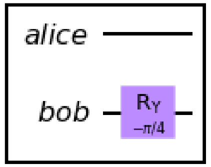
Answer: We don't need to.



State when x =0 and y = 0

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|0r_0\rangle - \sin\left(\frac{\pi}{8}\right)|0r_1\rangle + \sin\left(\frac{\pi}{8}\right)|1r_0\rangle + \cos\left(\frac{\pi}{8}\right)|1r_1\rangle\right]$$

$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \rightarrow$



$\rightarrow \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|00\rangle - \sin\left(\frac{\pi}{8}\right)|01\rangle + \sin\left(\frac{\pi}{8}\right)|10\rangle + \cos\left(\frac{\pi}{8}\right)|11\rangle\right]$

Circuit Implementation?

Step 1: create EPR pair before separating

$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$

Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

Counterclockwise Rotation

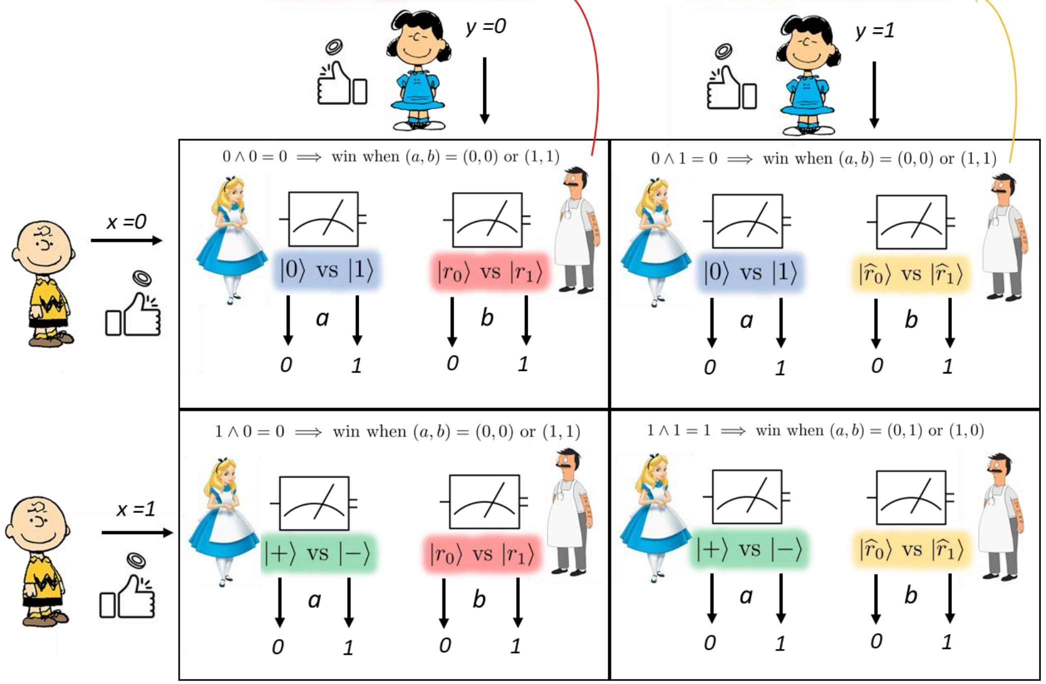
$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

Clockwise Rotation

$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$

$|\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$



Question 1: How do we apply a rotation to quantum state?

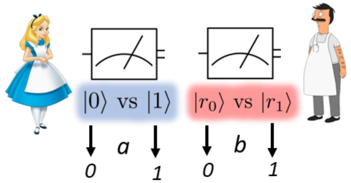
$$R_y(\theta) \longleftrightarrow \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Ry-gate (rotate state by theta/2 radians)

Question 2: How do we measure in alternate basis in Qiskit?

Answer: We don't need to.

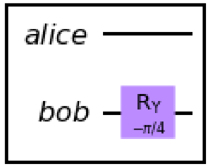
State when x =0 and y = 0



$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|0r_0\rangle - \sin\left(\frac{\pi}{8}\right)|0r_1\rangle + \sin\left(\frac{\pi}{8}\right)|1r_0\rangle + \cos\left(\frac{\pi}{8}\right)|1r_1\rangle\right]$$

equivalent if we interpret: $|0\rangle$ as $|r_0\rangle$ and $|1\rangle$ as $|r_1\rangle$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \rightarrow$$



$$\rightarrow \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|00\rangle - \sin\left(\frac{\pi}{8}\right)|01\rangle + \sin\left(\frac{\pi}{8}\right)|10\rangle + \cos\left(\frac{\pi}{8}\right)|11\rangle\right]$$

Circuit Implementation?

Step 1: create EPR pair before separating

$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$

Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

Counterclockwise Rotation

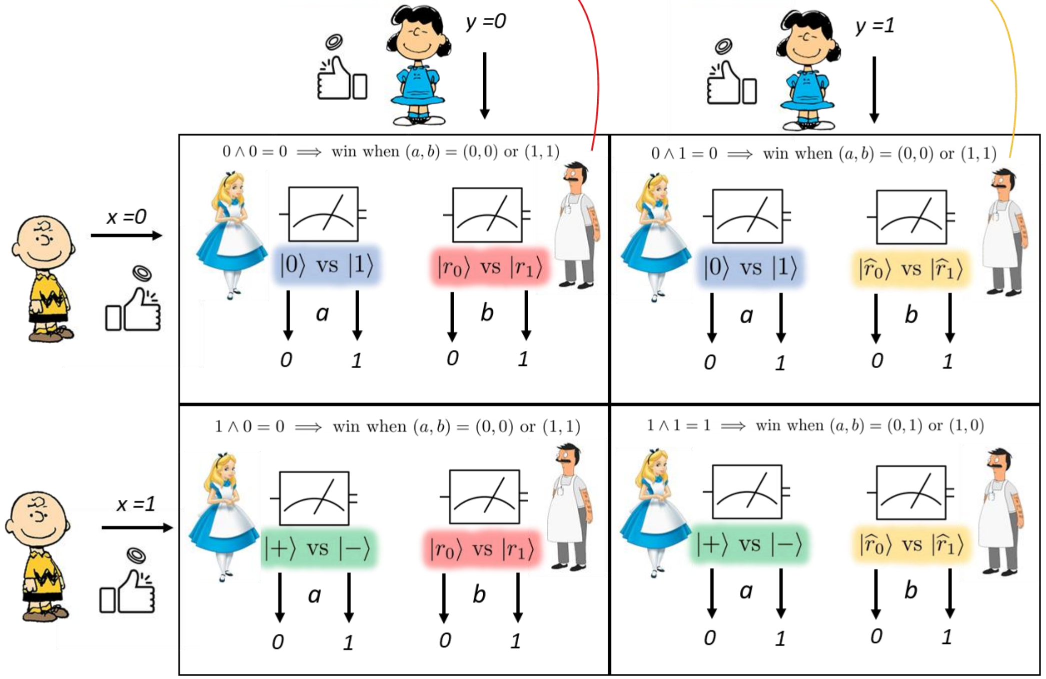
$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

Clockwise Rotation

$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$

$|\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$



Question 1: How do we apply a rotation to quantum state?

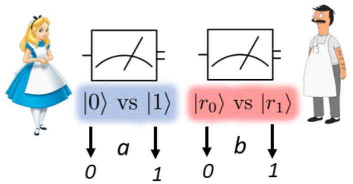
$$R_y(\theta) \longleftrightarrow \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Ry-gate (rotate state by theta/2 radians)

Question 2: How do we measure in alternate basis in Qiskit?

Answer: We don't need to.

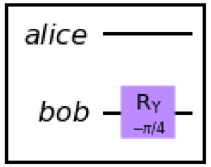
State when x =0 and y = 0



$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|0r_0\rangle - \sin\left(\frac{\pi}{8}\right)|0r_1\rangle + \sin\left(\frac{\pi}{8}\right)|1r_0\rangle + \cos\left(\frac{\pi}{8}\right)|1r_1\rangle\right]$$

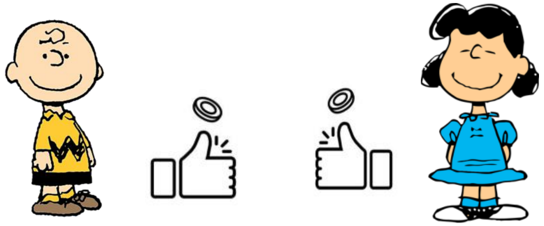
equivalent if we interpret: $|0\rangle$ as $|r_0\rangle$ and $|1\rangle$ as $|r_1\rangle$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \rightarrow$$



$$\rightarrow \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|00\rangle - \sin\left(\frac{\pi}{8}\right)|01\rangle + \sin\left(\frac{\pi}{8}\right)|10\rangle + \cos\left(\frac{\pi}{8}\right)|11\rangle\right]$$

Question 3: How do we simulate Charlie/Lucy coin flip in circuit ?



Circuit Implementation?

Step 1: create EPR pair before separating

$$|q_0, q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Recall: $= \frac{1}{\sqrt{2}}|++\rangle + \frac{1}{\sqrt{2}}|--\rangle$

Step 2: play the strategies according to table below
(where the goal will be to pick the optimal value of theta)

Counterclockwise Rotation

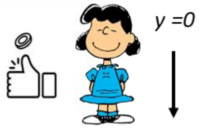
$|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

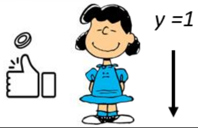
$|r_1\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

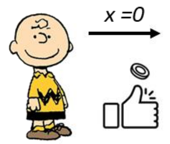
Clockwise Rotation

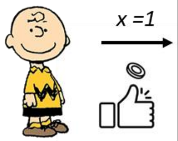
$|\hat{r}_0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$

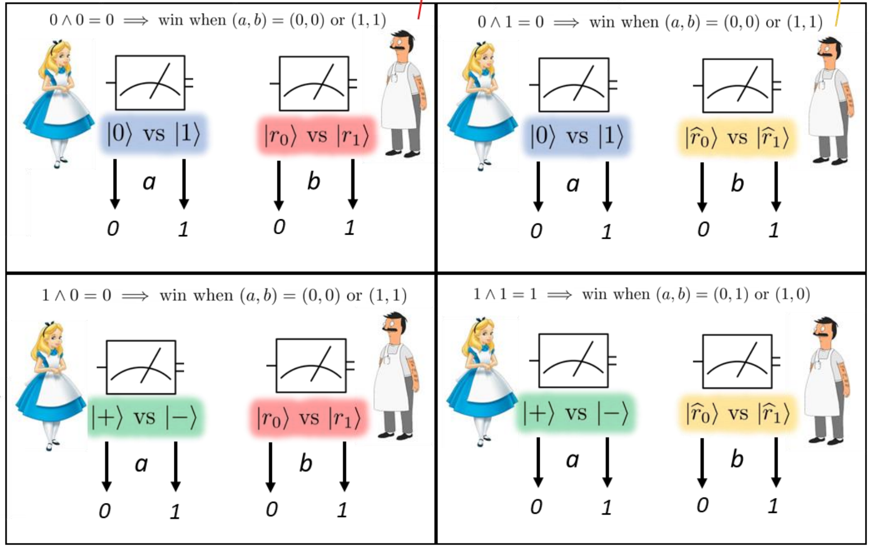
$|\hat{r}_1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$



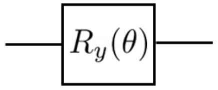








Question 1: How do we apply a rotation to quantum state?

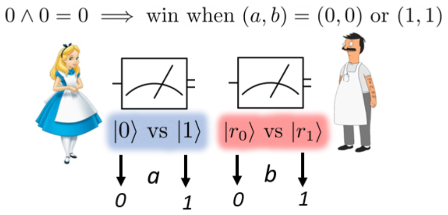


$$\longleftrightarrow \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Ry-gate (rotate state by theta/2 radians)

Question 2: How do we measure in alternate basis in Qiskit?

Answer: We don't need to.

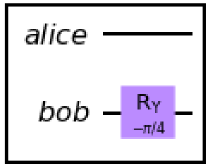


State when x =0 and y = 0

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|0r_0\rangle - \sin\left(\frac{\pi}{8}\right)|0r_1\rangle + \sin\left(\frac{\pi}{8}\right)|1r_0\rangle + \cos\left(\frac{\pi}{8}\right)|1r_1\rangle\right]$$

equivalent if we interpret: $|0\rangle$ as $|r_0\rangle$ and $|1\rangle$ as $|r_1\rangle$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \rightarrow$$



$$\rightarrow \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{8}\right)|00\rangle - \sin\left(\frac{\pi}{8}\right)|01\rangle + \sin\left(\frac{\pi}{8}\right)|10\rangle + \cos\left(\frac{\pi}{8}\right)|11\rangle\right]$$

Question 3: How do we simulate Charlie/Lucy coin flip in circuit ?

Answer: Something to think about :)