Bits vs Qubits and Quantum Advantage

CS 401: Quantum Computing
Dr. Kell, Spring 2023

High Level Review: How Classical Computers Work

(non-quantum)

Step 1: Pick the most convenient/efficient physical medium for storing and manipulating numbers.

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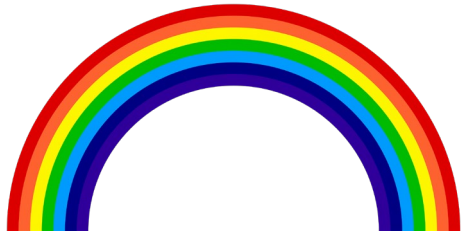
Letters/Symbols



Colors

3141592653589793238462643383

Hindu-Arabic Numbers

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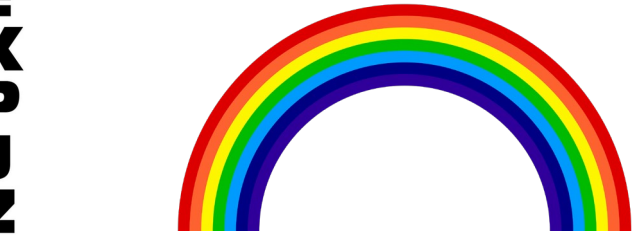

Letters/Symbols

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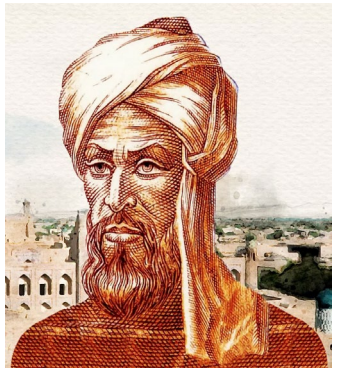
Hindu-Arabic Numbers

Invented 0-300 AD
In India



800 AD: Al Khwarizmi
(etymology of “algorithm”)
in Middle East

1200 AD: Fibonacci
In Europe

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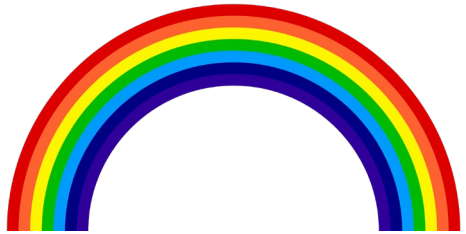

Colors

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Hindu-Arabic Numbers

Binary Numbers (2 digits instead of 10)

1 1 0 0 1 0 0 1

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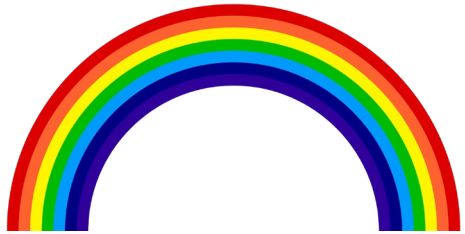

Colors

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Hindu-Arabic Numbers

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1 1 0 0 1 0 0 1

Lights Switches

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Colors

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Lights Bulbs

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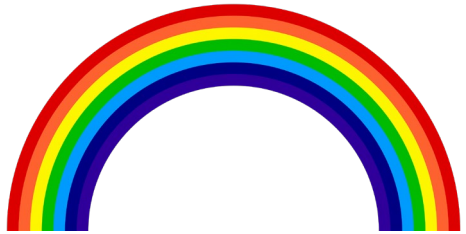


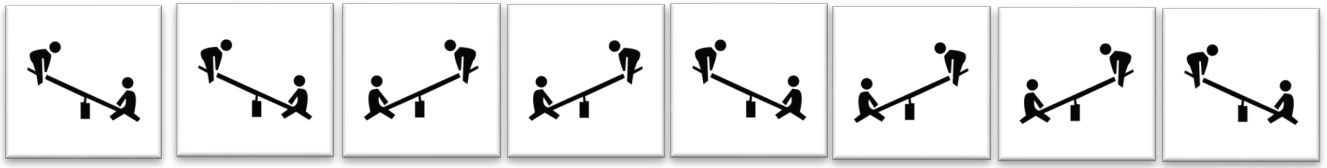
Colors

31415926535897932384626433 83

Hindu-Arabic Numbers

Binary Numbers (2 digits instead of 10)

1 1 0 0 1 0 0 1



Kids on See-Saw

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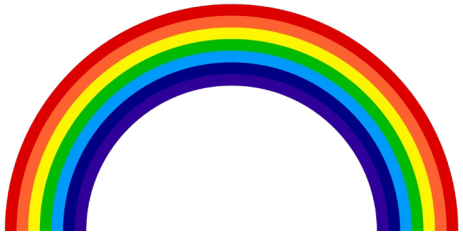

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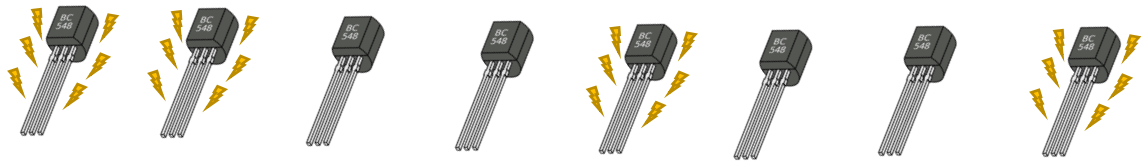

Computers Use: transistors + wires

(Classical Bits of Information)

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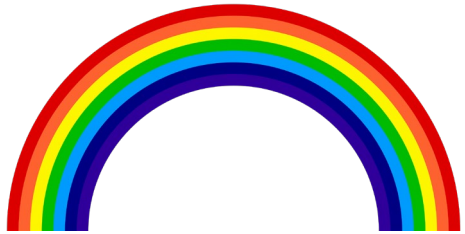

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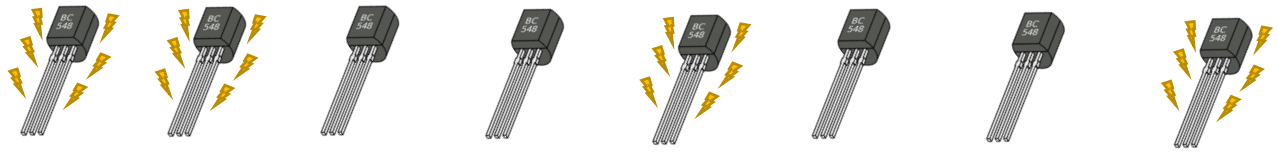
Computers Use: basically, mini light bulbs
(Classical Bits of Information)

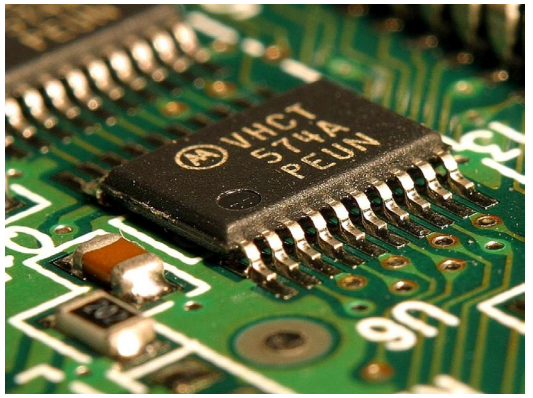
High Level Review: How Classical Computers Work

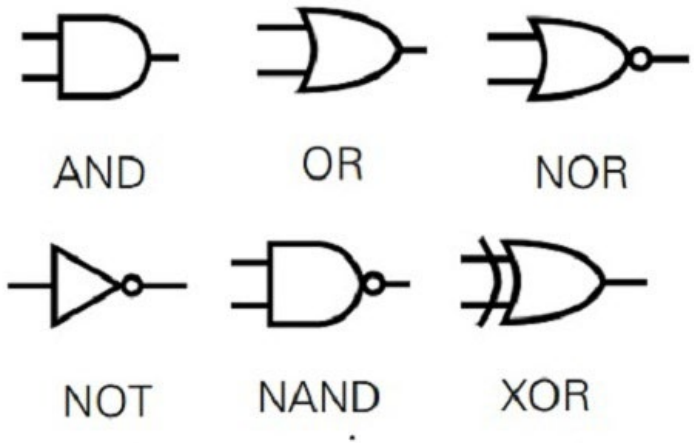
(non-quantum)

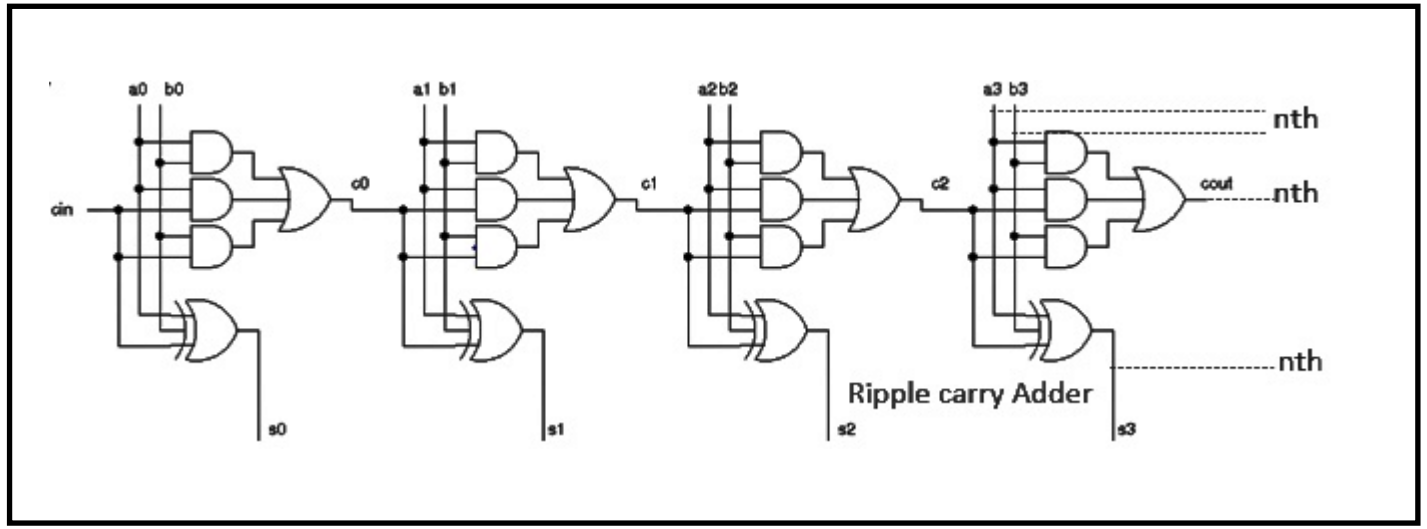
Step 2: “Automate” the process of thinking using physical information scheme.

1 1 0 0 1 0 0 1



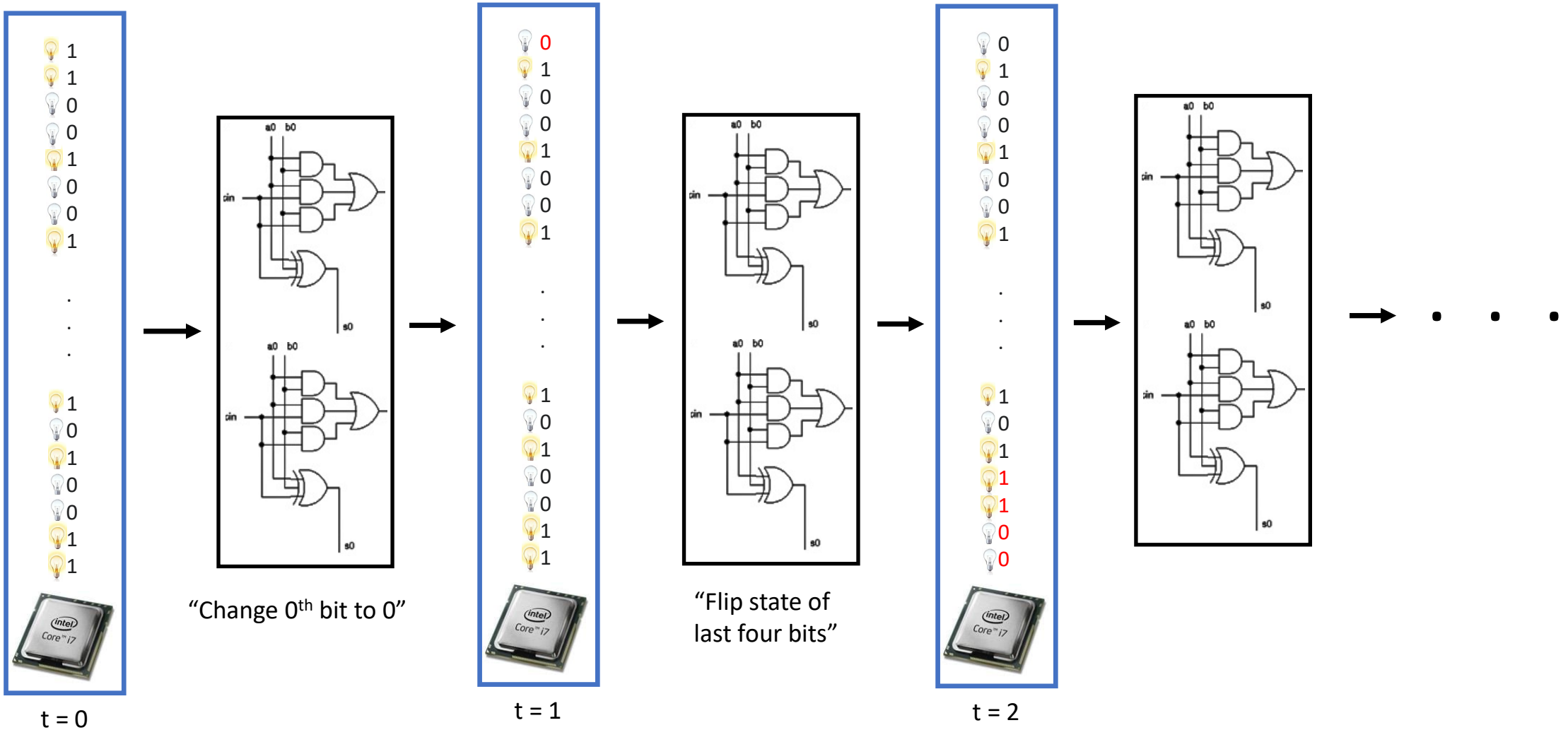






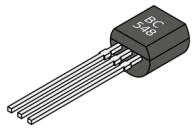
(Simplified) Evolution of Classical CPU Over Time

Entire State of CPU



Classical Bits versus Quantum Bits (“Qubits”)

Modern Classical Bits



Transistor Size ~ 10 nanometers
(0.00000001 meters)

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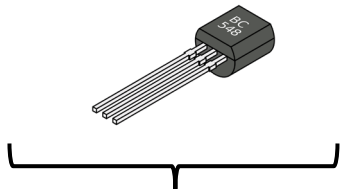

Transistor Size ~ 10 nanometers
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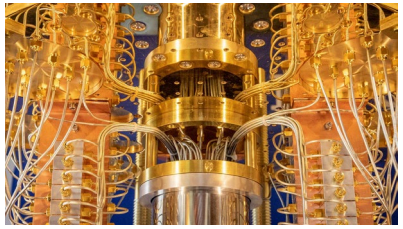
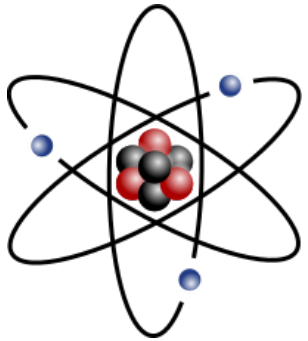
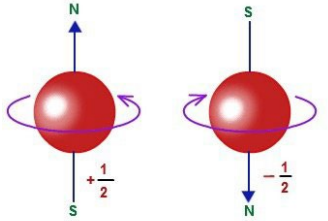
Qubit -> state of an atom or photon

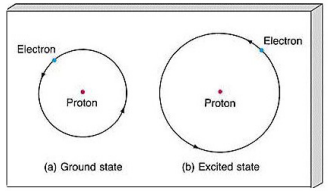
(Just make things even smaller)







“spin up” vs “spin down”



ground vs excited

Electron size ~ 10^{-18} meters
(0.000000000000000001 meters)

Canonical Problems with Quantum Advantage

Canonical Problems with Quantum Advantage

Problem 1: Factoring Integers

Input: integer x.
Output: non-trivial factors of x.

x = 54 ⟶ 2, 3, 6, 9, 18, 27

Best Classical Algorithm: O(2^n) for n bit numbers

Shor's Quantum Algorithm: O(poly(n))

Many cryptography schemes (e.g., RSA) rely on exponential runtime for the problem.

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Problem 2: Search Problem

Input: list L, target value
Output: index of target in L

L = [2, 1, 10, 4, 7, 9, 3]
target = 7
⟶ 4 (index of 7)

Many applications in cloud quantum computing, databases, etc.

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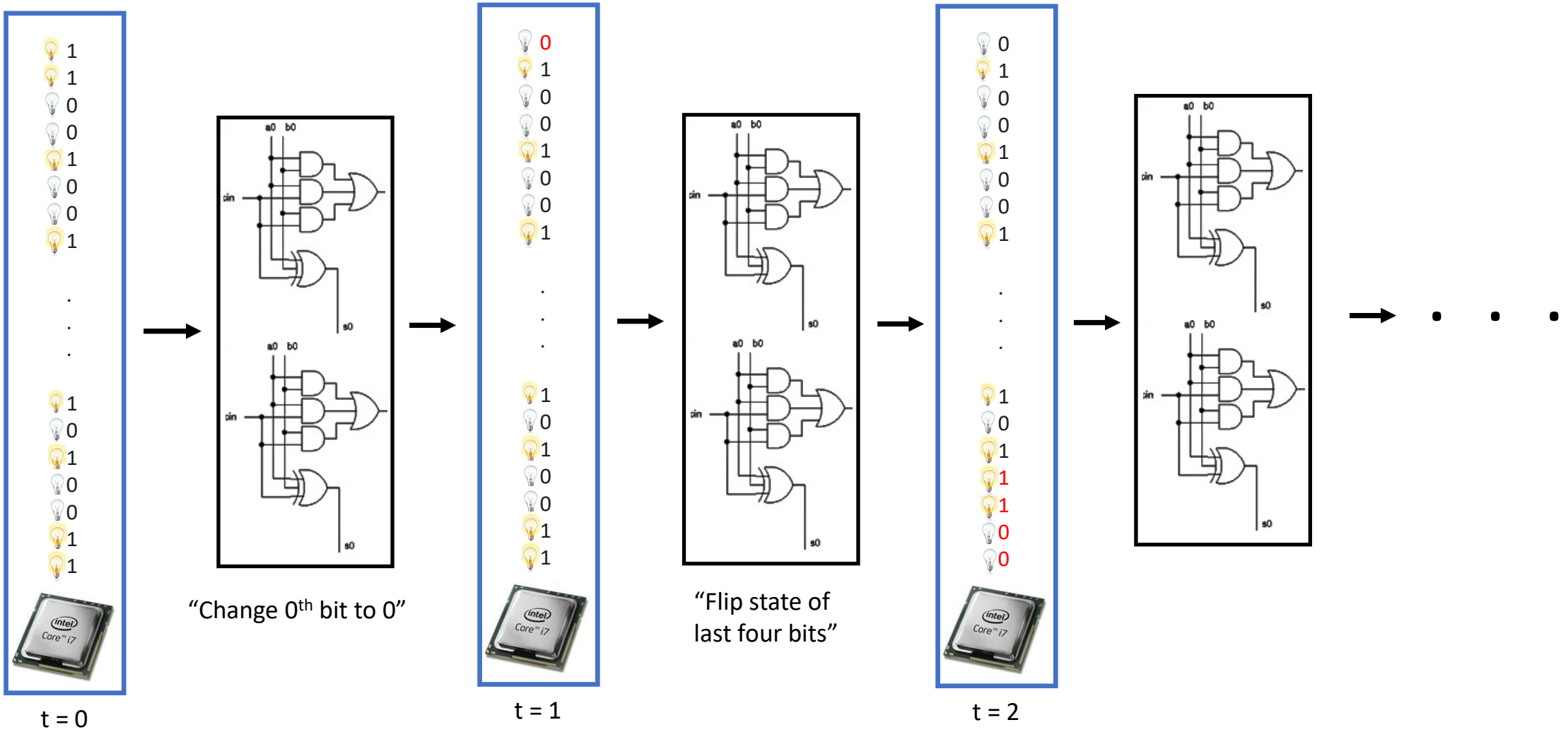
Best Possible Classical Algorithm: O(n)

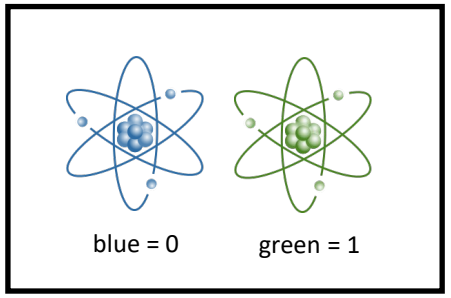
Grover's Quantum Algorithm: O($n^{1/2}$)

High Level: Why is this possible?

Entire State of CPU

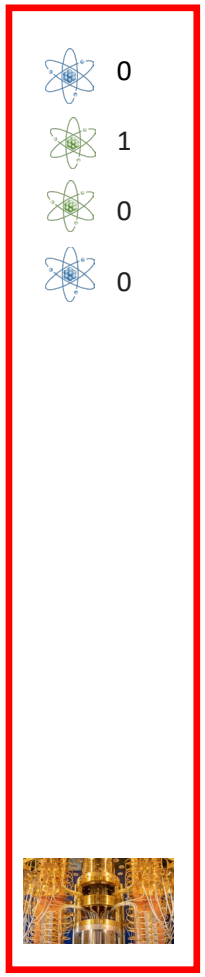


High Level: Why is this possible?

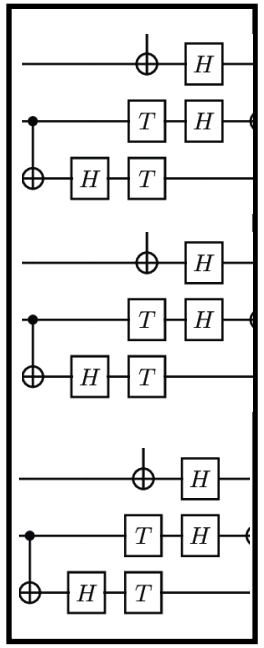


Qubit = some property of atom

State Quantum CPU

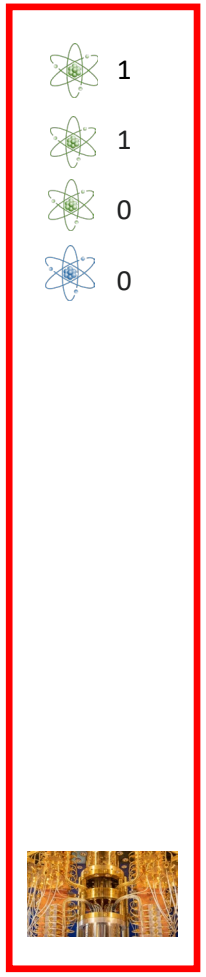


t = 0

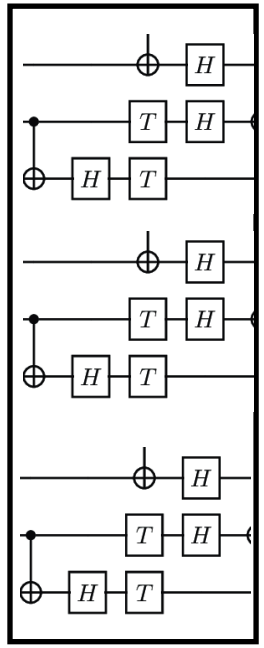


Quantum Logic Gates

State Quantum CPU



t = 1



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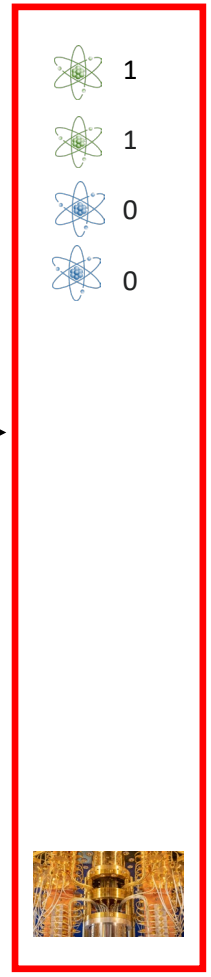
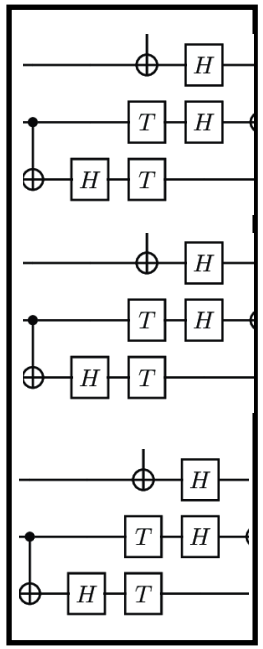


t = 2



Quantum Logic Gates

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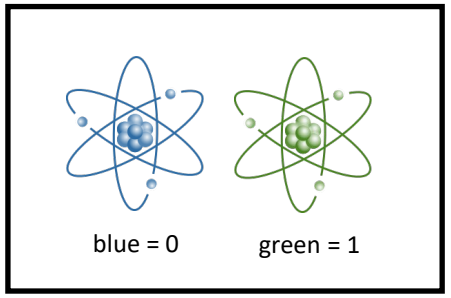
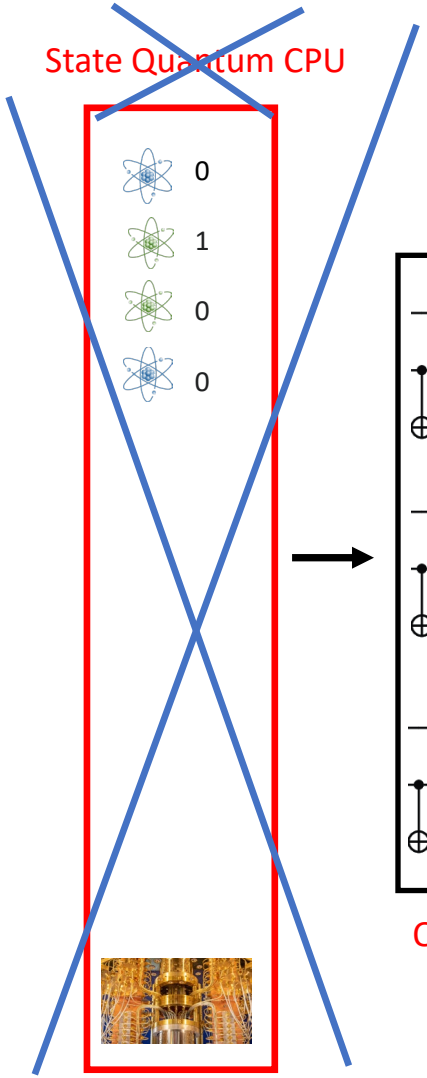


Qubit = some property of atom

State Quantum CPU



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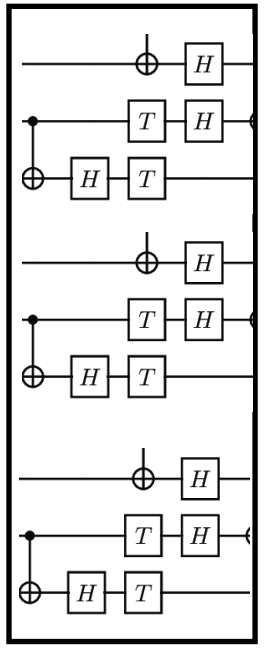

Quantum Logic Gates

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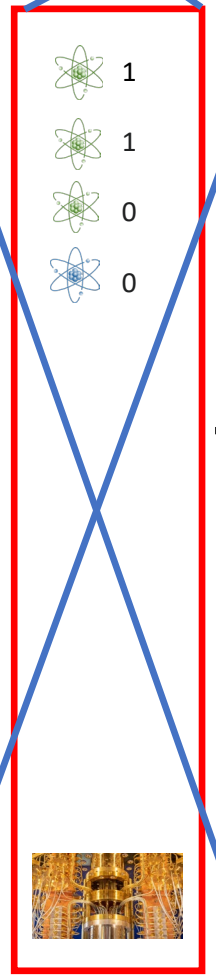

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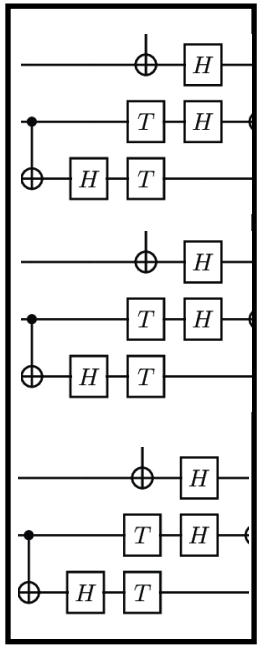

Quantum Logic Gates

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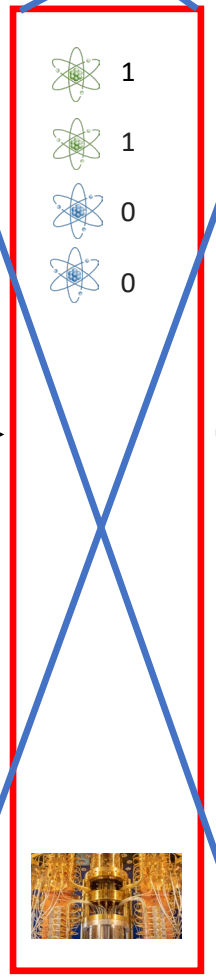

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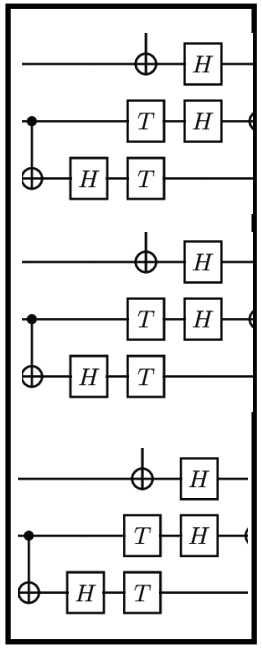

Quantum Logic Gates

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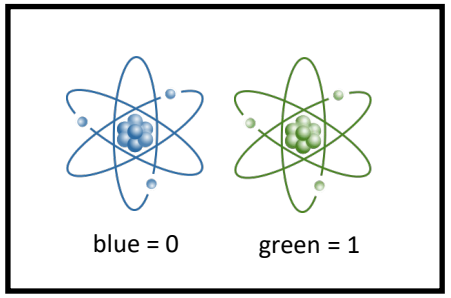

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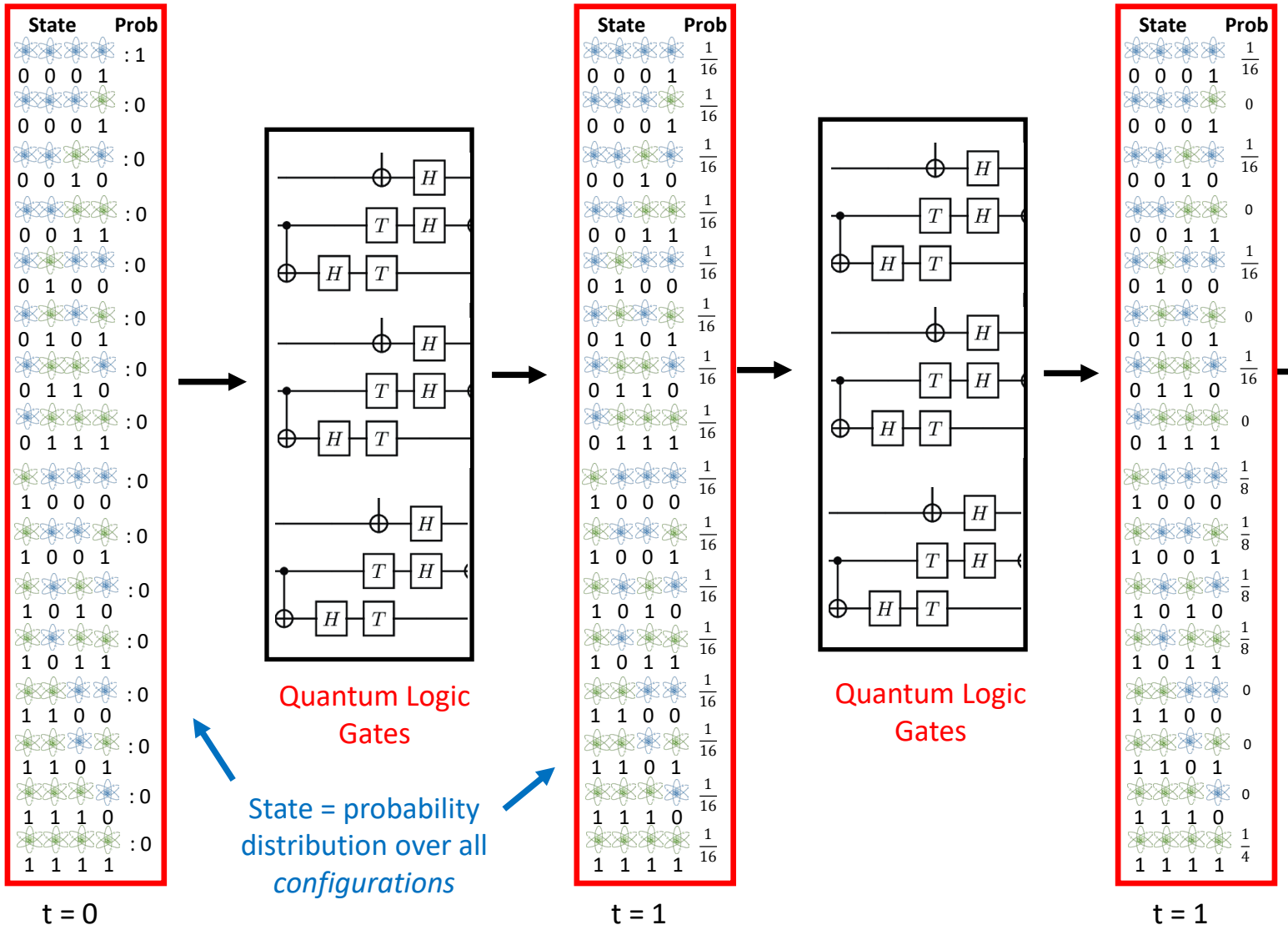
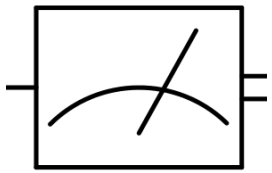




Measure/Observe

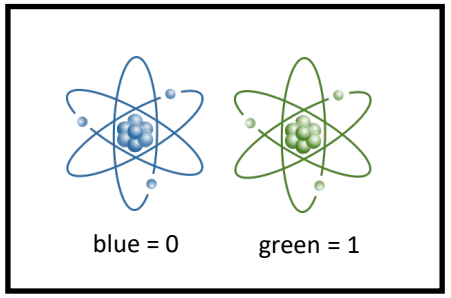
(Picks state according to probabilities)

Single outcome

1 0 1 0

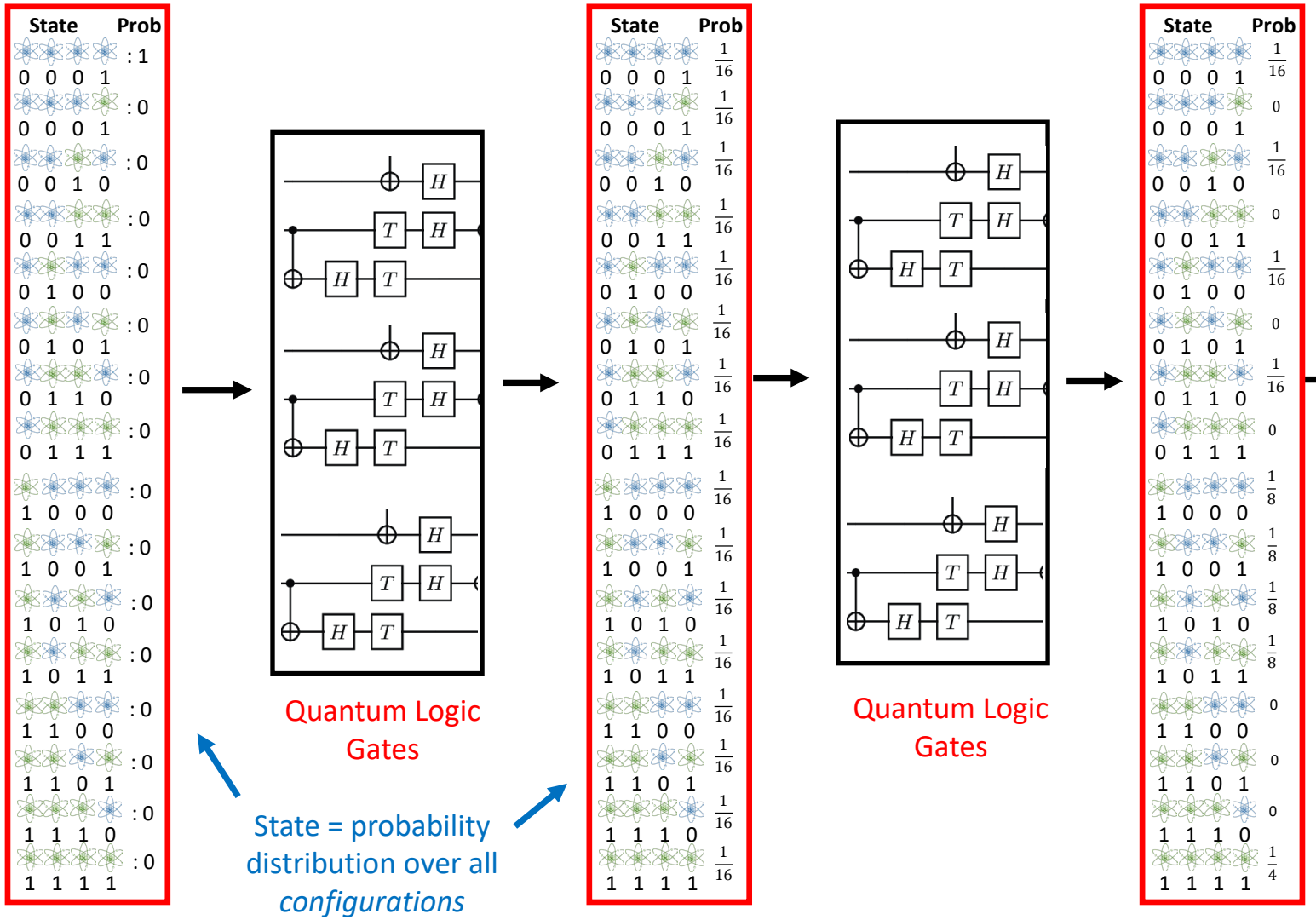
(had 1/8 chance)

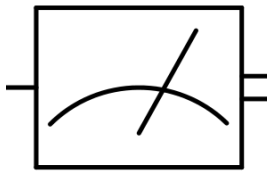
High Level: Why is this possible?



Qubit = some property of atom

State Quantum CPU





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Single outcome

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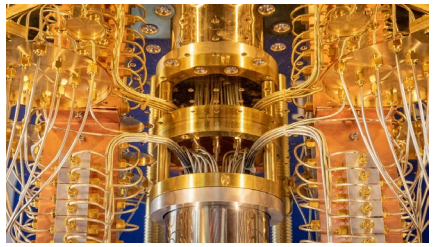
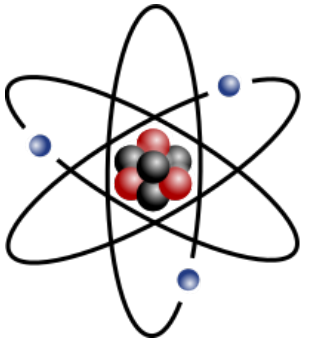
… Well, not exactly

- Probabilities can actually be *negative* or *complex* numbers.
- What happens if we measure some atoms/qubits but not others?

Conclusions





Key Takeaways

- **Quantum computers can be faster** because they get to manipulate and exponential amount of information in O(1) time.
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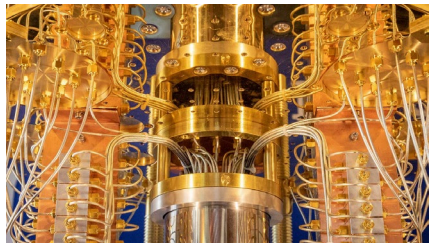

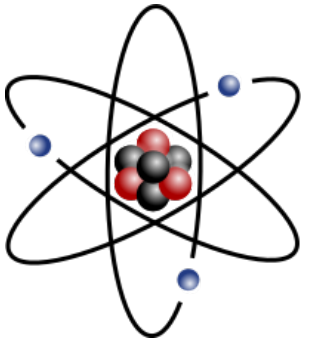

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Question: How does “Nature” keep track of and update so much information so quickly?

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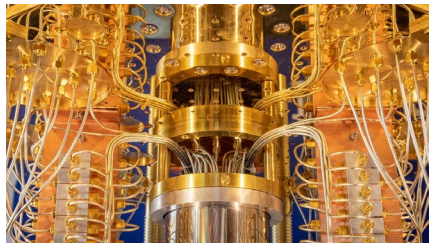

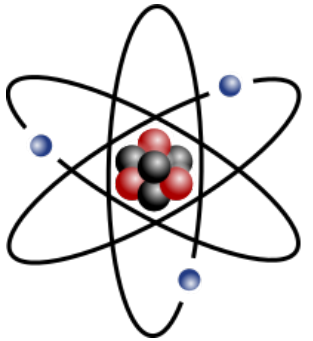

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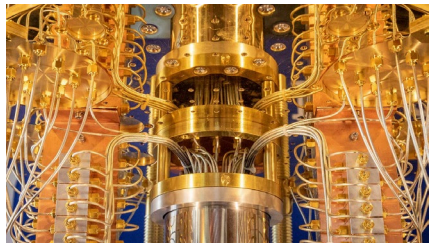

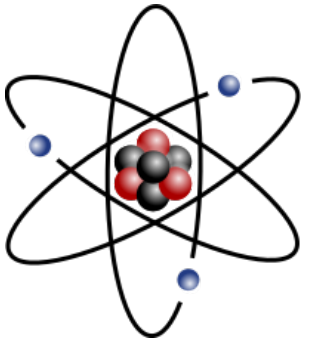

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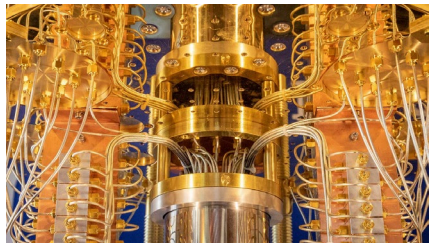

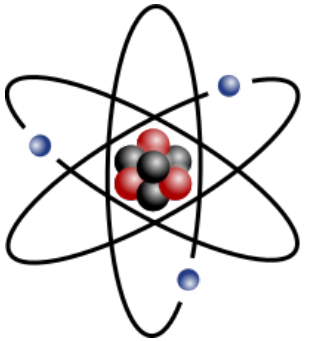

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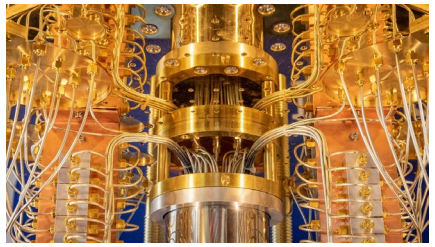

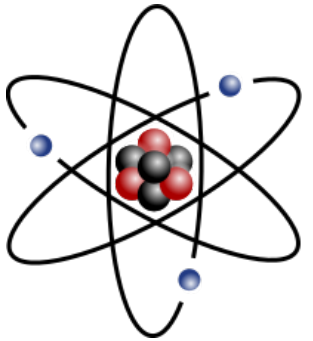

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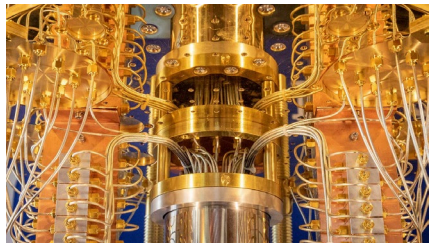

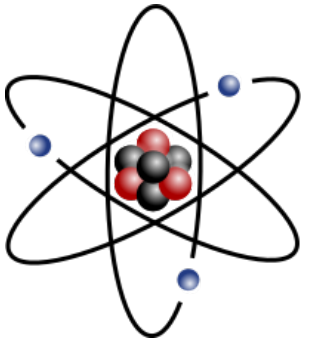

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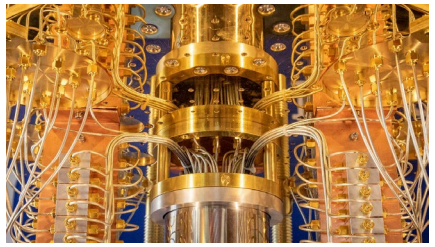

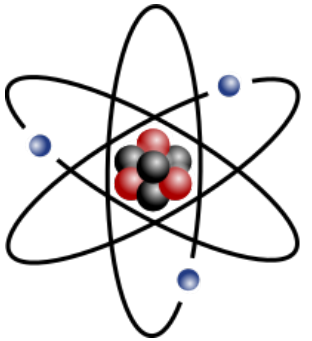

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  3. We live in a multiverse which interact *(many-worlds interpretation).*