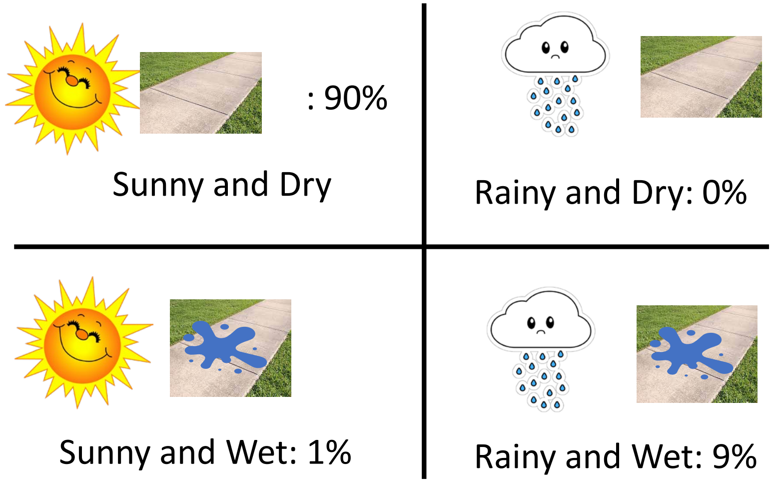
Lectures 9-11: Quantum Teleportation

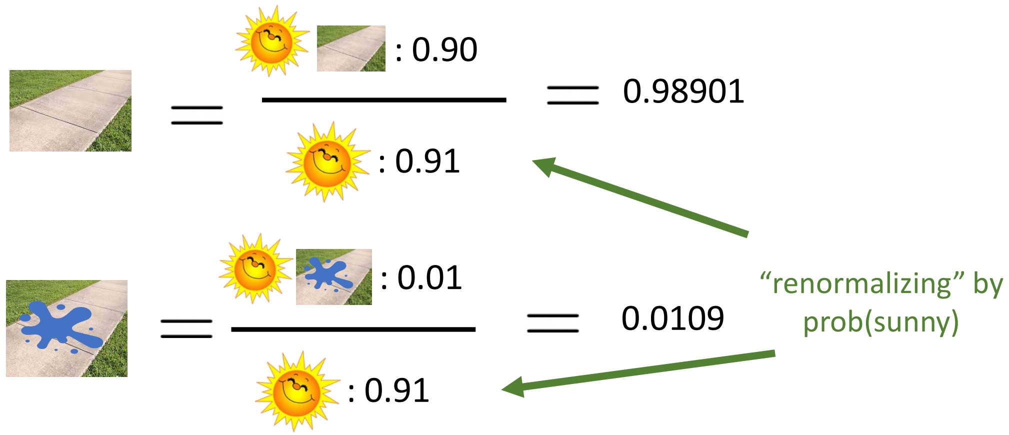
CS 401: Quantum Computing
Dr. Kell, Spring 2023

Quick Review: Renormalization

Classic Probability: probabilities for sunny/rainy and wet/dry.



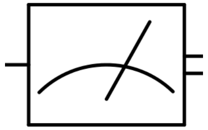
Given it's sunny, what's the probability the sidewalk is dry/wet?



Measurement in Quantum Mechanics: Same idea, but now we renormalize by squared amplitudes instead:

$$|a,b\rangle = \boxed{\frac{1}{\sqrt{2}}|00\rangle_{ab} + \frac{1}{\sqrt{4}}|01\rangle_{ab}} + \frac{1}{4}|10\rangle_{ab} + \frac{\sqrt{3}}{4}|11\rangle_{ab}$$

measure qubit a



suppose a = 0

Probabilities for measured state of Qubit a
(now a "classical outcome" once measured)

$$\boxed{|a\rangle = |0\rangle \text{ with prob: } \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{4}}\right)^2 = \frac{3}{4}}$$

$$|a\rangle = |1\rangle \text{ with prob: } \left(\frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2 = \frac{1}{4}$$

State of qubit b after observing a = 0

$$|b\rangle = \frac{\frac{1}{\sqrt{2}}}{\sqrt{3/4}}|0\rangle + \frac{\frac{1}{\sqrt{4}}}{\sqrt{3/4}}|1\rangle$$

$$= \frac{2}{\sqrt{6}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$$

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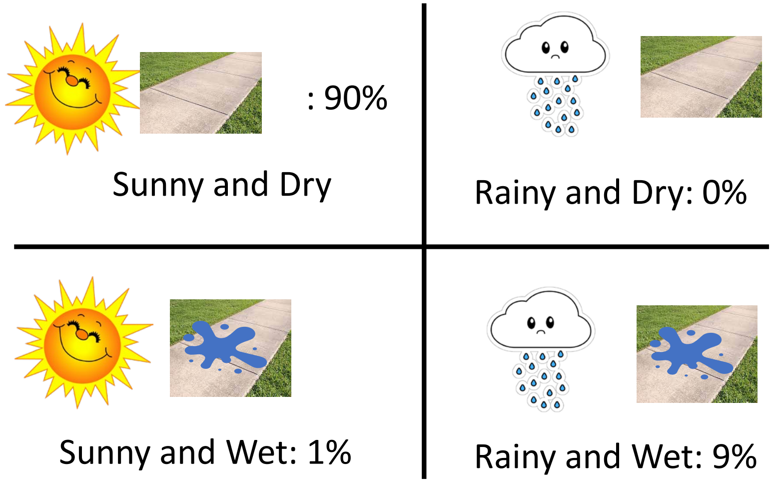

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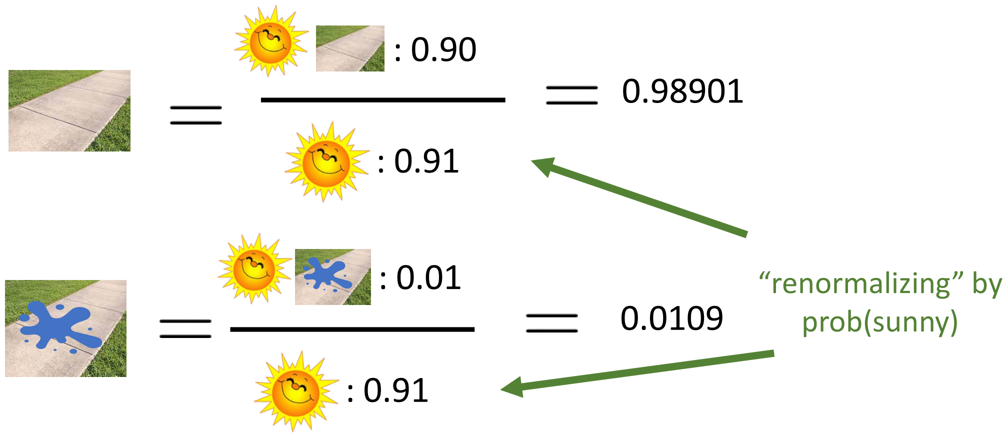


Measurement in Quantum Mechanics: Same idea, but now we renormalize by squared amplitudes instead:

In general, renormalization after measuring a.

$$|a,b\rangle = \boxed{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle} + \boxed{\alpha_{10}|10\rangle + \alpha_{11}|11\rangle}$$

measure a = 0

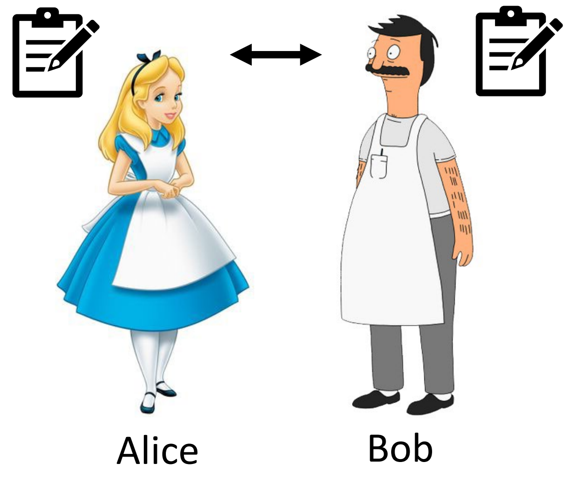
$$|b\rangle = \frac{\alpha_{00}}{\sqrt{\alpha_{00}^2 + \alpha_{01}^2}}|00\rangle + \frac{\alpha_{01}}{\sqrt{\alpha_{00}^2 + \alpha_{01}^2}}|01\rangle$$

measure a = 1

$$|b\rangle = \frac{\alpha_{10}}{\sqrt{\alpha_{10}^2 + \alpha_{11}^2}}|10\rangle + \frac{\alpha_{11}}{\sqrt{\alpha_{10}^2 + \alpha_{11}^2}}|11\rangle$$

Analogy: "Classical Teleportation"

1. Alice and Bob initially meet in-person. Can share information and trade notes.



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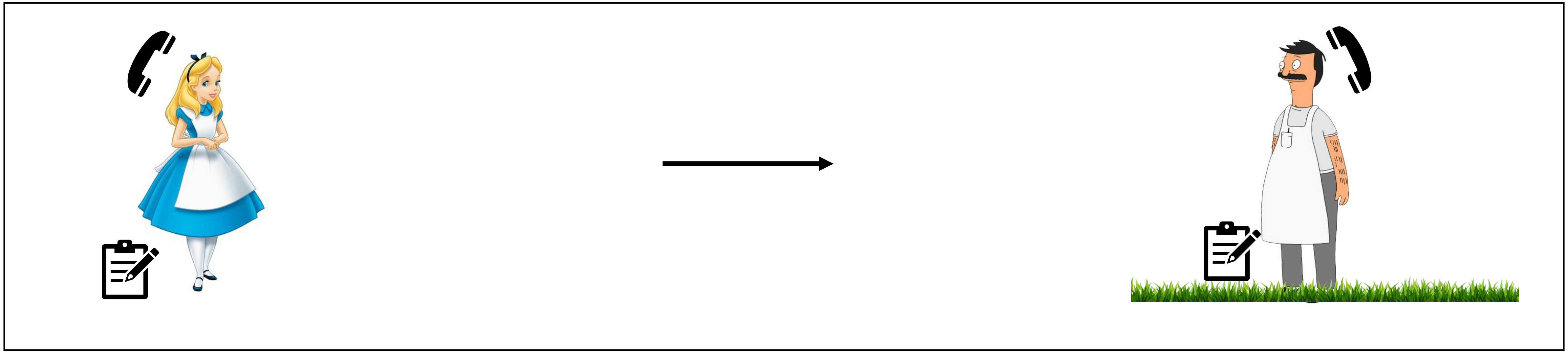
1. Alice and Bob initially meet in-person. Can share information and trade notes.

2. Bob moves to a remote location. They can at this point only communicate over phone (Bob’s location is unknown).



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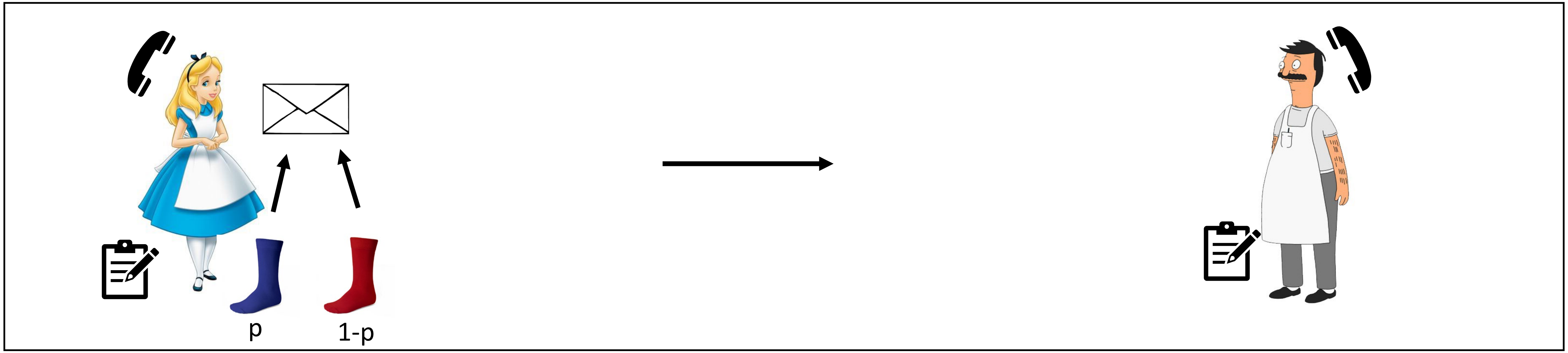

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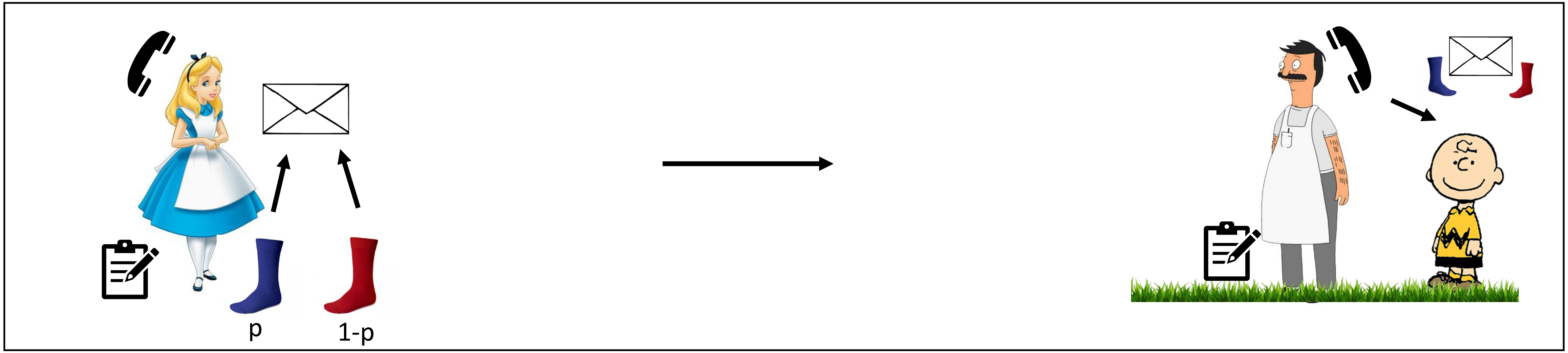
4. **Goal:** Alice wants to communicate information to Bob so he can pass along something (to his friend Charlie) that serves as a “proxy” for the envelope (i.e., another object that behaves like a “weighted coin” where prob(heads) = p and prob(tails) = 1-p.)



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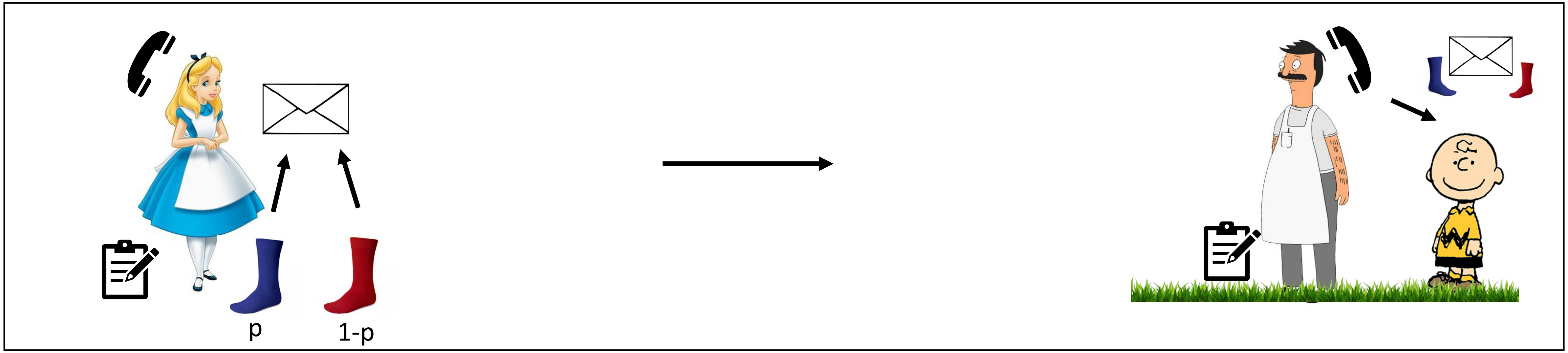

Analogy: "Classical Teleportation"

Question: Can this be accomplished without Alice opening the envelope?

Answer:

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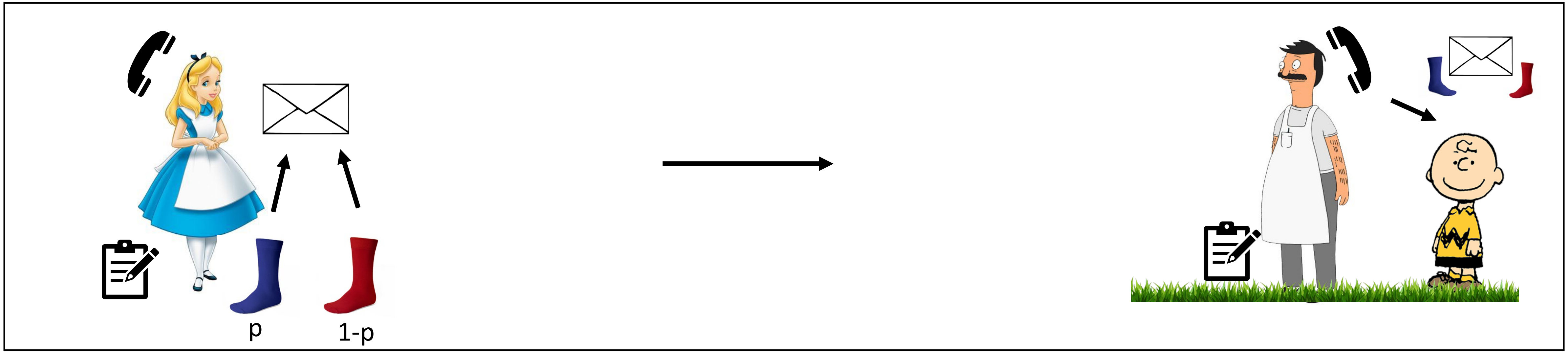

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Answer: Clearly no.

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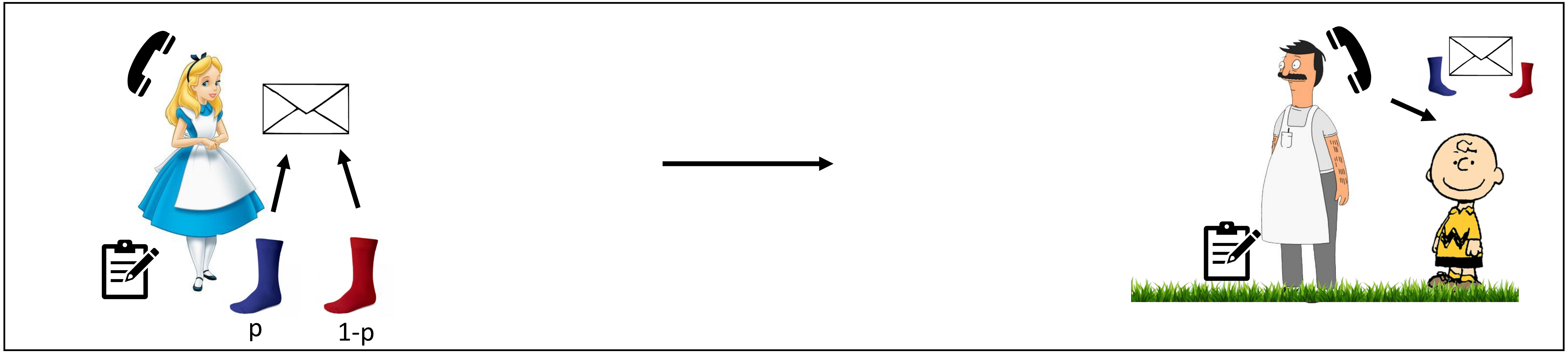

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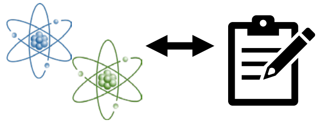
Answer: Clearly no… Unless we can use qubits!



Quantum Teleportation

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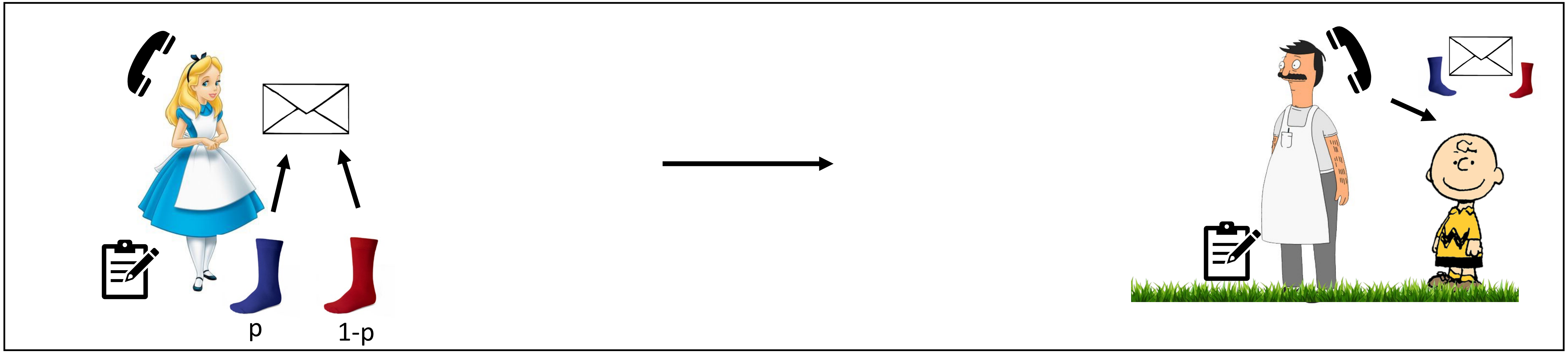

Quantum Teleportation Outline

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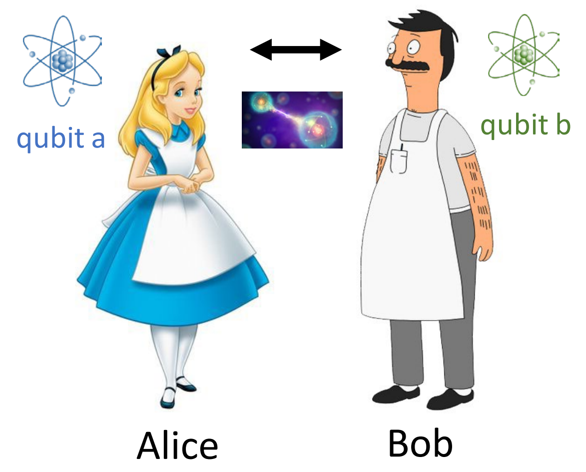

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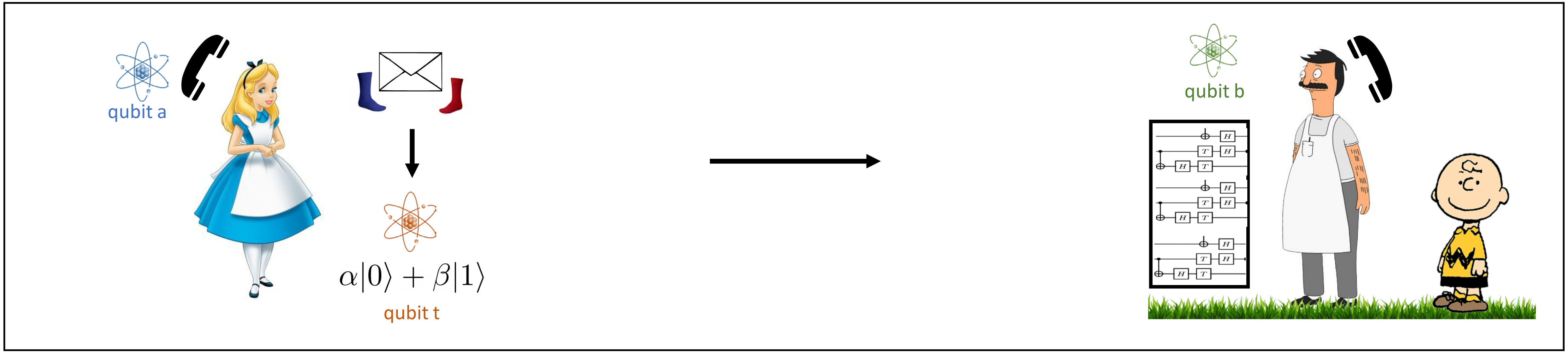

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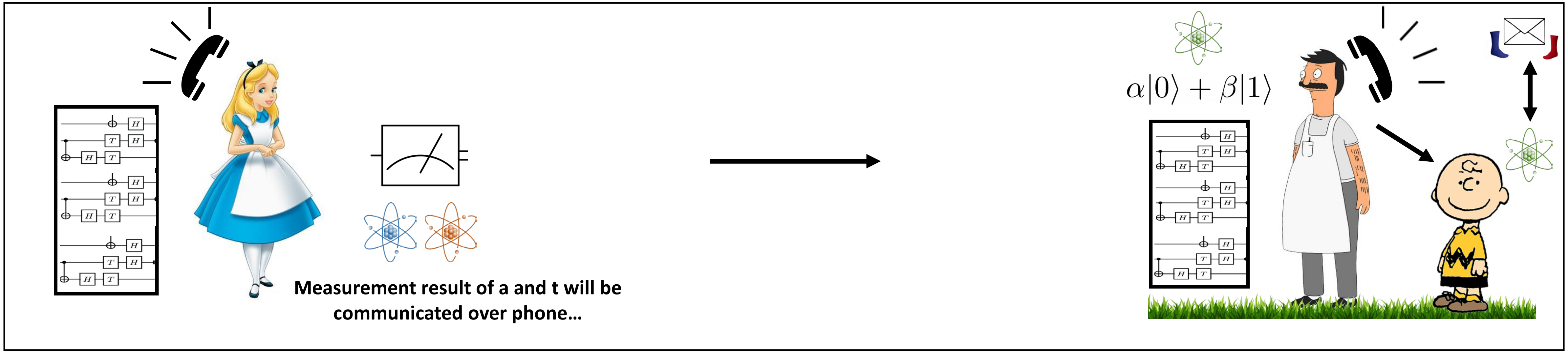

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   - Alice sends a and t through quantum gates and then measures both.
   - She then tells the result to Bob over the phone (four possibilities: 00, 01, 10, or 11).
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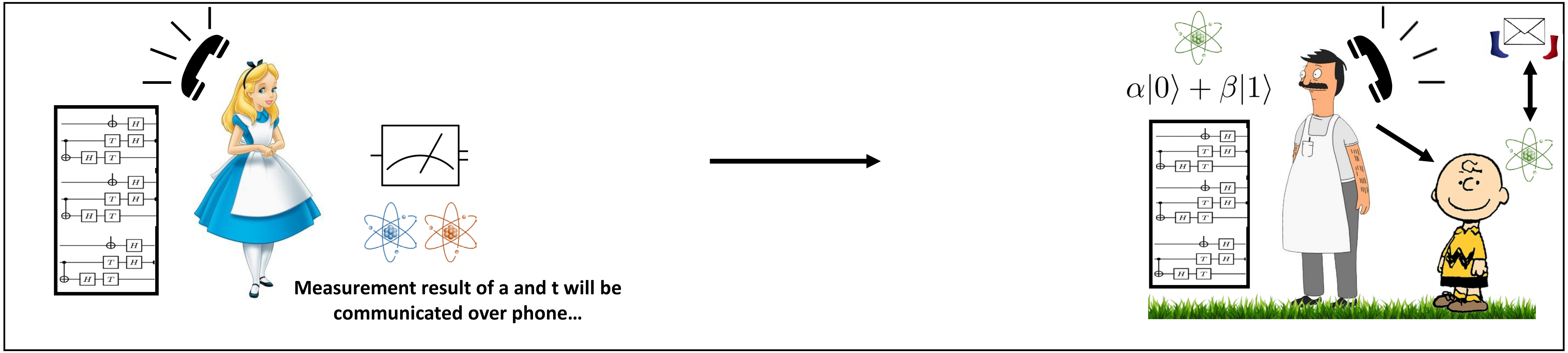
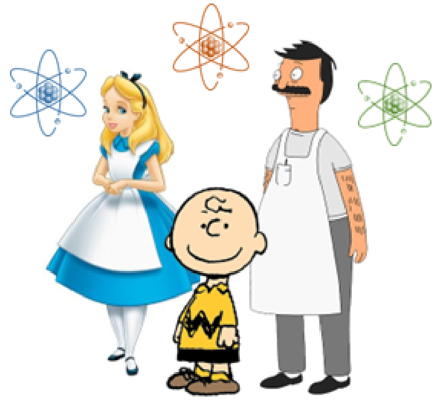


Quantum Teleportation: Alice's Computation



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$\alpha|0\rangle + \beta|1\rangle$

starting state of t

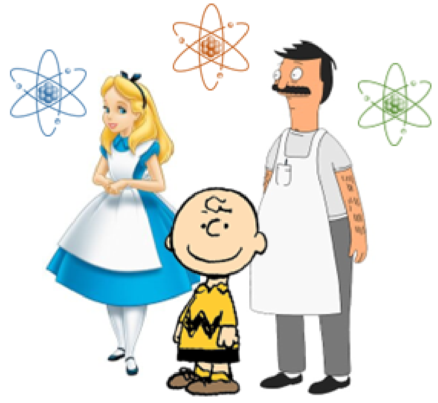
$|00\rangle$

starting state of ab

Step 1: Alice + Bob make a and b an EPR Pair

Step 2: Alice Quantum Gates

Quantum Teleportation: Alice's Computation



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$$\alpha|0\rangle + \beta|1\rangle$$

starting state of t

$$|00\rangle$$

starting state of ab

$$\otimes \downarrow$$

$$\alpha|000\rangle + \beta|100\rangle$$

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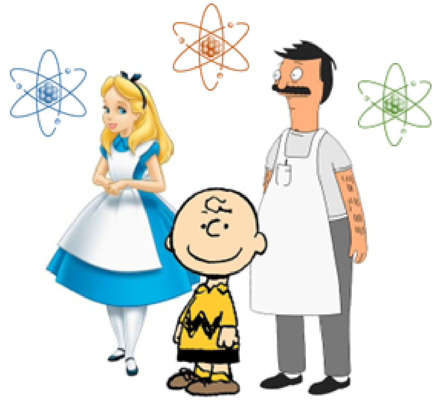

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$$\alpha|0\rangle + \beta|1\rangle$$

starting state of t

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starting state of ab

$$\otimes$$

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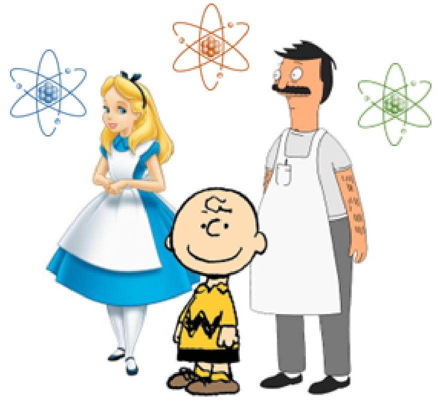
Step 1: Alice + Bob make a and b an EPR Pair

$$\frac{1}{\sqrt{2}}\left[\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right]$$

Step 2: Alice Quantum Gates

Quantum Teleportation: Alice’s Computation



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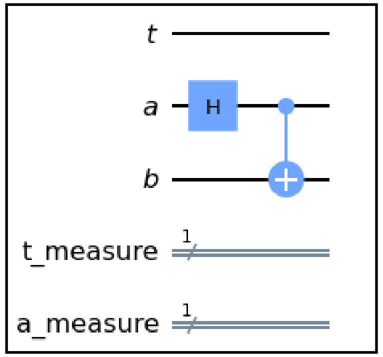

H gate on a, then CX(a,b)

→

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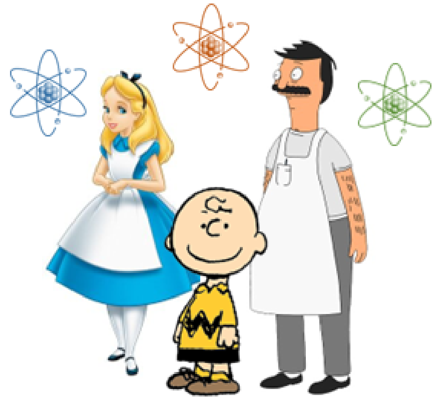

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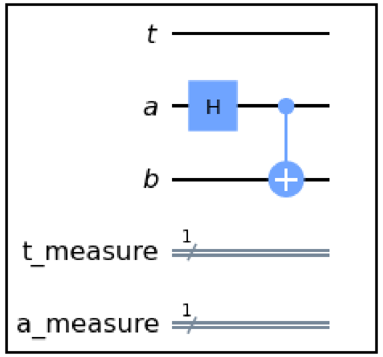

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Step 1: Alice + Bob make a and b an EPR Pair

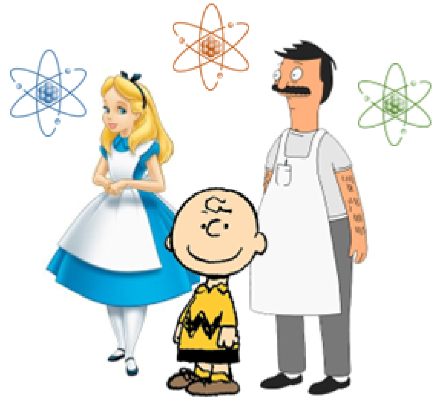
$$\frac{1}{\sqrt{2}}\left[\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\right]$$

bra-ket notation for “factoring out” common state

Step 2: Alice Quantum Gates

Quantum Teleportation: Alice’s Computation



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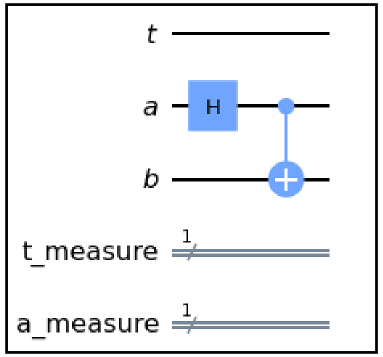

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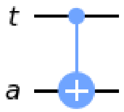
Use CX gate on t and a



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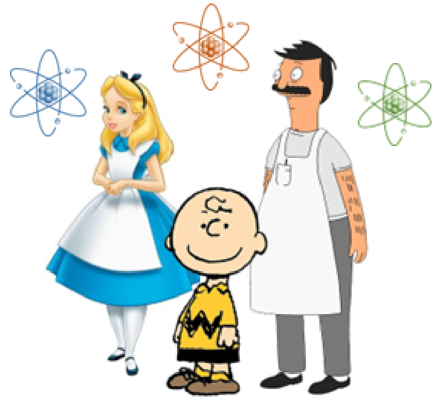
Quantum Teleportation: Alice's Computation



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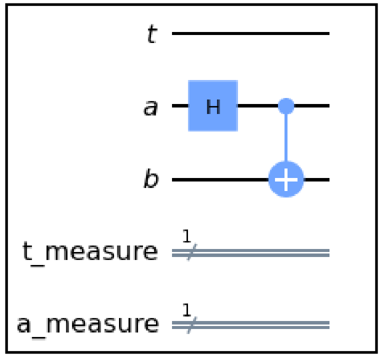

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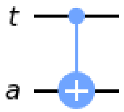

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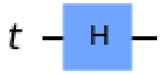
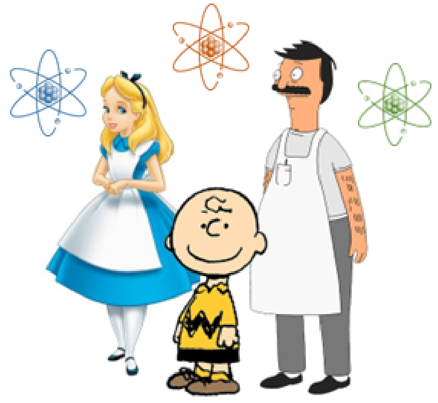


H gate on t

Quantum Teleportation: Alice’s Computation



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$$\alpha|0\rangle + \beta|1\rangle$$

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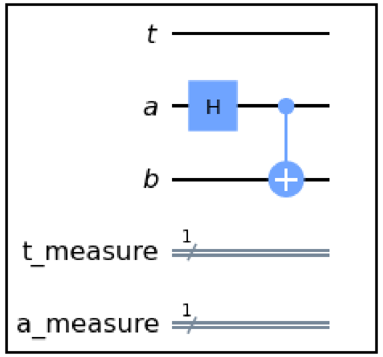

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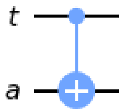
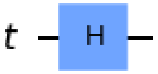


Step 2: Alice Quantum Gates

$$= \frac{1}{\sqrt{2}}\Big[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\Big]$$

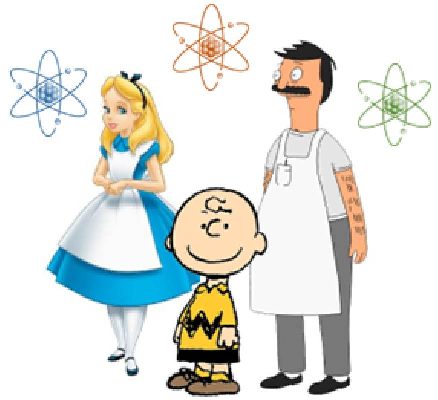


H gate on t

$$\frac{1}{\sqrt{2}}\Big[\frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|100\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\alpha}{\sqrt{2}}|111\rangle + \frac{\beta}{\sqrt{2}}|010\rangle - \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|001\rangle - \frac{\beta}{\sqrt{2}}|101\rangle\Big]$$

$$= \frac{1}{2}\Big[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\Big]$$

Quantum Teleportation: Alice's Computation



4. **Algorithm Outline**
- Alice sends a and t through quantum gates and then measures both.
- She then tells the result to Bob over the phone (four possibilities: 00, 01, 10, or 11).
- Bob then sends b through quantum gates that depend on the measurement outcome (four different operation sets for each outcome).

$$\alpha|0\rangle + \beta|1\rangle$$

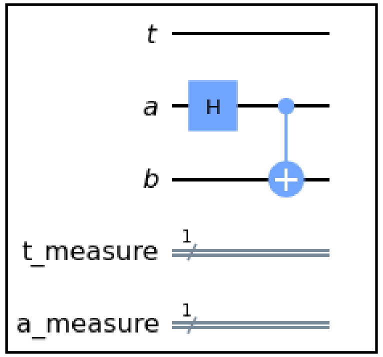
starting state of t

$$|00\rangle$$

starting state of ab

$$\otimes$$

$$\alpha|000\rangle + \beta|100\rangle$$



H gate on a, then CX(a,b)

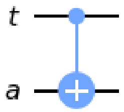
Step 1: Alice + Bob make a and b an EPR Pair

$$\frac{1}{\sqrt{2}}\Big[\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\Big]$$

$$= \frac{1}{\sqrt{2}}\Big[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\Big]$$

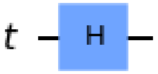
bra-ket notation for "factoring out" common state

Use CX gate on t and a



Step 2: Alice Quantum Gates

$$= \frac{1}{\sqrt{2}}\Big[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\Big]$$



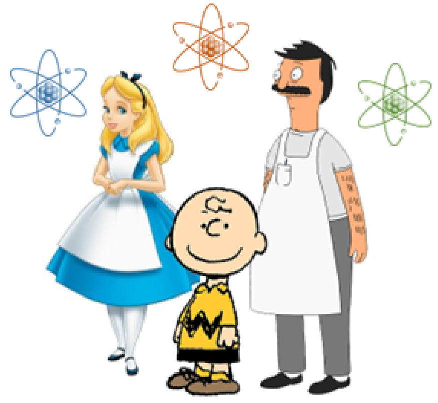
H gate on t

$$\frac{1}{\sqrt{2}}\left[\frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|100\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\alpha}{\sqrt{2}}|111\rangle + \frac{\beta}{\sqrt{2}}|010\rangle - \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|001\rangle - \frac{\beta}{\sqrt{2}}|101\rangle\right]$$

$$= \frac{1}{2}\Big[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\Big]$$

$$= \frac{1}{2}\Big[|00\rangle(\quad ? \quad) + |01\rangle(\quad ? \quad) + |10\rangle(\quad ? \quad) + |11\rangle(\quad ? \quad)\Big]$$

Quantum Teleportation: Alice’s Computation



4. **Algorithm Outline**
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- Bob then sends b through quantum gates that depend on the measurement outcome (four different operation sets for each outcome).

$$\alpha|0\rangle + \beta|1\rangle$$

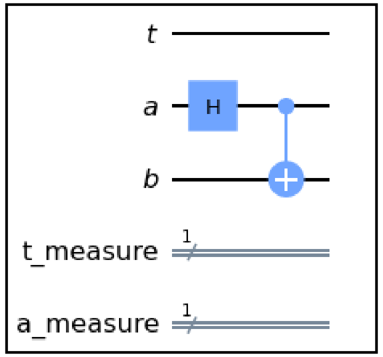
starting state of t

$$|00\rangle$$

starting state of ab

$$\otimes$$

$$\alpha|000\rangle + \beta|100\rangle$$



H gate on a, then CX(a,b)

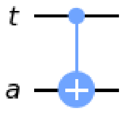
Step 1: Alice + Bob make a and b an EPR Pair

$$\frac{1}{\sqrt{2}}\left[\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\right]$$

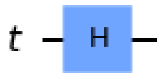
bra-ket notation for “factoring out” common state

Use CX gate on t and a



Step 2: Alice Quantum Gates

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\right]$$



H gate on t

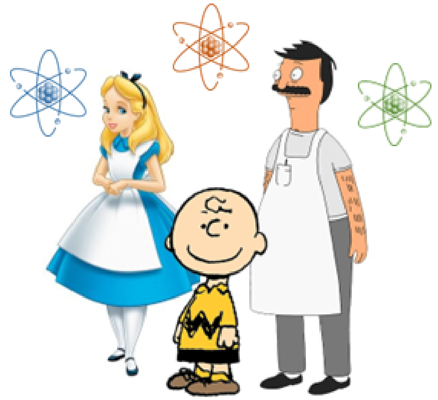
$$\frac{1}{\sqrt{2}}\left[\frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|100\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\alpha}{\sqrt{2}}|111\rangle + \frac{\beta}{\sqrt{2}}|010\rangle - \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|001\rangle - \frac{\beta}{\sqrt{2}}|101\rangle\right]$$

$$= \frac{1}{2}\left[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\right]$$

$$= \frac{1}{2}\left[|00\rangle(\quad ? \quad) + |01\rangle(\quad ? \quad) + |10\rangle(\quad ? \quad) + |11\rangle(\quad ? \quad)\right]$$

$$= \frac{1}{2}\left[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\beta|0\rangle + \alpha|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle)\right]$$

Quantum Teleportation: Alice's Computation



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$$\alpha|0\rangle + \beta|1\rangle$$

starting state of t

$$|00\rangle$$

starting state of ab

$$\otimes$$

$$\alpha|000\rangle + \beta|100\rangle$$

Step 1: Alice + Bob make a and b an EPR Pair

$$\frac{1}{\sqrt{2}}\Big[\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\Big]$$

$$= \frac{1}{\sqrt{2}}\Big[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\Big]$$

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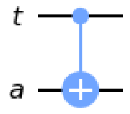
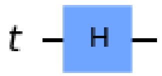
Use CX gate on t and a



Step 2: Alice Quantum Gates

$$= \frac{1}{\sqrt{2}}\Big[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\Big]$$



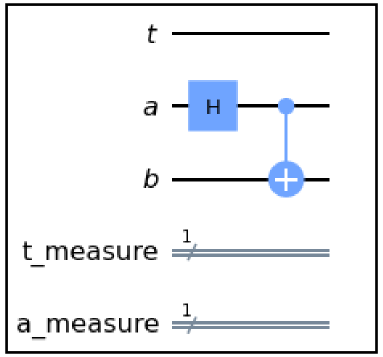
H gate on t

$$\frac{1}{\sqrt{2}}\left[\frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|100\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\alpha}{\sqrt{2}}|111\rangle + \frac{\beta}{\sqrt{2}}|010\rangle - \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|001\rangle - \frac{\beta}{\sqrt{2}}|101\rangle\right]$$

$$= \frac{1}{2}\Big[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\Big]$$

$$= \frac{1}{2}\Big[|00\rangle(\quad ? \quad) + |01\rangle(\quad ? \quad) + |10\rangle(\quad ? \quad) + |11\rangle(\quad ? \quad)\Big]$$

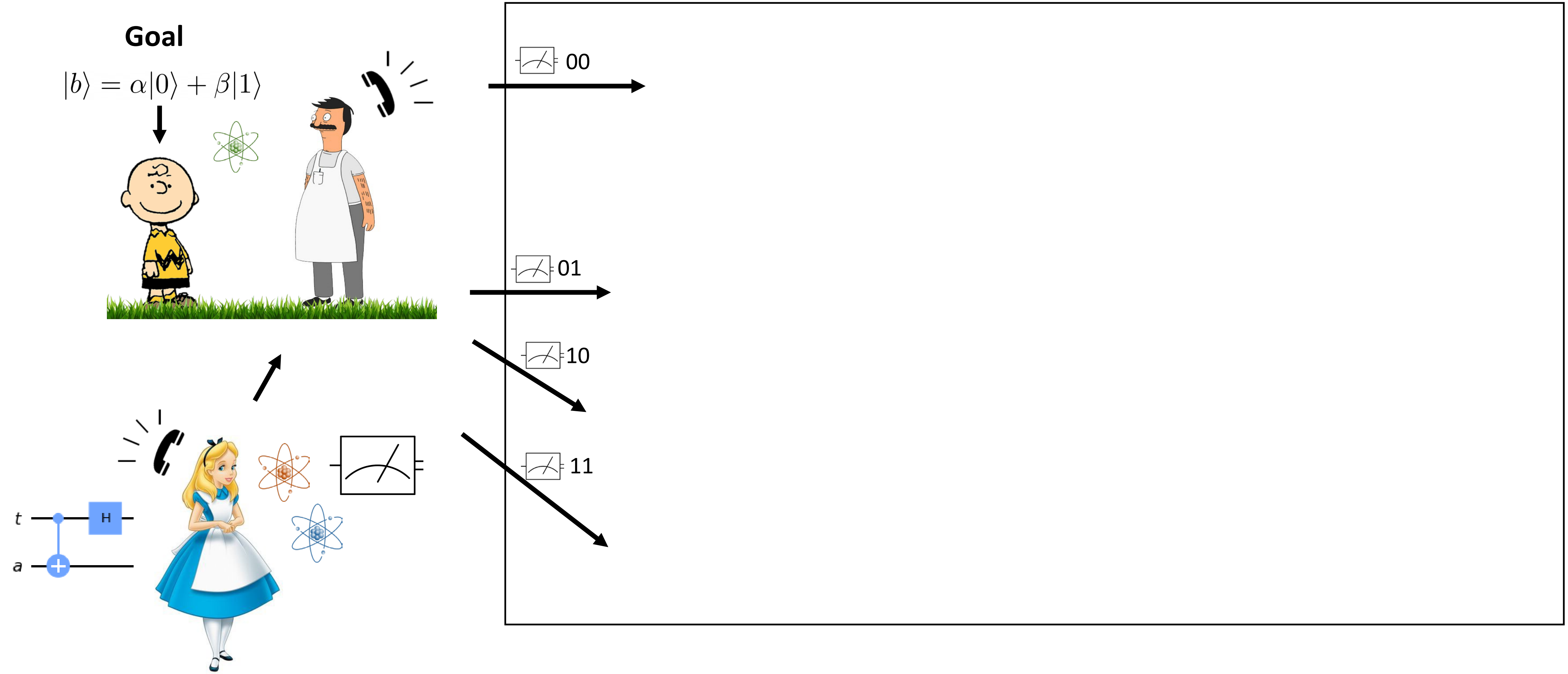
$$= \frac{1}{2}\Big[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\beta|0\rangle + \alpha|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle)\Big]$$



H gate on a, then CX(a,b)

When Bob hears result of Alice measurement, how should he proceed?

Quantum Teleportation: Bob's Computation



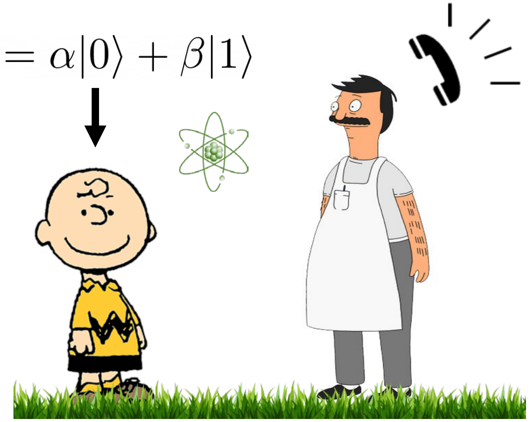
$$= \frac{1}{2}\Big[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\beta|0\rangle + \alpha|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle)\Big]$$

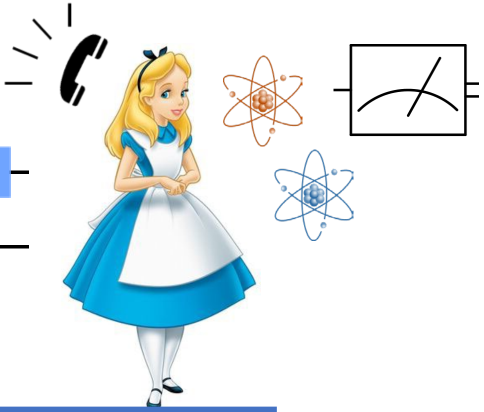
quantum state after Alice's computation

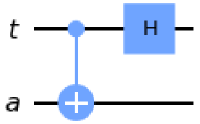
Quantum Teleportation: Bob’s Computation

Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$







00

$$\frac{\alpha/2}{\sqrt{(\alpha/2)^2+(\beta/2)^2}}|000\rangle + \frac{\beta/2}{\sqrt{(\alpha/2)^2+(\beta/2)^2}}|001\rangle$$

renormalization

01

10

11

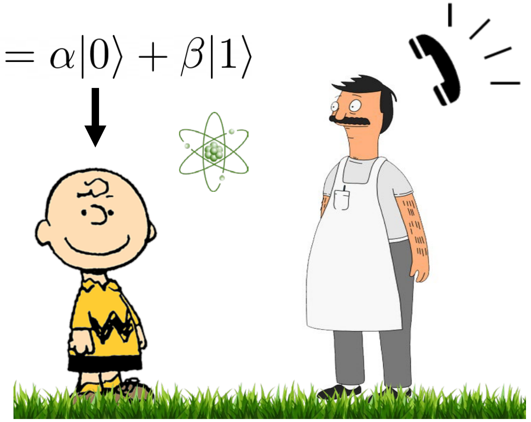
$$= \frac{1}{2}\left[\boxed{|00\rangle(\alpha|0\rangle+\beta|1\rangle)} + |01\rangle(\beta|0\rangle+\alpha|1\rangle) + |10\rangle(\alpha|0\rangle-\beta|1\rangle) + |11\rangle(-\beta|0\rangle+\alpha|1\rangle)\right]$$

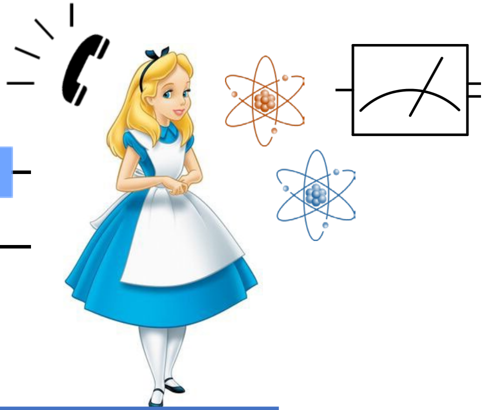
quantum state after Alice’s computation

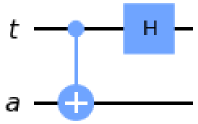
Quantum Teleportation: Bob's Computation

Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$







00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle$$

$$= \sqrt{\frac{\alpha^2+\beta^2}{4}}$$

01

10

11

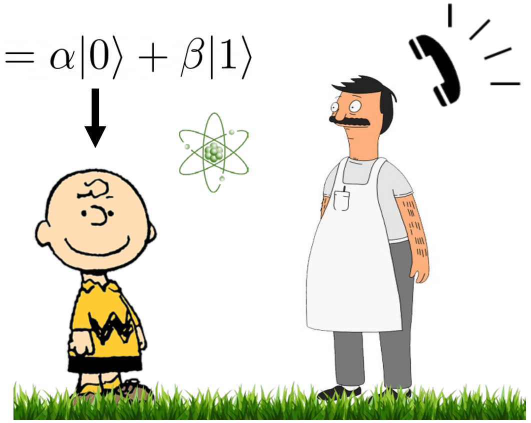
$$= \frac{1}{2}\left[\boxed{|00\rangle(\alpha|0\rangle+\beta|1\rangle)} + |01\rangle(\beta|0\rangle+\alpha|1\rangle) + |10\rangle(\alpha|0\rangle-\beta|1\rangle) + |11\rangle(-\beta|0\rangle+\alpha|1\rangle)\right]$$

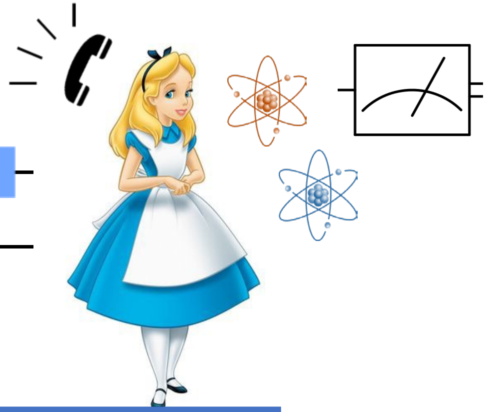
quantum state after Alice's computation

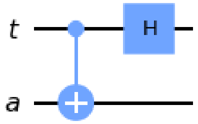
Quantum Teleportation: Bob’s Computation

Goal

$$|b\rangle = \alpha|0\rangle + \beta|1\rangle$$







00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle$$

$$= \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

01

10

11

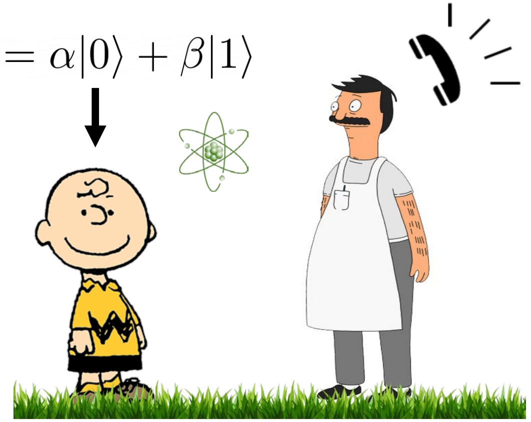
$$= \frac{1}{2}\left[\boxed{|00\rangle(\alpha|0\rangle + \beta|1\rangle)} + |01\rangle(\beta|0\rangle + \alpha|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle)\right]$$

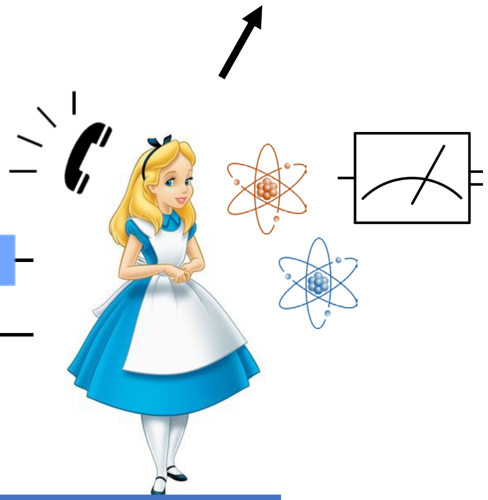
quantum state after Alice’s computation

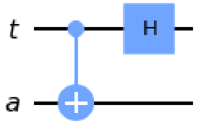
Quantum Teleportation: Bob’s Computation

Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$







00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle$$

01

10

11

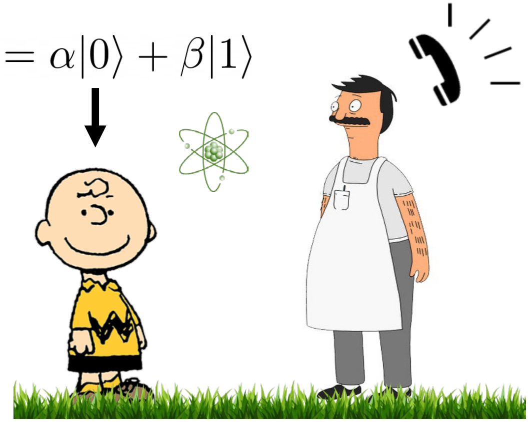
$$= \frac{1}{2}\left[\boxed{|00\rangle(\alpha|0\rangle + \beta|1\rangle)} + |01\rangle(\beta|0\rangle + \alpha|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle)\right]$$

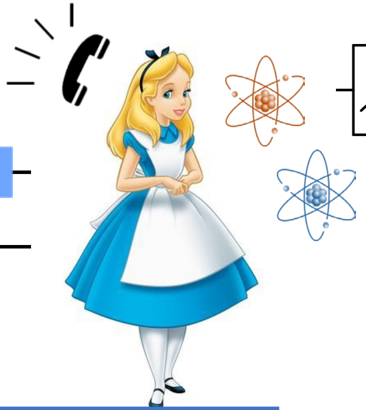
quantum state after Alice’s computation

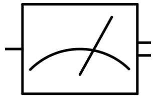
Quantum Teleportation: Bob’s Computation

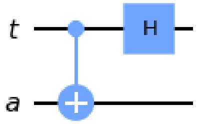
Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$









00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} \; = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle$$ ✓ (Bob does nothing)

01

10

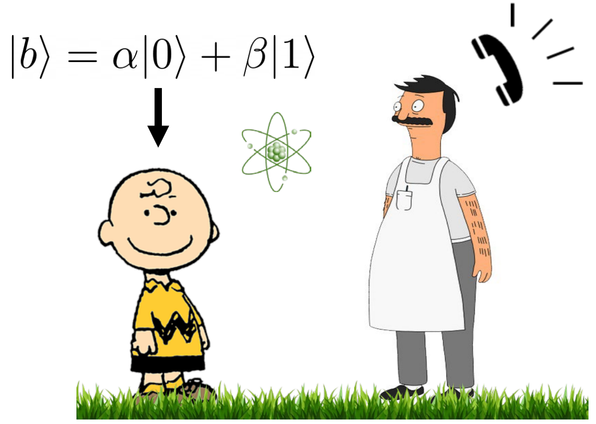
11

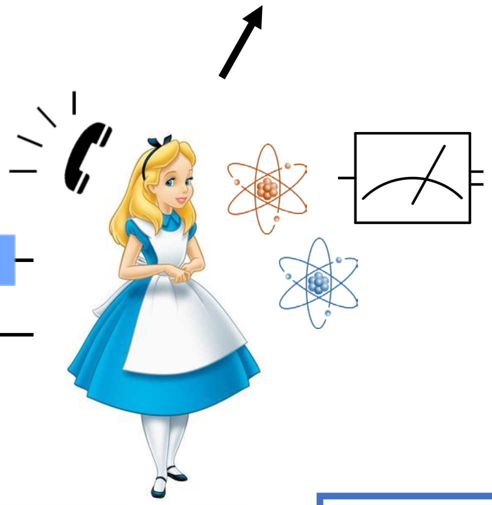
$$= \frac{1}{2}\left[\boxed{|00\rangle(\alpha|0\rangle + \beta|1\rangle)} + |01\rangle(\beta|0\rangle + \alpha|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle)\right]$$ ⟵ **quantum state after Alice’s computation**

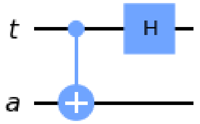
Quantum Teleportation: Bob's Computation

Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$







00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle$$ ✓ (Bob does nothing)

01

$$= \beta|010\rangle + \alpha|011\rangle \longrightarrow$$

10

11

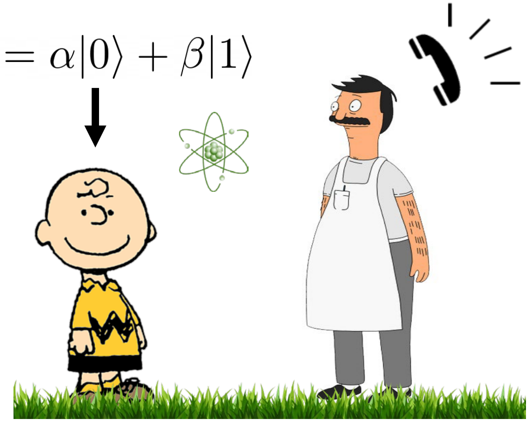
$$= \frac{1}{2}\left[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + \boxed{|01\rangle(\beta|0\rangle + \alpha|1\rangle)} + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle)\right] \longleftarrow$$

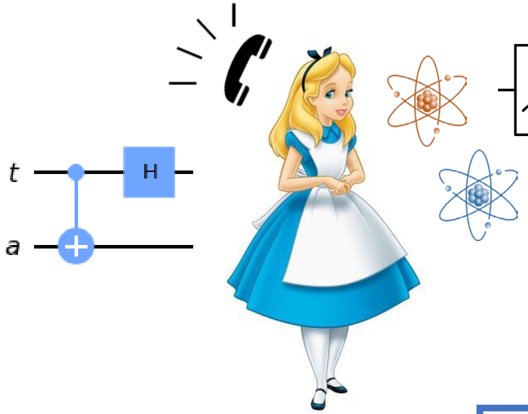
quantum state after Alice's computation

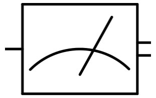
Quantum Teleportation: Bob's Computation

Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$







00 renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle$$ ✓ (Bob does nothing)

01 $= \beta|010\rangle + \alpha|011\rangle \longrightarrow$ b –[X]– $= \alpha|010\rangle + \beta|011\rangle$ ✓

10

11

$$= \frac{1}{2}\left[|00\rangle(\alpha|0\rangle+\beta|1\rangle) + \boxed{|01\rangle(\beta|0\rangle+\alpha|1\rangle)} + |10\rangle(\alpha|0\rangle-\beta|1\rangle) + |11\rangle(-\beta|0\rangle+\alpha|1\rangle)\right]$$ ⟵ **quantum state after Alice's computation**

Quantum Teleportation: Bob's Computation

Goal

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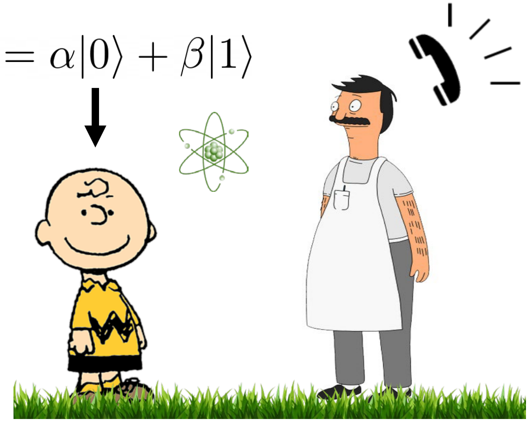
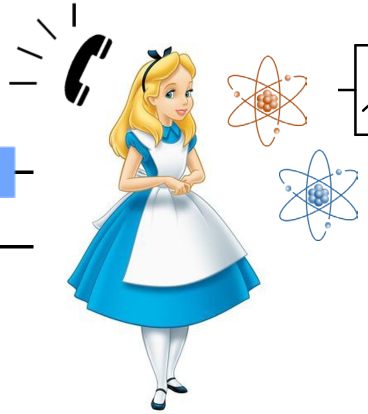
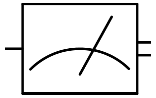
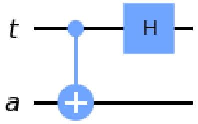








00

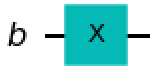
renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle$$ ✓ (Bob does nothing)

01

$$= \beta|010\rangle + \alpha|011\rangle \longrightarrow$$



$$= \alpha|010\rangle + \beta|011\rangle$$ ✓

10

$$= \alpha|100\rangle - \beta|101\rangle \longrightarrow$$

11

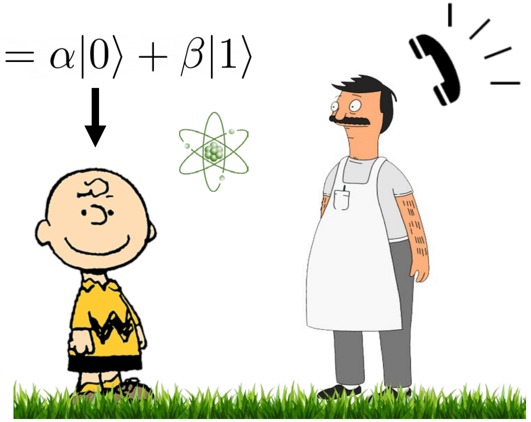
$$= \frac{1}{2}\left[|00\rangle(\alpha|0\rangle+\beta|1\rangle) + |01\rangle(\beta|0\rangle+\alpha|1\rangle) + \boxed{|10\rangle(\alpha|0\rangle-\beta|1\rangle)} + |11\rangle(-\beta|0\rangle+\alpha|1\rangle)\right] \longleftarrow$$

quantum state after Alice's computation

Quantum Teleportation: Bob’s Computation

Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$



00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle$$ ✓ (Bob does nothing)

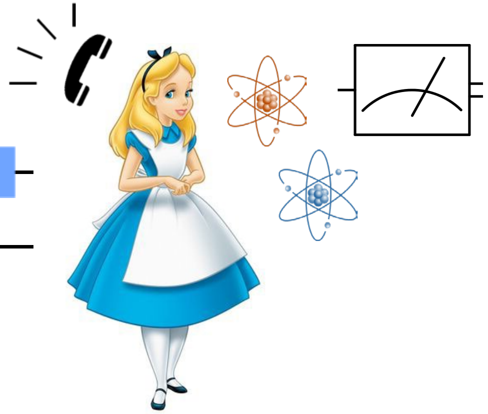
01

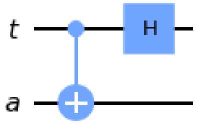
$$= \beta|010\rangle + \alpha|011\rangle \longrightarrow$$ b –[X]– $$= \alpha|010\rangle + \beta|011\rangle$$ ✓

10

$$= \alpha|100\rangle - \beta|101\rangle \longrightarrow$$ b –[Z]– $$= \alpha|100\rangle + \beta|101\rangle$$ ✓

11





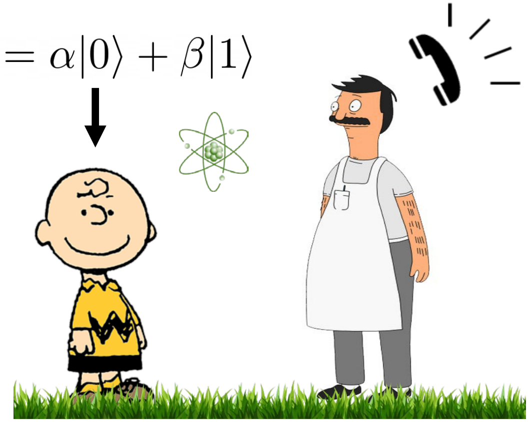
$$= \frac{1}{2}\Big[|00\rangle(\alpha|0\rangle+\beta|1\rangle) + |01\rangle(\beta|0\rangle+\alpha|1\rangle) + \boxed{|10\rangle(\alpha|0\rangle-\beta|1\rangle)} + |11\rangle(-\beta|0\rangle+\alpha|1\rangle)\Big] \longleftarrow$$

quantum state after Alice’s computation

Quantum Teleportation: Bob’s Computation

Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$



00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle \checkmark$$ (Bob does nothing)

01

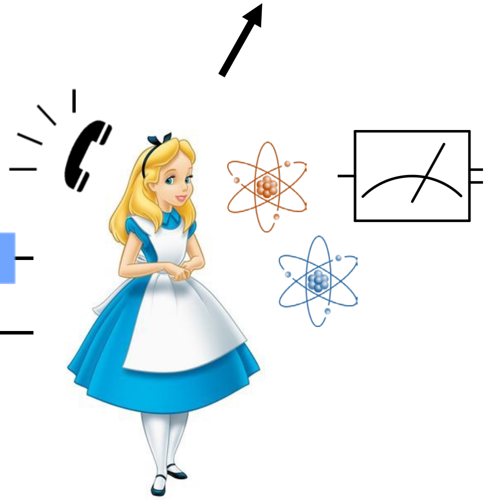
$$= \beta|010\rangle + \alpha|011\rangle \longrightarrow \quad b \text{ -[X]- } \quad = \alpha|010\rangle + \beta|011\rangle \checkmark$$

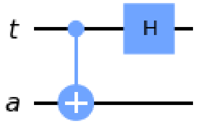
10

$$= \alpha|100\rangle - \beta|101\rangle \longrightarrow \quad b \text{ -[Z]- } \quad = \alpha|100\rangle + \beta|101\rangle \checkmark$$

11

$$= -\beta|110\rangle + \alpha|111\rangle \longrightarrow$$





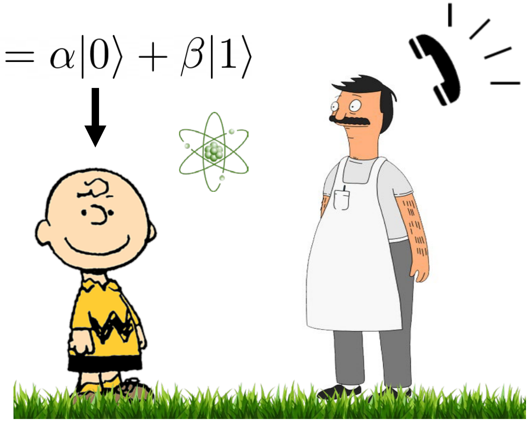
$$= \frac{1}{2}\left[|00\rangle(\alpha|0\rangle+\beta|1\rangle) + |01\rangle(\beta|0\rangle+\alpha|1\rangle) + |10\rangle(\alpha|0\rangle-\beta|1\rangle) + \boxed{|11\rangle(-\beta|0\rangle+\alpha|1\rangle)}\right]$$

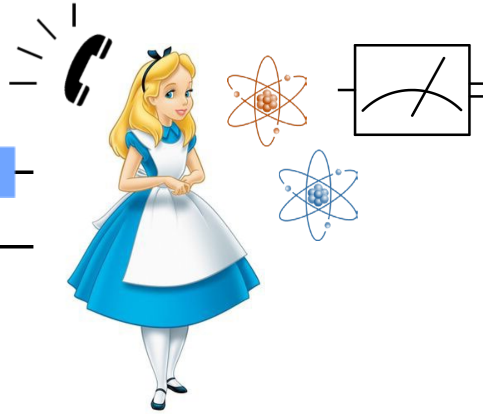
quantum state after Alice’s computation

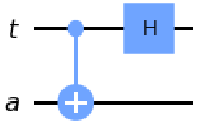
Quantum Teleportation: Bob's Computation

Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$







00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle$$ ✓ (Bob does nothing)

01

$$= \beta|010\rangle + \alpha|011\rangle \longrightarrow$$ b – X – $$= \alpha|010\rangle + \beta|011\rangle$$ ✓

10

$$= \alpha|100\rangle - \beta|101\rangle \longrightarrow$$ b – Z – $$= \alpha|100\rangle + \beta|101\rangle$$ ✓

11

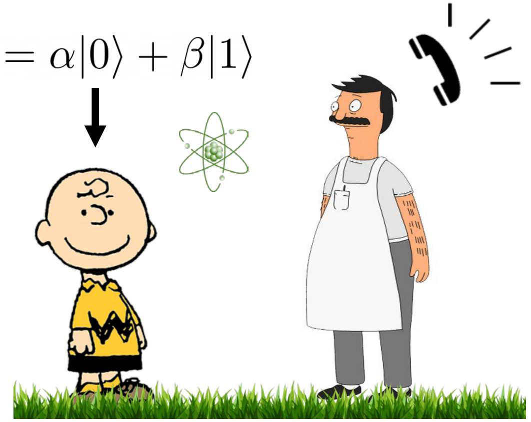
$$= -\beta|110\rangle + \alpha|111\rangle \longrightarrow$$ b – X – Z – $$= \alpha|110\rangle + \beta|111\rangle$$ ✓

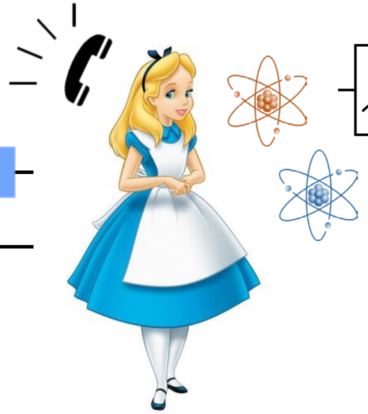
$$= \frac{1}{2}\left[|00\rangle(\alpha|0\rangle+\beta|1\rangle) + |01\rangle(\beta|0\rangle+\alpha|1\rangle) + |10\rangle(\alpha|0\rangle-\beta|1\rangle) + \boxed{|11\rangle(-\beta|0\rangle+\alpha|1\rangle)}\right]$$ ⟵ **quantum state after Alice's computation**

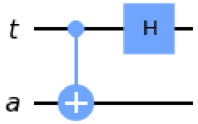
Quantum Teleportation: Bob's Computation

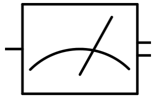
Goal

$|b\rangle = \alpha|0\rangle + \beta|1\rangle$









00

renormalization

$$\frac{\alpha/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|000\rangle + \frac{\beta/2}{\boxed{\sqrt{(\alpha/2)^2+(\beta/2)^2}}}|001\rangle \qquad = \sqrt{\frac{\alpha^2+\beta^2}{4}} = \frac{1}{2}$$

$$= \alpha|000\rangle + \beta|001\rangle$$ ✓ (Bob does nothing)

$$|b\rangle = \alpha|0\rangle + \beta|1\rangle$$

01

$$= \beta|010\rangle + \alpha|011\rangle \longrightarrow b \text{ -[X]- } = \alpha|010\rangle + \beta|011\rangle$$ ✓

10

$$= \alpha|100\rangle - \beta|101\rangle \longrightarrow b \text{ -[Z]- } = \alpha|100\rangle + \beta|101\rangle$$ ✓

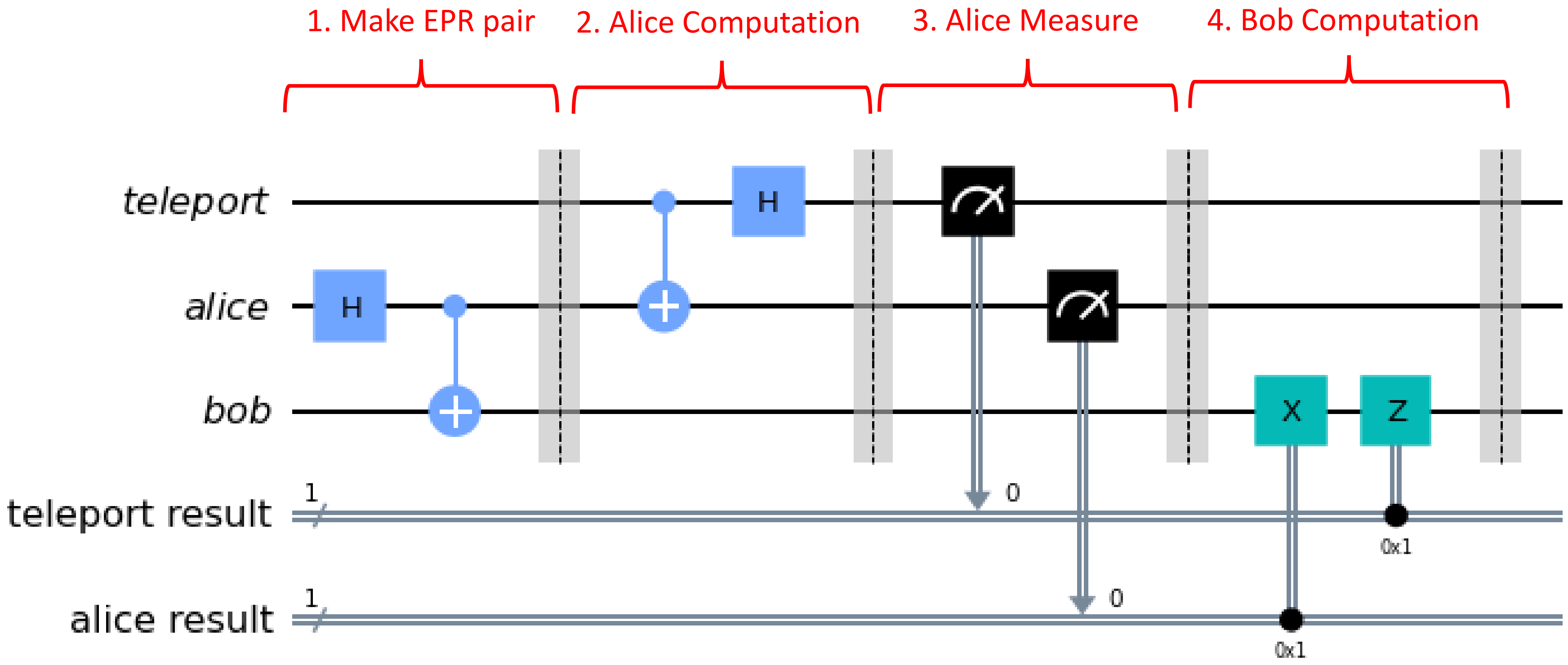
11

$$= -\beta|110\rangle + \alpha|111\rangle \longrightarrow b \text{ -[X]-[Z]- } = \alpha|110\rangle + \beta|111\rangle$$ ✓

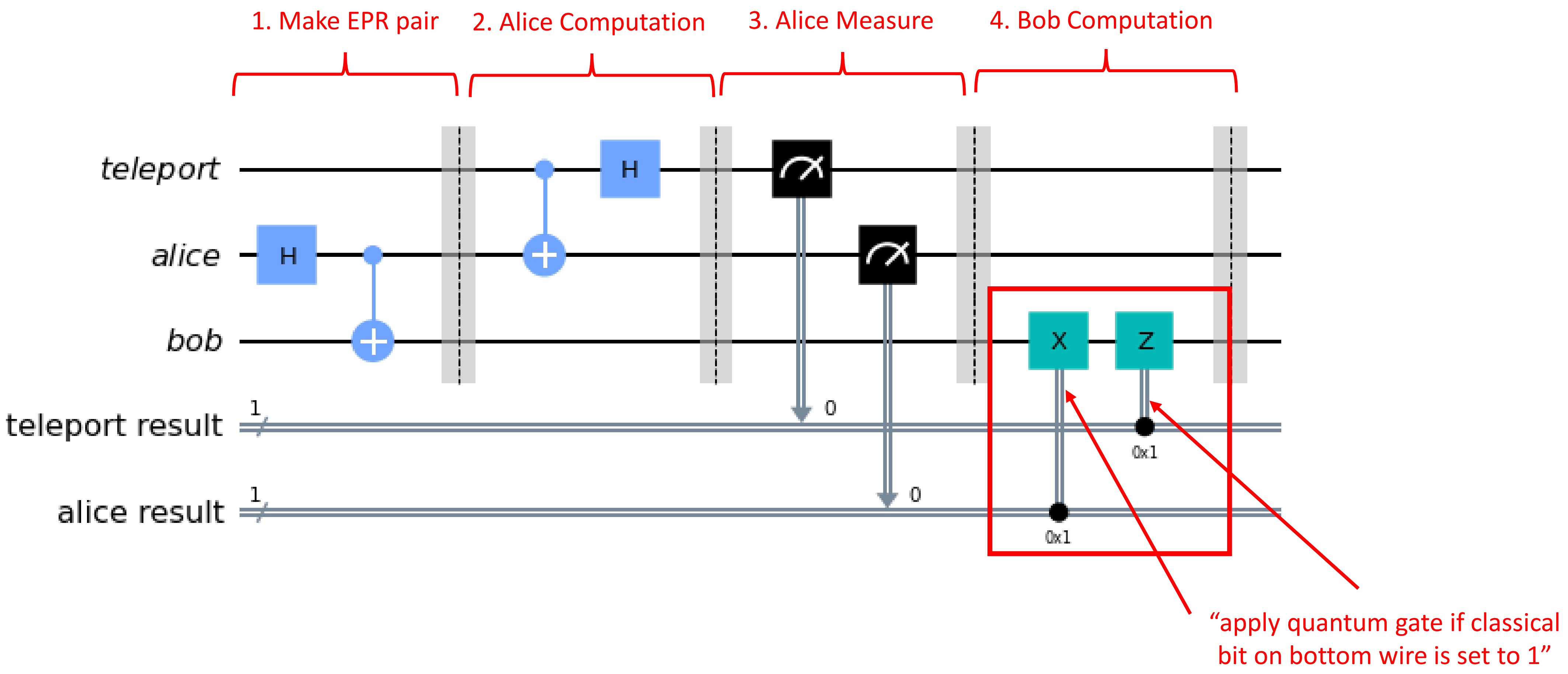
$$= \frac{1}{2}\Big[|00\rangle(\alpha|0\rangle+\beta|1\rangle) + |01\rangle(\beta|0\rangle+\alpha|1\rangle) + |10\rangle(\alpha|0\rangle-\beta|1\rangle) + \boxed{|11\rangle(-\beta|0\rangle+\alpha|1\rangle)}\Big] \longleftarrow$$

quantum state after Alice's computation

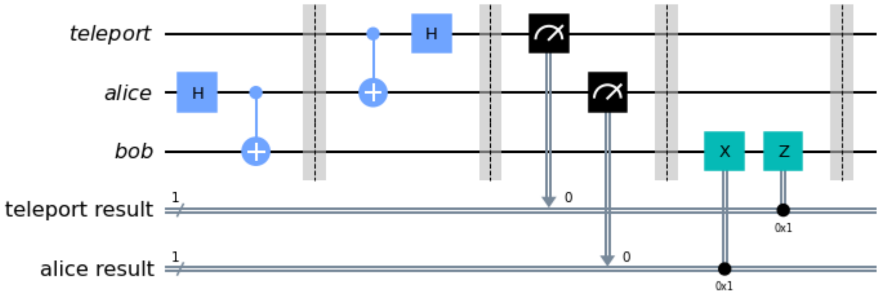
Overall Circuit



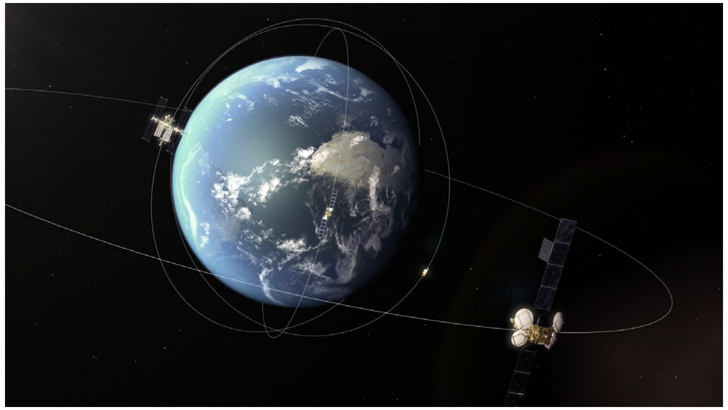
Overall Circuit

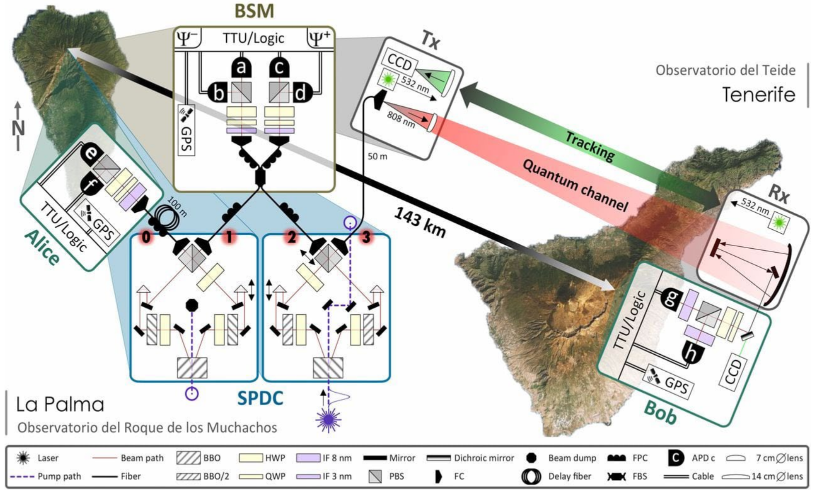


Experiments



First Proposed Theoretically in 1993 (Jozsa et. al)



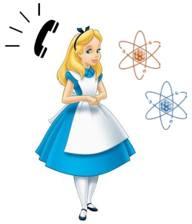


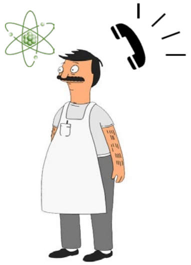
Canary Islands 2015 (143 km)

(courtesy of pnas.org)

First Experiments

Popescu and Zeilinger Groups 1997

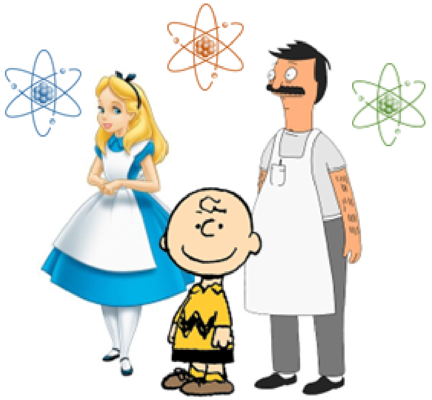




Danube River 2004 (600 meters)

Earth to Satellite 2017 (500 km)

Follow-up: Copying Qubits?



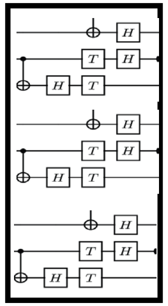
Goal: Copy quantum state of a qubit unto another.

qubit a

$\alpha|0\rangle + \beta|1\rangle \longrightarrow$

qubit b

$|0\rangle \longrightarrow$

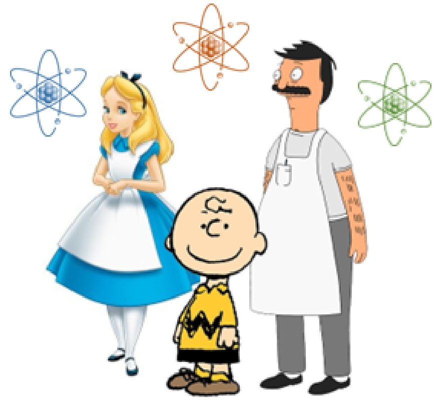


"Quantum Copying Circuit"

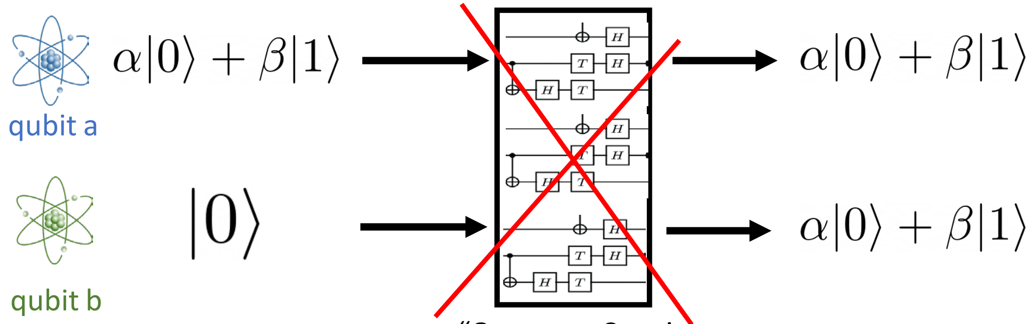
$\longrightarrow \alpha|0\rangle + \beta|1\rangle$

$\longrightarrow \alpha|0\rangle + \beta|1\rangle$

Follow-up: Copying Qubits?



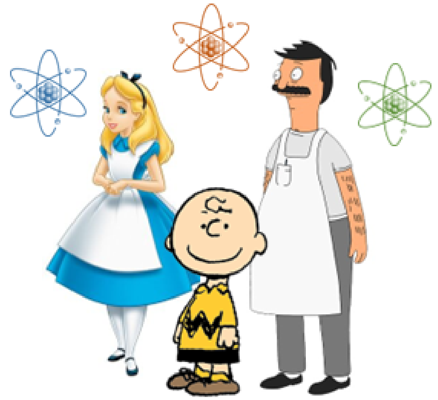
Goal: Copy quantum state of a qubit unto another.



"Quantum Copying Circuit"

No Cloning Theorem: Impossible to construct a quantum copying circuit (proof page 532 in NC).

Follow-up: Copying Qubits?



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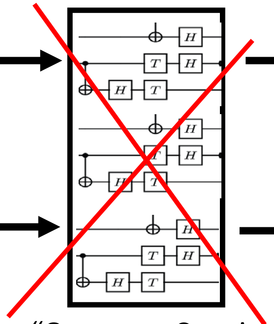


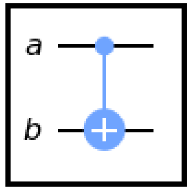
“Quantum Copying Circuit”

$\longrightarrow \alpha|0\rangle + \beta|1\rangle$

$\longrightarrow \alpha|0\rangle + \beta|1\rangle$

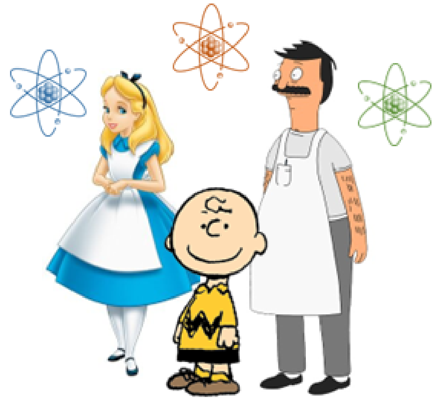
No Cloning Theorem: Impossible to construct a quantum copying circuit (proof page 532 in NC).

Tempting (but incorrect) Circuit: $\alpha|00\rangle + \beta|10\rangle \longrightarrow$



$\longrightarrow$

Follow-up: Copying Qubits?



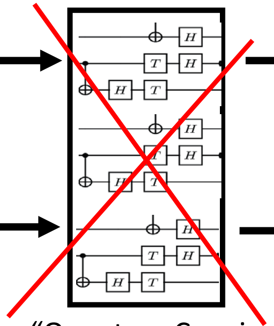
Goal: Copy quantum state of a qubit unto another.

qubit a

$\alpha|0\rangle + \beta|1\rangle \longrightarrow$

qubit b

$|0\rangle \longrightarrow$



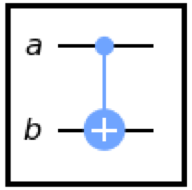
"Quantum Copying Circuit"

$\longrightarrow \alpha|0\rangle + \beta|1\rangle$

$\longrightarrow \alpha|0\rangle + \beta|1\rangle$

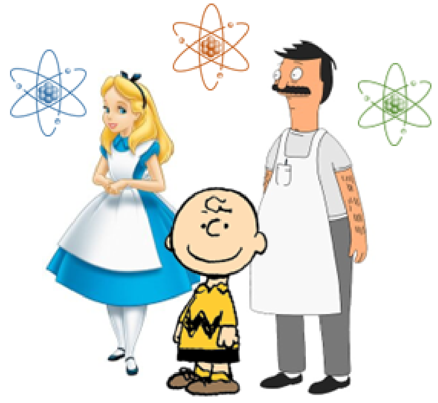
No Cloning Theorem: Impossible to construct a quantum copying circuit (proof page 532 in NC).

Tempting (but incorrect) Circuit: $\alpha|00\rangle + \beta|10\rangle \longrightarrow$



$\longrightarrow \alpha|00\rangle + \beta|11\rangle$

Follow-up: Copying Qubits?



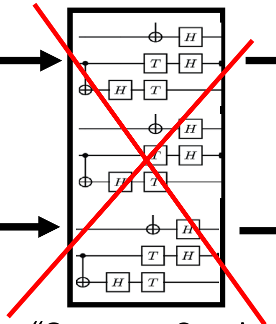
Goal: Copy quantum state of a qubit unto another.

qubit a

$\alpha|0\rangle + \beta|1\rangle$

qubit b

$|0\rangle$



"Quantum Copying Circuit"

$\alpha|0\rangle + \beta|1\rangle$

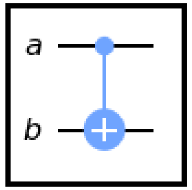
$\otimes$

$\alpha|0\rangle + \beta|1\rangle$

$= \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$

No Cloning Theorem: Impossible to construct a quantum copying circuit (proof page 532 in NC).

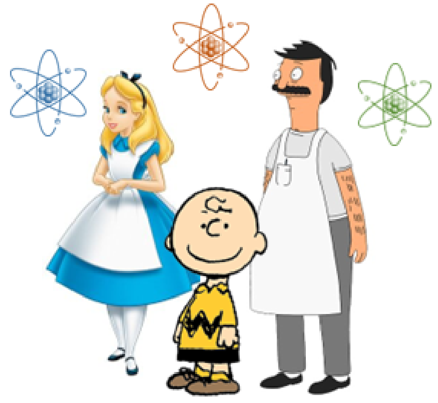
Tempting (but incorrect) Circuit: $\alpha|00\rangle + \beta|10\rangle$



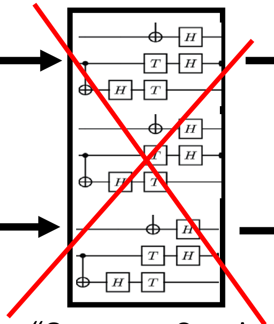
$\alpha|00\rangle + \beta|11\rangle \neq \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$

unless (alpha, beta) = (1, 0) or (0, 1)

Follow-up: Copying Qubits?



Goal: Copy quantum state of a qubit unto another.

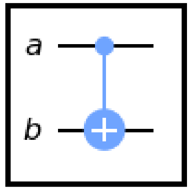
qubit a $\alpha|0\rangle + \beta|1\rangle$ →  → $\alpha|0\rangle + \beta|1\rangle$

$$\otimes \quad = \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$$

qubit b $|0\rangle$ → → $\alpha|0\rangle + \beta|1\rangle$

"Quantum Copying Circuit"

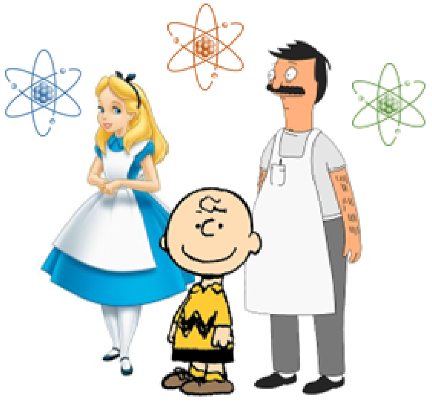
No Cloning Theorem: Impossible to construct a quantum copying circuit (proof page 532 in NC).

Tempting (but incorrect) Circuit: $\alpha|00\rangle + \beta|10\rangle$ →  → $\alpha|00\rangle + \beta|11\rangle \neq \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$

unless (alpha, beta) = (1, 0) or (0, 1)

Follow-up Question 1: But in QT circuit, aren't we essentially copying the teleportation qubit?

Follow-up: Copying Qubits?



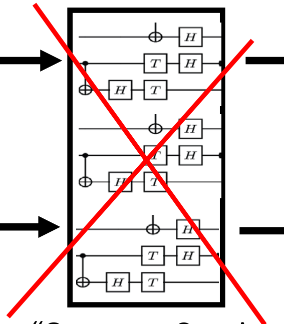
Goal: Copy quantum state of a qubit unto another.

qubit a

$\alpha|0\rangle + \beta|1\rangle \longrightarrow$

qubit b

$|0\rangle \longrightarrow$

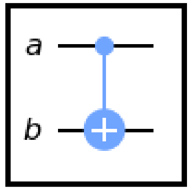


"Quantum Copying Circuit"

$$\longrightarrow \alpha|0\rangle + \beta|1\rangle \otimes \alpha|0\rangle + \beta|1\rangle = \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$$

No Cloning Theorem: Impossible to construct a quantum copying circuit (proof page 532 in NC).

Tempting (but incorrect) Circuit: $\alpha|00\rangle + \beta|10\rangle \longrightarrow$



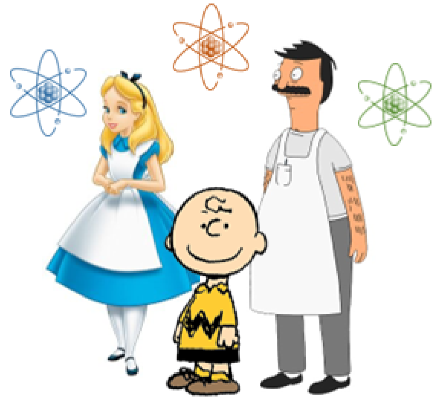
$$\longrightarrow \alpha|00\rangle + \beta|11\rangle \neq \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$$

unless (alpha, beta) = (1, 0) or (0, 1)

Follow-up Question 1: But in QT circuit, aren't we essentially copying the teleportation qubit?

No, because Alice must measure/destroy the original teleportation qubit in order to transfer its quantum state to Bob's qubit.

Follow-up: Copying Qubits?



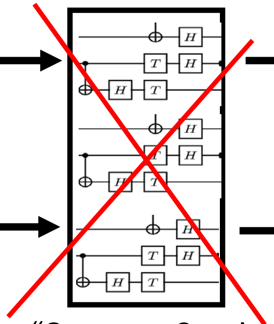
Goal: Copy quantum state of a qubit unto another.

qubit a

$\alpha|0\rangle + \beta|1\rangle$

qubit b

$|0\rangle$



"Quantum Copying Circuit"

$\alpha|0\rangle + \beta|1\rangle$

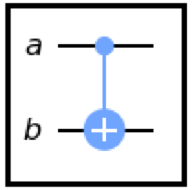
$\otimes$

$\alpha|0\rangle + \beta|1\rangle$

$= \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$

No Cloning Theorem: Impossible to construct a quantum copying circuit (proof page 532 in NC).

Tempting (but incorrect) Circuit: $\alpha|00\rangle + \beta|10\rangle \longrightarrow$



$\longrightarrow \alpha|00\rangle + \beta|11\rangle \neq \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$

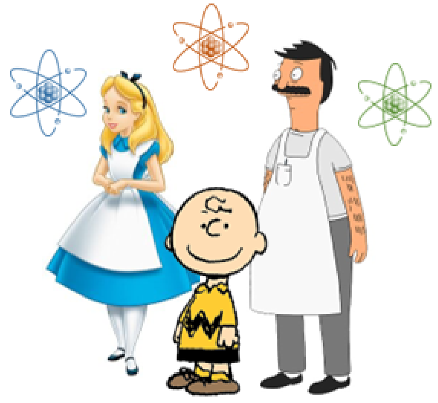
unless (alpha, beta) = (1, 0) or (0, 1)

Follow-up Question 1: But in QT circuit, aren't we essentially copying the teleportation qubit?

No, because Alice must measure/destroy the original teleportation qubit in order to transfer its quantum state to Bob's qubit.

Follow-up Question 2: … But isn't what distinguishes the quantum setting from the classical setting was that Alice doesn't need to "open the envelope" in the quantum setting?

Follow-up: Copying Qubits?



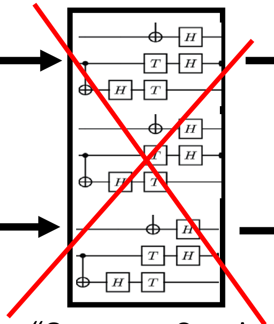
Goal: Copy quantum state of a qubit unto another.

qubit a

$\alpha|0\rangle + \beta|1\rangle \longrightarrow$

qubit b

$|0\rangle \longrightarrow$



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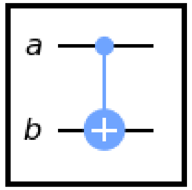

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Alice does need to in some sense "open the envelope" by measuring.
But quantum state of teleportation qubit remains unknown to all parties (Alice, Bob, and Charlie) instead of just Charlie.

Applications of Quantum Teleportation

Applications of Quantum Teleportation

Cryptography

Factoring Integers

Input: integer x.
Output: non-trivial factors of x.

x = 54 ⟶ 2, 3, 6, 9, 18, 27

Best Classical Algorithm: $O(2^n)$ for n bit numbers

Shor's Quantum Algorithm: O(poly(n))

Many cryptography schemes (e.g., RSA) rely on exponential runtime for the problem.

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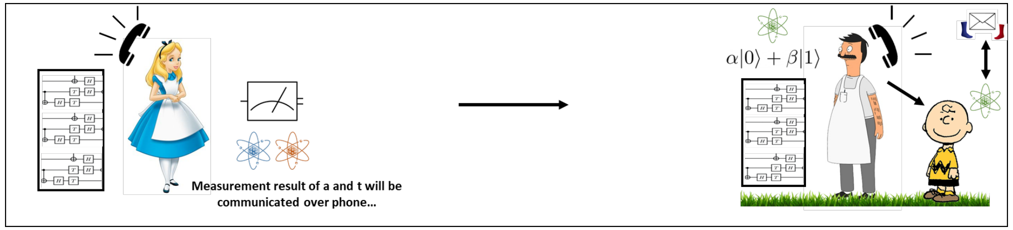

Quantum Key Distribution: Alice can transfer quantum information to Bob/Charlie without ever knowing its state.

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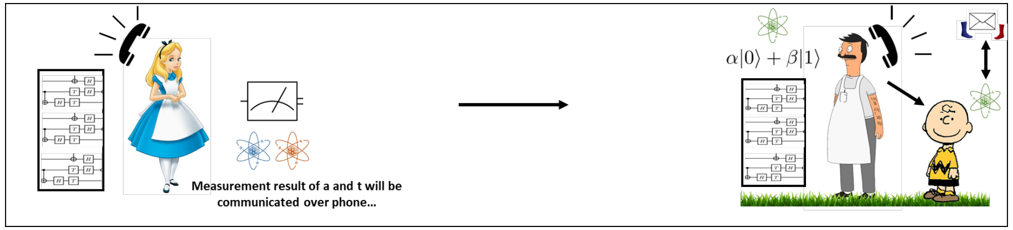

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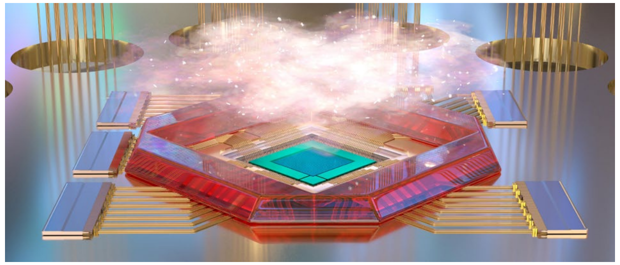
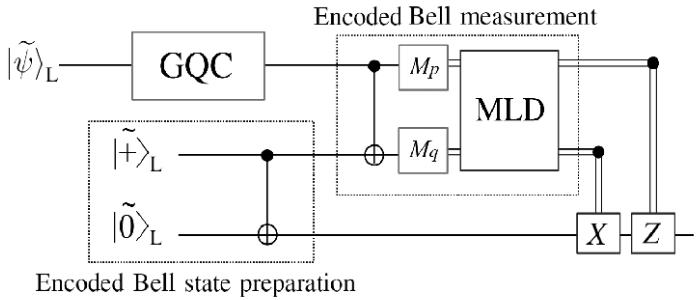
Quantum Error Correction





Similar circuits can be used to detect whether previous quantum gates did not perform operation correctly.