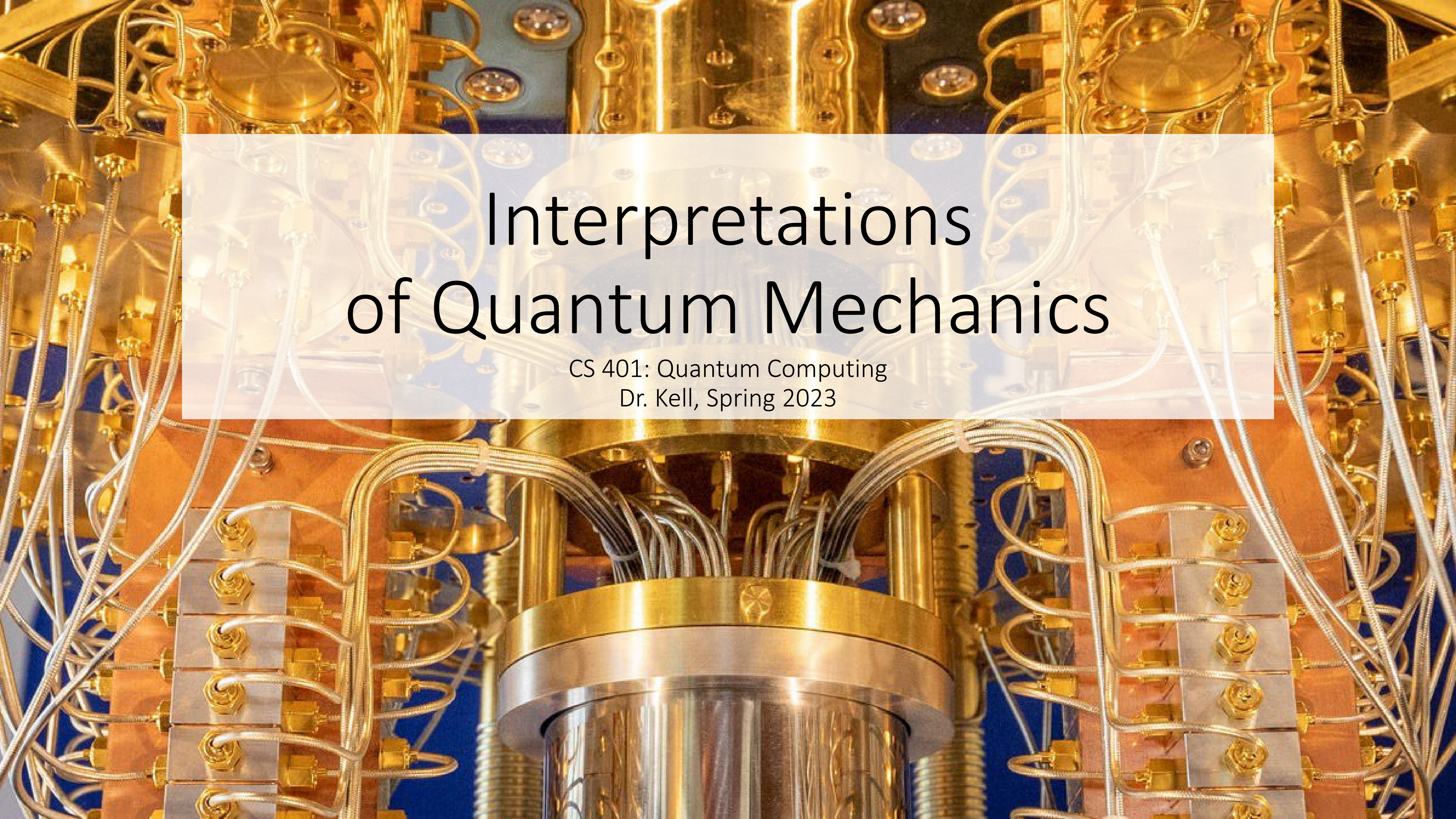
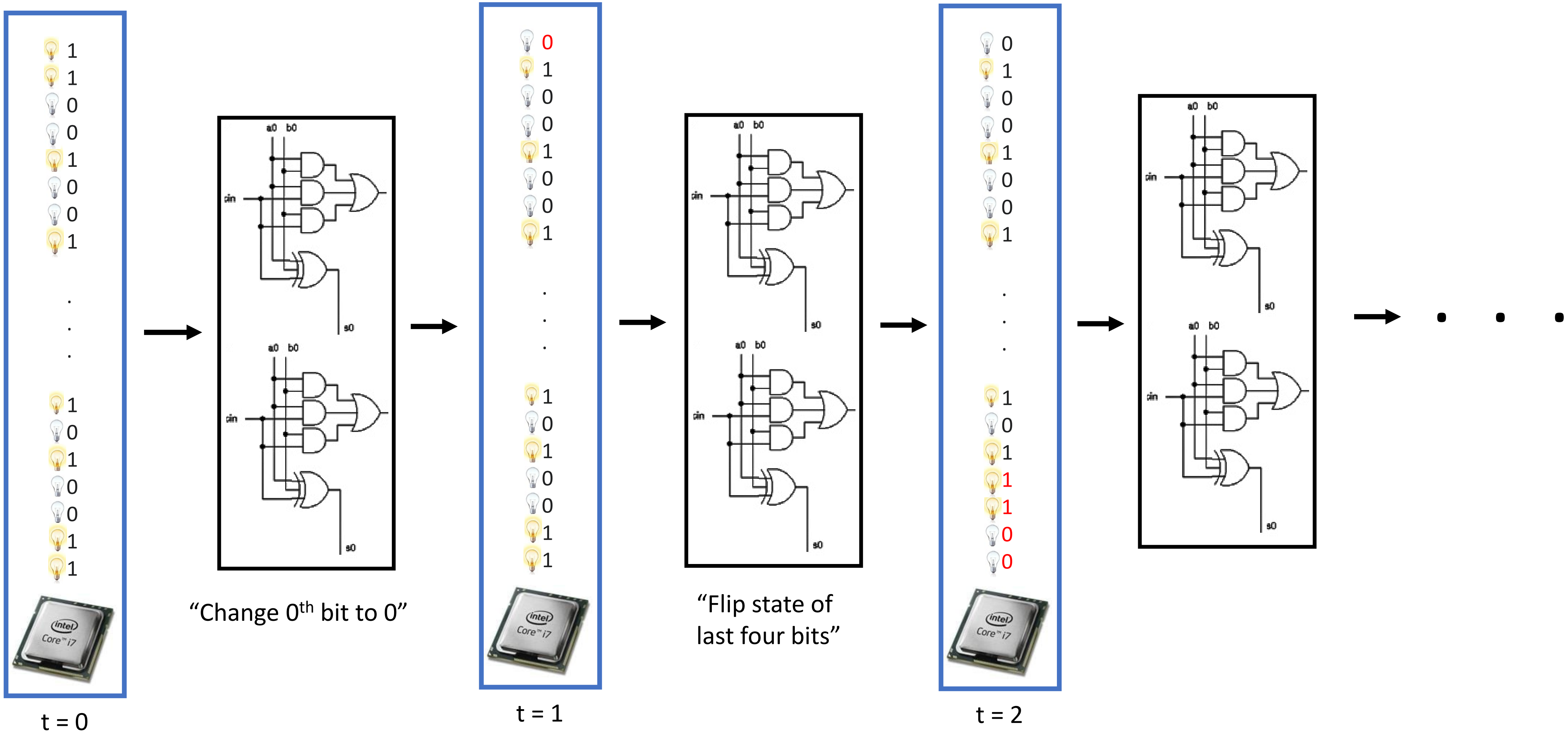


Interpretations of Quantum Mechanics

CS 401: Quantum Computing
Dr. Kell, Spring 2023

Review: Evolution of Classical versus Quantum Computers

Entire State of CPU

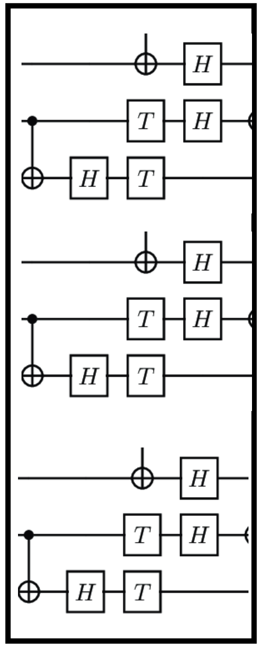


Review: Evolution of Classical versus Quantum Computers

State Quantum CPU

State	Prob
0 0 0 1	: 1
0 0 0 1	: 0
0 0 1 0	: 0
0 0 1 1	: 0
0 1 0 0	: 0
0 1 0 1	: 0
0 1 1 0	: 0
0 1 1 1	: 0
1 0 0 0	: 0
1 0 0 1	: 0
1 0 1 0	: 0
1 0 1 1	: 0
1 1 0 0	: 0
1 1 0 1	: 0
1 1 1 0	: 0
1 1 1 1	: 0

t = 0

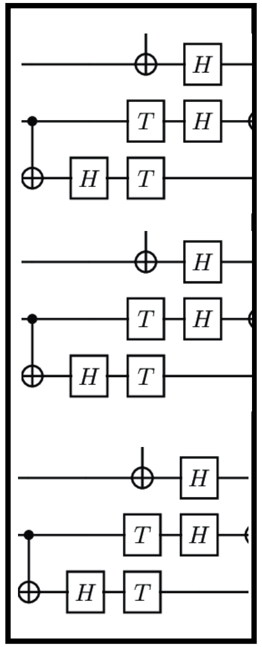


Quantum Logic Gates

State = probability distribution over all configurations

State	Prob
0 0 0 1	$\frac{1}{16}$
0 0 0 1	$\frac{1}{16}$
0 0 1 0	$\frac{1}{16}$
0 0 1 1	$\frac{1}{16}$
0 1 0 0	$\frac{1}{16}$
0 1 0 1	$\frac{1}{16}$
0 1 1 0	$\frac{1}{16}$
0 1 1 1	$\frac{1}{16}$
1 0 0 0	$\frac{1}{16}$
1 0 0 1	$\frac{1}{16}$
1 0 1 0	$\frac{1}{16}$
1 0 1 1	$\frac{1}{16}$
1 1 0 0	$\frac{1}{16}$
1 1 0 1	$\frac{1}{16}$
1 1 1 0	$\frac{1}{16}$
1 1 1 1	$\frac{1}{16}$

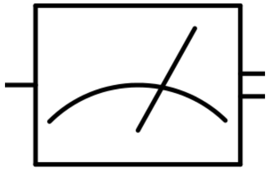
t = 1



Quantum Logic Gates

State	Prob
0 0 0 1	$\frac{1}{16}$
0 0 0 1	0
0 0 1 0	$\frac{1}{16}$
0 0 1 1	0
0 1 0 0	$\frac{1}{16}$
0 1 0 1	0
0 1 1 0	$\frac{1}{16}$
0 1 1 1	0
1 0 0 0	$\frac{1}{8}$
1 0 0 1	$\frac{1}{8}$
1 0 1 0	$\frac{1}{8}$
1 0 1 1	$\frac{1}{8}$
1 1 0 0	0
1 1 0 1	0
1 1 1 0	0
1 1 1 1	$\frac{1}{4}$

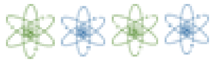
t = 2



Measure/Observe

(Picks state according to probabilities)

Single outcome



1 0 1 0

(had 1/8 chance)

… Well, not exactly

- Probabilities can actually be *negative* or *complex* numbers.
- What happens if we measure some atoms/qubits but not others?

Review: Evolution of Classical versus Quantum Computers

State Quantum CPU

State	Prob
0 0 0 1	: 1
0 0 0 1	: 0
0 0 1 0	: 0
0 0 1 1	: 0
0 1 0 0	: 0
0 1 0 1	: 0
0 1 1 0	: 0
0 1 1 1	: 0
1 0 0 0	: 0
1 0 0 1	: 0
1 0 1 0	: 0
1 0 1 1	: 0
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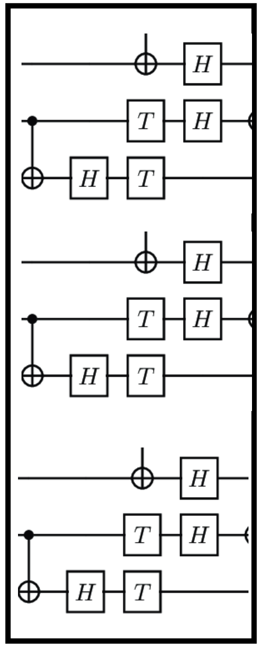
1 1 0 1	: 0
1 1 1 0	: 0
1 1 1 1	: 0

t = 0



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0 1 0 1	$\frac{1}{16}$
0 1 1 0	$\frac{1}{16}$
0 1 1 1	$\frac{1}{16}$
1 0 0 0	$\frac{1}{16}$
1 0 0 1	$\frac{1}{16}$
1 0 1 0	$\frac{1}{16}$
1 0 1 1	$\frac{1}{16}$
1 1 0 0	$\frac{1}{16}$
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1 1 1 0	$\frac{1}{16}$
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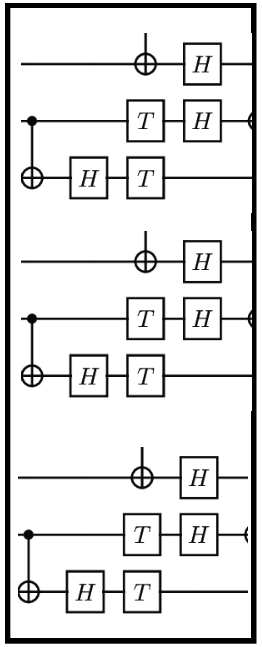
t = 1



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State	Prob
0 0 0 1	$\frac{1}{16}$
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0 0 1 0	$\frac{1}{16}$
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0 1 1 0	$\frac{1}{16}$
0 1 1 1	0
1 0 0 0	$\frac{1}{8}$
1 0 0 1	$\frac{1}{8}$
1 0 1 0	$\frac{1}{8}$
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1 1 0 0	0
1 1 0 1	0
1 1 1 0	0
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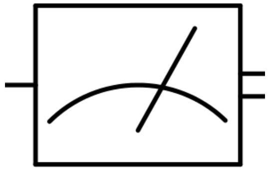

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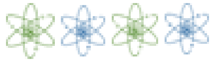

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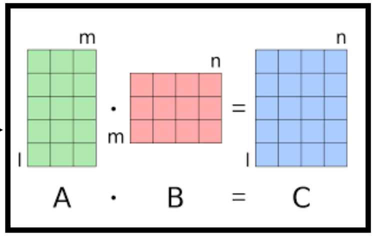
1 0 0 0	: 0
1 0 0 1	: 0
1 0 1 0	: 0
1 0 1 1	: 0
1 1 0 0	: 0
1 1 0 1	: 0
1 1 1 0	: 0
1 1 1 1	: 0

t = 0



Unitary Matrix Multiplication ("Unitary Evolution") Or

$$\Psi$$

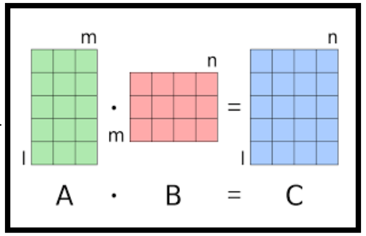
Evolution of the Wave Function (Psi)

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Schrodinger Equation

State	Prob
0 0 0 1	$\frac{1}{16}$
0 0 0 1	$\frac{1}{16}$
0 0 1 0	$\frac{1}{16}$
0 0 1 1	$\frac{1}{16}$
0 1 0 0	$\frac{1}{16}$
0 1 0 1	$\frac{1}{16}$
0 1 1 0	$\frac{1}{16}$
0 1 1 1	$\frac{1}{16}$
1 0 0 0	$\frac{1}{16}$
1 0 0 1	$\frac{1}{16}$
1 0 1 0	$\frac{1}{16}$
1 0 1 1	$\frac{1}{16}$
1 1 0 0	$\frac{1}{16}$
1 1 0 1	$\frac{1}{16}$
1 1 1 0	$\frac{1}{16}$
1 1 1 1	$\frac{1}{16}$

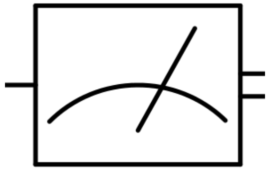
t = 1



$$\begin{bmatrix} \frac{1}{16} \\ 0 \\ \frac{1}{16} \\ 0 \\ \frac{1}{16} \\ 0 \\ \frac{1}{16} \\ 0 \\ \frac{1}{8} \\ \frac{1}{8} \\ \frac{1}{8} \\ \frac{1}{8} \\ 0 \\ 0 \\ 0 \\ \frac{1}{4} \end{bmatrix}$$

t = 2

16 dim vector (16x1 matrix) in "Hilbert space"



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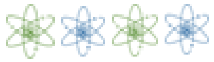

1 0 1 0

(had 1/8 chance)

… Well, not exactly

- Probabilities can actually be *negative* or *complex* numbers.
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Last Lecture's Takeaways

Key Takeaways

- **Quantum computers can be faster** because they get to manipulate and exponential amount of information in O(1) time.

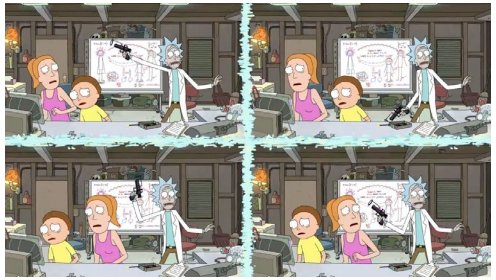
- **Quantum computers are not 100% superior** nor solve all hard problems trivially because:

    - The rules for how exponential number probabilities are updated are constrained to certain operations.

    - Even though we get to manipulate an exponential number of probabilities, we only see one outcome at the end.

Question: How does "Nature" keep track of and update so much information so quickly?

- **Answer:** Nobody knows.
- Three categories/types of answers:
    1. Who cares? Quantum physics works why ask the question?
    2. Quantum mechanics doesn't make sense, thus needs fixed.
    3. We live in a multiverse which interact *(many-worlds interpretation).*



“Sheet of Paper for each probability”

State Quantum CPU

State	Prob
0 0 0 1	: 1
0 0 0 1	: 0
0 0 1 0	: 0
0 0 1 1	: 0
0 1 0 0	: 0
0 1 0 1	: 0
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1 0 0 0	: 0
1 0 0 1	: 0
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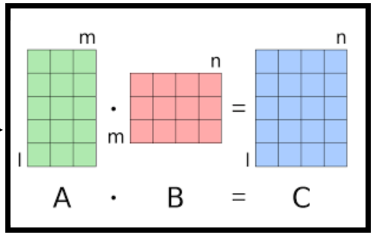
1 1 0 0	: 0
1 1 0 1	: 0
1 1 1 0	: 0
1 1 1 1	: 0

t = 0



Unitary Matrix Multiplication
(“Unitary Evolution”)

State	Prob
0 0 0 1	$\frac{1}{16}$
0 0 0 1	$\frac{1}{16}$
0 0 1 0	$\frac{1}{16}$
0 0 1 1	$\frac{1}{16}$
0 1 0 0	$\frac{1}{16}$
0 1 0 1	$\frac{1}{16}$
0 1 1 0	$\frac{1}{16}$
0 1 1 1	$\frac{1}{16}$
1 0 0 0	$\frac{1}{16}$
1 0 0 1	$\frac{1}{16}$
1 0 1 0	$\frac{1}{16}$
1 0 1 1	$\frac{1}{16}$
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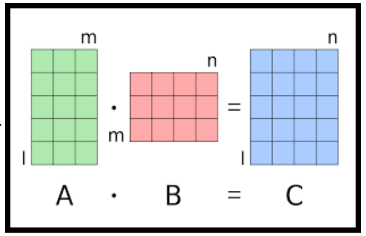
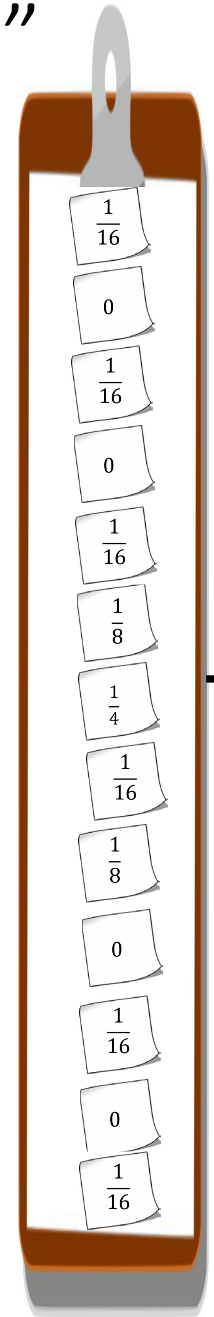
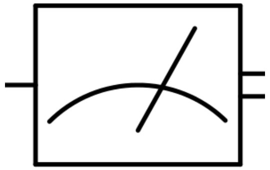
1 1 1 0	$\frac{1}{16}$
1 1 1 1	$\frac{1}{16}$

t = 1







Measure/Observe
(Picks state according to probabilities)

Single outcome

1 0 1 0

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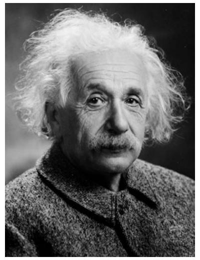
Don’t Feel Bad if this Seems Super Confusing

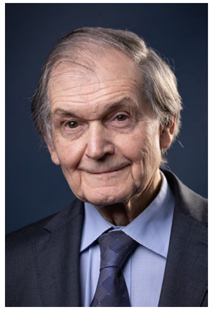
“If you think you understand quantum mechanics, you don’t understand quantum mechanics.”

- Richard Feynman



“All my attempts to adapt the theoretical foundation of physics to Quantum Theory failed completely. It was as if the ground had been pulled out from under one, with no firm foundation to be seen anywhere, upon which one could have built.”

- Albert Einstein



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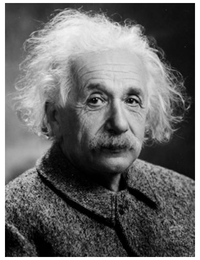

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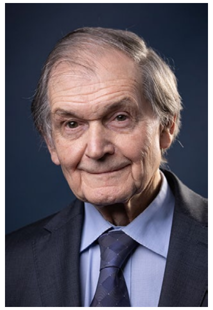

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… so pretty much everyone:

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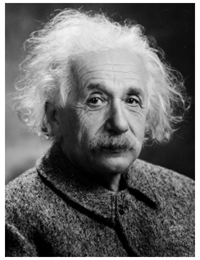

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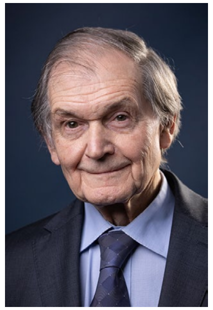

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- Roger Penrose

“In mathematics, you don’t understand things, you just get use to them.”

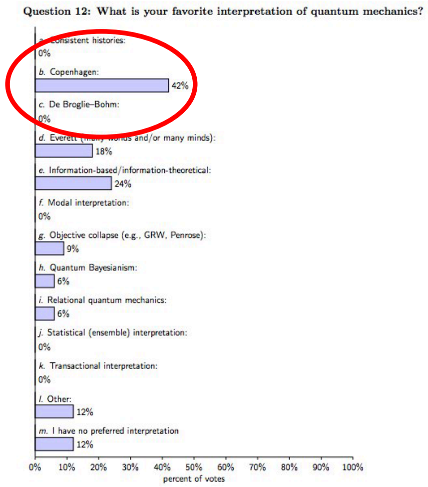
- John von Neumann

… so pretty much everyone:

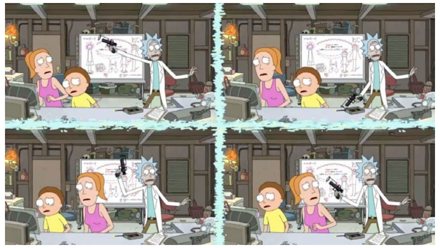
More Detail: “Interpretations Camps” of QM

Camp 1:
Who cares? Quantum mechanics works so why worry?

Plurality of physicist



Camp 3:
Many worlds interpretation



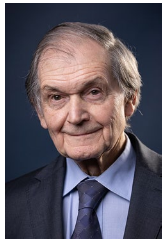
Everett

Deutsch

Carroll

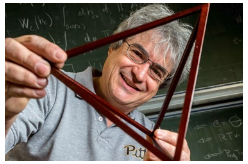
Tegmark

Camp 4:
Quantum mechanics needs modified
(or no current explanation is good)



Penrose

Smolin



Rovelli

Hossenfelder

Aaronson

Westmoreland

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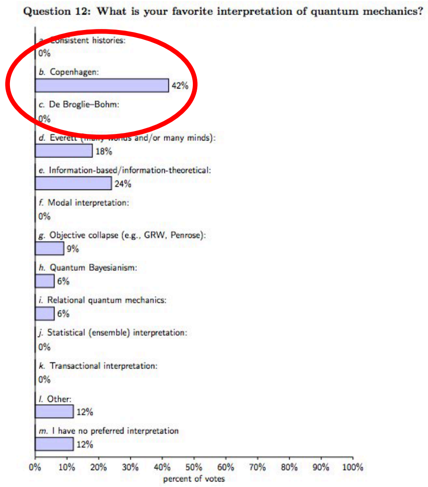

Camp 2:
Science is not concerned with what is “real.” In fact, QM implies Nature does not permit this.

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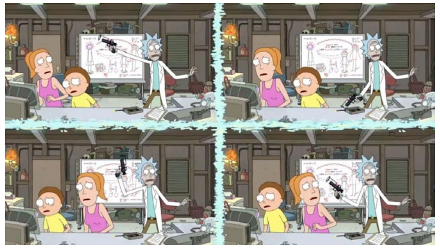

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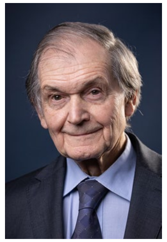

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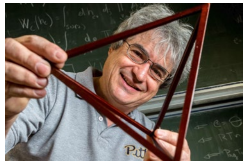

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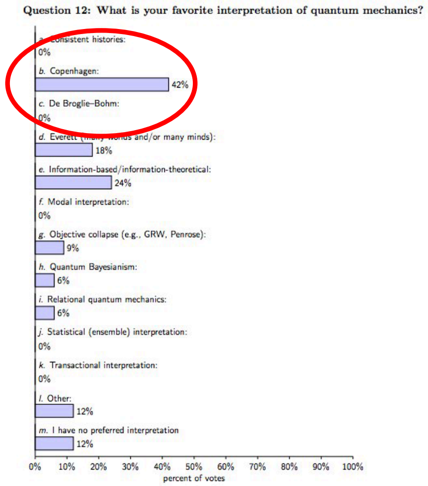

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Bohr

Heisenberg

Von Neumann

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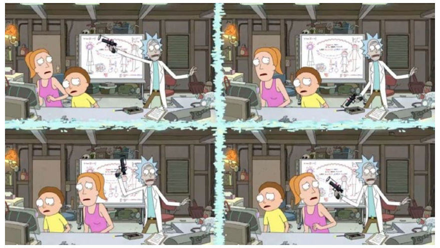

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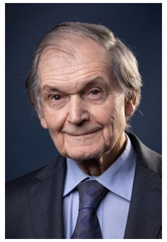

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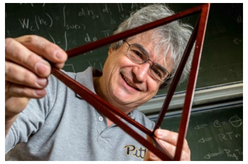

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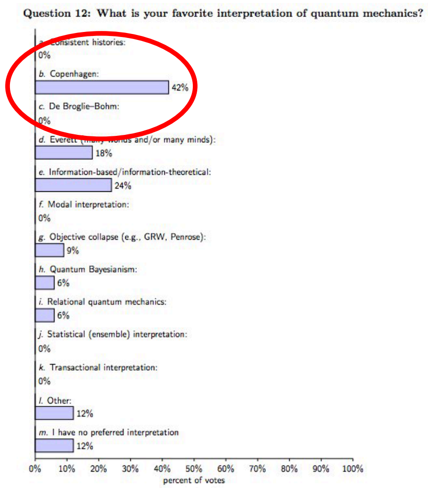

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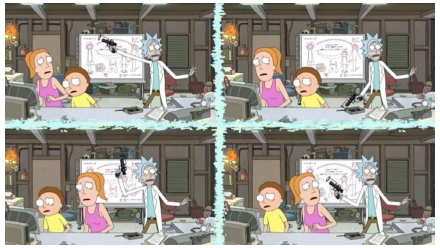

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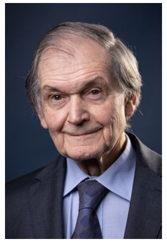

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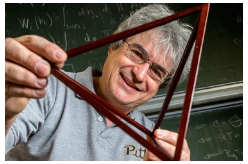

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Anti-Realism (epistemic) ← | → **Realism**

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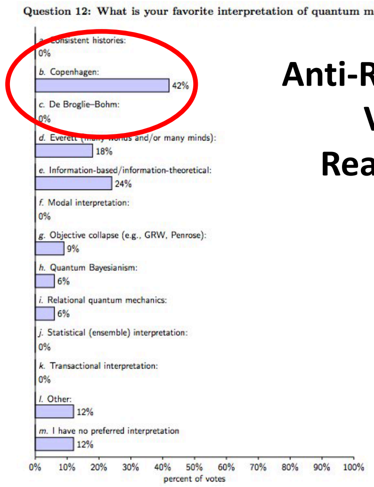


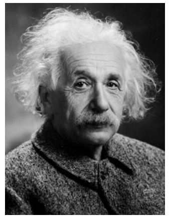
Anti-Realism
Vs
Realism

Camp 2:
Science is not concerned with what is "real." In fact, QM implies Nature does not permit this.


Bohr


Heisenberg

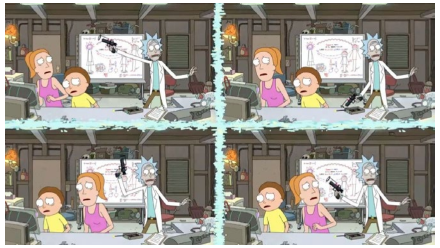

Von Neumann


Einstein


Hermann


Schrodinger

Camp 3:
Many worlds interpretation



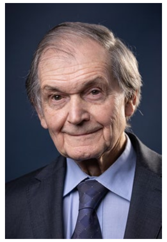

Everett


Deutsch

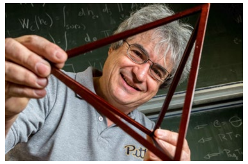

Carroll


Tegmark

Camp 4:
Quantum mechanics needs modified (or no current explanation is good)


Penrose


Smolin


Rovelli


Hossenfelder


Aaronson

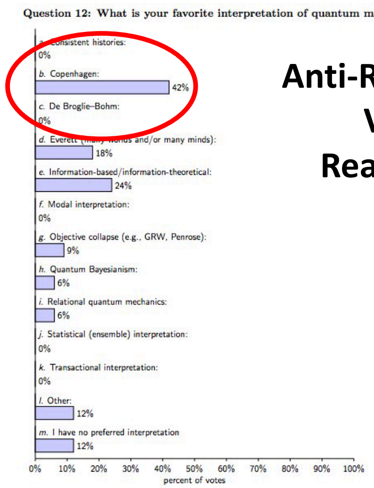

Westmoreland

More Detail: "Interpretations Camps" of QM

Anti-Realism (epistemic) ← | → **Realism**

Camp 1:
Who cares? Quantum mechanics works so why worry?

Plurality of physicist



Anti-Realism
Vs
Realism

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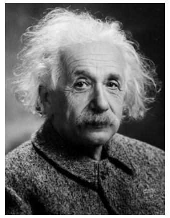
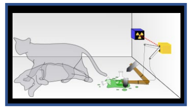


Einstein

You're wrong!

Hermann

Schrodinger



Schrodinger's Cat

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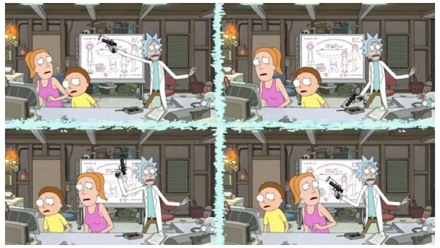

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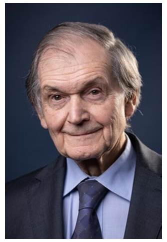

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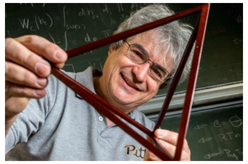

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Hossenfelder

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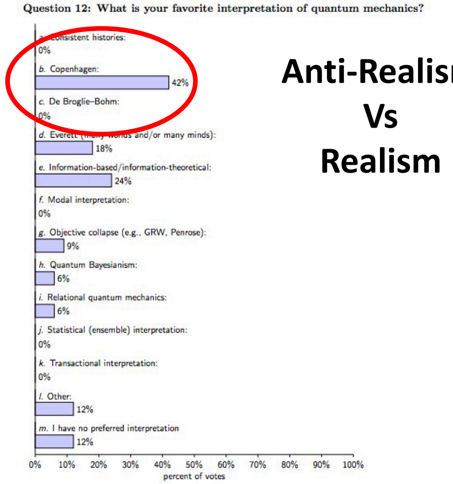

Anti-Realism Vs Realism

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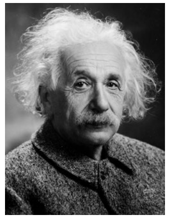

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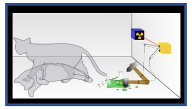
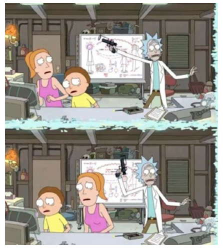


Schrodinger’s Cat

Can
Many worlds



Everett

Carroll

entropy

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Article

Interpretation of Quantum Theory: The Quantum “Grue-Bleen” Problem

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Abstract: We present a critique of the many-world interpretation of quantum mechanics, based on different “pictures” that describe the time evolution of an isolated quantum system. Without an externally imposed frame to restrict these possible pictures, the theory cannot yield non-trivial interpretational statements. This is analogous to Goodman’s famous “grue-bleen” problem of language and induction. Using a general framework applicable to many kinds of dynamical theories, we try to identify the kind of additional structure (if any) required for the meaningful interpretation of a theory. We find that the “grue-bleen” problem is not restricted to quantum mechanics, but also affects other theories including classical Hamiltonian mechanics. For all such theories, absent external frame information, an isolated system has no interpretation.

Keywords: interpretation of quantum mechanics; quantum foundations; many-worlds interpretation

check for updates

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1. Introduction

1. The Many-Worlds Interpretation

Any critique of the many-worlds interpretation of quantum mechanics ought to begin by praising it. In the simplest form of the interpretation, such as that presented by Everett in 1957 [1,2], the universe is regarded as a closed quantum system. Its state vector (Everett’s “universal wave function”) evolves unitarily according to an internal Hamiltonian. Measurements and the emergence of classical phenomena are described entirely by this evolution. “Observables” are simply dynamical variables described by operators. No separate “measurement process” or “wave function collapse” ideas are invoked.

Thus, consider a laboratory measurement of S_z on a spin-1/2 particle. This is nothing more than an interaction among the particle, the lab apparatus, and the conscious observer, all of which are subsystems of the overall quantum universe. Initially, the particle is in the state $|\psi_0\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$. The apparatus and the observer are in initial states $|0\rangle$ and $|\text{“ready”}\rangle$, respectively. Now the particle and the apparatus interact and become correlated:

$$|\psi_0\rangle \otimes |0\rangle \otimes |\text{“ready”}\rangle \longrightarrow \left(\alpha|\uparrow\rangle \otimes \left|+\tfrac{\hbar}{2}\right\rangle + \beta|\downarrow\rangle \otimes \left|-\tfrac{\hbar}{2}\right\rangle\right) \otimes |\text{“ready”}\rangle. \quad (1)$$

where $\left|+\frac{\hbar}{2}\right\rangle$ and $\left|-\frac{\hbar}{2}\right\rangle$ are apparatus states representing the two possible measurement results. The observer next interacts with the apparatus by reading its output, leading to a final state

$$\alpha|\uparrow\rangle \otimes \left|+\tfrac{\hbar}{2}\right\rangle \otimes |\text{“up”}\rangle + \beta|\downarrow\rangle \otimes \left|-\tfrac{\hbar}{2}\right\rangle \otimes |\text{“down”}\rangle. \quad (2)$$

The memory record of the observer (“up” or “down”) has become correlated to both the original spin and the reading on the apparatus. The two components of the superposition in Equation (2) are called “branches” or “worlds”. Since all subsequent evolution of the system is linear, the branches effectively evolve independently. The observer can condition predictions of the future behavior of the particle on his own memory record—for

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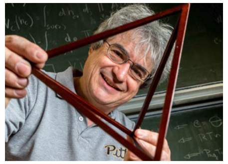

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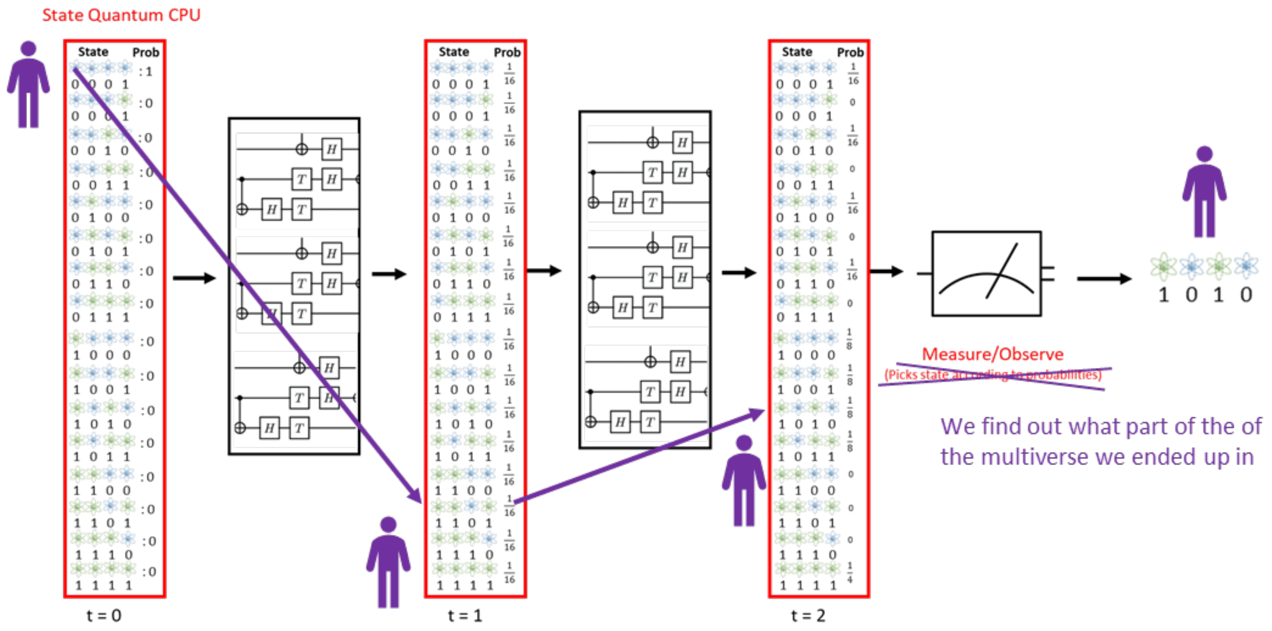
Overview of Many Worlds Interpretation



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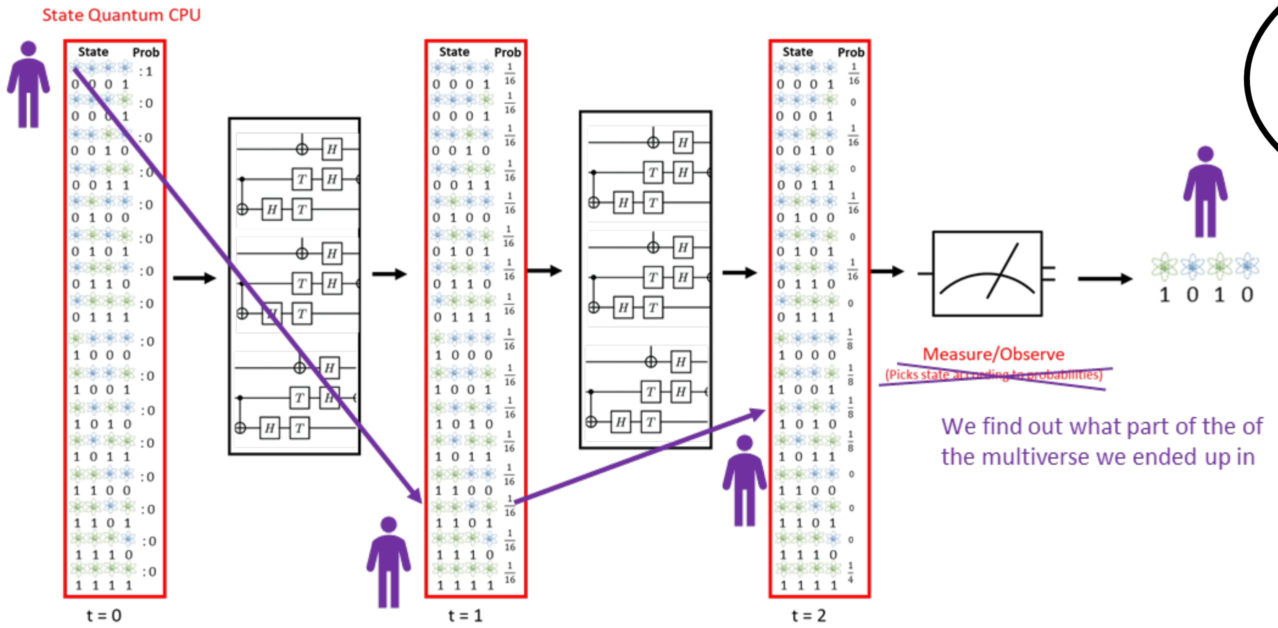

The earth orbits the sun, not the other way around.
This model gives a cleaner explanation and more closely matches observation.

We agree the model is useful for predicting things. But certainly, it's crazy to think the earth is *actually* moving around the sun.

Galileo

Catholic Church

Historical Analogy (circa 1600s)

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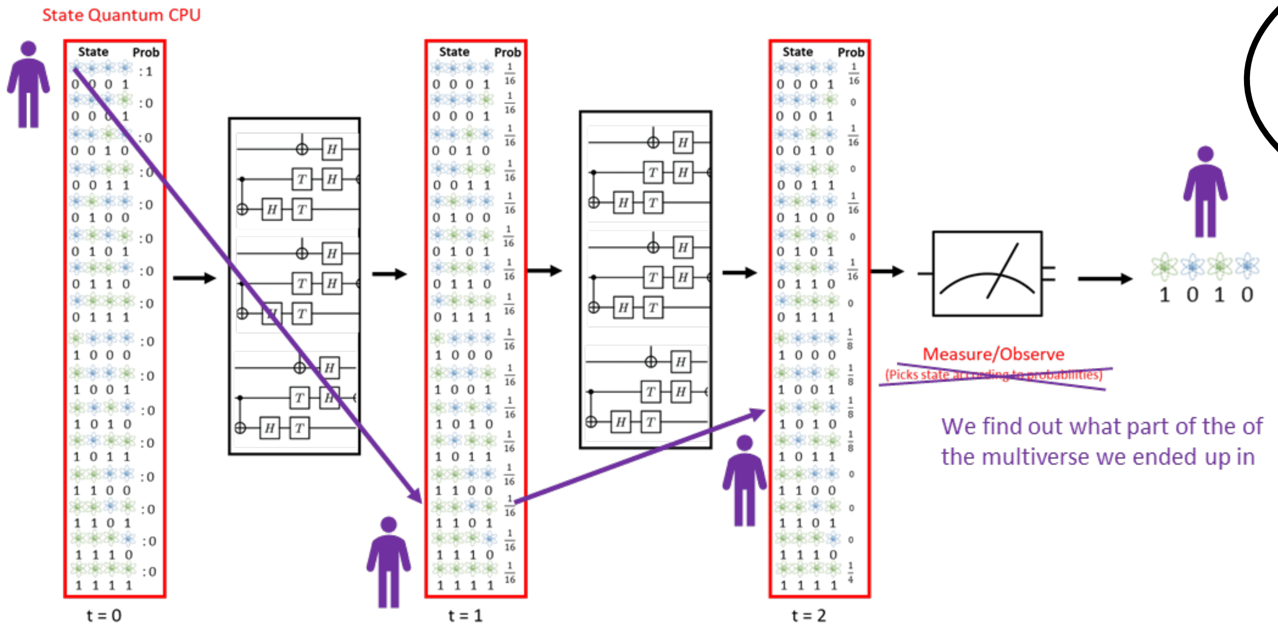

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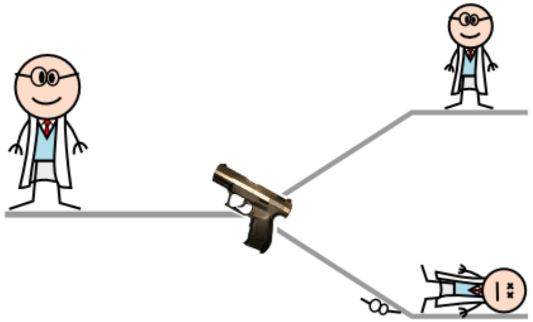
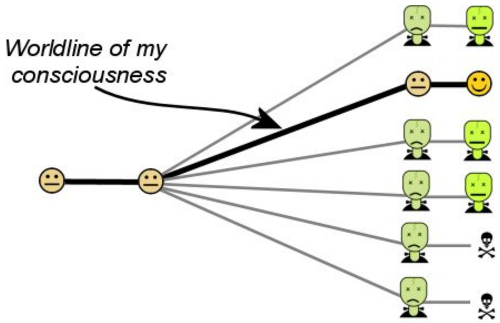
Question: Is it possible to test this with an experiment?

Yes!
(or maybe)

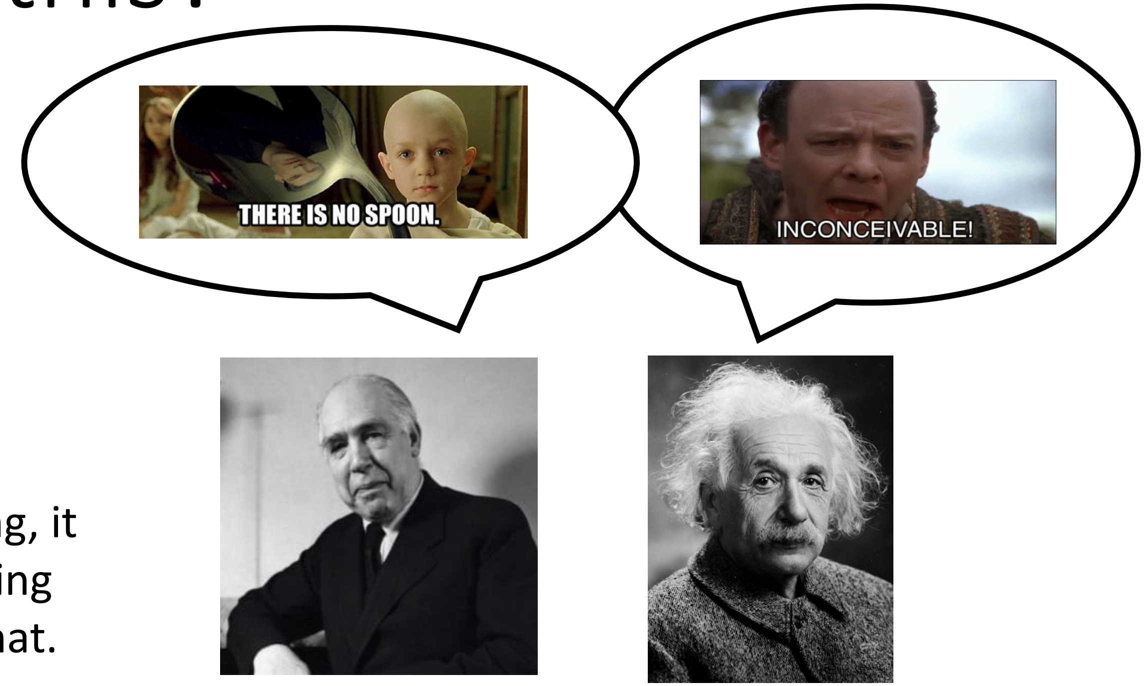




"Quantum Suicide" Experiment

Conclusions: Why talk about this?

- It's fun! (Hopefully, you found some of it interesting.)
- If you're confused on about what's happening in a quantum system/computer, you are not alone.
- Relating to our information conversation: If anti-realism is wrong, it seems like there must be kind of physical medium which is storing and transferring an exponential information, but it's unclear what.



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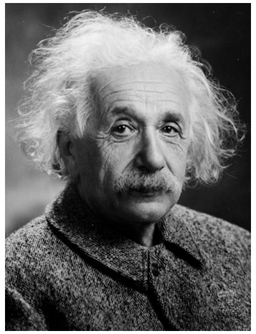

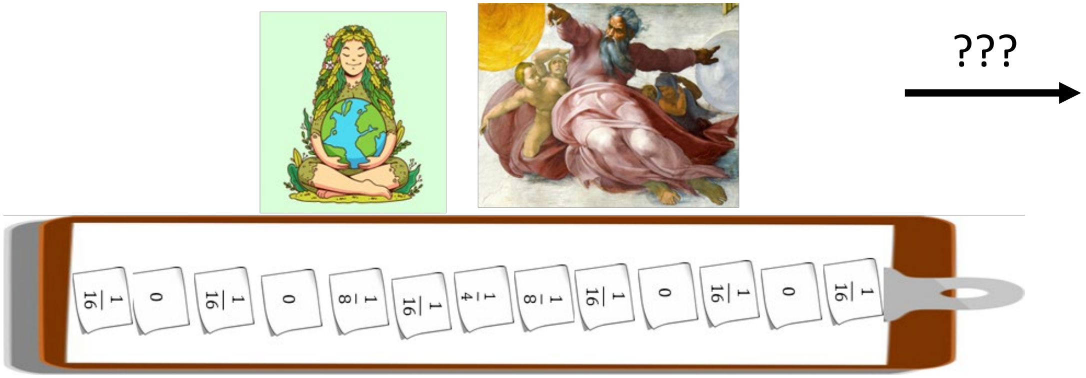

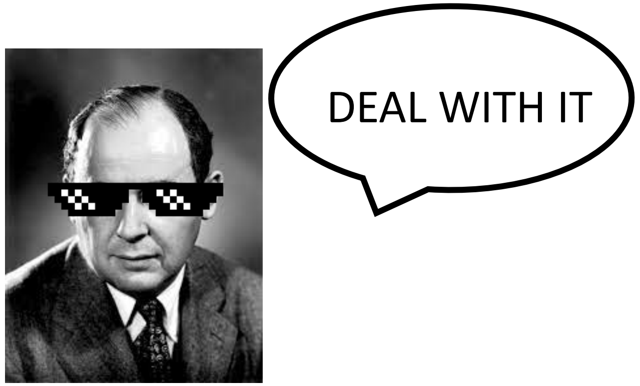

Canonical Problems with Quantum Advantage

Problem 1: Factoring Integers

Input: integer x.
Output: non-trivial factors of x.

x = 54 ⟶ 2, 3, 6, 9, 18, 27

Best Classical Algorithm: O(2^n) for n bit numbers

Shor's Quantum Algorithm: O(poly(n))

Many cryptography schemes (e.g., RSA) rely on exponential runtime for the problem.

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Problem 2: Search Problem

Input: list L, target value
Output: index of target in L

L = [2, 1, 10, 4, 7, 9, 3] ⟶ 4
target = 7 (index of 7)

Many applications in cloud quantum computing, databases, etc.

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Best Possible Classical Algorithm: O(n)

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Grover's Quantum Algorithm: O($n^{1/2}$)