Names: ____________________ ____________________ ____________________

Activity #1: Some Geometry (Solutions) **College Algebra**

1. Find an equation for the line passing through the point $(5, 2)$ and having slope $2/5$.

   Solution: Remember that to uniquely identify a line in the plane, we need two pieces of information. In this case we know two things about this line: its slope, and a point on the line. The simplest linear equation form to use here is the point-slope form: the line with slope m and passing through the point (h, k) is given by the equation
   $$\frac{y-k}{x-h} = m.$$
   Here we have $m = 2/5$ and $(h, k) = (5, 2)$. So this line is given by the equation
   $$\frac{y-2}{x-5} = \frac{2}{5}.$$
   We can solve this equation for y to find the slope-intercept form; this yields
   $$\boxed{y = \frac{2}{5}x}$$
   .

2. Find the slope between the points $(3, 4)$ and $(-2, 4)$.

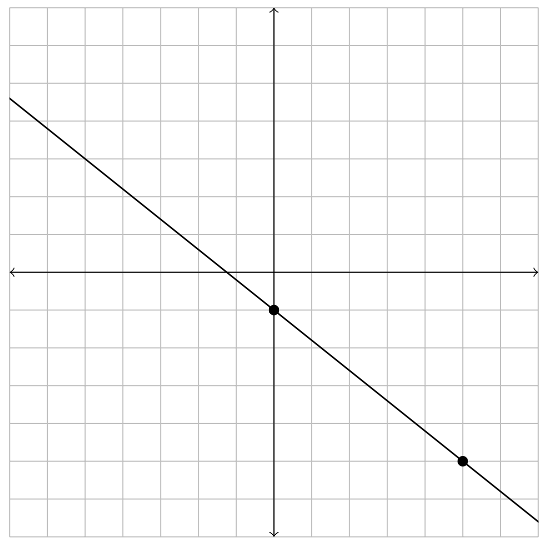
   Solution: Remember that the slope between the points (x_1, y_1) and (x_2, y_2) is
   $$\frac{y_2 - y_1}{x_2 - x_1}.$$
   Here, we have $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-2, 4)$, so that the slope between them is
   $$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(4) - (4)}{(-2) - (3)} = \frac{0}{-5}.$$
   So the slope between these points is $\boxed{0}$.

3. Find the distance between the points $(-5, -2)$ and $(-3, 3)$.

   Solution: Remember that the formula for the distance between (x_1, y_1) and (x_2, y_2) is
   $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$
   Here we have $(x_1, y_1) = (-5, -2)$ and $(x_2, y_2) = (-3, 3)$, so that the formula gives
   $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-3) - (-5))^2 + ((3) - (-2))^2} = \sqrt{(2)^2 + (5)^2} = \sqrt{29}.$$
   So the distance between these points is $\boxed{\sqrt{29}}$.

4. Plot the graph of the linear equation $y = -\frac{4}{5}x - 1$ on the plane below.

Solution: This line as slope $-4/5$ and y-intercept -1. We can use this information to find two points on the line and sketch as follows.



5. Find the slope between the points $(5, -7)$ and $(5, -1)$.

Solution: Remember that the slope between the points (x_1, y_1) and (x_2, y_2) is

$$\frac{y_2 - y_1}{x_2 - x_1}.$$

Here, we have $(x_1, y_1) = (5, -7)$ and $(x_2, y_2) = (5, -1)$, so that the slope between them is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (-7)}{(5) - (5)} = \frac{6}{0}.$$

We have a problem: the denominator of this fraction is zero. So the slope between these points is $\boxed{\text{undefined}}$.

6. Find the midpoint of the points $(2, -1)$ and $(-3, -7)$.

Solution: Remember that the midpoint of the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Here, we have $(x_1, y_1) = (2, -1)$ and $(x_2, y_2) = (-3, -7)$, so that the midpoint is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{(2) + (-3)}{2}, \frac{(-1) + (-7)}{2}\right) = \left(\frac{-1}{2}, \frac{-8}{2}\right).$$

So the midpoint is $\boxed{(-1/2, -4)}$.

7. Find an equation for the circle centered at $(6, -5)$ and having radius 1.

   Solution: Remember that the standard form equation of a circle centered at the point (h, k) and with radius r is
   $$(x - h)^2 + (y - k)^2 = r^2.$$
   Here we have $(h, k) = (6, -5)$ and $r = 1$; so this circle is given by the equation
   $$\boxed{(x - 6)^2 + (y + 5)^2 = 1}.$$

8. Find an equation for the circle centered at $(3, 1)$ and passing through $(-2, 5)$.

   Solution: The radius of this circle is the distance from the center, $(3, 1)$, to the point $(-2, 5)$. That distance is
   $$\sqrt{41}.$$
   Now the circle with center at (h, k) and radius r is given by the equation
   $$(x - h)^2 + (y - k)^2 = r^2.$$
   Thus this circle is given by the equation
   $$\boxed{(x - 3)^2 + (y - 1)^2 = 41}.$$

9. Find an equation for the line passing through the points $(4, -7)$ and $(-3, 7)$.

   Solution: We will find the point-slope form of this line. First, using the slope formula, we find that the slope of this line is
   $$m = \frac{(7) - (-7)}{(-3) - (4)} = -2.$$
   Since we know this line passes through (for instance) $(4, -7)$, using the point-slope formula, an equation for this line is
   $$\frac{y + 7}{x - 4} = -2.$$
   We can solve for y to get this equation in slope-intercept form as follows:
   $$\boxed{y = -2x + 1}$$

10. Convert the standard form linear equation
   $$4y + 6x = -5$$
   to slope-intercept form.

   Solution: To convert to slope-intercept form, we simply solve this equation for y to get
   $$\boxed{y = -\frac{3}{2}x - \frac{5}{4}}$$

11. Find an equation in slope-intercept form for the line passing through the point $(3, 2)$ and parallel to $y = \frac{1}{2}x+3$.

Solution: Let ℓ be the unknown line. Since ℓ is known to be parallel to $y = \frac{1}{2}x+3$, the slope of ℓ is $m = 1/2$. We also know that ℓ passes through the point $(3, 2)$. So ℓ is given by the point-slope form equation

$$\frac{y-2}{x-3} = 1/2.$$

Solving for y yields the slope-intercept equation

$$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$$