

Circles

Definition 1 (Circle). *Let o and x be points in an ordered geometry with a segment congruence.*

1. *The circle with center at o and passing through x is the set*

$$\mathcal{C}_o(x) = \{y \mid \overline{oy} \cong \overline{ox}\}.$$

2. *We say that a point z is interior to the circle $\mathcal{C}_o(x)$ if either $z = o$ or there is a point $y \in \mathcal{C}_o(x)$ such that $[ozy]$, and define the interior of $\mathcal{C}_o(x)$ to be the set*

$$\text{int } \mathcal{C}_o(x) = \{z \mid z = o \text{ or } [ozy] \text{ for some } y \in \mathcal{C}_o(x)\}.$$

3. *We say that a point z is exterior to the circle $\mathcal{C}_o(x)$ if there is a point $y \in \mathcal{C}_o(x)$ such that $[oyz]$, and define the exterior of $\mathcal{C}_o(x)$ to be the set*

$$\text{ext } \mathcal{C}_o(x) = \{z \mid [oyz] \text{ for some } y \in \mathcal{C}_o(x)\}.$$