Intro to Proof

Test 1

Form A

Spring 2016

Name: _____

Date: _____

READ THESE INSTRUCTIONS CAREFULLY!

- Circle or underline your final written answer.
- Justify your reasoning and show your work.
- If you run out of space, make a note and continue your work on the back of a page.

Write all your answers neatly on one or more separate sheets of paper.

1. We say that an integer n is *treven* if we have $n = 3k$ for some integer k. Similarly, we say that n is *spiffy* if $n = 3k + 1$ for some integer k, and that n is *righto* if $n = 3k + 2$ for some integer k. Use these definitions to prove the following.

   (a) If n is spiffy and m is righto, then $m + n$ is treven.

   (b) If n is treven and m is any integer, then mn is treven.

2. Prove that for all nonempty sets A, B, C, and D, we have

$$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D).$$

3. Recall that in class we defined the *symmetric difference* of two sets A and B to be

$$A \triangle B = (A \setminus B) \cup (B \setminus A).$$

   Prove that for all sets X, Y, and Z, we have

$$X \triangle (Y \triangle Z) = (X \triangle Y) \triangle Z.$$

4. Let A, B, and E be nonempty sets. Show that if $A \times E = B \times E$, then $A = B$.