Abstract I

Final Exam

Fall 2015

Name: ______________________________

Date: ______________________________

<u>READ THESE INSTRUCTIONS CAREFULLY!</u>

- Circle or underline your final written answer.
- Justify your reasoning and show your work.
- If you run out of space, make a note and continue your work on the back of a page.

1. Let R be a ring, with $S \subseteq R$ a subring and $I \subseteq R$ an ideal.

   (a) Show that $S \cap I$ is an ideal in S.

   (b) Show that I is an ideal in $S + I$.

   (c) Show that $S/(S \cap I) \cong (S + I)/I$. (Hint: show that the map $\varphi : S \to (S + I)/I$ is surjective with kernel $S \cap I$ and use the First Isomorphism Theorem.)

2. Let R be a ring. An element $x \in R$ is called *nilpotent* if $x^n = 0$ for some power n. For example, $\overline{2}$ is nilpotent in $\mathbb{Z}/(8)$ since $\overline{2}^3 = 0$.

   Show that if R is commutative then the set $N \subseteq R$ consisting of all the nilpotent elements is an ideal.

3. A ring element x is called *idempotent* if $x^2 = x$. For example, 0 is idempotent in any ring since $0^2 = 0$.

   (a) Determine which elements of $\mathbb{Z}/(30)$ are idempotent.

   (b) Determine which elements of $\mathbb{F}_3[x]/(x^2 - x)$ are idempotent.