Names: ____________________ ____________________ ____________________

Activity #4: Differentiation II (Solutions) **Calculus I**

1. Compute the derivative of the following function.

$$f(x) = (6x^2 + 7x + 7)^4$$

Solution: This f is a polynomial, so in principle we could expand f and differentiate term by term. However, computing the 4th power of a polynomial is not fun. Note that if we let $g(x) = 6x^2 + 7x + 7$ and $h(x) = x^4$, then $f = h \circ g$. Using the chain rule, we then have

$$\begin{aligned} \frac{d}{dx}(f(x)) &= \frac{d}{dx}((h \circ g)(x)) \\ &= h'(g(x))g'(x) \\ &= h'(6x^2 + 7x + 7)(12x + 7) \\ &= \boxed{4(6x^2 + 7x + 7)^3(12x + 7)} \end{aligned}$$

2. Compute the derivative of the following function.

$$f(x) = \sin(5x^2 + 2x + 5)$$

Solution: We can think of f as the composite of two simpler functions. In particular, $f = g \circ h$ where $g(x) = \sin(x)$ and $h(x) = 5x^2 + 2x + 5$. So we can use the chain rule as follows.

$$\begin{aligned} \frac{d}{dx}f(x) &= \frac{d}{dx}\sin(5x^2 + 2x + 5) \\ &= \cos(5x^2 + 2x + 5) \cdot \frac{d}{dx}(5x^2 + 2x + 5) \\ &= \boxed{\cos(5x^2 + 2x + 5)(10x + 2).} \end{aligned}$$

3. Compute the derivative of the following function.

$$f(x) = \frac{\sin(x)}{x^3 + 5x + 3}$$

Solution: Note that $f = \frac{g}{h}$, where $g(x) = \sin(x)$ and $h(x) = x^3 + 5x + 3$. Thus we can use the quotient rule as follows.

$$\begin{aligned} \frac{d}{dx}(f(x)) &= \frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) \\ &= \frac{h(x)g'(x) - g(x)h'(x)}{h(x)^2} \\ &= \boxed{\frac{(x^3 + 5x + 3)\cos(x) - \sin(x)(3x + 5)}{(x^3 + 5x + 3)^2}} \end{aligned}$$

4. Compute the derivative of the following function.

$$f(x) = \frac{(x^2 - 4x + 2)^4}{\sin(x)\cos(x)}$$

Hint: Use the quotient rule, then use the chain rule and the product rule (or a trig identity).

5. Compute the derivative of the following function.

$$f(x) = \sqrt[3]{x^3 + 6x - 4}$$

Hint: Use the chain rule.

6. Compute the second derivative of the following function.

$$y(x) = \left(1 + \frac{3}{x}\right)^5$$

Hint: Use the chain rule and the product rule.

7. Find an equation for the line tangent to

$$f(x) = \sqrt{x^2 - 2x + 8}$$

at $x = 2$.

Hint: Remember that the line passing through the point (h, k) and having slope m is given by the equation

$$\frac{y - k}{x - h} = m.$$