College Algebra

Test 3

Form A

Spring 2015

Name: ________________________________

Date: ________________________________

READ THESE INSTRUCTIONS CAREFULLY!

- Circle or underline your final written answer.
- Justify your reasoning and show your work.
- If you run out of space, make a note and continue your work on the back of a page.

Algebra Facts

Quadratic Formula

If a, b, and c are real numbers and $a \neq 0$, then the solutions of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Absolute Value

- If $|E| = F$, then either $E = F$ or $E = -F$.
- If $|E| \leq F$, then both $E \leq F$ and $E \geq -F$.
- If $|E| \geq F$, then either $E \geq F$ or $E \leq -F$.

Geometry Formulas

Given points (x_1, y_1) and (x_2, y_2), the distance between them is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

their midpoint is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right),$$

and the slope between them is

$$\frac{y_2 - y_1}{x_2 - x_1}.$$

Circles

The circle having center (h, k) and radius r is given by the equation

$$(x - h)^2 + (y - k)^2 = r^2$$

Lines

The standard form equation of a line looks like

$$ax + by + c = 0,$$

where a, b, and c are constants. The slope-intercept form is

$$y = mx + b,$$

where m is the slope of the line and b the y-intercept. The point-slope form is

$$y - y_0 = m(x - x_0),$$

where m is the slope and (x_0, y_0) is any point on the line.

Parabolas

The parabola with horizontal directrix, vertex at (h, k), and signed focal length p is given by the equation

$$y = \frac{1}{4p}(x - h)^2 + k.$$

This parabola opens up if $p > 0$ and down if $p < 0$.

Ellipses

The ellipse with foci at $(\pm c, 0)$ and major axis $2a$ is given by the equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

where $b^2 = c^2 - a^2$.

Transformations

$$\begin{array}{ccll}
x & \mapsto & x - h & \text{Horizontal Shift} \\
y & \mapsto & y - k & \text{Vertical Shift} \\
x & \mapsto & \frac{1}{a}x & \text{Horizontal Stretch} \\
y & \mapsto & \frac{1}{b}y & \text{Vertical Stretch}
\end{array}$$

1. Fill in the boxes to describe the long-term behavior of the following polynomial.

$$p(x) = 7x^6 + 13x^2 - x + 1$$

.

- As $x \to \infty$, $p(x) \to \boxed{}$

- As $x \to -\infty$, $p(x) \to \boxed{}$

2. The polynomial

$$p(x) = x^5 - 6x^4 + x^3 + 36x^2 - 20x - 48$$

has roots at $4; 2; -2; 3$. Completely factor $p(x)$ as a product of linear factors.

3. Construct a polynomial of degree 3 which has roots at -1, 1, and -2.

4. The polynomial

$$p(x) = x^4 - 4x^2 + 3$$

has a root at $\sqrt{3}$. Completely factor $p(x)$ as a product of linear factors.

5. Complete the square to find the standard form of the folloing parabola.

$$y = x^2 + 8x + 14$$

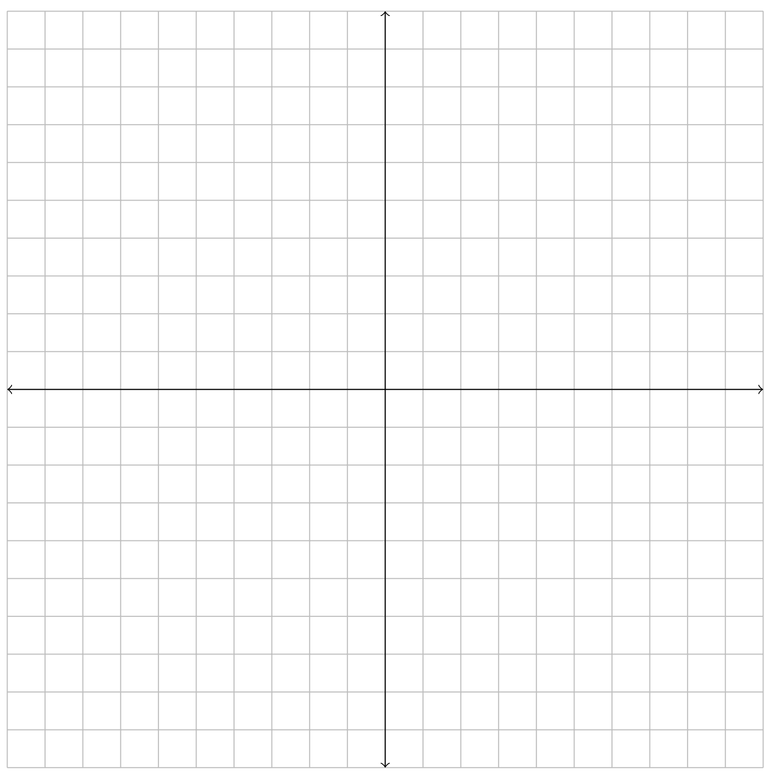
6. Find an equation for the parabola with horizontal directrix having vertex $(-3, 3)$ and focal length 3.

7. Find the domain of the following rational function.

$$f(x) = \frac{x^2 - 3x + 2}{x^3 - 5x^2 + 8x - 4}$$

8. Plot the following ellipse in the space provided.

$$\left(\frac{x-5}{5}\right)^2 + \left(\frac{y-4}{2}\right)^2 = 1$$

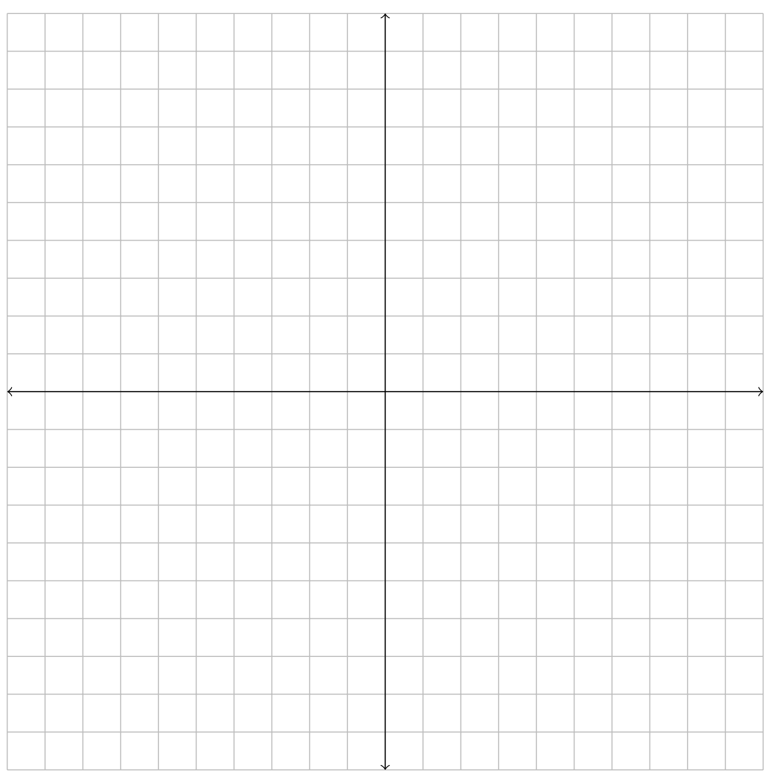


9. Find the long-term behavior asymptote of the following rational function.

$$f(x) = \frac{x^3 - 6x^2 + 11x - 6}{x + 5}$$

10. Plot the following parabola in the space provided.

$$y = \frac{1}{4}(x + 5)^2 + 3$$

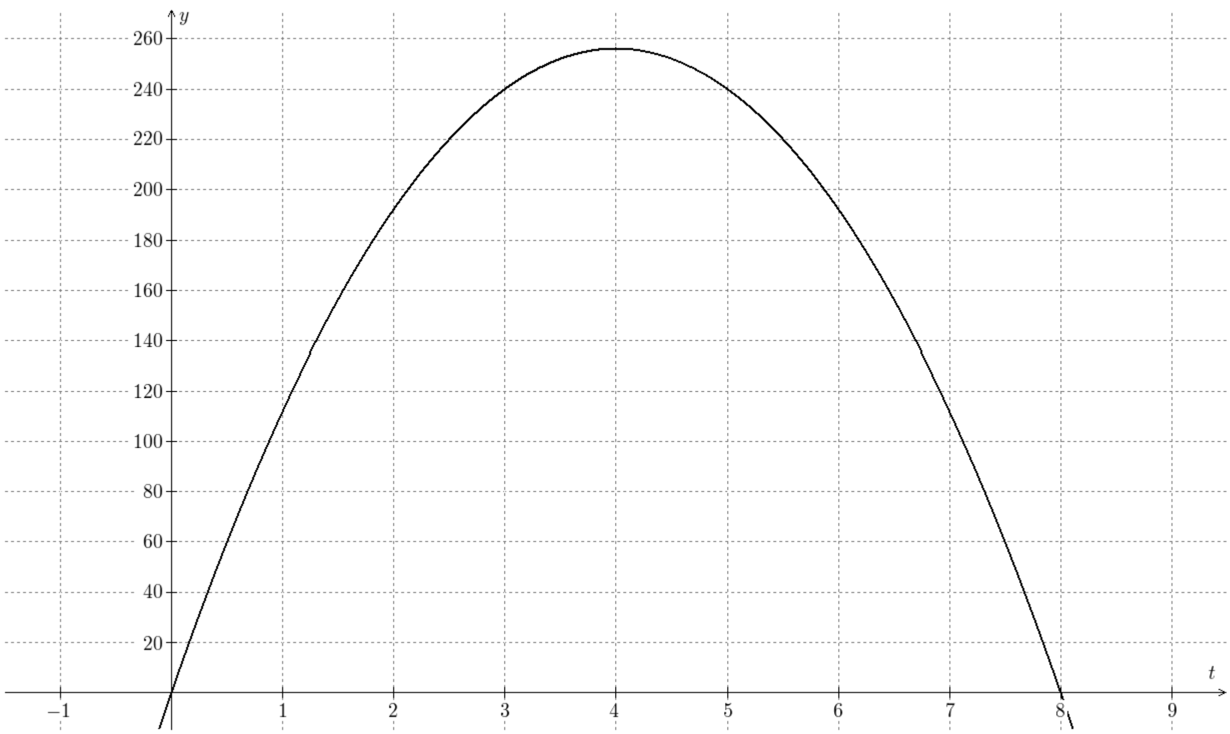


11. Suppose you and your friends create a catapult and launch a coconut with it.

(a) The height of the coconut t seconds after launch is given by $f(t) = 128t - 16t^2$. How long will the coconut be in the air? (Here we assume it will hit the ground when the height is 0).

(b) Viewing a graph of this function below, estimate the maximum height the coconut achieves.

Maximum Height:



(c) Recalling that the function of the height of the coconut is $f(t) = 128t - 16t^2$, use your knowledge about quadratic functions to determine the actual maximum height the coconut achieves.