Congruence

Definition 1 (Segment Congruence)**.** *Let $\mathcal{P}$ be an ordered geometry, and suppose we have an equivalence relation on pairs of points, denoted $\cong_s$. We call $\cong_s$ a* segment congruence *if the following properties are satisfied.*

SC1. $(x, y) \cong_s (y, x)$ *and* $(x, y) \cong_s (y, x)$ *for all points* x *and* y.

SC2. *If* $(x, y) \cong_s (z, w)$ *then* $(z, w) \cong_s (x, y)$ *for all points* x, y, z, *and* w.

SC3. *If* $(x, y) \cong_s (z, w)$ *and* $(z, w) \cong_s (u, v)$, *then* $(x, y) \cong_s (u, v)$ *for all* x, y, z, w, u, *and* v.

SC4. *If* $[xyz]$ *and* $[abc]$, *then if any two of the congruences*

$$(x, y) \cong_s (a, b), \quad (y, z) \cong_s (b, c), \quad (x, z) \cong_s (a, c)$$

hold, so does the third.

In this case, $\cong_s$ *is and equivalence relation on the set of* segments *in* $\mathcal{P}$, *and we write* $\overline{xy} \cong \overline{ab}$ *to mean* $(x, y) \cong_s (a, b)$.

Examples

Angle Congruence

Definition 2 (Angle Congruence)**.** *Let $\mathcal{P}$ be an ordered geometry, and suppose we have an equivalence relation on triples of points, denoted $\cong_a$. We call $\cong_a$ an* angle congruence *if the following properties are satisfied.*

AC1. $(a, o, b) \cong_a (b, o, a)$ *and* $(a, o, b) \cong_a (a, o, b)$ *for all points* a, o, *and* b.

AC2. *If* $(a, o, b) \cong_a (x, p, y)$, *then* $(x, p, y) \cong_a (a, o, b)$ *for all points* a, o, b, x, p, *and* y.

AC3. *If* $(a, o, b) \cong_a (x, p, y)$ *and* $(x, p, y) \cong_a (h, q, k)$, *then* $(a, o, b) \cong_a (h, q, k)$ *for all points* a, o, b, x, p, y, h, q, *and* k.

AC4. *If* $x \in \overrightarrow{oa}$ *and* $y \in \overrightarrow{ob}$ *and* x, y, *and* o *are distinct, then* $(a, o, b) \cong_a (x, o, y)$.