

The Line Separation Property

Definition 1 (Convexity). *Let \mathcal{P} be an incidence geometry with a betweenness relation $[\cdot \cdot \cdot]$. A non empty set S of points in \mathcal{P} is called convex if whenever $x, y \in S$ are distinct points, $\overline{xy} \subseteq S$.*

Definition 2 (Line Separation Property). *Let \mathcal{P} be an incidence geometry with a betweenness relation $[\cdot \cdot \cdot]$. We say that this geometry has the Line Separation Property if every line ℓ partitions the set of points not on ℓ into two nonempty, disjoint, convex sets, H_1 and H_2 , with the property that if $x \in H_1$ and $y \in H_2$ then $\overline{xy} \cap \ell = \{p\}$ for some point p . The sets H_1 and H_2 are called half-planes.*

Examples

Ordered Geometries

Definition 3 (Ordered Geometry). *Let \mathcal{P} be an incidence geometry with a betweenness relation $[\cdot \cdot \cdot]$. We say that \mathcal{P} (with this betweenness relation) is an Ordered Geometry if it has the Trichotomy Property, the 4-Point Property, the Interpolation Property, and the Line Separation Property.*