Names: ______________________ ______________________ ______________________

Activity #3: Differentiation I (Solutions) **Calculus I**

1. Compute the following derivative.

$$\frac{d}{dx}\left(13x^2 + 12x + 12\right)$$

Solution: Since this function is a polynomial, we can use the power rule on each term. The derivative is

$$\boxed{f'(x) = 26x + 12.}$$

2. Compute the derivative of the following function of t.

$$f(t) = \frac{4}{t^2} + \frac{8}{t} + 3t^3.$$

Solution: We start by moving all the variables to the numerators of fractions like so.

$$f(t) = 4t^{-2} + 8t^{-1} + 3t^3.$$

Now we can use the power rule on each term. In particular, we have

$$\boxed{f'(t) = -8t^{-3} - 8t^{-2} + 9t^2.}$$

3. Let $f(x) = x + \frac{6}{x}$.

   (a) Compute the derivative of f.
   (b) Find an equation for the line tangent to f at the point $(3, 5)$.

Solution: Note that

$$f(x) = x + 6x,$$

which we can differentiate term-by-term using the power rule to get

$$\boxed{f'(x) = -6x^{-2}.}$$

Recall that the line tangent to the graph of f at a point (u, v) has slope $f'(u)$ and passes through (u, v). Thus the slope of the tangent line to f at $x = 3$ is $f'(3) = -2/3$, and the line with this slope and passing through $(3, 5)$ is given by the equation

$$-2/3 = \frac{y - 5}{x - 3}$$

or, in slope-intercept form,

$$\boxed{y = -\frac{2}{3}x + 7.}$$

4. Compute the derivative of the following function.

$$f(x) = \frac{x^2 + 4x + 4}{2x - 7}$$

Solution: We can use the quotient rule on this function as follows.

$$\begin{aligned}
\frac{d}{dx} f(x) &= \frac{d}{dx} \frac{x^2 + 4x + 4}{2x - 7} \\
&= \frac{(2x-7) \cdot \frac{d}{dx}(x^2 + 4x + 4) - (x^2 + 4x + 4) \cdot \frac{d}{dx}(2x - 7)}{(2x-7)^2} \\
&= \frac{(2x-7)(2x+4) - (x^2 + 4x + 4)(2)}{(2x-7)^2} \\
&= \boxed{\frac{2x^2 - 14x - 36}{(2x-7)^2}}
\end{aligned}$$

5. Find the values of x at which the line tangent to

$$f(x) = x^3 + 7x^2 + 15x + 63$$

is horizontal.

Solution: Recall that a line is horizontal precisely when its slope is zero, and that the slope of the line tangent to f at c is given by $f'(c)$. So it suffices to find the solutions x of the equation $f'(x) = 0$. To this end, note that

$$f'(x) = 3x^2 + 14x^2 + 15.$$

This is a quadratic, which we can solve using our favorite method to find that f has a horizontal tangent line precisely when x is $\boxed{\text{-3 or -5/3.}}$