Name: ______________________

Homework #2: Union and Intersection **Intro to Proof**

Write all your answers neatly on one or more separate sheets of paper.

1. Define sets A, B, and C as follows.

$$\begin{aligned} A &= \{a, b, c, d, e, f\} \\ B &= \{a, c, e\} \\ C &= \{d, e, f\} \\ D &= \{a, b\} \end{aligned}$$

   Now find each of the following sets.

   (a) $D \cap (B \cup C)$

   (b) $(D \cap B) \cup C$

   (c) $A \cup (B \cap D)$

2. Suppose A, B, and C are sets such that $A \subseteq B$, $B \subseteq C$, and $C \subseteq A$. Show that $A = B = C$.

3. Can you find two specific sets A and B such that $A \in B$ and $A \subseteq B$ are both true?

4. Show that for all sets A and B, we have $A \cup (A \cap B) = A$.

5. Define two sets A and B as follows.

$$\begin{aligned} A &= \{n \in \mathbb{N} \mid n = 2m \text{ for some } m \in \mathbb{N}\} \\ B &= \{n \in \mathbb{N} \mid n = 3m \text{ for some } m \in \mathbb{N}\} \end{aligned}$$

   Show that

$$A \cap B = \{n \in \mathbb{N} \mid n = 6m \text{ for some } m \in \mathbb{N}\}.$$