Names: ____________________ ____________________ ____________________

Activity #1: Limits (Solutions) **Calculus I**

1. Compute the following limit.

$$\lim_{x\to 3}\left(3x^2+7x+4\right)$$

Solution: Since this expression is a polynomial, we can find the limit as x approaches 3 by evaluating at 3.

$$\begin{aligned}\lim_{x\to 3}\left(3x^2+7x+4\right) &= 3(3)^2+7(3)+4\\ &= 27+21+4\\ &= 52.\end{aligned}$$

2. Compute the following limit.

$$\lim_{x\to 8}\frac{x^2-15x+56}{x-8}$$

Solution: Note that this rational function is not defined at $x=8$. However, factoring the numerator we have

$$\begin{aligned}\lim_{x\to 8}\frac{x^2-15x+56}{x-8} &= \lim_{x\to 8}\frac{(x-8)(x-7)}{x-8}\\ &= \lim_{x\to 8}\frac{\cancel{(x-8)}(x-7)}{\cancel{x-8}}\\ &= \lim_{x\to 8}(x-7)\\ &= 1.\end{aligned}$$

3. Compute the following limit.

$$\lim_{x\to 5}\frac{x^2-25}{x-5}$$

Solution: Note that this expression is not defined if $x=5$. However, we can factor the numerator as a difference of squares and cancel.

$$\lim_{x\to 5}\frac{x^2-25}{x-5}=\lim_{x\to 5}\frac{(x-5)(x+5)}{x-5}=\lim_{x\to 5}(x+5)=10$$

4. Compute the following limit.

$$\lim_{x\to 49}\frac{x-49}{\sqrt{x}-7}$$

Solution: Note that this expression is not defined if $x=49$. However, we can factor the numerator as a difference of squares.

$$\lim_{x\to 49}\frac{x-49}{\sqrt{x}-7}=\lim_{x\to 49}\frac{(\sqrt{x}-7)(\sqrt{x}+7)}{\sqrt{x}-7}=\lim_{x\to 49}(\sqrt{x}+7)=14$$

5. Compute the following limit.

$$\lim_{x\to 33} \frac{\sqrt{x-8}-5}{x-33}$$

Solution: Note that this expression is not defined if $x = 33$. But also note that if we multiply the numerator by its radical conjugate, something nice happens:

$$\left(\sqrt{x-8}-5\right)\left(\sqrt{x-8}+5\right) = x-8-25 = x-33.$$

Let's try multiplying by 1, but write 1 as $\sqrt{x-8}+5$ over itself.

$$\begin{aligned} \lim_{x\to 33} \frac{\sqrt{x-8}-5}{x-33} &= \lim_{x\to 33} \left(\frac{\sqrt{x-8}-5}{x-33} \cdot \frac{\sqrt{x-8}+5}{\sqrt{x-8}+5} \right) \\ &= \lim_{x\to 33} \frac{x-33}{(x-33)(\sqrt{x-8}+5)} \\ &= \lim_{x\to 33} \frac{1}{\sqrt{x-8}+5} \\ &= \frac{1}{10} \end{aligned}$$

6. Compute the following limit.

$$\lim_{x\to 16} \frac{\sqrt{x-7}-3}{x-16}$$

Solution: Note that this expression is not defined if $x = 16$. But also note that if we multiply the numerator by its radical conjugate, something nice happens:

$$\left(\sqrt{x-7}-3\right)\left(\sqrt{x-7}+3\right) = x-7-9 = x-16.$$

Let's try multiplying by 1, but write 1 as $\sqrt{x-7}+3$ over itself.

$$\begin{aligned} \lim_{x\to 16} \frac{\sqrt{x-7}-3}{x-16} &= \lim_{x\to 16} \left(\frac{\sqrt{x-7}-3}{x-16} \cdot \frac{\sqrt{x-7}+3}{\sqrt{x-7}+3} \right) \\ &= \lim_{x\to 16} \frac{x-16}{(x-16)(\sqrt{x-7}+3)} \\ &= \lim_{x\to 16} \frac{1}{\sqrt{x-7}+3} \\ &= \frac{1}{6} \end{aligned}$$

7. Compute the following limit.

$$\lim_{x\to -1} \frac{x^3+x^2-9x-9}{x+1}$$

Solution: Note that this rational function is not defined if $x = -1$. However, -1 is a root of both the numerator and the denominator, so we can factor (either by grouping or using long or synthetic division) and cancel.

$$\begin{aligned} \lim_{x\to -1} \frac{x^3+x^2-9x-9}{x+1} &= \lim_{x\to -1} \frac{x^2(x+1)-9(x+1)}{x+1} \\ &= \lim_{x\to -1} \frac{(x+1)(x^2-9)}{x+1} \\ &= \lim_{x\to -1} (x^2-9) \\ &= -8 \end{aligned}$$

8. Compute the following limit.

$$\lim_{x \to 5} \frac{x^3 - 6x^2 + 3x + 10}{x - 5}$$

Solution: Note that this rational function is not defined if $x = 5$. However, 5 is a root of both the numerator and the denominator, so we can factor (using either long or synthetic division) and cancel.

$$\lim_{x \to 5} \frac{x^3 - 6x^2 + 3x + 10}{x - 5} = \lim_{x \to 5} \frac{(x-5)(x^2 - x - 2)}{x - 5} = \lim_{x \to 5} (x^2 - x - 2) = 18$$