

**Homework #1: Statements and Quantifiers****Intro to Proof**

Write all your answers neatly on one or more separate sheets of paper.

- Use truth tables (as we did in class) to establish the following rules of inference.
  - $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
  - $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$
  - $P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$
  - $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
- Determine whether each of the following arguments is valid. If so, give a derivation; if not, find a counterexample.
  - $$\begin{array}{l} P \wedge Q \\ (P \vee Q) \rightarrow R \\ \hline R \end{array}$$
  - $$\begin{array}{l} \neg X \rightarrow Y \\ \neg X \rightarrow Z \\ \hline \neg Z \rightarrow \neg Y \end{array}$$
  - $$\begin{array}{l} \neg A \rightarrow (B \rightarrow \neg C) \\ C \rightarrow \neg A \\ (\neg D \vee A) \rightarrow \neg \neg C \\ \neg D \\ \hline \neg B \end{array}$$

- Let  $P(x)$ ,  $Q(x)$ , and  $R(x)$  be statements defined as follows.

$$\begin{aligned} P(x) &= x \text{ has purple hair} \\ Q(x) &= x \text{ likes kiwifruit} \\ R(x) &= x \text{ has a pet frog} \end{aligned}$$

Translate the following statements into natural language.

- $\forall x.P(x)$
- $\exists x.Q(x)$
- $\forall x.(P(x) \wedge R(x))$
- $\exists x.(P(x) \rightarrow Q(x))$
- $\forall x.(R(x) \leftrightarrow \neg P(x))$