Intro to Proof

Test 2

Form A

Spring 2016

Name: _____

Date: _____

<u>READ THESE INSTRUCTIONS CAREFULLY!</u>

- Circle or underline your final written answer.

- Justify your reasoning and show your work.

- If you run out of space, make a note and continue your work on the back of a page.

Write all your answers neatly on one or more separate sheets of paper.

1. Compute the greatest common divisors of the following pairs of natural numbers using the Euclidean algorithm.

   (a) 104 and 21
   (b) 343 and 280
   (c) 578 and 442
   (d) 100083 and 10001

2. Use induction to prove that the following identity holds for all natural numbers n.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

3. Use induction to prove that the following identity holds for all natural numbers n. (You may find it helpful to use the identity in (2).)

$$\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$$

4. Use induction to show that $(n+1)(n+2)(n+3)$ is divisible by 6 for all natural numbers n.