

Angles

Definition 1 (Angle). Let \mathcal{P} be an ordered geometry and x , o , and y distinct points.

- The set

$$\angle xoy = \overrightarrow{ox} \cup \overrightarrow{oy}$$

is called the angle with vertex o and sides \overrightarrow{ox} and \overrightarrow{oy} .

- Suppose further that x , o , and y are not collinear. In this case, since \mathcal{P} is an ordered geometry, the lines \overleftrightarrow{ox} and \overleftrightarrow{oy} divide \mathcal{P} into half-planes. Let H_1 be the y half-plane of \overleftrightarrow{ox} , and let K_1 be the x half-plane of \overleftrightarrow{oy} . We define the interior of $\angle xoy$ to be the set

$$\text{int}\angle xoy = H_1 \cap K_1.$$

If x , y , and o are collinear, then the interior of $\angle xoy$ is not defined.

Definition 2 (Linear Pair, Vertical Pair). Suppose x , y , z , w , and o are distinct points in an ordered geometry.

- $\angle xoy$ and $\angle yoz$ are called an adjacent pair if $y \in \text{int}\angle xoz$.
- $\angle xoy$ and $\angle yoz$ are called a linear pair if $[xoz]$.
- $\angle xoy$ and $\angle zow$ are called a vertical pair if $[xoz]$ and $[yow]$.