The Plane Separation Property

Definition 1 (Convexity)**.** *Let $\mathcal{P}$ be an incidence geometry with a betweenness relation $[\cdots]$. A non empty set S of points in $\mathcal{P}$ is called* convex *if whenever $x, y \in S$ are distinct points, $\overline{xy} \subseteq S$.*

Definition 2 (Plane Separation Property)**.** *Let $\mathcal{P}$ be an incidence geometry with a betweenness relation $[\cdots]$. We say that this geometry has the* Plane Separation Property *if every line ℓ partitions the set of points not on ℓ into two nonempty, disjoint, convex sets, H_1 and H_2, with the property that if $x \in H_1$ and $y \in H_2$ then $\overline{xy} \cap \ell = \{p\}$ for some point p. The sets H_1 and H_2 are called* half-planes*.*

Examples

$\mathbb{R}^2$

Ordered Geometries

Definition 3 (Ordered Geometry)**.** *Let $\mathcal{P}$ be an incidence geometry with a betweenness relation $[\cdots]$. We say that $\mathcal{P}$ (with this betweenness relation) is an* Ordered Geometry *if it has the Trichotomy Property, the 4-Point Property, the Interpolation Property, and the Line Separation Property.*