

**Abstract Algebra Homework #2**

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1. Recall that if  $R$  is a ring and  $A$  a nonempty set, then  $R^A$  is the set of all functions  $f : A \rightarrow R$ . In class we defined a pointwise arithmetic on  $R^A$  as follows: given functions  $\alpha, \beta : A \rightarrow R$ , we define

$$(\alpha + \beta)(x) = \alpha(x) + \beta(x)$$

and

$$(\alpha\beta)(x) = \alpha(x)\beta(x).$$

- (a) Show that these operations make  $R^A$  into a ring.  
(b) Show that if  $R$  is commutative, then  $R^A$  is also commutative.
2. Suppose  $R$ ,  $S$ , and  $T$  are rings. Prove that

$$(R \oplus S) \oplus T \cong R \oplus (S \oplus T).$$

3. Suppose  $R_1$  and  $R_2$  are rings, and that  $S_1 \subseteq R_1$  and  $S_2 \subseteq R_2$  are subrings. Show that  $S_1 \oplus S_2$  is a subring of  $R_1 \oplus R_2$ .
4. Let  $R = \mathbb{Z}[i]$  be the ring of Gaussian integers.
- (a) Show that 29 is not irreducible in  $R$ . (Hint: Try to write 29 as a sum of squares.)  
(b) Find an irreducible factorization for 29 in  $R$ .  
(c) Show that 3 is irreducible in  $R$ . (Hint: Suppose  $3 = (a + bi)(c + di)$  is a nontrivial factorization, and consider the norm of both sides. Remember that the norm on  $\mathbb{Z}[i]$  is  $N(a + bi) = a^2 + b^2$ .)