Names: ____________ ____________ ____________

Activity #4: Functions (Solutions) **College Algebra**

1. Find the domain of the following function.

$$f(x) = \frac{8x^3 + x^2 + x + 1}{x^2 - 2x - 8}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

$$x^2 - 2x - 8 = 0.$$

This equation is a quadratic, and using our favorite solving strategy we see that its solutions are $x = 4$ and $x = -2$. So the domain of f is

$$\boxed{\text{all real numbers } \textit{except } 4 \text{ and } -2.}$$

2. Find the domain of the following function.

$$g(x) = \frac{1}{x^3 - 2x^2 - 63x}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

$$x^3 - 2x^2 - 63x = 0.$$

This equation is cubic, but it has no constant term, so we can factor out an x. That yields a quadratic which we can solve using our favorite strategy. We see that the solutions are $x = 0$, $x = -7$, and $x = 9$. So the domain of g is

$$\boxed{\text{all real numbers } \textit{except } 0, -7, \text{ and } 9.}$$

3. Find the domain of the following function.

$$f(x) = \sqrt{1x + 9}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a radical. This function will be defined as long as the expression in the radical is nonnegative. That is, at all solutions of the inequality

$$1x + 9 \geq 0.$$

Solving this inequality, we have $x \geq -9$. So the domain of f is

$$\boxed{\text{all real numbers } x \text{ such that } x \geq -9.}$$

4. Find the domain of the following function.

$$f(x) = \sqrt{|9x+9| - 8}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a radical. This function will be defined as long as the expression in the radical is nonnegative. That is, at all solutions of the inequality

$$|9x+9| - 8 \geq 0.$$

This is an absolute value inequality. Solving for the absolute value, we have

$$|9x+9| \geq 8.$$

This inequality can then be split into two like so:

$$9x+9 \geq 8 \quad \text{OR} \quad 9x+9 \leq -8.$$

The solution of this inequality is

$$x \geq -1/9 \quad \text{OR} \quad x \leq -17/9.$$

So the domain of f is

$$\boxed{\text{all real numbers } x \text{ such that } x \geq -1/9 \text{ or } x \leq -17/9.}$$

5. Evaluate the function

$$f(x) = 2x^3 + 4x + 6$$

at $x = 2$, $x = 0$, $x = -3$, and $x = 1/2$.

Solution: We have

$$\begin{aligned}
f(2) &= 2(2)^3 + 4(2) + 6 = \boxed{30} \\
f(0) &= 2(0)^3 + 4(0) + 6 = \boxed{6} \\
f(-3) &= 2(-3)^3 + 4(-3) + 6 = \boxed{-60} \\
f\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right) + 6 = \boxed{33/4}
\end{aligned}$$

6. Evaluate the function

$$f(x) = \begin{cases} 6x - 3 & \text{if } x \geq 5 \\ \frac{1}{x^2-2} & \text{if } x < 5 \end{cases}$$

at $x = 1$, $x = 9$, and $x = -7$.

Solution:

7. Find all solutions of the following inequality.

$$|3x + 5| + 13 \geq 27$$

Solution: First, solve for the absolute value expression by subtracting 13 from both sides.

$$|3x + 5| \geq 14.$$

This is an absolute value inequality of the form "absolute value greater than", so we can now rewrite as a compound inequality as follows.

$$3x + 5 \geq 14 \quad \text{OR} \quad 3x + 5 \leq -14.$$

Solving each of these for x, we have

$$x \leq -19/3 \quad \text{OR} \quad 3 \leq x.$$

In interval notation, the solution is $\boxed{(-\infty, -19/3] \cup [3, \infty)}$.