Betweenness and Linear Geometries

Definition 1 (Betweenness). *Let $\mathcal{P}$ be an incidence geometry. We say that a ternary relation $[\,\cdots]$ on the set of points of $\mathcal{P}$ is a* betweenness relation *if the following properties hold.*

  B1. If $[xyx]$, then $x = y$, for all points x and y.

  B2.

  B3.

Definition 2 (Segment, Ray). *Let x and y be distinct points in an incidence geometry $\mathcal{P} = (P, L)$.*

- *The set*
$$\overline{xy} = \{z \in P \mid z = x \text{ or } z = y \text{ or } [xzy]\}$$
  is called the segment *with* endpoints x *and* y. *If $z \in \overline{xy}$ and $z \neq x$ and $z \neq y$, we say that z is* interior *to $\overline{xy}$.*

- *The set*
$$\overrightarrow{xy} = \{z \in P \mid z = x \text{ or } z = y \text{ or } [xzy] \text{ or } [xyz]\}$$
  is called the ray *with* vertex x *toward* y.

Examples

  $\mathbb{R}^2$

  $\mathcal{A}$

The Trichotomy Property

The 4-Point Property