Circles

Definition 1 (Circle)**.** *Let o and x be points in an ordered geometry with a segment congruence.*

1. *The* circle *with* center *at o and passing through x is the set*

$$\mathcal{C}_o(x) = \{y \mid \overline{oy} \cong \overline{ox}\}.$$

2. *We say that a point z is* interior to *the circle $\mathcal{C}_o(x)$ if either $z = o$ or there is a point $y \in \mathcal{C}_o(x)$ such that $[ozy]$, and define the* interior *of $\mathcal{C}_o(x)$ to be the set*

$$\mathsf{int}\ \mathcal{C}_o(x) = \{z \mid z = o \text{ or } [ozy] \text{ for some } y \in \mathcal{C}_o(x)\}.$$

3. *We say that a point z is* exterior to *the circle $\mathcal{C}_o(x)$ if there is a point $y \in \mathcal{C}_o(x)$ such that $[oyz]$, and define the* exterior *of $\mathcal{C}_o(x)$ to be the set*

$$\mathsf{ext}\ \mathcal{C}_o(x) = \{z \mid [oyz] \text{ for some } y \in \mathcal{C}_o(x)\}.$$

Examples

$\mathbb{R}^2$ Let $O = (o_1, o_2)$ and $A = (a_1, a_2)$ be points in the Cartesian plane. Then $\mathcal{C}_O(A)$ consists of all points $X = (x_1, x_2)$ which satisfy the equation

$$(x_1 - o_1)^2 + (x_2 - o_2)^2 = (a_1 - o_1)^2 + (a_2 - o_2)^2.$$