

## Quadrilaterals

**Definition 1** (Quadrilateral). Let  $A$ ,  $B$ ,  $C$ , and  $D$  be points in a plane geometry. Then the set

$$Q(A, B, C, D) = \overline{AB} \cup \overline{BC} \cup \overline{CD} \cup \overline{DA}$$

is called the quadrilateral with vertices  $A$ ,  $B$ ,  $C$ , and  $D$  (in that order).

- $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{DA}$  are called the sides of the quadrilateral.
- $\overline{AC}$  and  $\overline{BD}$  are called the diagonals.
- $\angle ABC$ ,  $\angle BCD$ ,  $\angle CDA$ , and  $\angle DAB$  are the interior angles
- Sides  $\overline{AB}$  and  $\overline{CD}$  are said to be opposite, as are  $\overline{BC}$  and  $\overline{DA}$ .

**Definition 2.** Let  $A$ ,  $B$ ,  $C$ , and  $D$  be points.

- If any two of  $A$ ,  $B$ ,  $C$ , and  $D$  are equal, then  $Q(A, B, C, D)$  is said to be backtracking.
- $Q(A, B, C, D)$  is called self-intersecting if any point other than the vertices is on more than one side.
- $Q(A, B, C, D)$  is called degenerate if any three of its vertices are collinear.
- A quadrilateral which is not backtracking, self-intersecting, or degenerate is called simple.

**Definition 3.** We say that  $Q(A, B, C, D) \equiv Q(X, Y, Z, W)$  if  $\overline{AB} \equiv \overline{XY}$ ,  $\overline{BC} \equiv \overline{YZ}$ ,  $\overline{CD} \equiv \overline{ZW}$ ,  $\overline{DA} \equiv \overline{WX}$ ,  $\angle ABC \equiv \angle XYZ$ ,  $\angle BCD \equiv \angle YZW$ ,  $\angle CDA \equiv \angle ZWX$ , and  $\angle DAB \equiv \angle WXY$ .

**Proposition 1.** Quadrilateral congruence is an equivalence relation.

## Types of Quadrilaterals

**Definition 4.** A quadrilateral  $Q(A, B, C, D)$  is called

- equiangular if all its interior angles are congruent;
- a rectangle if all its interior angles are right;
- a kite if it has two pairs of congruent adjacent sides;
- equilateral (a.k.a. a rhombus) if all its sides are congruent;
- a trapezoid if one pair of opposite sides is parallel;
- a parallelogram if both pairs of opposite sides are parallel;
- cyclic if all its vertices lie on a common circle;
- tangential if all extended sides are tangent to a common circle;
- regular if it is both equilateral and equiangular.

Note: not all of these types of quadrilaterals are guaranteed to exist!