Contents

1 College Algebra

1.1 `ca/eqn/abs/linear.fvl`

1. $|-2x+9|+12=25$

2. $-5-|7x+4|=|7x+4|-5$

3. $|12x-18|+7=6$

1.2 `ca/eqn/quad/int.fvl`

1. $x^2 + 4x - 5 = 0$
2. $x^2 + 6x - 27 = 0$
3. $2x^2 + 3x - 14 = 0$
4. $x^2 + 4x + 1 = 0$

1.3 `ca/eqn/quad/quad-type.fvl`

1. $x^3 + 2x^2 - 48x = 0$
2. $2x^4 + 3x^2 - 5 = 0$
3. $|x^2 - x + 1| = 3$

1.4 `ca/eqn/rat/linear.fvl`

1. $\dfrac{x}{x+1} + 5 = \dfrac{6}{x+1}$

1.5 `ca/eqn/systems/linear.fvl`

1. Solve the following system of equations.

$$\left\{\begin{array}{rcl} -5y + x & = & 5 \\ -2y - 4x & = & 8 \end{array}\right.$$

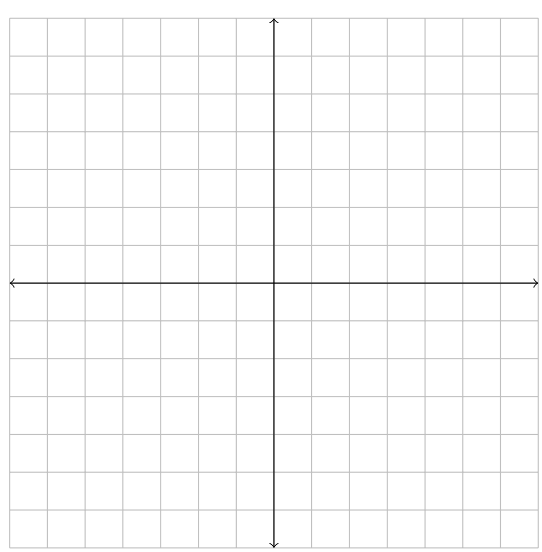
2. Solve the following system of equations.

$$\left\{\begin{array}{rcl} \frac{1}{3}y + \frac{1}{5}x & = & 8 \\ \\ \frac{2}{3}y + \frac{2}{5}x & = & 7 \end{array}\right.$$

1.6 `ca/eqn/transform/graph.fvl`

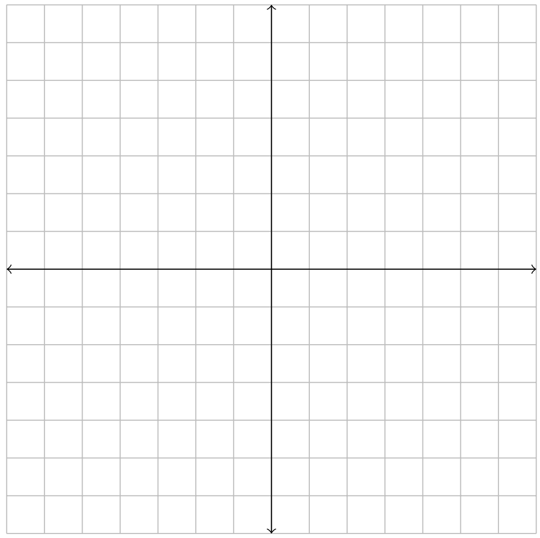
1. Sketch the graph of the following equation in the space provided.

$$(x+5)^2 + (y+1)^2 = 4$$



2. Sketch the graph of the following equation in the space provided.

$$\left(\frac{1}{3}(x+2)\right)^2 + (y+4)^2 = 4$$

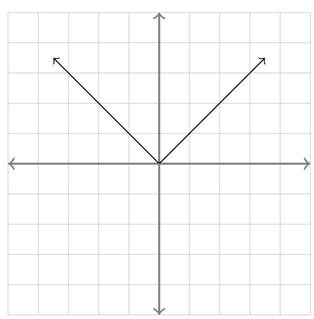


3. Fill in the boxes in the following statement.

   Replacing all the xs in an equation by $\boxed{}$ and all the ys by $\boxed{}$ will shift the equation's graph left by 7 units followed by a horizontal stretch by a factor of 2, and will shift the graph down by 6 units followed by a vertical stretch by a factor of 2.

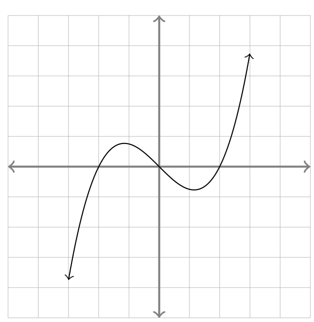
4. Graphically transform the following graph in the space provided.

   Shift right by 3 unit(s) and shift up by 2 unit(s).

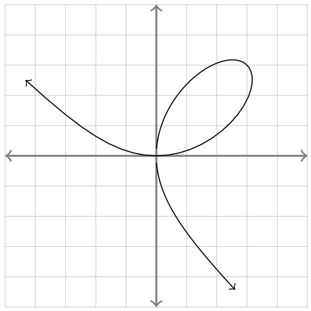


5. Graphically transform the following graph in the space provided.

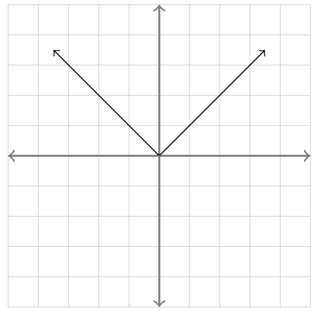
   Stretch horizontally by a factor of 2.



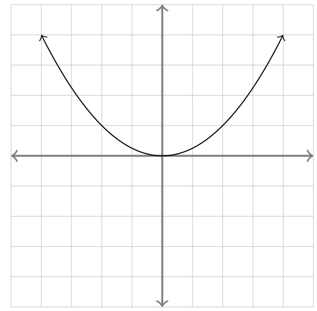
6. Determine whether or not the following graphs are symmetric across the x-axis, across the y-axis, or about the origin.



x-axis: yes/no
y-axis: yes/no
origin: yes/no



x-axis: yes/no
y-axis: yes/no
origin: yes/no



x-axis: yes/no
y-axis: yes/no
origin: yes/no

7. Determine whether or not the following equations are symmetric across the x-axis, across the y-axis, about the origin, or none of the three.

   (a) $y^3 = xy - 3$

   (b) $xy + y^2 = 2$

   (c) $y^3 - 1 = x^3 - 2$

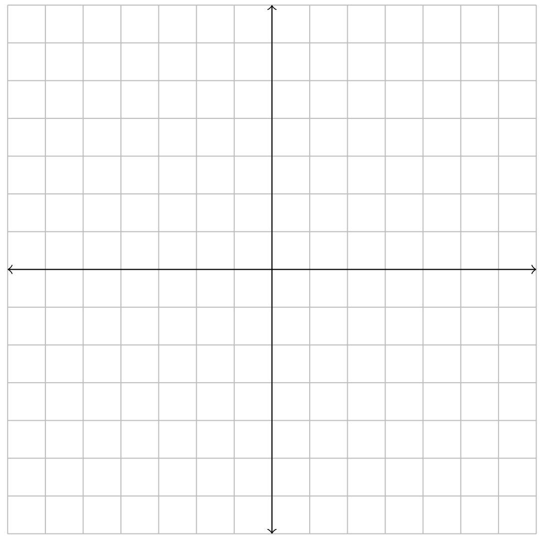
1.7 `ca/func/zeros.fvl`

1. Find the zeros of the following function: $f(x) = x^2 - 6x + 5$
2. Find the zeros of the following function: $f(x) = 2x^2 - 5x - 12$
3. Find the average rate of change of $f(x) = x^3 + 3x + 2$ from $x_1 = 1$ to $x_2 = 3$.

1.8 `ca/geom/lines.fvl`

1. Find an equation for the line passing through the point $(-6, -2)$ and having slope $1/3$.

2. Find the slope between the points $(4, -4)$ and $(-7, 6)$.
3. Plot the graph of the linear equation $y = \frac{4}{3}x - 3$ on the plane below.



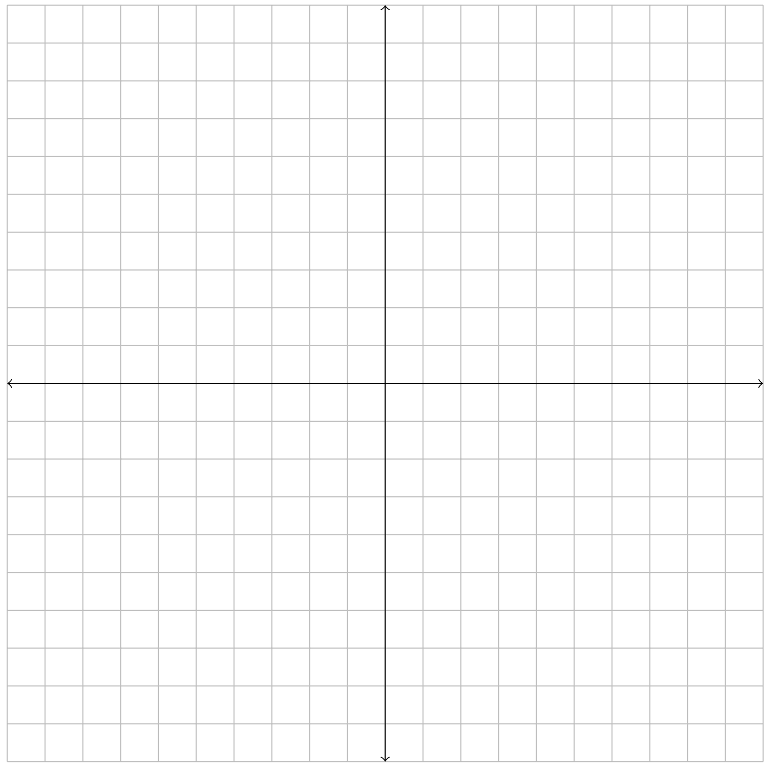
1.9 `ca/geom/circles.fvl`

1. Find the distance between the points $(6, -3)$ and $(-3, 4)$.
2. Find an equation for the circle centered at $(1, -4)$ and having radius 3.
3. Find an equation for the circle centered at $(5, -7)$ and passing through $(-1, 1)$.

1.10 `ca/geom/ellipses.fvl`

1. Plot the following ellipse in the space provided.

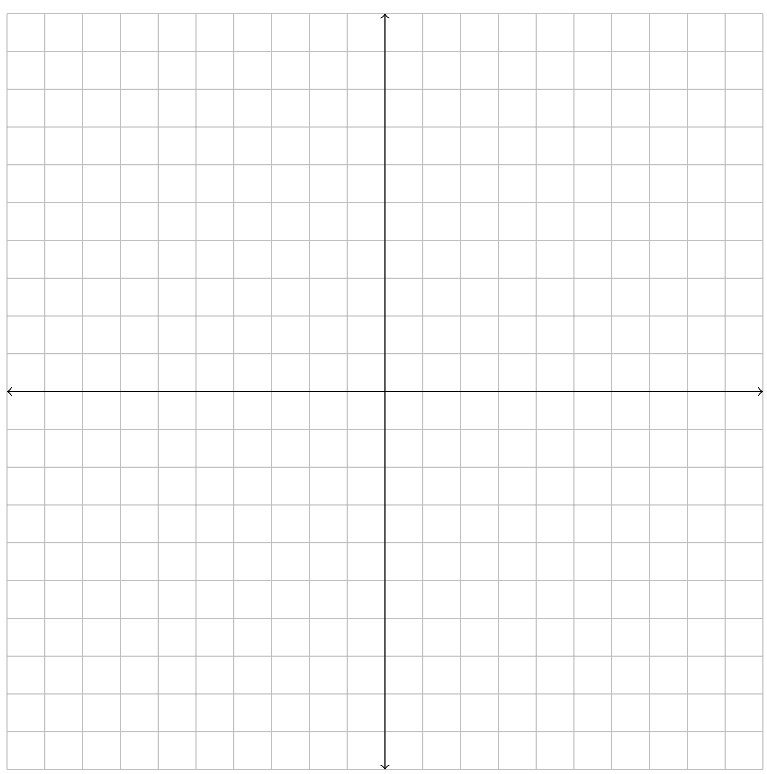
$$\left(\frac{x-5}{4}\right)^2 + \left(\frac{y-5}{2}\right)^2 = 1$$



1.11 `ca/geom/parabolas.fvl`

1. Find an equation for the parabola with horizontal directrix having vertex $(4, 7)$ and focal length -2.
2. Plot the following parabola in the space provided.

$$y = \frac{1}{8}(x+4)^2 - 3$$



3. Complete the square to find the standard form of the folloing parabola.

$$y = x^2 + 10x + 27$$

1.12 `ca/ineq/abs.fvl`

1. $|3x - 6| + 6 < 9$
2. $|2x - 7| + 12 > 27$
3. $2|3x + 2| + 9 < 14$
4. $2| - 3x + 8| + 11 > 16$

1.13 `ca/poly/factor/conjugate.fvl`

1. The polynomial
$$p(x) = 2x^5 + 5x^4 - 5x^3 - 17x^2 - 3x + 6$$
has a root at $\sqrt{3}$. Completely factor $p(x)$ as a product of linear factors.
2. The polynomial
$$p(x) = 3x^5 - x^4 - 15x^3 + 5x^2 + 18x - 6$$
has a root at $\sqrt{2}$. Completely factor $p(x)$ as a product of linear factors.
3. The polynomial
$$p(x) = x^4 - 3x^2 + 2$$
has a root at $\sqrt{2}$. Completely factor $p(x)$ as a product of linear factors.
4. The polynomial
$$p(x) = 2x^5 - 9x^4 + 2x^3 - 31x^2 - 24x + 20$$
has a root at $2i$. Completely factor $p(x)$ as a product of linear factors.

1.14 `ca/poly/factor/synthdiv.fvl`

1. The polynomial
$$p(x) = x^5 - 5x^4 - 7x^3 + 53x^2 - 18x - 72$$
has roots at $-1; 3; 2; 4$. Completely factor $p(x)$ as a product of linear factors.
2. The polynomial
$$p(x) = x^5 - 8x^4 + 25x^3 - 38x^2 + 28x - 8$$
has roots at 1 and 2. Find the multiplicity of these roots.
3. Construct a polynomial of degree 3 which has roots at 1, -1, and -2.
4. Find the list of candidate roots of the polynomial
$$p(x) = 5x^3 - 2x^2 - 5x + 9$$
given by the Rational Root Theorem. **Do not factor.**
5. Factor the following polynomial.
$$p(x) = 4x^4 + 4x^3 - 13x^2 - 19x - 6$$
6. Factor the following polynomial.
$$p(x) = 6x^5 - 31x^4 + 60x^3 - 55x^2 + 24x - 4$$
7. Factor the following polynomial.
$$p(x) = 36x^7 + 120x^6 - 35x^5 - 297x^4 + 11x^3 + 173x^2 + 12x - 20$$
Hint: Draw a graph of p using a computer or calculator.

1.15 `ca/poly/division.fvl`

1. Using polynomial long division, find the quotient and remainder when

$$a(x) = x^5 - 4x^4 + 14x^2 - 17x + 6$$

is divided by

$$b(x) = x^3 - 2x^2 - 5x + 6.$$

2. Use synthetic division to find the quotient and remainder when

$$a(x) = x^5 - x^4 - 5x^3 + 5x^2 + 4x - 4$$

is divided by $b(x) = x + 2$.

1.16 `ca/poly/endbehavior.fvl`

1. Fill in the boxes to describe the long-term behavior of the following polynomial.

$$p(x) = 3x^3 - 2x + 1$$

.

- As $x \to \infty$, $p(x) \to \boxed{}$
- As $x \to -\infty$, $p(x) \to \boxed{}$

1.17 `ca/ratfun/asymptotes.fvl`

1. Find the long-term behavior asymptote of the following rational function.

$$f(x) = \frac{x^3 - 4x^2 + x + 6}{x - 4}$$

2. Find the domain of the following rational function.

$$f(x) = \frac{x^2 - x - 2}{x^3 - 3x^2 + 4}$$

3. Consider the following rational function.

$$f(x) = \frac{(x-1)^3(x-2)^4(x-3)^2}{(x-1)^7(x-2)^2(x-3)^1}$$

For each point c not in the domain of f, determine whether f has a hole or a vertical asymptote at c.

4. Find the long-term behavior and vertical asymptotes of the rational function

$$f(x) = \frac{x^5 - 4x^4 - 15x^3 + 46x^2 + 44x - 120}{x^3 - 7x - 6}.$$

2 Calculus

2.1 `calc/diff/2nd.fvl`

1. Compute the second derivative of the following function.

$$y(x) = \left(1 + \frac{1}{x}\right)^3$$

2.2 `calc/diff/chain.fvl`

1. Compute the derivative of the following function.

$$f(x) = (5x^2 + 3x + 7)^7$$

2. Compute the derivative of the following function.

$$f(x) = \sqrt[2]{x^3 + 7x - 3}$$

3. Compute the derivative of the following function.

$$f(x) = \sin(2x^2 + 5x + 4)$$

2.3 `calc/diff/implicit.fvl`

1. Compute $\frac{dy}{dx}$ by implicit differentiation.

$$y^3 + 2xy - 2x^2 = 3$$

2. Find equations for the tangent and normal lines to the curve

$$2xy + 4x + 5y = 3$$

at the point $(1, -1/7)$.

2.4 `calc/diff/laurent.fvl`

1. Compute the derivative of the following function of t.

$$f(t) = \frac{5}{t^3} + \frac{7}{t} + 6t^5.$$

2.5 `calc/diff/poly.fvl`

1. Compute the following derivative.

$$\frac{d}{dx}\left(3x^2 + 11x + 14\right)$$

Solution: Since this function is a polynomial, we can use the power rule on each term. The derivative is $6x + 11$.

2.6 `calc/diff/quot.fvl`

1. Compute the derivative of the following function.

$$f(x) = \frac{x^2 + 3x + 3}{3x - 9}$$

2. Compute the derivative of the following function.

$$f(x) = \frac{\sin(x)}{x^3 + 5x + 2}$$

3. Compute the derivative of the following function.

$$f(x) = \frac{(x^2 - 2x + 4)^2}{\sin(x)\cos(x)}$$

2.7 `calc/diff/tangent.fvl`

1. Find the values of x at which the line tangent to

$$f(x) = x^3 + 6x^2 + 9x + 61$$

   is horizontal.

2. Let $f(x) = x + \frac{4}{x}$.

   (a) Compute the derivative of f.
   (b) Find an equation for the line tangent to f at the point $(2, 4)$.

3. Find an equation for the line tangent to

$$f(x) = \sqrt{x^2 + 4x + 9}$$

   at $x = -3$.

2.8 `calc/lim/cts.fvl`

1. Let $f(x)$ be the function

$$f(x) = \begin{cases} \frac{x-b}{b+2} & \text{if } x < 0 \\ x^2 + b & \text{if } x \geq 0. \end{cases}$$

   Find the value(s) of the constant b such that $f(x)$ is continuous everywhere.

   Solution: Remember that $\lim\limits_{x\to 0} f(x)$ exists precisely when the one-sided limits $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to 0^-} f(x)$ exist and are equal to one another. In this case, we have

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x^2 + b) = b$$

   and

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} \frac{x-b}{b+2} = \frac{-b}{b+2}.$$

   Setting these equal, we have

$$b = \frac{-b}{b+2}.$$

   The values for b we want are precisely the solutions of this equation. Clearing denominators, we have

$$b^2 + 2b = -b,$$

   and solving for zero we have

$$b^2 + 3b = 0 \qquad \text{which factors as} \qquad b(b+3) = 0.$$

   So $b = 0$ or $b = -3$.

2.9 `calc/lim/diffquot.fvl`

1. Compute the limit of the difference quotient

$$\lim_{x\to t} \frac{f(x) - f(t)}{x - t}$$

   when $f(x) = 7x + 3$ and $t = 5$.

Solution: We have

$$\begin{aligned}
\lim_{x\to 5}\frac{f(x)-f(5)}{x-5} &= \lim_{x\to 5}\frac{(7x+3)-(7\cdot 5+3)}{x-5} \\
&= \lim_{x\to 5}\frac{7x-7\cdot 5}{x-5} \\
&= \lim_{x\to 5}\frac{7(x-5)}{x-5} \\
&= \lim_{x\to 5} 7 \\
&= 7
\end{aligned}$$

2. Compute the limit of the difference quotient

$$\lim_{x\to t}\frac{f(x)-f(t)}{x-t}$$

when $f(x) = 8x^2 + 3x + 10$ and $t = 4$.

Solution: We have

$$\begin{aligned}
\lim_{x\to 4}\frac{f(x)-f(4)}{x-4} &= \lim_{x\to 4}\frac{(8x^2+3x+10)-(8(4)^2+3(4)+10)}{x-4} \\
&= \lim_{x\to 4}\frac{8x^2-8(4)^2+3x-3(4)+10-10}{x-4} \\
&= \lim_{x\to 4}\frac{8(x^2-4^2)+3(x-4)}{x-4} \\
&= \lim_{x\to 4}\frac{8(x-4)(x+4)+3(x-4)}{x-4} \\
&= \lim_{x\to 4}\frac{(x-4)(8(x+4)+3)}{x-4} \\
&= \lim_{x\to 4}(8(x+4)+3) \\
&= 8(4+4)+3 \\
&= 67
\end{aligned}$$

3. Compute the limit of the difference quotient

$$\lim_{x\to t}\frac{f(x)-f(t)}{x-t}$$

when $f(x) = \sqrt{x+6}$ and $t = 1$.

Solution: We have

$$\begin{aligned}
\lim_{x\to 1}\frac{f(x)-f(1)}{x-1} &= \lim_{x\to 1}\frac{\sqrt{x+6}-\sqrt{7}}{x-1} \\
&= \lim_{x\to 1}\left(\frac{\sqrt{x+6}-\sqrt{7}}{x-1}\cdot\frac{\sqrt{x+6}+\sqrt{7}}{\sqrt{x+6}+\sqrt{7}}\right) \\
&= \lim_{x\to 1}\frac{(x+6)-(7)}{(x-1)\left(\sqrt{x+6}+\sqrt{7}\right)} \\
&= \lim_{x\to 1}\frac{x-1}{(x-1)\left(\sqrt{x+6}+\sqrt{7}\right)} \\
&= \lim_{x\to 1}\frac{1}{\sqrt{x+6}+\sqrt{7}} \\
&= \frac{1}{2\sqrt{7}}
\end{aligned}$$

2.10 `calc/lim/infinity.fvl`

1. Compute the following limit.

$$\lim_{x\to\infty} \frac{\sqrt{6x^2+6x+7}}{4x+3}$$

Solution: We cannot use the limit laws here because the limits of the numerator and denominator do not exist. However, note that for $x \neq 0$ we have $1 = (1/|x|)/(1/|x|)$.

$$\begin{aligned}
\lim_{x\to\infty} \frac{\sqrt{6x^2+6x+7}}{4x+3} &= \lim_{x\to\infty} \left(\frac{\sqrt{6x^2+6x+7}}{4x+3} \cdot \frac{\frac{1}{|x|}}{\frac{1}{|x|}} \right) \\
&= \lim_{x\to\infty} \left(\frac{\sqrt{6x^2+6x+7}}{4x+3} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}} \right) \\
&= \lim_{x\to\infty} \frac{\frac{\sqrt{6x^2+6x+7}}{\sqrt{x^2}}}{\frac{4x+3}{x}} \\
&= \lim_{x\to\infty} \frac{\sqrt{\frac{6x^2+6x+7}{x^2}}}{\frac{4x+3}{x}} \\
&= \lim_{x\to\infty} \frac{\sqrt{6+\frac{6}{x}+\frac{7}{x^2}}}{4+\frac{3}{x}} \\
&= \frac{\lim\limits_{x\to\infty} \sqrt{6+\dfrac{6}{x}+\dfrac{7}{x^2}}}{\lim\limits_{x\to\infty} \left(4+\dfrac{3}{x}\right)} \\
&= \frac{\sqrt{6}}{4}
\end{aligned}$$

2. Compute the following limit.

$$\lim_{x\to\infty} \frac{9x+3}{8x+4}$$

Solution: We cannot use the limit laws to break up this quotient because the limits of the numerator and denominator do not exist as $x \to \infty$. However, note that for $x \neq 0$, we have $1 = (1/x)/(1/x)$. Multiplying by 1 in this form, we have

$$\begin{aligned}
\lim_{x\to\infty} \frac{9x+3}{8x+4} &= \lim_{x\to\infty} \frac{9x+3}{8x+4} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to\infty} \frac{\frac{9x+3}{x}}{\frac{8x+4}{x}} \\
&= \lim_{x\to\infty} \frac{9+\frac{3}{x}}{8+\frac{4}{x}}.
\end{aligned}$$

Now the limits of the numerator and denominator do exist as $x \to \infty$, and we have

$$\begin{aligned}
\lim_{x\to\infty} \frac{9x+3}{8x+4} &= \frac{\lim_{x\to\infty}\left(9+\frac{3}{x}\right)}{\lim_{x\to\infty}\left(8+\frac{4}{x}\right)} \\
&= \frac{9}{8}
\end{aligned}$$

2.11 `calc/lim/poly.fvl`

1. Compute the following limit.

$$\lim_{x\to 2}\left(5x^2+8x+4\right)$$

Solution: Since this expression is a polynomial, we can find the limit as x approaches 2 by evaluating at 2.

$$\begin{aligned}\lim_{x\to 2}\left(5x^2+8x+4\right) &= 5(2)^2+8(2)+4\\ &= 20+16+4\\ &= 40.\end{aligned}$$

2.12 `calc/lim/rat.fvl`

1. Compute the following limit.

$$\lim_{x\to 4}\frac{x^3-64}{x-4}$$

Solution: Note that this expression is not defined if $x=4$ because of the $x-4$ in the denominator. However, we can factor the numerator as a difference of cubes and cancel.

$$\lim_{x\to 4}\frac{x^3-64}{x-4}=\lim_{x\to 4}\frac{(x-4)(x^2+4x+16)}{x-4}=\lim_{x\to 4}(x^2+4x+16)=48$$

2. Compute the following limit.

$$\lim_{x\to 2}\frac{x^2-4}{x-2}$$

Solution: Note that this expression is not defined if $x=2$. However, we can factor the numerator as a difference of squares and cancel.

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=\lim_{x\to 2}(x+2)=4$$

3. Compute the following limit.

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}$$

Solution: Note that this expression is not defined if $x=9$. However, we can factor the numerator as a difference of squares.

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}=\lim_{x\to 9}\frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3}=\lim_{x\to 9}(\sqrt{x}+3)=6$$

4. Compute the following limit.

$$\lim_{x\to 22}\frac{\sqrt{x-6}-4}{x-22}$$

Solution: Note that this expression is not defined if $x=22$. But also note that if we multiply the numerator by its radical conjugate, something nice happens:

$$\left(\sqrt{x-6}-4\right)\left(\sqrt{x-6}+4\right)=x-6-16=x-22.$$

Let's try multiplying by 1, but write 1 as $\sqrt{x-6}+4$ over itself.

$$\begin{aligned}\lim_{x\to 22}\frac{\sqrt{x-6}-4}{x-22} &= \lim_{x\to 22}\left(\frac{\sqrt{x-6}-4}{x-22}\cdot\frac{\sqrt{x-6}+4}{\sqrt{x-6}+4}\right)\\ &= \lim_{x\to 22}\frac{x-22}{(x-22)(\sqrt{x-6}+4)}\\ &= \lim_{x\to 22}\frac{1}{\sqrt{x-6}+4}\\ &= \frac{1}{8}\end{aligned}$$

5. Compute the following limit.

$$\lim_{x\to 1} \frac{x^{1/3}-1}{x^{1/2}-1}$$

Solution: Note that although this expression is continuous, it is not defined at $x = 1$. Let's start by writing all exponents with a common denominator:

$$\lim_{x\to 1} \frac{x^{2/6}-1}{x^{3/6}-1}.$$

If we substitute $y = x^{1/6}$, we have

$$\frac{x^{2/6}-1}{x^{3/6}-1} = \frac{y^2-1}{y^3-1} = \frac{(y-1)(y+1)}{(y-1)(y^2+y+1)} = \frac{y+1}{y^2+y+1} = \frac{x^{1/6}+1}{x^{1/3}+x^{1/6}+1}$$

provided $y \neq 1$ (and hence $x \neq 1$). Note that we used the difference of squares and difference of cubes factoring formulas. Now we have

$$\begin{aligned} \lim_{x\to 1} \frac{x^{1/3}-1}{x^{1/2}-1} &= \lim_{x\to 1} \frac{x^{1/6}+1}{x^{1/3}+x^{1/6}+1} \\ &= \frac{1^{1/6}+1}{1^{1/3}+1^{1/6}+1} \\ &= 2/3. \end{aligned}$$

6. Compute the following limit.

$$\lim_{x\to -1} \frac{x^2-1}{x+1}$$

Solution: Note that this rational function is not defined at $x = -1$. However, factoring the numerator we have

$$\begin{aligned} \lim_{x\to -1} \frac{x^2-1}{x+1} &= \lim_{x\to -1} \frac{(x+1)(x-1)}{x+1} \\ &= \lim_{x\to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} \\ &= \lim_{x\to -1} (x-1) \\ &= -2. \end{aligned}$$

7. Compute the following limit.

$$\lim_{x\to -4} \frac{x^3+4x^2-9x-36}{x+4}$$

Solution: Note that this rational function is not defined if $x = -4$. However, -4 is a root of both the numerator and the denominator, so we can factor (either by grouping or using long or synthetic division) and cancel.

$$\begin{aligned} \lim_{x\to -4} \frac{x^3+4x^2-9x-36}{x+4} &= \lim_{x\to -4} \frac{x^2(x+4)-9(x+4)}{x+4} \\ &= \lim_{x\to -4} \frac{(x+4)(x^2-9)}{x+4} \\ &= \lim_{x\to -4} (x^2-9) \\ &= 7 \end{aligned}$$

8. Compute the following limit.

$$\lim_{x\to -5} \frac{x^3+4x^2-11x-30}{x+5}$$

Solution: Note that this rational function is not defined if $x = -5$. However, -5 is a root of both the numerator and the denominator, so we can factor (using either long or synthetic division) and cancel.

$$\lim_{x\to -5} \frac{x^3+4x^2-11x-30}{x+5} = \lim_{x\to -5} \frac{(x+5)(x^2-x-6)}{x+5} = \lim_{x\to -5} (x^2-x-6) = 24$$

9. Compute the following limit.

$$\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4}$$

Solution: This expression is not defined if $x = a$. However, the denominator factors as a difference of squares and cancel as follows:

$$\begin{aligned}\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4} &= \lim_{x\to a} \frac{x^2-a^2}{(x^2-a^2)(x^2+a^2)} \\ &= \lim_{x\to a} \frac{1}{x^2+a^2} \\ &= \frac{1}{2a^2}\end{aligned}$$

2.13 `calc/lim/trig.fvl`

1. Compute the following limit.

$$\lim_{x\to 0} \frac{\sin(8x)}{x}$$

Solution: Note that

$$\begin{aligned}\lim_{x\to 0} \frac{\sin(8x)}{x} &= \lim_{x\to 0} \frac{k\sin(8x)}{8x} \\ &= 8\lim_{x\to 0} \frac{\sin(8x)}{8x} \\ &= 8\lim_{x\to 0} \operatorname{sinc}(8x) \\ &= 8\operatorname{sinc}\left(\lim_{x\to 0} 8x\right) \quad \text{(since sinc is continuous)} \\ &= 8\operatorname{sinc}(0) \\ &= 8.\end{aligned}$$

2. Compute the following limit.

$$\lim_{x\to 0} \frac{4x^2+12x+\sin x}{x}$$

Solution: This expression is not defined if $x = 0$. However, we can split this fraction like so:

$$\begin{aligned}\lim_{x\to 0} \frac{4x^2+12x+\sin(x)}{x} &= \lim_{x\to 0} \left(\frac{4x^2+12x}{x} + \frac{\sin x}{x}\right) \\ &= \lim_{x\to 0} \left(4x+12+\frac{\sin x}{x}\right).\end{aligned}$$

Recall that the limit of a sum is the sum of limits, *provided* the limit of each summand exists. In this case they do, and we have

$$
\begin{aligned}
&= \lim_{x\to 0}(4x+12) + \lim_{x\to 0}\frac{\sin x}{x} \\
&= 12+1 \\
&= 13.
\end{aligned}
$$

2.14 `calc/appl/findFunction.fvl`

1. The function $f(x) = ax^2 + bx + c$ passes through the point $(2, 2)$ and is tangent to the line $y = -2x - 6$ at the point $(3, -12)$. Find a, b, and c.

   Solution: We have three constraints on f, each of which we can use to find an equation that a, b, and c must satisfy. Since f passes through $(2, 2)$, we have $f(2) = 2$; that is,

   $$c + 2b + 4a = 2.$$

   Similarly, since f passes through $(3, -12)$ we have $f(3) = -12$; that is,

   $$c + 3b + 9a = -12.$$

   Finally, if f is tangent to the line $y = -2x - 6$ at $x = 3$ then the derivative of f at 3 is the slope of that line, so we have $f'(3) = -2$; that is,

   $$b + 6a = -2.$$

   We now have a system of three equations in three unknowns whose solutions (if they exist) are values of a, b, and c which satisfy these constraints. This system is represented by the following matrix.

   $$\left[\begin{array}{ccc|c} 4 & 2 & 1 & 2 \\ 9 & 3 & 1 & -12 \\ 6 & 1 & 0 & -2 \end{array}\right]$$

   We can solve this system in a few different ways, such as elimination by substitution or addition. Using Gauss-Jordan elimination, this matrix is row-equivalent to the matrix

   $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -74 \\ 0 & 0 & 1 & 102 \end{array}\right]$$

   from which we can read off the values of a, b, and c. Specifically, $a = 12$, $b = -74$, and $c = 102$.

3 Statistics

3.1 `stat/expectVal.fvl`

1. You have an opportunity to pay \$4 to draw a single card from a standard deck. If the card is a heart you win \$15, and otherwise you get nothing. What is the expected value of this game?

   Solution: -0.25

2. You have an opportunity to draw a single card from a standard deck. If you draw a king, you win \$5, and otherwise you get nothing. What is the largest amount of money you should be willing to pay to play this game?

3. You have an opportunity to pay \$3 to roll two 6-sided dice of different colors. If you roll doubles, you win \$6; if you roll two numbers whose sum is 3, you win \$51; otherwise you get nothing. What is the expected value of this game?

   Solution: 0.83

4. The probability of winning the South Central Lottery with a single ticket, costing \$2, is $1/9000$. Suppose the jackpot is \$7000. What is the expected value of a single lottery ticket?

   Solution: -1.22

3.2 `stat/compute.fvl`

1. Find the mean of the following list of numbers.

$$4,\ 2,\ 0,\ 5,\ 3,\ 2,\ 4,\ 3$$

Solution: Remember that to find the mean of n numbers x_1, x_2, ..., x_n, we add them up and divide by the number of numbers. In this case,

$$\begin{aligned} \frac{4+2+0+5+3+2+4+3}{8} &= \frac{23}{8} \\ &\approx 2.8. \end{aligned}$$

2. Find the mean of the following list of numbers.

$$11,\ 5,\ 5,\ 14,\ 12,\ 5,\ 8,\ 14$$

Solution: Remember that to find the mean of n numbers x_1, x_2, ..., x_n, we add them up and divide by the number of numbers. In this case,

$$\begin{aligned} \frac{11+5+5+14+12+5+8+14}{8} &= \frac{74}{8} \\ &\approx 9.2. \end{aligned}$$

3. Find the mean of the following list of numbers.

$$76,\ 52,\ 76,\ 52,\ 86,\ 88,\ 82,\ 86,\ 51,\ 68$$

Solution: Remember that to find the mean of n numbers x_1, x_2, ..., x_n, we add them up and divide by the number of numbers. In this case,

$$\begin{aligned} \frac{76+52+76+52+86+88+82+86+51+68}{10} &= \frac{717}{10} \\ &\approx 71.7. \end{aligned}$$

4. Find the mean deviation of the following list of numbers.

$$7,\ 6,\ 2,\ 3,\ 6,\ 7$$

Solution: Remember that the mean deviation of x_1, x_2, ..., x_n is

$$\frac{1}{n}\sum_{i=1}^{n} |x_i - \overline{x}|,$$

where $\overline{x}$ is the mean of the x_i. In this case the mean is $\overline{x} = 5.16$. Then the mean deviation is

$$\begin{aligned} & \frac{1}{6}\big(|7-5.16| + |6-5.16| + |2-5.16| + |3-5.16| + |6-5.16| + |7-5.16|\big) \\ = \; & \frac{1}{6}\big(1.83 + 0.83 + 3.16 + 2.16 + 0.83 + 1.83\big) \\ = \; & 1.77 \end{aligned}$$

5. Find the mean deviation of the following list of numbers.

$$12,\ 11,\ 15,\ 14,\ 11$$

Solution: Remember that the mean deviation of x_1, x_2, …, x_n is

$$\frac{1}{n}\sum_{i=1}^{n} |x_i - \overline{x}|,$$

where $\overline{x}$ is the mean of the x_i. In this case the mean is $\overline{x} = 12.60$. Then the mean deviation is

$$\begin{aligned}
& \frac{1}{5}\big(|12 - 12.60| + |11 - 12.60| + |15 - 12.60| + |14 - 12.60| + |11 - 12.60|\big) \\
= \; & \frac{1}{5}\big(0.60 + 1.60 + 2.40 + 1.40 + 1.60\big) \\
= \; & 1.52
\end{aligned}$$

6. Find the standard deviation of the following list of numbers.

$$5,\ 12,\ 7,\ 7,\ 8$$

Solution: Remember that the standard deviation of x_1, x_2, …, x_n is

$$\sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}},$$

where $\overline{x}$ is the mean of the x_i. In this case the mean is $\overline{x} = 7.80$. Then the standard deviation is

$$\begin{aligned}
& \sqrt{\frac{1}{4}\big((5 - 7.80)^2 + (12 - 7.80)^2 + (7 - 7.80)^2 + (7 - 7.80)^2 + (8 - 7.80)^2\big)} \\
= \; & \sqrt{\frac{1}{4}\big(7.84 + 17.64 + 0.64 + 0.64 + 0.04\big)} \\
= \; & 2.31
\end{aligned}$$

7. Suppose we have collected the following list of numbers.

$$8,\ 5,\ 8,\ 5,\ 8,\ 7,\ 7,\ 7,\ 6,\ 8$$

Compute the z-scores of 4 and 14 with respect to this list.

Solution: Remember that the z-score of a particular number x with respect to a list of numbers is

$$z = \frac{x - \overline{x}}{s},$$

where $\overline{x}$ is the mean and s the standard deviation. In this case we can see that $\overline{x} = 6.9$ and $s = 1.135$, so that the z-score of 4 is

$$\frac{4 - 6.9}{1.135} = -2.553$$

and of 14 is

$$\frac{14 - 6.9}{1.135} = 6.251.$$

8. Suppose we have collected the following list of numbers.

$$14,\ 4,\ 8,\ 5,\ 22,\ 1,\ 6,\ 13,\ 24,\ 15$$

Compute the z-scores of 3 and 25 with respect to this list.

Solution: Remember that the z-score of a particular number x with respect to a list of numbers is

$$z = \frac{x - \overline{x}}{s},$$

where $\overline{x}$ is the mean and s the standard deviation. In this case we can see that $\overline{x} = 11.2$ and $s = 7.332$, so that the z-score of 3 is

$$\frac{3 - 11.2}{7.332} = -1.118$$

and of 25 is

$$\frac{25 - 11.2}{7.332} = 1.882.$$

9. Find the coefficient of variation of the following list of numbers.

$$6, 6, 5, 8, 6, 8, 3$$

Solution: Remember that the coefficient of variation of a list of numbers is $100\% \cdot s/\overline{x}$, where s is the standard deviation and $\overline{x}$ the mean, expressed as a percentage. In this case the mean is $\overline{x} = 6.0$ and the standard deviation is $s = 1.60$, so the coefficient of variation is $100\% \cdot s/\overline{x} = 26\%$.

10. Find the coefficient of variation of the following list of numbers.

$$18, 20, 17, 18, 19, 20$$

Solution: Remember that the coefficient of variation of a list of numbers is $100\% \cdot s/\overline{x}$, where s is the standard deviation and $\overline{x}$ the mean, expressed as a percentage. In this case the mean is $\overline{x} = 18.6$ and the standard deviation is $s = 1.10$, so the coefficient of variation is $100\% \cdot s/\overline{x} = 5\%$.

11. Compute the mean and standard deviation of the following discrete random variable.

x	0	1	2	3
$P(x)$	1/16	3/8	7/16	1/8

3.3 `stat/probability.fvl`

1. Suppose we roll two 6-sided dice, one red and one blue, whose faces are numbered from 1 to 6. What is the probability that we roll two numbers whose sum is exactly 10?

2. Suppose we roll two 6-sided dice, one pink and one green, with faces labeled 1 through 6. Compute the probability of the following events.

   (a) The dice show the same number.
   (b) The sum of the numbers on the dice is exactly 3.

3. Suppose we roll a single 12-sided die with faces labeled 1 through 12.

   (a) What is the sample space of this experiment?
   (b) Find the probabilities of the following events.
      i. Roll a 8
      ii. Roll a number divisible by 6
      iii. Roll a number greater than 6

4. Suppose we roll a single 20-sided die, whose faces are numbered from 1 to 20. What is the probability that we roll a number strictly less than 10?

   Solution: 9/20

5. Three cards are drawn from a standard deck without replacement. What is the probability that all three have the same face value? (E.g. all three are aces.)

6. Two cards are drawn from a standard deck without replacement. What is the probability that both have the same face value? (E.g. both are aces.)

7. Suppose we select a single card from a standard deck. Compute the probability of the following events.

   (a) The card is a 8.
   (b) The card is red.
   (c) The card is a heart.
   (d) The card is a face card (Jack, Queen, or King).

8. Suppose we draw a single card from a standard 52-card deck. What is the probability that we draw either a diamond or a face card?

   Solution: This is an event of the form E or F, where E is the event "draw a diamond" and F the event "draw a face card". There are 13 diamonds, 12 face cards, and 3 diamonds which are also face cards. Using the sum rule, we can say

$$\begin{aligned} P(E \text{ or } F) &= P(E) + P(F) - P(E \text{ and } F) \\ &= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} \\ &= \frac{11}{26} \end{aligned}$$

9. Suppose we draw a single card from a standard deck. What is the probability that the card is either a spade or a face card?

10. Suppose we flip a coin four times in a row, to get a sequence of coin flips. For example, if we flip heads, then tails, then heads, then heads, the result is (H, T, H, H). Write down the sample space for this experiment. Then compute the probability of the following events.

    (a) We flip tails four times.
    (b) We flip exactly two heads.
    (c) We flip at least three tails.
    (d) The first two flips are tails.

11. Suppose E and F are events of some experiment such that $P(E) = 0.3$, $P(F) = 0.2$, and $P(E \text{ OR } F) = 0.4$. What is $P(E \text{ AND } F)$?

3.4 `stat/word.fvl`

1. A survey was conducted to determine the study habits and final grades of statistics students. 291 stats students were asked whether or not they passed their stats class and whether they studied alone or with others. The results of the survey are collected in the following table.

	Pass	Fail
Study Alone	91	47
Study with Others	123	30

   Use this data to answer the following.

   (a) What is the probability that a given student passed statistics, given that they studied alone?

   (b) What is the probability that a given student studied alone, given that they passed statistics?

   Solution:

2. The Mathematics section of the ACT consists of 60 multiple choice questions, each with 5 possible answers. If a student decides to answer all of the questions randomly, we can model the number of correct answers as a binomial random variable.

(a) What are the median and standard deviation of the resulting distribution?

(b) Our friend claims to have guessed randomly at all 60 math questions on the ACT, and ends up with a score of 9 correct answers. Is this an unusual outcome? Why?

3. Mars Inc., the manufacturer of M&M candy, claims that 15% of plain M&Ms are red. To test this claim, we will collect a sample of 100 M&Ms and count the number of red ones.

   (a) If we model the number of red M&Ms in our sample using a binomial random variable, what are the mean and standard deviation of the resulting distribution?

   (b) Suppose we find that our sample has 9 red M&Ms. Is this an unusually large or small number? Why?

   Solution:

4. According to data from the U.S. Defense Department, in the first nine months of 2011 there were an average of 608 IED attacks per month worldwide. Using this information we can model the number of IED attacks in a given month using a Poisson random variable.

   (a) Find the standard deviation of the resulting distribution.

   (b) Suppose that in a given month there are 644 IED attacks worldwide. Is this an unusually large or small number? Why?

3.5 `stat/match.fvl`

1. Match each sampling method to its description.

   ____ Random Sampling

   ____ Simple Random Sampling

   ____ Convenience Sampling

   ____ Stratified Sampling

   ____ Cluster Sampling

   ____ Self-Selected Sampling

   A. Select individuals which are easy to find.

   B. Allow individuals to choose whether or not to be in the sample.

   C. Each individual has an equal chance of being selected.

   D. Each subset of a given size has an equal chance of being selected.

   E. Divide the population into subpopulations, then choose
   emphall individuals from
   emphsome subpopulations.

   F. Divide the population into subpopulations, then choose
   emphsome individuals from
   emphall subpopulations.

2. Match each word or phrase to its definition.

____ placebo effect

____ double-blind

____ histogram

____ random sampling error

____ measure of center

____ frequency table

____ nonrandom sampling error

____ time-series chart

____ scatterplot

____ longitudinal study

____ measure of dispersion

____ lurking variable

A. A way to summarize numerical data by counting the frequency of values in each of several intervals.

B. A way to quantify the 'middle' of a data set.

C. When we choose a sample which is very different from a given population, due to the use of an unsound sampling method.

D. A way to quantify how 'spread out' a data set is.

E. When we choose a sample using a sound sampling method, but the resulting sample is very different from the given population due to chance.

F. A type of bar chart, where each bar represents a range of values and the height of each bar represents the frequency of values in that range.

G. A factor which we do not measure while collecting data, but which affects the factors we do measure.

H. The observation that merely receiving treatment – even a sugar pill or saline injection – can cause some patients to get better.

I. A way to visualize data by plotting it as points on an xy-plane.

J. A medical study in which neither the patients nor the doctors know whether or not the treatment being given contains an active ingredient.

K. A method of collecting data from the same sample at regular intervals over a period of time.

L. A line graph whose x-axis represents a continuous variable, such as time.