Name: ______________________

Calculus 1: Review (Test 1)

1. Find the mean of the following list of numbers.

$$5,\ 5,\ 11,\ 11,\ 12,\ 11,\ 11,\ 10$$

Solution: Remember that to find the mean of n numbers x_1, x_2, …, x_n, we add them up and divide by the number of numbers. In this case,

$$\begin{aligned} \frac{5+5+11+11+12+11+11+10}{8} &= \frac{76}{8} \\ &\approx 9.5. \end{aligned}$$

2. Find the mean of the following list of numbers.

$$3,\ 2,\ 3,\ 2,\ 3,\ 1,\ 2,\ 2$$

Solution: Remember that to find the mean of n numbers x_1, x_2, …, x_n, we add them up and divide by the number of numbers. In this case,

$$\begin{aligned} \frac{3+2+3+2+3+1+2+2}{8} &= \frac{18}{8} \\ &\approx 2.2. \end{aligned}$$

3. Find the mean deviation of the following list of numbers.

$$3,\ 8,\ 4,\ 8,\ 6,\ 5$$

Solution: Remember that the mean deviation of x_1, x_2, …, x_n is

$$\frac{1}{n}\sum_{i=1}^{n} |x_i - \overline{x}|,$$

where $\overline{x}$ is the mean of the x_i. In this case the mean is $\overline{x} = 5.66$. Then the mean deviation is

$$\begin{aligned} & \frac{1}{6}\big(|3-5.66| + |8-5.66| + |4-5.66| + |8-5.66| + |6-5.66| + |5-5.66|\big) \\ = & \frac{1}{6}\big(2.66 + 2.33 + 1.66 + 2.33 + 0.33 + 0.66\big) \\ = & 1.66 \end{aligned}$$

4. Find the standard deviation of the following list of numbers.

$$11, 8, 7, 8, 11$$

Solution: Remember that the standard deviation of x_1, x_2, $\ldots$, x_n is

$$\sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}},$$

where $\overline{x}$ is the mean of the x_i. In this case the mean is $\overline{x} = 9.00$. Then the standard deviation is

$$\begin{aligned}
& \sqrt{\frac{1}{4}\big((11-9.00)^2 + (8-9.00)^2 + (7-9.00)^2 + (8-9.00)^2 + (11-9.00)^2\big)} \\
= \; & \sqrt{\frac{1}{4}\big(4.00 + 1.00 + 4.00 + 1.00 + 4.00\big)} \\
= \; & 1.67
\end{aligned}$$

5. Suppose we have collected the following list of numbers.

$$15, 19, 4, 7, 11, 8, 21, 14, 18, 10$$

Compute the z-scores of 2 and 24 with respect to this list.

Solution: Remember that the z-score of a particular number x with respect to a list of numbers is

$$z = \frac{x - \overline{x}}{s},$$

where $\overline{x}$ is the mean and s the standard deviation. In this case we can see that $\overline{x} = 12.7$ and $s = 5.330$, so that the z-score of 2 is

$$\frac{2 - 12.7}{5.330} = -2.007$$

and of 24 is

$$\frac{24 - 12.7}{5.330} = 2.120.$$

6. Suppose we have collected the following list of numbers.

$$6, 9, 10, 6, 8, 12, 8, 7, 8, 9$$

Compute the z-scores of 2 and 13 with respect to this list.

Solution: Remember that the z-score of a particular number x with respect to a list of numbers is

$$z = \frac{x - \overline{x}}{s},$$

where $\overline{x}$ is the mean and s the standard deviation. In this case we can see that $\overline{x} = 8.3$ and $s = 1.734$, so that the z-score of 2 is

$$\frac{2 - 8.3}{1.734} = -3.631$$

and of 13 is

$$\frac{13 - 8.3}{1.734} = 2.709.$$

7. Find the coefficient of variation of the following list of numbers.

$$16,\ 17,\ 19,\ 19,\ 14,\ 20$$

Solution: Remember that the coefficient of variation of a list of numbers is $100\% \cdot s/\overline{x}$, where s is the standard deviation and $\overline{x}$ the mean, expressed as a percentage. In this case the mean is $\overline{x} = 17.5$ and the standard deviation is $s = 2.06$, so the coefficient of variation is $100\% \cdot s/\overline{x} = 11\%$.

8. Suppose we roll a single 20-sided die, whose faces are numbered from 1 to 20. What is the probability that we roll a number strictly less than 4?

Solution: The die has 3 faces with numbers strictly less than 4. The probability of rolling one of these numbers is

$$P(E) = \frac{\#\text{ of ways to roll a number between 1 and 3}}{\#\text{ of possible outcomes}} = \frac{3}{20}.$$

So the probability is $\boxed{3/20}$.

9. Suppose we draw a single card from a standard 52-card deck. What is the probability that we draw either a club or a face card?

Solution: This is an event of the form "E or F", where E is the event "draw a club" and F the event "draw a face card". There are 13 clubs, 12 face cards, and 3 clubs which are also face cards. Using the sum rule, we can say

$$\begin{aligned} P(E \text{ or } F) &= P(E) + P(F) - P(E \text{ and } F) \\ &= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} \\ &= \frac{11}{26}. \end{aligned}$$

So the probability is $\boxed{11/26}$.

10. Suppose we roll two 6-sided dice, one orange and one blue, whose faces are numbered from 1 to 6. What is the probability that we roll two numbers whose sum is exactly 8?

Solution: One outcome of this experiment is an ordered pair of numbers, (a, b), where a is the number on the orange die and b the number on the blue die. Each die will come up a number between 1 and 6 (inclusive). We can visualize all possible outcomes as an array with one row for each outcome of the orange die and one column for each outcome of the blue die as follows.

		blue					
		1	2	3	4	5	6
orange	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Now the probability of rolling two numbers whose sum is exactly 8 is

$$P(E) = \frac{\#\text{ of ways to roll two numbers whose sum is 8}}{\#\text{ of possible outcomes}}.$$

The total number of outcomes is the number of cells in the body of the table above: $6 \times 6 = 36$. The number of outcomes which add to 8 is 5. So the probability is $\boxed{5/36}$.

11. Suppose we roll two 6-sided dice, one pink and one purple, with faces labeled 1 through 6. Compute the probability of the following events.

    (a) The dice show the same number.
    (b) The sum of the numbers on the dice is exactly 5.

    Solution: Each outcome of this experiment can be represented by an ordered pair of numbers (a, b), where a is the number on the pink die and b the number on the purple die. There are 36 possible outcomes. There are exactly 6 outcomes where these numbers are equal; namely, $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$, $(5, 5)$, and $(6, 6)$. So the probability of the dice showing the same face is $6/36 = \boxed{1/6}$. There are 4 ways to roll two numbers which add to 5, so the probability of rolling two numbers which add to 5 is $\boxed{1/9}$.

12. A survey was conducted to determine the study habits and final grades of statistics students. 268 stats students were asked whether or not they passed their stats class and whether they studied alone or with others. The results of the survey are collected in the following table.

	Pass	Fail
Study Alone	84	24
Study with Others	139	21

    Use this data to answer the following.

    (a) What is the probability that a given student passed statistics, given that they studied alone?
    (b) What is the probability that a given student studied alone, given that they passed statistics?

    Solution:

13. Match each sampling method to its description.

____ Random Sampling	A. Divide the population into subpopulations, then choose emphall individuals from emphsome subpopulations.
____ Simple Random Sampling	B. Each individual has an equal chance of being selected.
____ Convenience Sampling	C. Allow individuals to choose whether or not to be in the sample.
____ Stratified Sampling	D. Divide the population into subpopulations, then choose emphsome individuals from emphall subpopulations.
____ Cluster Sampling	E. Select individuals which are easy to find.
____ Self-Selected Sampling	F. Each subset of a given size has an equal chance of being selected.