

unless A and B happen to be the statement analogous to part of \cap , is not true.

$\{2, 3\}$ and $C = \{x, y\}$ and $D = \{2\}$ and $C \cap D = \{y\}$, and so verify that Theorem 3.3.8 (v) $\{2\}$ and $C \cap D = \{y\}$, and so further hand, we have $A \times C = \{(2, y), (2, z), (3, y), (3, z)\}$, Thus $(A \cap B) \times (C \cap D) =$

ulation. We then have $A \cup B = \{2, 3\}$ and $C \cup D = \{y, z\}$, Using $A \times C$ and $B \times D$ we have $(A \times C) \cup (B \times D) = \{(2, y), (2, z), (3, y), (3, z)\}$. Thus $(A \cup B) \times (C \cup D) = \{(2, y), (2, z), (3, y), (3, z)\}$. We see between the situation in this is schematically in Figure 3.3.4 \diamond

$\times (C \cup D)$



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$\{3, 4\}$. Find each of the following

$- B$,
 $- A$.

3.3.2. Let $C = \{a, b, c, d, e, f\}$ and $D = \{a, c, e\}$ and $E = \{d, e, f\}$ and $F = \{a, b\}$. Find each of the following sets.

- | | |
|------------------------|---------------------------------|
| (1) $C - (D \cup E)$. | (4) $F \cap (D \cup E)$. |
| (2) $(C - D) \cup E$. | (5) $(F \cap D) \cup E$. |
| (3) $F - (C - E)$. | (6) $(C - D) \cup (F \cap E)$. |

3.3.3. Let $X = [0, 5)$ and $Y = [2, 4]$ and $Z = (1, 3]$ and $W = (3, 5)$ be intervals in \mathbb{R} . Find each of the following sets.

- | | |
|------------------|---------------------------|
| (1) $Y \cup Z$. | (4) $X \times W$. |
| (2) $Z \cap W$. | (5) $(X \cap Y) \cup Z$. |
| (3) $Y - W$. | (6) $X - (Z \cup W)$. |

3.3.4. Let

$$G = \{n \in \mathbb{Z} \mid n = 2m \text{ for some } m \in \mathbb{Z}\}$$

$$H = \{n \in \mathbb{Z} \mid n = 3k \text{ for some } k \in \mathbb{Z}\}$$

$$I = \{n \in \mathbb{Z} \mid n^2 \text{ is odd}\}$$

$$J = \{n \in \mathbb{Z} \mid 0 \leq n \leq 10\}.$$

Find each of the following sets.

- | | |
|------------------|------------------------|
| (1) $G \cup I$. | (4) $J - G$. |
| (2) $G \cap I$. | (5) $I - H$. |
| (3) $G \cap H$. | (6) $J \cap (G - H)$. |

3.3.5. Given two sets A and B , consider the sets $A - B$ and $B - A$. Are they necessarily disjoint? Give a proof or a counterexample.

3.3.6. [Used in Section 3.3.] Prove Theorem 3.3.2 parts (i) – (iii) and (vi) – (ix).

3.3.7. [Used in Section 3.3.] Prove Theorem 3.3.5 parts (i) – (vi).

3.3.8. [Used in Section 3.3.] Prove Theorem 3.3.8 parts (i), (ii), (iv), (v).

3.3.9. Let X be a set, and let $A, B, C \subseteq X$. Suppose that $A \cap B = A \cap C$, and that $(X - A) \cap B = (X - A) \cap C$. Show that $B = C$.

3.3.10. Let A, B and C be sets. Show that $(A - B) \cap C = (A \cap C) - B = (A \cap C) - (B \cap C)$.

3.3.11. For real numbers a, b and c we know that $a - (b - c) = (a - b) + c$. Discover and prove a formula for $A - (B - C)$, where A, B and C are sets.