Name: ______________________

College Algebra: Review (Test 1)

1. Find all solutions of the following equation.

$$|-20x - 19| + 8 = 6$$

Solution: First, we subtract 8 from both sides, which gives us

$$|-20x - 19| = -2.$$

But remember that the absolute value always gives a positive number or zero; any solution of this equation would give us a number whose absolute value is negative. So this equation has $\boxed{\text{no solution}}$.

2. Find an equation for the line passing through the points $(4, 5)$ and $(-3, -5)$.

Solution: We will find the point-slope form of this line. First, using the slope formula, we find that the slope of this line is

$$m = \frac{(-5) - (5)}{(-3) - (4)} = 10/7.$$

Since we know this line passes through (for instance) $(4, 5)$, using the point-slope formula, an equation for this line is

$$\frac{y - 5}{x - 4} = 10/7.$$

We can solve for y to get this equation in slope-intercept form as follows:

$$\boxed{y = \frac{10}{7}x - \frac{5}{7}}$$

3. Convert the standard form linear equation

$$-6y + 6x = -7$$

to slope-intercept form.

Solution: To convert to slope-intercept form, we simply solve this equation for y to get

$$\boxed{y = x + \frac{7}{6}}$$

4. Find all solutions of the following equation.

$$\frac{x}{x-2} + 2 = \frac{2}{x-2}$$

Solution: First, we can clear the denominators of this equation by multiplying both sides by a common denominator; in this case, multiplying by $x - 2$ will work. That gives the equation

$$x + 2(x - 2) = 2.$$

(Remember! Multiplying by a variable may introduce extraneous solutions, so we will have to check our answers at the end.) Expanding this out and combining like terms gives $x = 2$. But plugging this value in for x in the original equation gives a zero in the denominator of a fraction. This is bad! So in fact this equation has $\boxed{\text{no solution}}$.

5. Find all solutions of the following equation.

$$\frac{1}{5} + \frac{1}{x-3} = 1$$

Solution: Usually, the first thing we do to an equation with fractions is clear the denominator. That would work here. But instead, let's subtract 1/5 from both sides first:

$$\frac{1}{x-3} = \frac{4}{5}.$$

When we have two fractions equal to each other, we can flip them both over, like so:

$$\frac{x-3}{1} = \frac{5}{4}.$$

From here we can add 3 and simplify, to get $x = 17/4$. Now while solving this equation, we implicitly multiplied both sides by a variable (when we flipped the fractions over). So we need to make sure that this x is not an extraneous solution. It isn't, so we have the solution $\boxed{x = 17/4}$.

6. Find all solutions of the following equation.

$$x^3 - 12x^2 + 32x = 0$$

Solution: Note that the terms of this polynomial have a common factor, namely x. Un-distributing this common factor gives the equation

$$x(x^2 - 12x + 32) = 0,$$

which factors further as

$$x(x - 8)(x - 4) = 0.$$

By the Zero Product Property, the solutions of this equation are $\boxed{x = 8,\ x = 4,\ \text{and } x = 0}$.

7. Find all solutions of the following equation.

$$2x^4 - 9x^2 + 10 = 0$$

Solution: This is a degree 4 polynomial. But note that if we make the substitution $y = x^2$, we can rewrite our equation as

$$2y^2 - 9y + 10 = 0,$$

which is quadratic. Now this equation factors as

$$(2y - 5)(y - 2) = 0,$$

and thus has two solutions: $y = 5/2$ or $y = 2$. Then $x^2 = 5/2$ or $x^2 = 2$, so that $\boxed{x = \pm\sqrt{5/2} \text{ or } x = \pm\sqrt{2}}$.

8. Find an equation for the circle centered at $(6, -1)$ and having radius 6.

Solution: Remember that the standard form equation of a circle centered at the point (h, k) and with radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

Here we have $(h, k) = (6, -1)$ and $r = 6$; so this circle is given by the equation

$$\boxed{(x - 6)^2 + (y + 1)^2 = 36}.$$

9. Find the midpoint of the points $(7, 3)$ and $(-7, 4)$.

Solution: Remember that the midpoint of the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Here, we have $(x_1, y_1) = (7, 3)$ and $(x_2, y_2) = (-7, 4)$, so that the midpoint is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{(7) + (-7)}{2}, \frac{(3) + (4)}{2}\right) = \left(\frac{0}{2}, \frac{7}{2}\right).$$

So the midpoint is $\boxed{(0, 7/2)}$.

10. Find all solutions of the following equation.

$$x^2 - 12x + 32 = 0$$

Solution: We can solve this equation by factoring. We have

$$x^2 - 12x + 32 = (x - 8)(x - 4) = 0,$$

which is true if $\boxed{x = 8 \text{ or } x = 4}$.

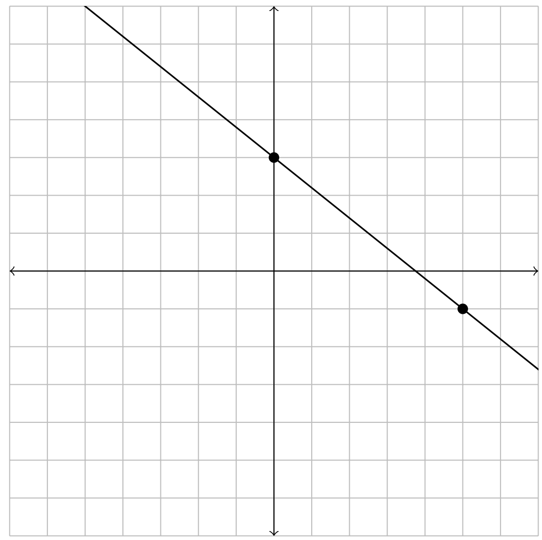
11. Find an equation for the line passing through the point $(7, -7)$ and having slope $2/5$.

    Solution: Remember that to uniquely identify a line in the plane, we need two pieces of information. In this case we know two things about this line: its slope, and a point on the line. The simplest linear equation form to use here is the point-slope form: the line with slope m and passing through the point (h, k) is given by the equation
    $$\frac{y-k}{x-h} = m.$$
    Here we have $m = 2/5$ and $(h, k) = (7, -7)$. So this line is given by the equation
    $$\frac{y+7}{x-7} = \frac{2}{5}.$$
    We can solve this equation for y to find the slope-intercept form; this yields
    $$\boxed{y = \frac{2}{5}x - \frac{49}{5}}$$
    .

12. Find all solutions of the following equation.
    $$\frac{x}{x-1} + 3 = \frac{9}{x-1}$$
    Solution: First, we can clear the denominators of this equation by multiplying both sides by a common denominator; in this case, multiplying by $x - 1$ will work. That gives the equation
    $$x + 3(x-1) = 9.$$
    (Remember! Multiplying by a variable may introduce extraneous solutions, so we will have to check our answers at the end.) Expanding this out and combining like terms gives $x = 3$. Since plugging this value in for x in the original equation does not make any denominators equal to zero, it is a solution. So $\boxed{x = 3}$.

13. Plot the graph of the linear equation $y = -\frac{4}{5}x + 3$ on the plane below.

    Solution: This line as slope $-4/5$ and y-intercept 3. We can use this information to find two points on the line and sketch as follows.



14. Find the distance between the points $(1, -1)$ and $(2, 1)$.

    Solution: Remember that the formula for the distance between (x_1, y_1) and (x_2, y_2) is

    $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

    Here we have $(x_1, y_1) = (1, -1)$ and $(x_2, y_2) = (2, 1)$, so that the formula gives

    $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((2) - (1))^2 + ((1) - (-1))^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5}.$$

    So the distance between these points is $\boxed{\sqrt{5}}$.

15. Find all solutions of the following equation.

    $$|3x + 5| + 10 = 21$$

    Solution: First we get the absolute value by itself on one side of the equation.

    $$|3x + 5| = 11$$

    Now we can split this equation into two: we have

    $$3x + 5 = 11 \quad \text{or} \quad 3x + 5 = -11.$$

    Solving these equations separately gives two solutions: $\boxed{x = 2 \text{ or } -16/3}$.

16. Find an equation in slope-intercept form for the line passing through the point $(1, 2)$ and parallel to $y = \frac{1}{2}x - 4$.

Solution: Let ℓ be the unknown line. Since ℓ is known to be parallel to $y = \frac{1}{2}x - 4$, the slope of ℓ is $m = 1/2$. We also know that ℓ passes through the point $(1, 2)$. So ℓ is given by the point-slope form equation

$$\frac{y-2}{x-1} = 1/2.$$

Solving for y yields the slope-intercept equation

$$\boxed{y = \frac{1}{2}x + \frac{3}{2}}$$

17. Find all solutions of the following equation.

$$x^2 - 3x - 4 = 0$$

Solution: We can solve this equation by factoring. We have

$$x^2 - 3x - 4 = (x+1)(x-4) = 0,$$

which is true if $\boxed{x = -1 \text{ or } x = 4}$.

18. Find all solutions of the following equation.

$$-6 - |4x + 9| = |4x + 9| - 6$$

Solution: First we can add 6 to both sides:

$$-|4x + 9| = |4x + 9|$$

From here we *could* split the equation in two, as we usually do with absolute value equations. That would give us two simpler equations, but each would still have an absolute value in it. Let's do something different: notice that the "stuff" in the absolute value signs is the same on both sides of this equation, so by adding $|4x + 9|$ to both sides, we can "combine like terms" like so:

$$2|4x + 9| = 0.$$

Dividing by 2 then gives us

$$|4x + 9| = 0,$$

which we can now split up as

$$4x + 9 = 0 \quad \text{or} \quad 4x + 9 = 0.$$

These equations are the same, so we really only need to solve one of them to get $\boxed{x = -9/4}$.

19. Find all solutions of the following equation.

$$|x^2 - 3x + 5| = 3$$

Solution: This absolue value equation is in the form $|A| = B$, so we can split it into two. In this case, we have the two equations

$$x^2 - 3x + 5 = 3 \qquad \text{or} \qquad x^2 - 3x + 5 = -3.$$

These simplify as

$$x^2 - 3x + 2 = 0 \qquad \text{or} \qquad x^2 - 3x + 8 = 0.$$

The first equation factors as

$$(x - 1)(x - 2) = 0,$$

giving two solutions, $x = 1$ and $x = 2$. The second quadratic can be solved using the quadratic formula, giving two more solutions: $x = (3 \pm \sqrt{-23})/2$. Thus we have four solutions altogether:

$$\boxed{x = 1 \text{ or } x = 2 \text{ or } x = \frac{3 \pm \sqrt{-23}}{2}}.$$