College Algebra

Test 3

Form A

Spring 2016

Name: ______________________________

Date: ______________________________

READ THESE INSTRUCTIONS CAREFULLY!

- Circle or underline your final written answer.
- Justify your reasoning and show your work.
- If you run out of space, make a note and continue your work on the back of a page.

1. Find all solutions of the following equation.

$$x^3 - 11x^2 + 24x = 0$$

Solution: Note that the terms of this polynomial have a common factor, namely x. Un-distributing this common factor gives the equation

$$x(x^2 - 11x + 24) = 0,$$

which factors further as

$$x(x - 8)(x - 3) = 0.$$

By the Zero Product Property, the solutions of this equation are $\boxed{x = 8,\ x = 3, \text{ and } x = 0}$.

2. Find all solutions of the following equation.

$$2x^4 - 5x^2 + 2 = 0$$

Solution: This is a degree 4 polynomial. But note that if we make the substitution $y = x^2$, we can rewrite our equation as

$$2y^2 - 5y + 2 = 0,$$

which is quadratic. Now this equation factors as

$$(2y - 1)(y - 2) = 0,$$

and thus has two solutions: $y = 1/2$ or $y = 2$. Then $x^2 = 1/2$ or $x^2 = 2$, so that $\boxed{x = \pm\sqrt{1/2} \text{ or } x = \pm\sqrt{2}}$.

3. Fill in the boxes to describe the long-term behavior of the following polynomial.

$$p(x) = 3x^3 - 2x + 1$$

.

As $x \to \infty$, $p(x) \to \boxed{}$, and as $x \to -\infty$, $p(x) \to \boxed{}$

Solution: Remember that to find the long term behavior of a polynomial, we need to know the *sign* (positive or negative) of the leading coefficient and the *parity* (even or odd) of the degree. In this case the degree (largest exponent) is 3 and the leading coefficient (coefficient on the highest-degree term) is 1.

Since $1 > 0$ and 3 is odd, as $x \to \infty$, $p(x) \to \boxed{\infty}$ and as $x \to -\infty$, $p(x) \to \boxed{-\infty}$.

4. Use synthetic division to find the quotient and remainder when

$$a(x) = x^5 - 6x^4 + 10x^3 - 11x + 6$$

is divided by $b(x) = x - 1$.

5. Find the list of candidate roots of the polynomial

$$p(x) = 5x^3 + 4x^2 - 5x + 9$$

given by the Rational Root Theorem. **Do not factor.**

Solution:

6. Construct a polynomial of degree 3 which has roots at 1, 2, and -1.

Solution: Remember that c is a root of $p(x)$ precisely when $x - c$ is a factor of $p(x)$. (This is called the Factor Theorem.) That means that we can force a polynomial to have a given root, say 1, by making $x - 1$ a factor.

In this case, a polynomial will have these roots if it has the factors $x - 1$, $x - 2$, and $x + 1$. For example,

$$\boxed{(x-1)(x-2)(x+1)}$$

works.

7. The polynomial

$$p(x) = x^5 + 2x^4 - 10x^3 - 20x^2 + 9x + 18$$

has roots at $-3; 1; -2; 3$. Completely factor $p(x)$ as a product of linear factors.

Solution: Using synthetic division we can see that $p(x)$ factors as

$$\boxed{(x+1)(x+2)(x+3)(x-1)(x-3)}.$$

8. The polynomial

$$p(x) = x^7 - 11x^6 + 51x^5 - 129x^4 + 192x^3 - 168x^2 + 80x - 16$$

has roots at 1 and 2. Find the multiplicity of these roots.

Solution: Recall that the *multiplicity* of a root c of a polynomial is the number of times $x - c$ appears as a factor. We can use synthetic division repeatedly to determine this number. In this case, we have

$$p(x) = (x-1)^3(x-2)^4;$$

so 1 has multiplicity $\boxed{3}$ and 2 has multiplicity $\boxed{4}$.

Bonus. The polynomial

$$p(x) = 2x^5 - 7x^4 - 3x^3 + 33x^2 - 35x + 10$$

has a root at $\sqrt{5}$. Completely factor $p(x)$ as a product of linear factors.