Names: ____________________ ____________________ ____________________

Activity #4: Functions (Solutions) **College Algebra**

1. Find the domain of the following function.

$$f(x) = \frac{4x^3 + x^2 + 4x + 8}{x^2 - 16}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

$$x^2 - 16 = 0.$$

This equation is a quadratic, and using our favorite solving strategy we see that its solutions are $x = 4$ and $x = -4$. So the domain of f is

$$\boxed{\text{all real numbers } \textit{except } 4 \text{ and } -4.}$$

2. Find the domain of the following function.

$$g(x) = \frac{1}{x^3 + 2x^2 - 48x}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

$$x^3 + 2x^2 - 48x = 0.$$

This equation is cubic, but it has no constant term, so we can factor out an x. That yields a quadratic which we can solve using our favorite strategy. We see that the solutions are $x = 0$, $x = -8$, and $x = 6$. So the domain of g is

$$\boxed{\text{all real numbers } \textit{except } 0, -8, \text{ and } 6.}$$

3. Find the domain of the following function.

$$f(x) = \sqrt{2x + 2}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a radical. This function will be defined as long as the expression in the radical is nonnegative. That is, at all solutions of the inequality

$$2x + 2 \geq 0.$$

Solving this inequality, we have $x \geq -1$. So the domain of f is

$$\boxed{\text{all real numbers } x \text{ such that } x \geq -1.}$$

4. Find the domain of the following function.

$$f(x) = \sqrt{|1x+5| - 4}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a radical. This function will be defined as long as the expression in the radical is nonnegative. That is, at all solutions of the inequality

$$|1x+5| - 4 \geq 0.$$

This is an absolute value inequality. Solving for the absolute value, we have

$$|1x+5| \geq 4.$$

This inequality can then be split into two like so:

$$1x+5 \geq 4 \quad \text{OR} \quad 1x+5 \leq -4.$$

The solution of this inequality is

$$x \geq -1 \quad \text{OR} \quad x \leq -9.$$

So the domain of f is

$$\boxed{\text{all real numbers } x \text{ such that } x \geq -1 \text{ or } x \leq -9.}$$

5. Evaluate the function

$$f(x) = 5x^3 + 6x + 4$$

at $x = 2$, $x = 0$, $x = -3$, and $x = 1/2$.

Solution: We have

$$\begin{aligned}
f(2) &= 5(2)^3 + 6(2) + 4 = \boxed{56} \\
f(0) &= 5(0)^3 + 6(0) + 4 = \boxed{4} \\
f(-3) &= 5(-3)^3 + 6(-3) + 4 = \boxed{-149} \\
f\left(\frac{1}{2}\right) &= 5\left(\frac{1}{2}\right)^3 + 6\left(\frac{1}{2}\right) + 4 = \boxed{61/8}
\end{aligned}$$

6. Evaluate the function

$$f(x) = \begin{cases} 6x - 3 & \text{if } x \geq 5 \\ \frac{1}{x^2-2} & \text{if } x < 5 \end{cases}$$

at $x = 1$, $x = 9$, and $x = -7$.

Solution:

7. Find all solutions of the following inequality.

$$|4x - 9| + 6 \geq 9$$

Solution: First, solve for the absolute value expression by subtracting 6 from both sides.

$$|4x - 9| \geq 3.$$

This is an absolute value inequality of the form "absolute value greater than", so we can now rewrite as a compound inequality as follows.

$$4x - 9 \geq 3 \quad \text{OR} \quad 4x - 9 \leq -3.$$

Solving each of these for x, we have

$$x \leq 3/2 \quad \text{OR} \quad 3 \leq x.$$

In interval notation, the solution is $\boxed{(-\infty, 3/2] \cup [3, \infty)}$.