Names: ______________________ ______________________ ______________________

Activity #2: Probability (Solutions) **Statistics**

1. Suppose we roll a single 20-sided die, whose faces are numbered from 1 to 20. What is the probability that we roll a number strictly less than 9?

   Solution: The die has 8 faces with numbers strictly less than 9. The probability of rolling one of these numbers is
   $$P(E) = \frac{\text{\# of ways to roll a number between 1 and 8}}{\text{\# of possible outcomes}} = \frac{8}{20}.$$
   So the probability is $\boxed{2/5}$.

2. Suppose we draw a single card from a standard 52-card deck. What is the probability that we draw either a spade or a face card?

   Solution: This is an event of the form "E or F", where E is the event "draw a spade" and F the event "draw a face card". There are 13 spades, 12 face cards, and 3 spades which are also face cards. Using the sum rule, we can say
   $$\begin{aligned} P(E \text{ or } F) &= P(E) + P(F) - P(E \text{ and } F) \\ &= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} \\ &= \frac{11}{26}. \end{aligned}$$
   So the probability is $\boxed{11/26}$.

3. Suppose we roll two 6-sided dice, one pink and one purple, whose faces are numbered from 1 to 6. What is the probability that we roll two numbers whose sum is exactly 10?

   Solution: One outcome of this experiment is an ordered pair of numbers, (a, b), where a is the number on the pink die and b the number on the purple die. Each die will come up a number between 1 and 6 (inclusive). We can visualize all possible outcomes as an array with one row for each outcome of the pink die and one column for each outcome of the purple die as follows.

		purple					
		1	2	3	4	5	6
pink	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

   Now the probability of rolling two numbers whose sum is exactly 10 is
   $$P(E) = \frac{\text{\# of ways to roll two numbers whose sum is 10}}{\text{\# of possible outcomes}}.$$
   The total number of outcomes is the number of cells in the body of the table above: $6 \times 6 = 36$. The number of outcomes which add to 10 is 3. So the probability is $\boxed{1/12}$.

4. Suppose we roll a single 12-sided die with faces labeled 1 through 12.

   (a) What is the sample space of this experiment?
   (b) Find the probabilities of the following events.
      i. Roll a 5
      ii. Roll a number greater than 8

   Solution: The sample space of this experiment is the set of whole numbers from 1 to 12. That is,

   $$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}.$$

   The probability of rolling exactly 5 is $\boxed{1/12}$. There are 4 faces labeled with numbers greater than 8, so the probability of rolling a number greater than 8 is $\boxed{1/3}$.

5. Suppose we roll two 6-sided dice, one pink and one purple, with faces labeled 1 through 6. Compute the probability of the following events.

   (a) The dice show the same number.
   (b) The sum of the numbers on the dice is exactly 4.

   Solution: Each outcome of this experiment can be represented by an ordered pair of numbers (a, b), where a is the number on the pink die and b the number on the purple die. There are 36 possible outcomes. There are exactly 6 outcomes where these numbers are equal; namely, $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$, $(5, 5)$, and $(6, 6)$. So the probability of the dice showing the same face is $6/36 = \boxed{1/6}$. There are 3 ways to roll two numbers which add to 4, so the probability of rolling two numbers which add to 4 is $\boxed{1/12}$.

6. Suppose we select a single card from a standard deck. Compute the probability of the following events.

   (a) The card is a 5.
   (b) The card is red.
   (c) The card is a spade.
   (d) The card is a face card (Jack, Queen, or King).

   Solution: There are 52 cards in a standard deck. Four of these have rank 5, so the probability of drawing one is $4/52 = \boxed{1/13}$. There are 26 red cards, and the probability of drawing one of these is $26/52 = \boxed{1/2}$. Thirteen of the cards are spades, so the probability of drawing one is $13/52 = \boxed{1/4}$. There are 12 face cards, so the probability of drawing one is $12/52 = \boxed{3/13}$.

7. Suppose we flip a coin four times in a row, to get a sequence of coin flips. For example, if we flip heads, then tails, then heads, then heads, the result is (H, T, H, H). Write down the sample space for this experiment. Then compute the probability of the following events.

   (a) We flip tails four times.
   (b) We flip exactly two heads.
   (c) We flip at least three tails.
   (d) The first two flips are tails.

   Solution: This experiment has 16 total outcomes:

(H,H,H,H)	(H,H,H,T)	(H,H,T,H)	(H,H,T,T)
(H,T,H,H)	(H,T,H,T)	(H,T,T,H)	(H,T,T,T)
(T,H,H,H)	(T,H,H,T)	(T,H,T,H)	(T,H,T,T)
(T,T,H,H)	(T,T,H,T)	(T,T,T,H)	(T,T,T,T)

Exactly one outcome consists of four Ts, so the probability of this event is $\boxed{1/16}$. Six outcomes consist of exactly two heads:

$$(H,H,T,T),(H,T,H,T),(H,T,T,H),(T,H,H,T),(T,H,T,H),(T,T,H,H).$$

So the probability of flipping exactly two heads is $6/16 = \boxed{3/8}$. There are five ways to flip at least three tails: four ways to flip exactly three tails,

$$(T,T,T,H),(T,T,H,T),(T,H,T,T),(H,T,T,T)$$

and one way to flip four tails, (T,T,T,T). So the probability of flipping at least three tails is $\boxed{5/16}$. There are four possible outcomes where the first two flips are both tails:

$$(T,T,T,T),(T,T,T,H),(T,T,H,T),(T,T,H,H).$$

So the probability of flipping tails on both of the first two flips is $4/16 = \boxed{1/4}$.

8. Suppose E and F are events of some experiment such that $P(E) = 0.3$, $P(F) = 0.3$, and $P(E \text{ OR } F) = 0.5$. What is $P(E \text{ AND } F)$?

Solution: Remember that the Sum Formula for probabilities states that for any two events E and F, we have

$$P(E \text{ OR } F) = P(E) + P(F) - P(E \text{ AND } F).$$

Here, we know three of the terms in this equation, and thus can solve for the fourth.

$$0.5 = 0.3 + 0.3 - P(E \text{ AND } F)$$

so that $\boxed{P(E \text{ AND } F) = 0.1}$.