(13) $\neg S(a)$	(12), (9), Modus Toll. Pon.
(14) $(\exists x \text{ in } U)[\neg S(x)]$	(13), Existential Gen.

Note that in line (4) we chose some letter "a" that was not in use prior to this line, since we are using Existential Instantiation. We needed to use this rule of inference at that point in the derivation in order to remove the quantifier in line (3) of the premises, thus allowing us to use the rules of inference given in Section 1.4 (which did not involve quantifiers). In lines (7) and (10) we were free to use the same letter "a" as in line (4), since Universal Instantiation allows us to choose anything in U that we want.

Exercises

1.5.1. Suppose that the possible values of x are all people. Let $Y(x) =$ "x has green hair," let $Z(x) =$ "x likes pickles" and $W(x) =$ "x has a pet frog." Translate the following statements into words.

(1) $(\forall x)Y(x)$.
(2) $(\exists x)Z(x)$.
(3) $(\forall x)[W(x) \wedge Z(x)]$.
(4) $(\exists x)[Y(x) \rightarrow W(x)]$.
(5) $(\forall x)[W(x) \leftrightarrow \neg Z(x)]$.

1.5.2. Suppose that the possible values of x and y are all cars. Let $L(x, y) =$ "x is as fast as y," let $M(x, y) =$ "x is as expensive as y" and $N(x, y) =$ "x is as old as y." Translate the following statements into words.

(1) $(\exists x)(\forall y)L(x, y)$.
(2) $(\forall x)(\exists y)M(x, y)$.
(3) $(\exists y)(\forall x)[L(x, y) \vee N(x, y)]$.
(4) $(\forall y)(\exists x)[\neg M(x, y) \rightarrow L(x, y)]$.

1.5.3. Suppose that the possible values of x are all cows. Let $P(y) =$ "y is brown," let $Q(y) =$ "y is four years old" and $R(y) =$ "y has white spots." Translate the following statements into symbols.

(1) There is a brown cow.

(2) All cows are four years old.

(3) There is a brown cow with white spots.

(4) All four year old cows have white spots.

(5) There exists a cow such that if it is four years old, then it has no white spots.

(6) All cows are brown if and only if they are not four years old.

(7) There are no brown cows.