Name: ______________________

College Algebra: Review (Test 3)

1. Find all solutions of the following equation.

$$x^3 - 14x^2 + 45x = 0$$

Solution: Note that the terms of this polynomial have a common factor, namely x. Un-distributing this common factor gives the equation

$$x(x^2 - 14x + 45) = 0,$$

which factors further as

$$x(x-9)(x-5) = 0.$$

By the Zero Product Property, the solutions of this equation are $\boxed{x = 9,\ x = 5, \text{ and } x = 0}$.

2. Find all solutions of the following equation.

$$2x^4 - 13x^2 + 6 = 0$$

Solution: This is a degree 4 polynomial. But note that if we make the substitution $y = x^2$, we can rewrite our equation as

$$2y^2 - 13y + 6 = 0,$$

which is quadratic. Now this equation factors as

$$(2y-1)(y-6) = 0,$$

and thus has two solutions: $y = 1/2$ or $y = 6$. Then $x^2 = 1/2$ or $x^2 = 6$, so that $\boxed{x = \pm\sqrt{1/2} \text{ or } x = \pm\sqrt{6}}$.

3. Compute the following product.

$$(x-1)(x+1)(2x+1)$$

Solution: Using the distributive property (or FOIL), the product is $\boxed{x^3 - x^2 - x + 1}$.

4. Fill in the boxes to describe the long-term behavior of the following polynomial.

$$p(x) = 2x^4 + 5x^2 - 3$$

.

As $x \to \infty$, $p(x) \to \boxed{}$, and as $x \to -\infty$, $p(x) \to \boxed{}$

Solution: Remember that to find the long term behavior of a polynomial, we need to know the *sign* (positive or negative) of the leading coefficient and the *parity* (even or odd) of the degree. In this case the degree (largest exponent) is 4 and the leading coefficient (coefficient on the highest-degree term) is -3.

Since $-3 > 0$ and 4 is even, as $x \to \infty$, $p(x) \to \boxed{\infty}$ and as $x \to -\infty$, $p(x) \to \boxed{\infty}$.

5. Fill in the boxes to describe the long-term behavior of the following polynomial.

$$p(x) = -7x^6 + 13x^2 - x + 1$$

.

As $x \to \infty$, $p(x) \to \boxed{}$, and as $x \to -\infty$, $p(x) \to \boxed{}$

Solution: Remember that to find the long term behavior of a polynomial, we need to know the *sign* (positive or negative) of the leading coefficient and the *parity* (even or odd) of the degree. In this case the degree (largest exponent) is 6 and the leading coefficient (coefficient on the highest-degree term) is 1.

Since $1 < 0$ and 6 is even, as $x \to \infty$, $p(x) \to \boxed{-\infty}$ and as $x \to -\infty$, $p(x) \to \boxed{-\infty}$.

6. Use synthetic division to find the quotient and remainder when

$$a(x) = x^5 - 6x^4 + 10x^3 - 11x + 6$$

is divided by $b(x) = x - 1$.

7. Find the list of candidate roots of the polynomial

$$p(x) = 5x^3 + 3x^2 - 4x + 4$$

given by the Rational Root Theorem. **Do not factor.**

Solution:

8. Construct a polynomial of degree 3 which has roots at 2, 1, and -1.

Solution: Remember that c is a root of $p(x)$ precisely when $x - c$ is a factor of $p(x)$. (This is called the Factor Theorem.) That means that we can force a polynomial to have a given root, say 1, by making $x - 1$ a factor.

In this case, a polynomial will have these roots if it has the factors $x - 2$, $x - 1$, and $x + 1$. For example,

$$\boxed{(x-2)(x-1)(x+1)}$$

works.

9. Construct a polynomial having roots at 5, 3, and -4.

   Solution: Remember that c is a root of $p(x)$ precisely when $x - c$ is a factor of $p(x)$. (This is called the Factor Theorem.) That means that we can force a polynomial to have a given root, say 1, by making $x - 1$ a factor.

   In this case, a polynomial will have these roots if it has the factors $x - 5$, $x - 3$, and $x + 4$. For example,

   $$\boxed{(x-5)(x-3)(x+4)}$$

   works.

10. Construct a polynomial having roots at 1, -2, and 1/2.

    Solution: Remember that c is a root of $p(x)$ precisely when $x - c$ is a factor of $p(x)$. (This is called the Factor Theorem.) That means that we can force a polynomial to have a given root, say 1, by making $x - 1$ a factor.

    For fraction roots, like 1/2, we can do a little better. Instead of using the factor $x - \frac{1}{2}$, we can use $2x - 1$. To see why, multiply both sides of the equation $x - \frac{1}{2} = 0$ by 2.

    In this case, a polynomial will have these roots if it has the factors $x - 1$, $x + 2$, and $2x - 1$. For example,

    $$\boxed{(x-1)(x+2)(2x-1)}$$

    works.

11. The polynomial

    $$p(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$$

    has roots at $1; -1; 2; -2$. Completely factor $p(x)$ as a product of linear factors.

    Solution: Using synthetic division we can see that $p(x)$ factors as

    $$\boxed{(x-3)(x-1)(x+1)(x-2)(x+2)}.$$

12. The polynomial

    $$p(x) = 2x^5 + 3x^4 - 21x^3 - 29x^2 + 27x + 18$$

    has roots at $-3; 3; 1$. Completely factor $p(x)$ as a product of linear factors.

    Solution: Using synthetic division we can see that $p(x)$ factors as

$$\boxed{(x-3)(x+2)(x+3)(x-1)(2x+1)}.$$

13. The polynomial

$$p(x) = x^7 - 10x^6 + 42x^5 - 96x^4 + 129x^3 - 102x^2 + 44x - 8$$

has roots at 1 and 2. Find the multiplicity of these roots.

Solution: Recall that the *multiplicity* of a root c of a polynomial is the number of times $x - c$ appears as a factor. We can use synthetic division repeatedly to determine this number. In this case, we have

$$p(x) = (x-1)^4(x-2)^3;$$

so 1 has multiplicity $\boxed{4}$ and 2 has multiplicity $\boxed{3}$.

14. Factor the following polynomial.

$$p(x) = 4x^4 + 8x^3 - 7x^2 - 11x + 6$$

Solution: