Names: ______________________ ______________________ ______________________

Activity #8: Quadratic-ish Equations (Solutions) **College Algebra**

1. Find all solutions of the following equation.

$$x^3 - 16x^2 + 63x = 0$$

Solution: Note that the terms of this polynomial have a common factor, namely x. Un-distributing this common factor gives the equation

$$x(x^2 - 16x + 63) = 0,$$

which factors further as

$$x(x-9)(x-7) = 0.$$

By the Zero Product Property, the solutions of this equation are $\boxed{x = 9,\ x = 7, \text{ and } x = 0}$.

2. Find all solutions of the following equation.

$$2x^4 - 7x^2 + 6 = 0$$

Solution: This is a degree 4 polynomial. But note that if we make the substitution $y = x^2$, we can rewrite our equation as

$$2y^2 - 7y + 6 = 0,$$

which is quadratic. Now this equation factors as

$$(2y-3)(y-2) = 0,$$

and thus has two solutions: $y = 3/2$ or $y = 2$. Then $x^2 = 3/2$ or $x^2 = 2$, so that $\boxed{x = \pm\sqrt{3/2} \text{ or } x = \pm\sqrt{2}}$.

3. Compute the following product.

$$(x-1)(x-1)(x+1)$$

Solution: Using the distributive property (or FOIL), the product is $\boxed{x^3 - x^2 - x + 1}$.

4. Compute the following product.

$$(x+1)(x-1)(2x-1)$$

Solution: Using the distributive property (or FOIL), the product is $\boxed{x^3 + x^2 - x - 1}$.

5. Fill in the boxes to describe the long-term behavior of the following polynomial.

$$p(x) = 3x^3 - 2x + 1$$

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As $x \to \infty$, $p(x) \to \boxed{}$, and as $x \to -\infty$, $p(x) \to \boxed{}$

Solution: Remember that to find the long term behavior of a polynomial, we need to know the *sign* (positive or negative) of the leading coefficient and the *parity* (even or odd) of the degree. In this case the degree (largest exponent) is 3 and the leading coefficient (coefficient on the highest-degree term) is 1.

Since $1 > 0$ and 3 is odd, as $x \to \infty$, $p(x) \to \boxed{\infty}$ and as $x \to -\infty$, $p(x) \to \boxed{-\infty}$.

6. Fill in the boxes to describe the long-term behavior of the following polynomial.

$$p(x) = -3x^3 + 5x^2 + 1$$

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As $x \to \infty$, $p(x) \to \boxed{}$, and as $x \to -\infty$, $p(x) \to \boxed{}$

Solution: Remember that to find the long term behavior of a polynomial, we need to know the *sign* (positive or negative) of the leading coefficient and the *parity* (even or odd) of the degree. In this case the degree (largest exponent) is 3 and the leading coefficient (coefficient on the highest-degree term) is 1.

Since $1 < 0$ and 3 is odd, as $x \to \infty$, $p(x) \to \boxed{-\infty}$ and as $x \to -\infty$, $p(x) \to \boxed{\infty}$.

7. Use synthetic division to find the quotient and remainder when

$$a(x) = x^5 - x^4 - 9x^3 + 5x^2 + 16x - 12$$

is divided by $b(x) = x - 3$.