Name: ______________________

**Homework #2** **Abstract Algebra**

1. Recall that if $R$ is a ring and $A$ a nonempty set, then $R^A$ is the set of all functions $f : A \to R$. In class we defined a pointwise arithmetic on $R^A$ as follows: given functions $\alpha, \beta : A \to R$, we define

$$(\alpha + \beta)(x) = \alpha(x) + \beta(x)$$

and

$$(\alpha\beta)(x) = \alpha(x)\beta(x).$$

   (a) Show that these operations make $R^A$ into a ring.

   (b) Show that if $R$ is commutative, then $R^A$ is also commutative.

2. Suppose $R$, $S$, and $T$ are rings. Prove that

$$(R \oplus S) \oplus T \cong R \oplus (S \oplus T).$$

3. Suppose $R_1$ and $R_2$ are rings, and that $S_1 \subseteq R_1$ and $S_2 \subseteq R_2$ are subrings. Show that $S_1 \oplus S_2$ is a subring of $R_1 \oplus R_2$.

4. Let $R = \mathbb{Z}[i]$ be the ring of Gaussian integers.

   (a) Show that 29 is not irreducible in $R$. (Hint: Try to write 29 as a sum of squares.)

   (b) Find an irreducible factorization for 29 in $R$.

   (c) Show that 3 is irreducible in $R$. (Hint: Suppose $3 = (a + bi)(c + di)$ is a nontrivial factorization, and consider the norm of both sides. Remember that the norm on $\mathbb{Z}[i]$ is $N(a + bi) = a^2 + b^2$.)