Names: ____________________ ____________________ ____________________

Activity #4: Some Geometry (Solutions) **College Algebra**

1. Find an equation for the line passing through the point $(4, 2)$ and having slope -2/3.

   Solution: Remember that to uniquely identify a line in the plane, we need two pieces of information. In this case we know two things about this line: its slope, and a point on the line. The simplest linear equation form to use here is the point-slope form: the line with slope m and passing through the point (h, k) is given by the equation
   $$\frac{y-k}{x-h} = m.$$
   Here we have $m = -2/3$ and $(h, k) = (4, 2)$. So this line is given by the equation
   $$\frac{y-2}{x-4} = \frac{-2}{3}.$$
   We can solve this equation for y to find the slope-intercept form; this yields
   $$\boxed{y = -\frac{2}{3}x + \frac{14}{3}}$$
   .

2. Find the slope between the points $(2, -4)$ and $(-1, 7)$.

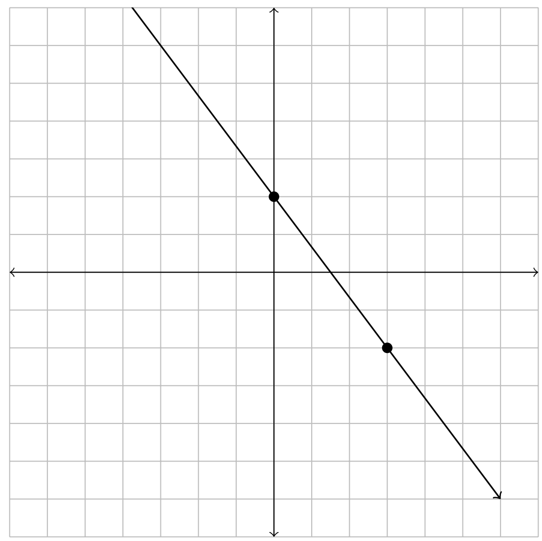
   Solution: Remember that the slope between the points (x_1, y_1) and (x_2, y_2) is
   $$\frac{y_2 - y_1}{x_2 - x_1}.$$
   Here, we have $(x_1, y_1) = (2, -4)$ and $(x_2, y_2) = (-1, 7)$, so that the slope between them is
   $$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(7) - (-4)}{(-1) - (2)} = \frac{11}{-3}.$$
   So the slope between these points is $\boxed{\text{-11/3}}$.

3. Find the distance between the points $(-2, -2)$ and $(-3, -5)$.

   Solution: Remember that the formula for the distance between (x_1, y_1) and (x_2, y_2) is
   $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$
   Here we have $(x_1, y_1) = (-2, -2)$ and $(x_2, y_2) = (-3, -5)$, so that the formula gives
   $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-3) - (-2))^2 + ((-5) - (-2))^2} = \sqrt{(-1)^2 + (-3)^2} = \sqrt{10}.$$
   So the distance between these points is $\boxed{\sqrt{10}}$.

4. Plot the graph of the linear equation $y = -\frac{4}{3}x + 2$ on the plane below.

Solution: This line as slope $-4/3$ and y-intercept 2. We can use this information to find two points on the line and sketch as follows.



5. Find the slope between the points $(2, 1)$ and $(2, -1)$.

Solution: Remember that the slope between the points (x_1, y_1) and (x_2, y_2) is

$$\frac{y_2 - y_1}{x_2 - x_1}.$$

Here, we have $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (2, -1)$, so that the slope between them is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (1)}{(2) - (2)} = \frac{-2}{0}.$$

We have a problem: the denominator of this fraction is zero. So the slope between these points is $\boxed{\text{undefined}}$.

6. Find the midpoint of the points $(6, -5)$ and $(-1, 1)$.

Solution: Remember that the midpoint of the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Here, we have $(x_1, y_1) = (6, -5)$ and $(x_2, y_2) = (-1, 1)$, so that the midpoint is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{(6) + (-1)}{2}, \frac{(-5) + (1)}{2}\right) = \left(\frac{5}{2}, \frac{-4}{2}\right).$$

So the midpoint is $\boxed{(5/2, -2)}$.

7. Find an equation for the circle centered at $(1, 3)$ and having radius 4.

   Solution: Remember that the standard form equation of a circle centered at the point (h, k) and with radius r is
   $$(x-h)^2 + (y-k)^2 = r^2.$$
   Here we have $(h, k) = (1, 3)$ and $r = 4$; so this circle is given by the equation
   $$\boxed{(x-1)^2 + (y-3)^2 = 16}.$$

8. Find an equation for the circle centered at $(5, 2)$ and passing through $(-5, -4)$.

   Solution: The radius of this circle is the distance from the center, $(5, 2)$, to the point $(-5, -4)$. That distance is
   $$\sqrt{136}.$$
   Now the circle with center at (h, k) and radius r is given by the equation
   $$(x-h)^2 + (y-k)^2 = r^2.$$
   Thus this circle is given by the equation
   $$\boxed{(x-5)^2 + (y-2)^2 = 136}.$$

9. Find an equation for the line passing through the points $(2, 3)$ and $(-7, 1)$.

   Solution: We will find the point-slope form of this line. First, using the slope formula, we find that the slope of this line is
   $$m = \frac{(1)-(3)}{(-7)-(2)} = 2/9.$$
   Since we know this line passes through (for instance) $(2, 3)$, using the point-slope formula, an equation for this line is
   $$\frac{y-3}{x-2} = 2/9.$$
   We can solve for y to get this equation in slope-intercept form as follows:
   $$\boxed{y = \frac{2}{9}x + \frac{23}{9}}$$

10. Convert the standard form linear equation
    $$-4y + 4x = -2$$
    to slope-intercept form.

Solution: To convert to slope-intercept form, we simply solve this equation for y to get

$$\boxed{y = x + \frac{1}{2}}$$

11. Find an equation in slope-intercept form for the line passing through the point $(1, 3)$ and parallel to $y = \frac{1}{2}x - 1$.

    Solution: Let ℓ be the unknown line. Since ℓ is known to be parallel to $y = \frac{1}{2}x - 1$, the slope of ℓ is $m = 1/2$. We also know that ℓ passes through the point $(1, 3)$. So ℓ is given by the point-slope form equation

    $$\frac{y-3}{x-1} = 1/2.$$

    Solving for y yields the slope-intercept equation

    $$\boxed{y = \frac{1}{2}x + \frac{5}{2}}$$