Names: ______________________ ______________________ ______________________

Activity #9: Polynomials (Solutions) **College Algebra**

1. Construct a polynomial of degree 3 which has roots at 1, -1, and -2.

   Solution: Remember that c is a root of $p(x)$ precisely when $x - c$ is a factor of $p(x)$. (This is called the Factor Theorem.) That means that we can force a polynomial to have a given root, say 1, by making $x - 1$ a factor.

   In this case, a polynomial will have these roots if it has the factors $x - 1$, $x + 1$, and $x + 2$. For example,

   $$\boxed{(x-1)(x+1)(x+2)}$$

   works.

2. Construct a polynomial having roots at -4, 5, and 1.

   Solution: Remember that c is a root of $p(x)$ precisely when $x - c$ is a factor of $p(x)$. (This is called the Factor Theorem.) That means that we can force a polynomial to have a given root, say 1, by making $x - 1$ a factor.

   In this case, a polynomial will have these roots if it has the factors $x + 4$, $x - 5$, and $x - 1$. For example,

   $$\boxed{(x+4)(x-5)(x-1)}$$

   works.

3. Construct a polynomial having roots at 5, 3, and 1/2.

   Solution: Remember that c is a root of $p(x)$ precisely when $x - c$ is a factor of $p(x)$. (This is called the Factor Theorem.) That means that we can force a polynomial to have a given root, say 1, by making $x - 1$ a factor.

   For fraction roots, like 1/2, we can do a little better. Instead of using the factor $x - \frac{1}{2}$, we can use $2x - 1$. To see why, multiply both sides of the equation $x - \frac{1}{2} = 0$ by 2.

   In this case, a polynomial will have these roots if it has the factors $x - 5$, $x - 3$, and $2x - 1$. For example,

   $$\boxed{(x-5)(x-3)(2x-1)}$$

   works.

4. The polynomial

   $$p(x) = x^5 - 4x^4 - 5x^3 + 20x^2 + 4x - 16$$

   has roots at $1; -1; 2; -2$. Completely factor $p(x)$ as a product of linear factors.

   Solution: Using synthetic division we can see that $p(x)$ factors as

$$\boxed{(x-2)(x+2)(x-4)(x+1)(x-1)}.$$

5. The polynomial

$$p(x) = 2x^5 - 5x^4 - 29x^3 + 41x^2 + 99x + 36$$

has roots at $3; -1; -3$. Completely factor $p(x)$ as a product of linear factors.

Solution: Using synthetic division we can see that $p(x)$ factors as

$$\boxed{(x-3)(x-4)(x+3)(x+1)(2x+1)}.$$

6. The polynomial

$$p(x) = x^7 - 11x^6 + 51x^5 - 129x^4 + 192x^3 - 168x^2 + 80x - 16$$

has roots at 1 and 2. Find the multiplicity of these roots.

Solution: Recall that the *multiplicity* of a root c of a polynomial is the number of times $x - c$ appears as a factor. We can use synthetic division repeatedly to determine this number. In this case, we have

$$p(x) = (x-1)^3(x-2)^4;$$

so 1 has multiplicity $\boxed{3}$ and 2 has multiplicity $\boxed{4}$.

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