

ements about Irene is a tautology,  
quite intuitively reasonable. It is  
licated (and unintuitive) tautologies  
uth table of the statement  $((P \wedge Q)$

$\rightarrow$	$(P$	$\rightarrow$	$(Q$	$\rightarrow$	$R))$
T	T	T	T	T	T
T	T	F	T	F	F
T	T	T	F	T	T
T	T	T	F	T	F
T	F	T	T	T	T
T	F	T	T	F	F
T	F	T	F	T	T
T	F	T	F	T	F
11	9	10	6	8	7

at  $((P \wedge Q) \rightarrow R) \rightarrow (P \rightarrow (Q$   
hether each of  $P$ ,  $Q$  and  $R$  are true  
ogy. Suppose we let  $P$  = "Sam is  
"Sam and Warren eat pasta." Then  
at if Sam and Warren are both sad  
if Sam is sad, then if Warren is sad  
e reader can verify with a truth table  
 $\wedge Q$  is always false.

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ions are statements?

nk about traveling to Spain.

will be unhappy.

me.

ions are statements?

$$(1) 4 < 3.$$

$$(2) \text{ If } x \geq 2 \text{ then } x^3 \geq 1.$$

$$(3) y < 7.$$

$$(4) x + y = z.$$

$$(5) (a + b)^2 = a^2 + 2ab + b^2.$$

$$(6) a^2 + b^2 = c^2.$$

$$(7) \text{ If } w = 3 \text{ then } z^w \neq 0.$$

**1.2.3.** Let  $P$  = "I like fruit," let  $Q$  = "I do not like cereal" and  $R$  = "I know how to cook an omelette." Translate the following statements into words.

$$(1) P \wedge Q.$$

$$(2) Q \vee R.$$

$$(3) \neg R.$$

$$(4) \neg(P \vee Q).$$

$$(5) \neg P \vee \neg Q.$$

$$(6) \neg P \vee Q.$$

$$(7) (R \wedge P) \vee Q.$$

$$(8) R \wedge (P \vee Q).$$

**1.2.4.** Let  $X$  = "I am happy," let  $Y$  = "I am watching a movie" and  $Z$  = "I am eating spaghetti." Translate the following statements into words.

$$(1) Z \rightarrow X.$$

$$(2) X \leftrightarrow Y.$$

$$(3) (Y \vee Z) \rightarrow X.$$

$$(4) Y \vee (Z \rightarrow X).$$

$$(5) (Y \rightarrow \neg X) \wedge (Z \rightarrow \neg X).$$

$$(6) (X \wedge \neg Y) \leftrightarrow (Y \vee Z).$$

**1.2.5.** Let  $X$  = "Fred has red hair," let  $Y$  = "Fred has a big nose" and  $R$  = "Fred likes to eat figs." Translate the following statements into symbols.

(1) Fred does not like to eat figs.

(2) Fred has red hair, and does not have a big nose.

(3) Fred has red hair or he likes to eat figs.

(4) Fred likes to eat figs, and he has red hair or he has a big nose.

(5) Fred likes to eat figs and he has red hair, or he has a big nose.

(6) It is not the case that Fred has a big nose or he has red hair.

(7) It is not the case that Fred has a big nose, or he has red hair.

(8) Fred has a big nose and red hair, or he has a big nose and likes to eat figs.

**1.2.6.** Let  $E$  = "The house is blue," let  $F$  = "The house is 30 years old" and  $G$  = "The house is ugly." Translate the following statements into symbols.

(1) If the house is 30 years old, then it is ugly.

(2) If the house is blue, then it is ugly or it is 30 years old.

(3) If the house is blue then it is ugly, or it is 30 years old.

(4) The house is not ugly if and only if it is 30 years old.

**1.2.11.** Make a truth table for each of the following statements.

- |                                  |  |
|----------------------------------|--|
| (1) $P \wedge \neg Q$ .          | (4) $(A \vee B) \wedge (A \vee C)$ .       |
| (2) $(R \vee S) \wedge \neg R$ . | (5) $(P \wedge R) \vee \neg(Q \wedge S)$ . |
| (3) $X \vee (\neg Y \vee Z)$ .   |  |

**1.2.12.** Make a truth table for each of the following statements.

- |   |   |
|---|---|
| (1) $X \rightarrow \neg Y$ .                | (4) $(E \leftrightarrow F) \rightarrow (E \leftrightarrow G)$ . |
| (2) $(R \rightarrow S) \leftrightarrow R$ . | (5) $(P \rightarrow R) \vee \neg(Q \leftrightarrow S)$ .        |
| (3) $\neg M \rightarrow (N \wedge L)$ .     |   |

**1.2.13.** Which of the following statements is a tautology, which is a contradiction and which is neither?

- (1)  $P \vee (\neg P \wedge Q)$ .
- (2)  $(X \vee Y) \leftrightarrow (\neg X \rightarrow Y)$ .
- (3)  $(A \wedge \neg B) \wedge (\neg A \vee B)$ .
- (4)  $(Z \vee (\neg Z \vee W)) \wedge \neg(W \wedge U)$ .
- (5)  $(L \rightarrow (M \rightarrow N)) \rightarrow (M \rightarrow (L \rightarrow N))$ .
- (6)  $((X \leftrightarrow Z) \wedge (X \leftrightarrow Y)) \wedge X$ .
- (7)  $((P \leftrightarrow \neg Q) \wedge P) \wedge Q$ .

**1.2.14.** Which of the following statements is a tautology, which is a contradiction and which is neither?

- (1) If John eats a blueberry pizza, then he either eats a blueberry pizza or he does not.
- (2) If John either eats a blueberry pizza or he does not, then he eats a blueberry pizza.
- (3) If pigs have wings and pigs do not have wings, then the sun sets in the east.
- (4) If Ethel goes to the movies then Agnes will eat a cake, and Agnes does not eat cake, and Ethel goes to the movies.
- (5) Rabbits eat cake or pie, and if rabbits eat pie then they eat cake.
- (6) The cow is green or the cow is not green, iff the goat is blue and the goat is not blue.

**1.2.15.** Let  $P$  be a statement, let  $TA$  be a tautology, and let  $CO$  be a contradiction.

- (1) Show that  $P \vee TA$  is a tautology.
- (2) Show that  $P \wedge CO$  is a contradiction.