Name: ______________________

College Algebra: Review (Test 2)

1. Find an equation for the line passing through the point $(3, -6)$ and having slope -2/5.

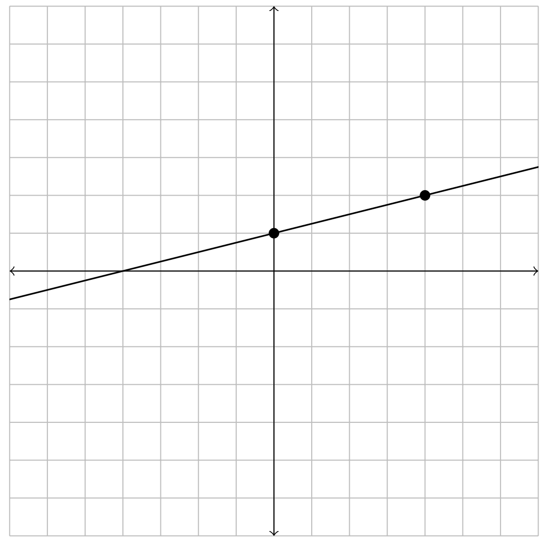
   Solution: Remember that to uniquely identify a line in the plane, we need two pieces of information. In this case we know two things about this line: its slope, and a point on the line. The simplest linear equation form to use here is the point-slope form: the line with slope m and passing through the point (h, k) is given by the equation
   $$\frac{y-k}{x-h} = m.$$
   Here we have $m = -2/5$ and $(h, k) = (3, -6)$. So this line is given by the equation
   $$\frac{y+6}{x-3} = \frac{-2}{5}.$$
   We can solve this equation for y to find the slope-intercept form; this yields
   $$\boxed{y = -\frac{2}{5}x - \frac{24}{5}}$$
   .

2. Find the distance between the points $(-5, 5)$ and $(-2, 4)$.

   Solution: Remember that the formula for the distance between (x_1, y_1) and (x_2, y_2) is
   $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$
   Here we have $(x_1, y_1) = (-5, 5)$ and $(x_2, y_2) = (-2, 4)$, so that the formula gives
   $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-2) - (-5))^2 + ((4) - (5))^2} = \sqrt{(3)^2 + (-1)^2} = \sqrt{10}.$$
   So the distance between these points is $\boxed{\sqrt{10}}$.

3. Plot the graph of the linear equation $y = \frac{1}{4}x + 1$ on the plane below.

   Solution: This line as slope $1/4$ and y-intercept 1. We can use this information to find two points on the line and sketch as follows.



4. Find the slope between the points $(6, 3)$ and $(6, -4)$.

   Solution: Remember that the slope between the points (x_1, y_1) and (x_2, y_2) is

   $$\frac{y_2 - y_1}{x_2 - x_1}.$$

   Here, we have $(x_1, y_1) = (6, 3)$ and $(x_2, y_2) = (6, -4)$, so that the slope between them is

   $$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4) - (3)}{(6) - (6)} = \frac{-7}{0}.$$

   We have a problem: the denominator of this fraction is zero. So the slope between these points is $\boxed{\text{undefined}}$.

5. Find the midpoint of the points $(3, 3)$ and $(-3, 5)$.

   Solution: Remember that the midpoint of the points (x_1, y_1) and (x_2, y_2) is

   $$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

   Here, we have $(x_1, y_1) = (3, 3)$ and $(x_2, y_2) = (-3, 5)$, so that the midpoint is

   $$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{(3) + (-3)}{2}, \frac{(3) + (5)}{2}\right) = \left(\frac{0}{2}, \frac{8}{2}\right).$$

   So the midpoint is $\boxed{(0, 4)}$.

6. Find an equation for the circle centered at $(3, -2)$ and passing through $(-3, -1)$.

   Solution: The radius of this circle is the distance from the center, $(3, -2)$, to the point $(-3, -1)$. That distance is

   $$\sqrt{37}.$$

   Now the circle with center at (h, k) and radius r is given by the equation

   $$(x - h)^2 + (y - k)^2 = r^2.$$

   Thus this circle is given by the equation

   $$\boxed{(x - 3)^2 + (y + 2)^2 = 37}.$$

7. Find an equation for the line passing through the points $(5, 3)$ and $(-3, 6)$.

   Solution: We will find the point-slope form of this line. First, using the slope formula, we find that the slope of this line is

   $$m = \frac{(6) - (3)}{(-3) - (5)} = -3/8.$$

   Since we know this line passes through (for instance) $(5, 3)$, using the point-slope formula, an equation for this line is

   $$\frac{y - 3}{x - 5} = -3/8.$$

   We can solve for y to get this equation in slope-intercept form as follows:

   $$\boxed{y = -\frac{3}{8}x + \frac{39}{8}}$$

8. Convert the standard form linear equation

   $$-6y + x = -4$$

   to slope-intercept form.

   Solution: To convert to slope-intercept form, we simply solve this equation for y to get

   $$\boxed{y = \frac{1}{6}x + \frac{2}{3}}$$

9. Find an equation in slope-intercept form for the line passing through the point $(2, 4)$ and parallel to $y = \frac{1}{2}x - 3$.

   Solution: Let ℓ be the unknown line. Since ℓ is known to be parallel to $y = \frac{1}{2}x - 3$, the slope of ℓ is $m = 1/2$. We also know that ℓ passes through the point $(2, 4)$. So ℓ is given by the point-slope form equation

   $$\frac{y - 4}{x - 2} = 1/2.$$

Solving for y yields the slope-intercept equation

$$\boxed{y = \frac{1}{2}x + 3}$$

10. Evaluate the function

$$f(x) = 4x^3 + 6x + 3$$

at $x = 2$, $x = 0$, $x = -3$, and $x = 1/2$.

Solution: We have

$$\begin{aligned}
f(2) &= 4(2)^3 + 6(2) + 3 = \boxed{47} \\
f(0) &= 4(0)^3 + 6(0) + 3 = \boxed{3} \\
f(-3) &= 4(-3)^3 + 6(-3) + 3 = \boxed{-123} \\
f\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^3 + 6\left(\frac{1}{2}\right) + 3 = \boxed{13/2}
\end{aligned}$$

11. Evaluate the function

$$f(x) = \begin{cases} 5x - 3 & \text{if } x \geq 4 \\ \dfrac{1}{x^2 - 3} & \text{if } x < 4 \end{cases}$$

at $x = 8$, $x = 1$, and $x = -2$.

Solution: This is a *piecewise defined* function, so remember that before we can evaluate f at a particular x we have to test x against the guards.

First we'll find $f(8)$. Since $8 \geq 4$, we use the first branch of f. So

$$f(8) = 5 \cdot 8 - 3 = \boxed{37}.$$

Next we'll find $f(1)$. Since $1 < 4$, we use the second branch of f. So

$$f(1) = 5 \cdot 1 - 3 = \boxed{2}.$$

Finally, we'll find $f(-2)$. Since $-2 < 4$, we use the second branch of f. So

$$f(-2) = 5 \cdot (-2) - 3 = \boxed{-13}.$$

12. Let $f(x) = 5x + 2$ and $g(x) = x^2 - 4$. Compute the following.

(a) $(f \circ g)(-2)$
(b) $(g \circ f)(-2)$

(c) $(f \circ g)(x)$

Solution: Recall that $(f \circ g)(x) = f(g(x))$ for all x. So we have the following.

$$(f \circ g)(-2) = f(g(-2)) = f(0) = \boxed{2}$$

$$(g \circ f)(-2) = g(f(-2)) = g(-8) = \boxed{60}$$

$$(f \circ g)(x) = f(x^2 - 4) = \boxed{-18}$$

13. Find the domain of the following function.

$$f(x) = \frac{2x^3 + x^2 + 6x + 8}{x^2 + x - 12}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

$$x^2 + x - 12 = 0.$$

This equation is a quadratic, and using our favorite solving strategy we see that its solutions are $x = 3$ and $x = -4$. So the domain of f is

$$\boxed{\text{all real numbers } \textit{except } 3 \text{ and } -4.}$$

14. Find the domain of the following function.

$$f(x) = \sqrt{2x + 9}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a radical. This function will be defined as long as the expression in the radical is nonnegative. That is, at all solutions of the inequality

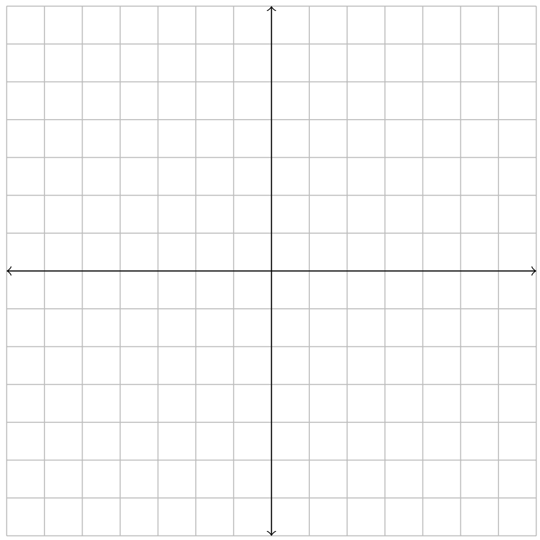
$$2x + 9 \geq 0.$$

Solving this inequality, we have $x \geq -9/2$. So the domain of f is

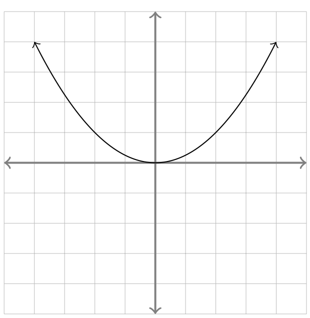
$$\boxed{\text{all real numbers } x \text{ such that } x \geq -9/2.}$$

15. Sketch the graph of the following equation in the space provided.

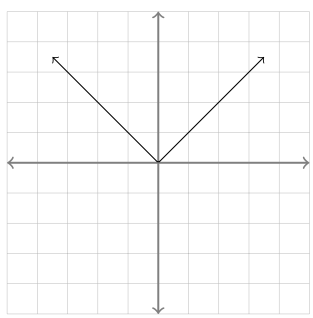
$$(x+3)^2 + (y-5)^2 = 1$$



16. Determine whether or not the following graphs are symmetric across the x-axis, across the y-axis, or about the origin.



x-axis: yes/no
y-axis: yes/no
origin: yes/no



x-axis: yes/no
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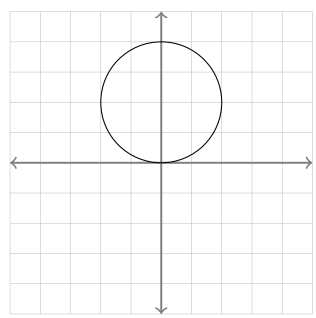
origin: yes/no



x-axis: yes/no
y-axis: yes/no
origin: yes/no

17. Determine whether or not the following equations are symmetric across the x-axis, across the y-axis, about the origin, or none of the three.

(a) $y^3 - 1 = x^3 - 2$

(b) $y^3 = xy - 3$

(c) $y^2 = x^3 - x$