College Algebra

Test 2

Form A

Spring 2017

Name: ______________________________

Date: ______________________________

READ THESE INSTRUCTIONS CAREFULLY!

- Circle or underline your final written answer.
- Justify your reasoning and show your work.
- If you run out of space, make a note and continue your work on the back of a page.

Algebra Facts

Quadratic Formula

If a, b, and c are real numbers and $a \neq 0$, then the solutions of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Absolute Value

- If $|E| = F$, then either $E = F$ or $E = -F$.
- If $|E| \leq F$, then both $E \leq F$ and $E \geq -F$.
- If $|E| \geq F$, then either $E \geq F$ or $E \leq -F$.

Geometry Formulas

Given points (x_1, y_1) and (x_2, y_2), the distance between them is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

their midpoint is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right),$$

and the slope between them is

$$\frac{y_2 - y_1}{x_2 - x_1}.$$

Circles

The circle having center (h, k) and radius r is given by the equation

$$(x - h)^2 + (y - k)^2 = r^2$$

Lines

The **standard form** equation of a line looks like

$$ax + by + c = 0,$$

where a, b, and c are constants. The **slope-intercept form** is

$$y = mx + b,$$

where m is the slope of the line and b the y-intercept. The **point-slope form** is

$$y - y_0 = m(x - x_0),$$

where m is the slope and (x_0, y_0) is any point on the line.

Transformations

$$\begin{array}{ccll}
x & \mapsto & x - h & \text{Horizontal Shift} \\
y & \mapsto & y - k & \text{Vertical Shift} \\
 & & & \\
x & \mapsto & \dfrac{1}{a}x & \text{Horizontal Stretch} \\
 & & & \\
y & \mapsto & \dfrac{1}{b}y & \text{Vertical Stretch}
\end{array}$$

1. (10 pts.) Find an equation for the line passing through the point $(-6, 1)$ and having slope $1/3$.

Solution: Remember that to uniquely identify a line in the plane, we need two pieces of information. In this case we know two things about this line: its slope, and a point on the line. The simplest linear equation form to use here is the point-slope form: the line with slope m and passing through the point (h, k) is given by the equation

$$\frac{y-k}{x-h} = m.$$

Here we have $m = 1/3$ and $(h, k) = (-6, 1)$. So this line is given by the equation

$$\frac{y-1}{x+6} = \frac{1}{3}.$$

We can solve this equation for y to find the slope-intercept form; this yields

$$\boxed{y = \frac{1}{3}x + 3}$$

.

2. (10 pts.) Find the distance between the points $(3, 1)$ and $(4, 1)$.

Solution: Remember that the formula for the distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

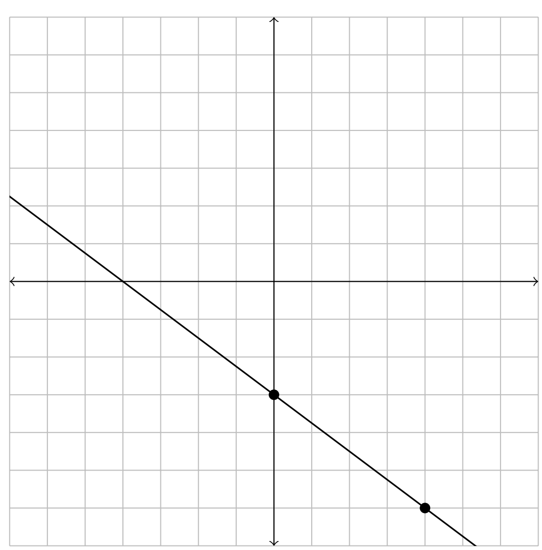
Here we have $(x_1, y_1) = (3, 1)$ and $(x_2, y_2) = (4, 1)$, so that the formula gives

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((4) - (3))^2 + ((1) - (1))^2} = \sqrt{(1)^2 + (0)^2} = \sqrt{1}.$$

So the distance between these points is $\boxed{\sqrt{1}}$.

3. (10 pts.) Plot the graph of the linear equation $y = -\frac{3}{4}x - 3$ on the plane below.

Solution: This line as slope $-3/4$ and y-intercept -3. We can use this information to find two points on the line and sketch as follows.



4. (10 pts.) Find an equation for the line passing through the points $(4,1)$ and $(-5,4)$.

Solution: We will find the point-slope form of this line. First, using the slope formula, we find that the slope of this line is

$$m = \frac{(4)-(1)}{(-5)-(4)} = -1/3.$$

Since we know this line passes through (for instance) $(4,1)$, using the point-slope formula, an equation for this line is

$$\frac{y-1}{x-4} = -1/3.$$

We can solve for y to get this equation in slope-intercept form as follows:

$$\boxed{y = -\frac{1}{3}x + \frac{7}{3}}$$

5. (10 pts.) Convert the standard form linear equation

$$7y + 3x = -1$$

to slope-intercept form.

Solution: To convert to slope-intercept form, we simply solve this equation for y to get

$$\boxed{y = -\frac{3}{7}x - \frac{1}{7}}$$

6. (10 pts.) Find an equation in slope-intercept form for the line passing through the point $(4,2)$ and parallel to $y = \frac{1}{2}x - 1$.

Solution: Let ℓ be the unknown line. Since ℓ is known to be parallel to $y = \frac{1}{2}x - 1$, the slope of ℓ is $m = 1/2$. We also know that ℓ passes through the point $(4,2)$. So ℓ is given by the point-slope form equation

$$\frac{y-2}{x-4} = 1/2.$$

Solving for y yields the slope-intercept equation

$$\boxed{y = \frac{1}{2}x}$$

7. (10 pts.) Let $f(x) = 6x + 2$ and $g(x) = x^2 - 4$. Compute the following.

(a) $(f \circ g)(-1)$
(b) $(g \circ f)(-1)$
(c) $(f \circ g)(x)$

Solution: Recall that $(f \circ g)(x) = f(g(x))$ for all x. So we have the following.

$$(f \circ g)(-1) = f(g(-1)) = f(-3) = \boxed{-16}$$

$$(g \circ f)(-1) = g(f(-1)) = g(-4) = \boxed{12}$$

$$(f \circ g)(x) = f(x^2 - 4) = \boxed{-22}$$

8. (10 pts.) Find the domain of the following function.

$$f(x) = \frac{2x^3 + x^2 + 3x + 8}{x^2 - 4}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

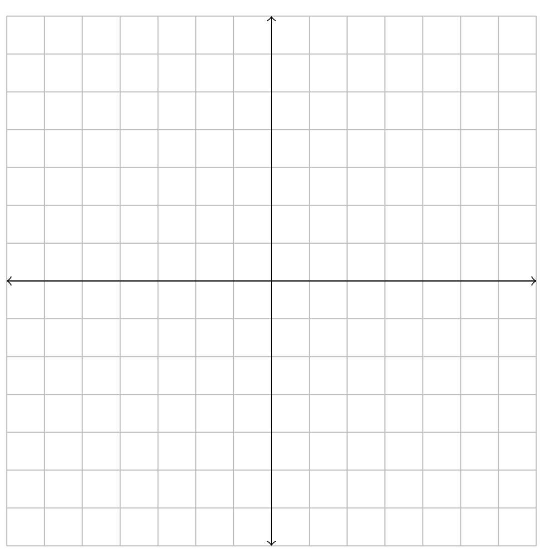
$$x^2 - 4 = 0.$$

This equation is a quadratic, and using our favorite solving strategy we see that its solutions are $x = 2$ and $x = -2$. So the domain of f is

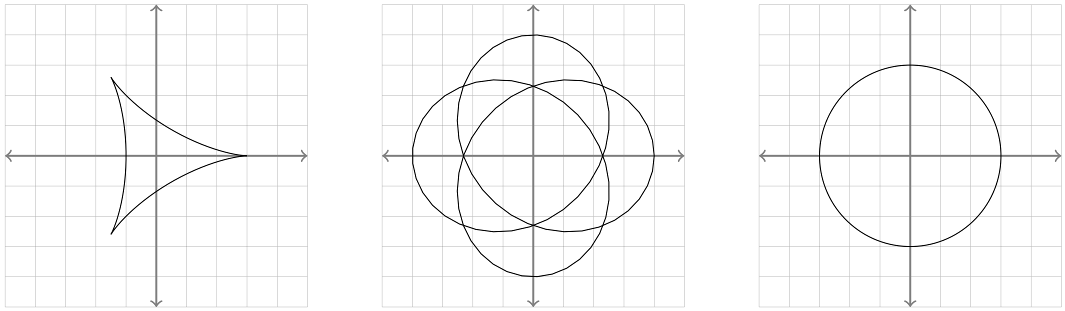
$$\boxed{\text{all real numbers } \textit{except } 2 \text{ and } -2.}$$

9. (10 pts.) Sketch the graph of the following equation in the space provided.

$$(x-3)^2 + (y-4)^2 = 1$$



10. (10 pts.) Determine whether or not the following graphs are symmetric across the x-axis, across the y-axis, or about the origin.



x-axis: yes/no
y-axis: yes/no
origin: yes/no

x-axis: yes/no
y-axis: yes/no
origin: yes/no

x-axis: yes/no
y-axis: yes/no
origin: yes/no

(Bonus.) Find the inverse of $f(x) = 2x + 5$.

Solution: If a function has an inverse, we can *sometimes* find it by swapping the roles of x and $f(x)$ and then solving for $f(x)$. In this case, we write our equation as

$$x = 2f(x) + 5$$

and then solve for $f(x)$ as

$$\boxed{f(x) = \frac{x-5}{2}}$$