Name: ______________________

College Algebra: Review (Test 1)

1. Find all solutions of the following equation.

$$|10x + 8| + 5 = 1$$

Solution: First, we subtract 5 from both sides, which gives us

$$|10x + 8| = -4.$$

But remember that the absolute value always gives a positive number or zero; any solution of this equation would give us a number whose absolute value is negative. So this equation has $\boxed{\text{no solution}}$.

2. Find all solutions of the following inequality.

$$|-3x + 6| + 15 \leq 30$$

Solution: First, solve for the absolute value expression by subtracting 15 from both sides.

$$|-3x + 6| \leq 15.$$

This is an absolute value inequality of the form "absolute value less than", so we can now rewrite as a compound inequality as follows.

$$-3x + 6 \leq 15 \quad \text{AND} \quad -3x + 6 \geq -15.$$

Solving each of these for x, we have

$$7 \geq x \quad \text{AND} \quad x \geq -3.$$

(Remember to change the direction of the inequality when dividing by -3!) In interval notation, the solution is $\boxed{[-3, 7]}$.

3. Find all solutions of the following equation.

$$\frac{x}{x-5} + 5 = \frac{5}{x-5}$$

Solution: First, we can clear the denominators of this equation by multiplying both sides by a common denominator; in this case, multiplying by $x - 5$ will work. That gives the equation

$$x + 5(x - 5) = 5.$$

(Remember! Multiplying by a variable may introduce extraneous solutions, so we will have to check our answers at the end.) Expanding this out and combining like terms gives $x = 5$. But plugging this value in for x in the original equation gives a zero in the denominator of a fraction. This is bad! So in fact this equation has $\boxed{\text{no solution}}$.

4. Find all solutions of the following equation.

$$\frac{6}{7} + \frac{1}{x-3} = 1$$

Solution: Usually, the first thing we do to an equation with fractions is clear the denominator. That would work here. But instead, let's subtract 6/7 from both sides first:

$$\frac{1}{x-3} = \frac{1}{7}.$$

When we have two fractions equal to each other, we can flip them both over, like so:

$$\frac{x-3}{1} = \frac{7}{1}.$$

From here we can add 3 and simplify, to get $x = 10$. Now while solving this equation, we implicitly multiplied both sides by a variable (when we flipped the fractions over). So we need to make sure that this x is not an extraneous solution. It isn't, so we have the solution $\boxed{x = 10}$.

5. Find all solutions of the following inequality.

$$|-3x-5| + 5 \geq 15$$

Solution: First, solve for the absolute value expression by subtracting 5 from both sides.

$$|-3x-5| \geq 10.$$

This is an absolute value inequality of the form "absolute value greater than", so we can now rewrite as a compound inequality as follows.

$$-3x - 5 \geq 10 \quad \text{OR} \quad -3x - 5 \leq -10.$$

Solving each of these for x, we have

$$x \geq 5/3 \quad \text{OR} \quad -5 \geq x.$$

(Remember to change the direction of the inequality when dividing by -3!) In interval notation, the solution is $\boxed{(-\infty, -5] \cup [5/3, \infty)}$.

6. Find all solutions of the following equation.

$$2x^2 - 3x - 20 = 0$$

Solution: We can solve this equation by factoring. We have

$$2x^2 - 3x - 20 = (2x+5)(x-4) = 0,$$

which is true if $\boxed{x = -5/2 \text{ or } x = 4}$. (We can also use the quadratic formula.)

7. Find all solutions of the following equation.

$$x^2 - 6x + 6 = 0$$

Solution: We can try factoring this equation, but there are no two integers whose sum is -6 and whose product is 6. Instead we will use the quadratic formula as follows.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(6)}}{2} \\ &= \frac{6 \pm \sqrt{12}}{2} \\ &= \frac{6 \pm 2\sqrt{3}}{2} \\ &= 3 \pm 1\sqrt{3} \end{aligned}$$

Thus $\boxed{x = 3 \pm 1\sqrt{3}}$.

8. Find all solutions of the following equation.

$$x^2 - 13x + 36 = 0$$

Solution: We can solve this equation by factoring. We have

$$x^2 - 13x + 36 = (x - 9)(x - 4) = 0,$$

which is true if $\boxed{x = 9 \text{ or } x = 4}$.

9. Find all solutions of the following equation.

$$3x^2 - 11x + 10 = 0$$

Solution: We can solve this equation by factoring. We have

$$3x^2 - 11x + 10 = (3x - 5)(x - 2) = 0,$$

which is true if $\boxed{x = 5/3 \text{ or } x = 2}$. (We can also use the quadratic formula.)

10. Find all solutions of the following equation.

$$\frac{x}{x-4} + 3 = \frac{1}{x-4}$$

Solution: First, we can clear the denominators of this equation by multiplying both sides by a common denominator; in this case, multiplying by $x - 4$ will work. That gives the equation

$$x + 3(x - 4) = 1.$$

(Remember! Multiplying by a variable may introduce extraneous solutions, so we will have to check our answers at the end.) Expanding this out and combining like terms gives $x = 13/4$. Since plugging this value in for x in the original equation does not make any denominators equal to zero, it is a solution. So $\boxed{x = 13/4}$.

11. Find all solutions of the following equation.

$$|5x + 7| + 8 = 25$$

Solution: First we get the absolute value by itself on one side of the equation.

$$|5x + 7| = 17$$

Now we can split this equation into two: we have

$$5x + 7 = 17 \quad \text{or} \quad 5x + 7 = -17.$$

Solving these equations separately gives two solutions: $\boxed{x = 2 \text{ or } -24/5}$.

12. Find all solutions of the following equation.

$$x^2 - 5x + 6 = 0$$

Solution: We can solve this equation by factoring. We have

$$x^2 - 5x + 6 = (x - 2)(x - 3) = 0,$$

which is true if $\boxed{x = 2 \text{ or } x = 3}$.

13. Find all solutions of the following equation.

$$-6 - |2x - 5| = |2x - 5| - 6$$

Solution: First we can add 6 to both sides:

$$-|2x - 5| = |2x - 5|$$

From here we *could* split the equation in two, as we usually do with absolute value equations. That would give us two simpler equations, but each would still have an absolute value in it. Let's do something different: notice that the "stuff" in the absolute value signs is the same on both sides of this equation, so by adding $|2x - 5|$ to both sides, we can "combine like terms" like so:

$$2|2x - 5| = 0.$$

Dividing by 2 then gives us

$$|2x - 5| = 0,$$

which we can now split up as

$$2x - 5 = 0 \quad \text{or} \quad 2x - 5 = 0.$$

These equations are the same, so we really only need to solve one of them to get $\boxed{x = 5/2}$.

14. Find all solutions of the following equation.

$$x^2 - 8x + 4 = 0$$

Solution: We can try factoring this equation, but there are no two integers whose sum is -8 and whose product is 4. Instead we will use the quadratic formula as follows.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2} \\ &= \frac{8 \pm \sqrt{48}}{2} \\ &= \frac{8 \pm 4\sqrt{3}}{2} \\ &= 4 \pm 2\sqrt{3} \end{aligned}$$

Thus $\boxed{x = 4 \pm 2\sqrt{3}}$.