Names: ____________________ ____________________ ____________________

Activity #6: Domains (Solutions) **College Algebra**

1. Find the domain of the following function.

$$f(x) = \frac{5x^3 + x^2 + 7x + 8}{x^2 + x - 12}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

$$x^2 + x - 12 = 0.$$

This equation is a quadratic, and using our favorite solving strategy we see that its solutions are $x = 3$ and $x = -4$. So the domain of f is

$$\boxed{\text{all real numbers } \textit{except } 3 \text{ and } -4.}$$

2. Find the domain of the following function.

$$g(x) = \frac{1}{x^3 - 11x^2 + 28x}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

$$x^3 - 11x^2 + 28x = 0.$$

This equation is cubic, but it has no constant term, so we can factor out an x. That yields a quadratic which we can solve using our favorite strategy. We see that the solutions are $x = 0$, $x = 7$, and $x = 4$. So the domain of g is

$$\boxed{\text{all real numbers } \textit{except } 0, 7, \text{ and } 4.}$$

3. Find the domain of the following function.

$$g(x) = \frac{1}{x^3 - x^2 - 56x}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a denominator. This function will be defined as long as that denominator is not zero. That is, at all real numbers *except* the solutions of the equation

$$x^3 - x^2 - 56x = 0.$$

This equation is cubic, but it has no constant term, so we can factor out an x. That yields a quadratic which we can solve using our favorite strategy. We see that the solutions are $x = 0$, $x = -7$, and $x = 8$. So the domain of g is

$$\boxed{\text{all real numbers } \textit{except } 0, -7, \text{ and } 8.}$$

4. Find the domain of the following function.

$$f(x) = \sqrt{7x+1}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a radical. This function will be defined as long as the expression in the radical is nonnegative. That is, at all solutions of the inequality

$$7x + 1 \geq 0.$$

Solving this inequality, we have $x \geq -1/7$. So the domain of f is

$$\boxed{\text{all real numbers } x \text{ such that } x \geq -1/7.}$$

5. Find the domain of the following function.

$$f(x) = \sqrt{|9x+7| - 6}$$

Solution: Remember that two bad things can happen which may cause a number *not* to be in the domain of a function; variables in denominators and variables in radicals. Here we have a variable in a radical. This function will be defined as long as the expression in the radical is nonnegative. That is, at all solutions of the inequality

$$|9x + 7| - 6 \geq 0.$$

This is an absolute value inequality. Solving for the absolute value, we have

$$|9x + 7| \geq 6.$$

This inequality can then be split into two like so:

$$9x + 7 \geq 6 \quad \text{OR} \quad 9x + 7 \leq -6.$$

The solution of this inequality is

$$x \geq -1/9 \quad \text{OR} \quad x \leq -13/9.$$

So the domain of f is

$$\boxed{\text{all real numbers } x \text{ such that } x \geq -1/9 \text{ or } x \leq -13/9.}$$