Solution

Since y is 20,000, we solve the following equation to find x:

$$20{,}000 = 355.9x + 11{,}075.3$$

$$8924.7 = 355.9x \qquad \text{Subtract 11,075.3 from each side.}$$

$$25.08 \approx x \qquad \text{Divide each side by 355.9.}$$

So to the nearest year the median income reaches $20,000 in 25 years after 1990, or 2015.

TRY THIS. In what year (to the nearest year) will the median income be $25,000? ■

FOR THOUGHT...   True or False? Explain.

1. The number 3 is in the solution set to $5(4 - x) = 2x - 1$ T

2. The equation $3x - 1 = 8$ is equivalent to $3x - 2 = 7$. T

3. The equation $x + \sqrt{x} = -2 + \sqrt{x}$ is equivalent to $x = -2$. F

4. The solution set to $x - x = 7$ is the empty set. T

5. The equation $12x = 0$ is an inconsistent equation. F

6. The equation $x - 0.02x = 0.98x$ is an identity. T

7. The equation $|x| = -8$ is equivalent to $x = 8$ or $x = -8$. F

8. The equations $\dfrac{x}{x-5} = \dfrac{5}{x-5}$ and $x = 5$ are equivalent. F

9. To solve $-\dfrac{2}{3}x = \dfrac{3}{4}$, we should multiply each side by $-\dfrac{2}{3}$. F

10. If a and b are real numbers, then $ax + b = 0$ has a solution. F

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1.1 EXERCISES

CONCEPTS

Fill in the blank.

1. A statement that two algebraic expressions are equal is a(n) _____ . equation

2. An equation of the form $ax + b = 0$ with $a \neq 0$ is a(n) _____ equation. linear

3. Equations with the same solution set are _____ equations. equivalent

4. The set of all solutions to an equation is the _____ . solution set

5. A(n) _____ is an equation that is satisfied by all real numbers for which both sides are defined. identity

6. A(n) _____ is an equation that has no solution. inconsistent equation

7. A(n) _____ is an equation that has at least one solution but is not an identity. conditional equation

8. A number that satisfies the final equation but not the original equation is a(n) _____ . extraneous root

SKILLS

Determine whether each given number is a solution to the equation following it. (Example 1)

9. $3, 2x - 4 = 9$ No

10. $-2, \dfrac{1}{x} - \dfrac{1}{2} = -1$ Yes

11. $-3, (x - 1)^2 = 16$ Yes

12. $4, \sqrt{3x + 4} = -4$ No

Solve each equation and check your answer. (Example 1)

13. $3x - 5 = 0$ $\{5/3\}$

14. $-2x + 3 = 0$ $\{3/2\}$

15. $-3x + 6 = 12$ $\{-2\}$

16. $5x - 3 = -13$ $\{-2\}$

17. $8x - 6 = 1 - 6x$ $\{1/2\}$

18. $4x - 3 = 6x - 1$ $\{-1\}$

19. $7 + 3x = 4(x - 1)$ $\{11\}$

20. $-3(x - 5) = 4 - 2x$ $\{11\}$

21. $-\dfrac{3}{4}x = 18$ $\{-24\}$

22. $\dfrac{2}{3}x = -9$ $\{-27/2\}$

23. $\dfrac{x}{2} - 5 = -12 - \dfrac{2x}{3}$ $\{-6\}$

24. $\dfrac{x}{4} - 3 = \dfrac{x}{2} + 3$ $\{-24\}$

25. $\dfrac{3}{2}x + \dfrac{1}{3} = \dfrac{1}{4}x - \dfrac{1}{6}$ $\{-2/5\}$

26. $\dfrac{x}{2} + \dfrac{x}{5} = \dfrac{x}{6} - \dfrac{1}{3}$ $\{-5/8\}$

Solve each equation. Identify each equation as an identity, an inconsistent equation, or a conditional equation. (Example 2)

27. $3(x - 6) = 3x - 18$
R, identity

28. $2a + 3a = 6a$
$\{0\}$, conditional

29. $2x + 3x = 4x$
$\{0\}$, conditional

30. $4(y - 1) = 4y - 4$
R, identity

31. $2(x + 3) = 3(x - 1)$
$\{9\}$, conditional

32. $2(x + 1) = 3x + 2$
$\{0\}$, conditional

33. $3(x - 6) = 3x + 18$
$\varnothing$, inconsistent

34. $2x + 3x = 5x + 1$
$\varnothing$, inconsistent

35. $\dfrac{3x}{x} = 3$ $\{x \mid x \neq 0\}$, identity

36. $\dfrac{x(x + 2)}{x + 2} = x$
$\{x \mid x \neq -2\}$, identity

Solve each equation involving rational expressions. Identify each equation as an identity, an inconsistent equation, or a conditional equation. (Example 3)

37. $\dfrac{1}{w - 1} - \dfrac{1}{2w - 2} = \dfrac{1}{2w - 2}$ $\{w \mid w \neq 1\}$, identity

38. $\dfrac{1}{x} + \dfrac{1}{x - 3} = \dfrac{9}{x^2 - 3x}$ $\{6\}$, conditional

39. $\dfrac{1}{x} - \dfrac{1}{3x} = \dfrac{1}{2x} + \dfrac{1}{6x}$
$\{x \mid x \neq 0\}$, identity

40. $\dfrac{1}{5x} - \dfrac{1}{4x} + \dfrac{1}{3x} = -\dfrac{17}{60}$
$\{-1\}$, conditional

41. $\dfrac{z + 2}{z - 3} = \dfrac{5}{-3}$
$\{9/8\}$, conditional

42. $\dfrac{2x - 3}{x - 4} = \dfrac{5}{x - 4}$
$\varnothing$, inconsistent

43. $\dfrac{1}{x - 3} - \dfrac{1}{x + 3} = \dfrac{6}{x^2 - 9}$
$\{x \mid x \neq 3 \text{ and } x \neq -3\}$, identity

44. $\dfrac{4}{x - 1} - \dfrac{9}{x + 1} = \dfrac{3}{x^2 - 1}$
$\{2\}$, conditional

45. $4 + \dfrac{6}{y - 3} = \dfrac{2y}{y - 3}$
$\varnothing$, inconsistent

46. $\dfrac{x}{x + 6} - 3 = 1 - \dfrac{6}{x + 6}$
$\varnothing$, inconsistent

47. $\dfrac{t}{t + 3} + 4 = \dfrac{2}{t + 3}$
$\{-2\}$, conditional

48. $\dfrac{3x}{x + 1} - 5 = \dfrac{x - 11}{x + 1}$
$\{2\}$, conditional

Use a calculator to help you solve each equation. Round each approximate answer to three decimal places. (Example 4)

49. $0.27x - 3.9 = 0.48x + 0.29$ $\{-19.952\}$

50. $x - 2.4 = 0.08x + 3.5$ $\{6.413\}$

51. $0.06(x - 3.78) = 1.95$ $\{36.28\}$

52. $0.86(3.7 - 2.3x) = 4.9$ $\{-0.869\}$

53. $2a + 1 = -\sqrt{17}$ $\{-2.562\}$

54. $3c + 4 = \sqrt{38}$ $\{0.721\}$

55. $\dfrac{0.001}{y - 0.333} = 3$ $\{1/3\}$

56. $1 + \dfrac{0.001}{t - 1} = 0$ $\{0.999\}$

57. $\dfrac{x}{0.376} + \dfrac{x}{0.135} = 2$
$\{0.199\}$

58. $\dfrac{1}{x} + \dfrac{5}{6.72} = 10.379$ $\{0.104\}$

59. $(x + 3.25)^2 = (x - 4.1)^2$ $\{0.425\}$

60. $0.25(2x - 1.6)^2 = (x - 0.9)^2$ $\{0.85\}$

61. $(2.3 \times 10^6)x + 8.9 \times 10^5 = 1.63 \times 10^4$ $\{-0.380\}$

62. $(-3.4 \times 10^{-9})x + 3.45 \times 10^{-8} = 1.63 \times 10^4$
$\{-4.794 \times 10^{12}\}$

Solve each absolute value equation. Use the summary on page 84. (Example 5)

63. $|x| = 8$ $\{-8, 8\}$

64. $|x| = 2.6$ $\{-2.6, 2.6\}$

65. $|x - 4| = 8$ $\{-4, 12\}$

66. $|x - 5| = 3.6$ $\{1.4, 8.6\}$

67. $|x - 6| = 0$ $\{6\}$

68. $|x - 7| = 0$ $\{7\}$

69. $|x + 8| = -3$ $\varnothing$

70. $|x + 9| = -6$ $\varnothing$

71. $|2x - 3| = 7$ $\{-2, 5\}$

72. $|3x + 4| = 12$
$\{-16/3, 8/3\}$

73. $\dfrac{1}{2}|x - 9| = 16$ $\{-23, 41\}$

74. $\dfrac{2}{3}|x + 4| = 8$ $\{-16, 8\}$

75. $2|x + 5| - 10 = 0$
$\{-10, 0\}$

76. $6 - 4|x + 3| = -2$
$\{-5, -1\}$

77. $8|3x - 2| = 0$ $\{2/3\}$

78. $5|6 - 3x| = 0$ $\{2\}$

79. $2|x| + 7 = 6$ $\varnothing$

80. $5 + 3|x - 4| = 0$ $\varnothing$

Solve each equation. (Examples 1–5)

81. $x - 0.05x = 190$ $\{200\}$

82. $x + 0.1x = 121$ $\{110\}$

83. $0.1x - 0.05(x - 20) = 1.2$ $\{4\}$

84. $0.03x - 0.2 = 0.2(x + 0.03)$ $\{-103/85\}$

85. $(x + 2)^2 = x^2 + 4$ $\{0\}$

86. $(x - 3)^2 = x^2 - 9$ $\{3\}$

87. $(2x - 3)^2 = (2x + 5)^2$ $\{-1/2\}$

88. $(3x - 4)^2 + (4x + 1)^2 = (5x + 2)^2$ $\{13/36\}$

89. $\dfrac{x}{2} + 1 = \dfrac{1}{4}(x - 6)$
$\{-10\}$

90. $-\dfrac{1}{6}(x + 3) = \dfrac{1}{4}(3 - x)$
$\{15\}$

91. $\dfrac{y - 3}{2} + \dfrac{y}{5} = 3 - \dfrac{y + 1}{6}$
$\{5\}$

92. $\dfrac{y - 3}{5} - \dfrac{y - 4}{2} = 5$
$\{-12\}$

93. $5 + 7|x + 6| = 19$
$\{-8, -4\}$

94. $9 - |2x - 3| = 6$
$\{0, 3\}$

95. $9 - 4|2x - 3| = 9$ $\{3/2\}$

96. $-7 - |3x + 1| = |3x + 1| - 7$ $\{-1/3\}$

97. $8 - 5|5x + 1| = 12$ $\varnothing$

98. $5|7 - 3x| + 2 = 4|7 - 3x| - 1$ $\varnothing$

99. $\dfrac{3}{x - 2} + \dfrac{4}{x + 2} = \dfrac{7x - 2}{x^2 - 4}$ $\{x \mid x \neq 2 \text{ and } x \neq -2\}$, identity

100. $\dfrac{2}{x-1} - \dfrac{3}{x+2} = \dfrac{8-x}{x^2+x-2}$ $\varnothing$, inconsistent

101. $\dfrac{4}{x+3} - \dfrac{3}{2-x} = \dfrac{7x+1}{x^2+x-6}$

$\{x \mid x \ne -3 \text{ and } x \ne 2\}$, identity

102. $\dfrac{3}{x} - \dfrac{4}{1-x} = \dfrac{7x-3}{x^2-x}$ **103.** $\dfrac{x-2}{x-3} = \dfrac{x-3}{x-4}$

$\{x \mid x \ne 0 \text{ and } x \ne 1\}$, identity $\varnothing$, inconsistent

104. $\dfrac{y-1}{y+4} = \dfrac{y+1}{y-2}$ $\{-1/4\}$, conditional

MODELING

Solve each problem. (Example 6)

105. *Working Mothers* The percentage of working mothers y can be modeled by the equation

$$y = 0.0102x + 0.644$$

where x is the number of years since 1990 (U.S. Census Bureau, www.census.gov).

a. Use the accompanying graph to estimate the year in which 70% of mothers were in the work force. About 1995

b. Is the percentage of mothers in the labor force increasing or decreasing? Increasing

c. Use the equation to find the year in which 95% of mothers will be in the labor force. 2020
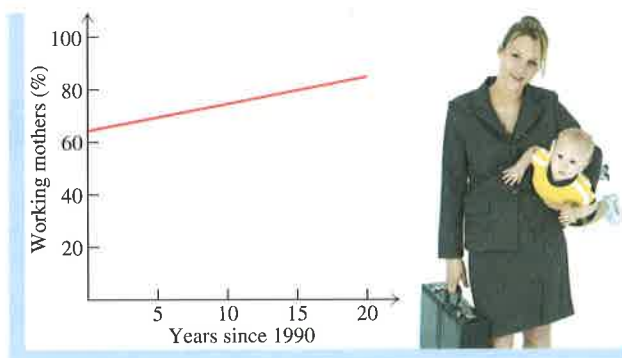

Figure for Exercises 105 and 106

106. *Working Mothers* Use the equation in the previous exercise to find the year in which 64.4% of mothers were in the labor force. 1990

107. *Cost Accounting* An accountant has been told to distribute a bonus to the employees that is 15% of the company's net income. Since the bonus is an expense to the accountant, it must be subtracted from the income to determine the net income. If the company has an income of $140,000 before the bonus, then the accountant must solve

$$B = 0.15(140,000 - B)$$

to find the bonus B. Find B. $18,260.87

108. *Corporate Taxes* For a class C corporation in Louisiana, the amount of state income tax S is deductible on the federal return and the amount of federal income tax F is deductible on the state return. With $200,000 taxable income and a 30% federal tax rate, the federal tax is $0.30(200,000 - S)$. If the state tax rate is 6% then the state tax satisfies

$$S = 0.06(200,000 - 0.30(200,000 - S)).$$

Find the state tax S and the federal tax F. $8553.97, $57,433.81

109. *Production Cost* An automobile manufacturer, who spent $500 million to develop a new line of cars, wants the cost of development and production to be $12,000 per vehicle. If the production costs are $10,000 per vehicle, then the cost per vehicle for development and production of x vehicles is $(10,000x + 500,000,000)/x$ dollars. Solve the equation

$$\frac{10,000x + 500,000,000}{x} = 12,000$$

to find the number of vehicles that must be sold so that the cost of development and production is $12,000 per vehicle. 250,000
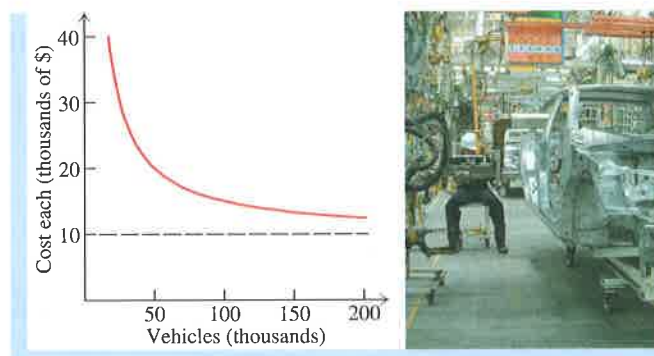

Figure for Exercise 109

110. *Harmonic Mean* The *harmonic mean (HM)* of the numbers $x_1, x_2, \ldots, x_n$ is defined as

$$HM = \frac{n}{\dfrac{1}{x_1} + \dfrac{1}{x_2} + \cdots + \dfrac{1}{x_n}}.$$

The accompanying table shows the gross domestic product (GDP) for the top five countries in trillions of U.S. dollars (www.worldbank.org).

a. What is the harmonic mean for the GDP for these five countries? $4.96 trillion

b. If Brazil is included, the harmonic mean for the six countries is $4.26 trillion. What is the GDP for Brazil? $2.49 trillion

Table for Exercise 110

Country	GDP ($ trillions)
U.S.	15.1
China	7.3
Japan	5.9
Germany	3.6
France	2.8

111. *Inscribed Circle* A right triangle has sides of length 1 and 2 as shown in the accompanying figure. Find the exact radius of the inscribed circle. $(\sqrt{3}-1)/2$

Figure for Exercise 111

112. *Inscribed Circle* A right triangle has sides of length 1 and 1 as shown in the accompanying figure. Find the exact radius of the inscribed circle. $(2-\sqrt{2})/2$
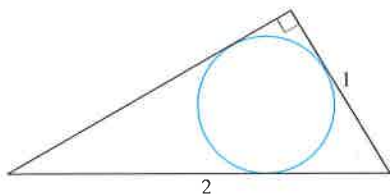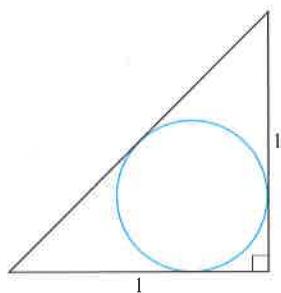

Figure for Exercise 112

WRITING/DISCUSSION

113. *Definitions* Without looking back in the text, write the definitions of linear equation, identity, and inconsistent equation. Use complete sentences.

114. *Cooperative Learning* Each student in a small group should write a linear equation, an identity, an inconsistent equation, and an equation that has an extraneous root. The group should solve each equation and determine whether each equation is of the required type.

REVIEW

115. Which of the numbers in the set $\{-3, -\pi, 0, \sqrt{3}, 7\}$ are integers? $-3, 0, 7$

116. Evaluate $5 \cdot 4 - 3 \cdot 2^3$. -4

117. Simplify $-5(x-4) - 4(6-3x)$. $7x - 4$

118. Evaluate the expression $\dfrac{2^{-2} - 3^{-1}}{5^{-1} - 4^{-1}}$. $5/3$

119. Write $\sqrt{\dfrac{5}{3}}$ in simplified form. $\sqrt{15}/3$

120. Find the domain of the expression $\dfrac{x-4}{x}$. $(-\infty, 0) \cup (0, \infty)$

OUTSIDE THE BOX

121. *Nines* Nine people applying for credit at the Highway 99 Loan Company listed nine different incomes, each containing a different number of digits. Each of the nine incomes was a whole number of dollars and the maximum income was a nine-digit number. The loan officer found that the arithmetic mean of the nine incomes was $123,456,789. What are the nine incomes? $9, $99, $999, $9999, $99,999, $999,999, $9,999,999, $99,999,999, $999,999,999

122. *Summing Factorials* What is the units digit in the sum

$$1! + 2! + 3! + 4! + \cdots + 1775! + 1776!?\ 3$$

1.1 POP QUIZ

Solve each equation and identify each as an identity, inconsistent equation, or a conditional equation.

1. $7x - 6 = 0$ $\{6/7\}$, conditional

2. $\dfrac{1}{4}x - \dfrac{1}{6} = \dfrac{1}{3}$ $\{2\}$, conditional

3. $3(x-9) = 3x - 27$
R, identity

4. $|w - 1| = 6$
$\{-5, 7\}$, conditional

5. $2(x+6) = 2x + 6$
$\varnothing$, inconsistent

6. $(x+1)^2 = x^2 + 1$
$\{0\}$, conditional