College Algebra

Test 1

Form A

Spring 2017

Name: ______________________________

Date: ______________________________

READ THESE INSTRUCTIONS CAREFULLY!

- Circle or underline your final written answer.
- Justify your reasoning and show your work.
- If you run out of space, make a note and continue your work on the back of a page.

Algebra Facts

Quadratic Formula

If a, b, and c are real numbers and $a \neq 0$, then the solutions of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Absolute Value

- If $|E| = F$, then either $E = F$ or $E = -F$.
- If $|E| \leq F$, then both $E \leq F$ and $E \geq -F$.
- If $|E| \geq F$, then either $E \geq F$ or $E \leq -F$.

1. (10 pts.) Find all solutions of the following equation.

$$|-15x+12|+7=5$$

Solution: First, we subtract 7 from both sides, which gives us

$$|-15x+12|=-2.$$

But remember that the absolute value always gives a positive number or zero; any solution of this equation would give us a number whose absolute value is negative. So this equation has $\boxed{\text{no solution}}$.

2. (10 pts.) Find all solutions of the following inequality.

$$|-4x-8|+9\leq 21$$

Solution: First, solve for the absolute value expression by subtracting 9 from both sides.

$$|-4x-8|\leq 12.$$

This is an absolute value inequality of the form ”absolute value less than”, so we can now rewrite as a compound inequality as follows.

$$-4x-8\leq 12 \quad \text{AND} \quad -4x-8\geq -12.$$

Solving each of these for x, we have

$$1\geq x \quad \text{AND} \quad x\geq -5.$$

(Remember to change the direction of the inequality when dividing by -4!) In interval notation, the solution is $\boxed{[-5,1]}$.

3. (10 pts.) Find all solutions of the following equation.

$$\frac{x}{x-6}+6=\frac{6}{x-6}$$

Solution: First, we can clear the denominators of this equation by multiplying both sides by a common denominator; in this case, multiplying by $x-6$ will work. That gives the equation

$$x+6(x-6)=6.$$

(Remember! Multiplying by a variable may introduce extraneous solutions, so we will have to check our answers at the end.) Expanding this out and combining like terms gives $x=6$. But plugging this value in for x in the original equation gives a zero in the denominator of a fraction. This is bad! So in fact this equation has $\boxed{\text{no solution}}$.

4. (10 pts.) Find all solutions of the following equation.

$$\frac{3}{5} + \frac{1}{x-4} = 1$$

Solution: Usually, the first thing we do to an equation with fractions is clear the denominator. That would work here. But instead, let's subtract 3/5 from both sides first:

$$\frac{1}{x-4} = \frac{2}{5}.$$

When we have two fractions equal to each other, we can flip them both over, like so:

$$\frac{x-4}{1} = \frac{5}{2}.$$

From here we can add 4 and simplify, to get $x = 13/2$. Now while solving this equation, we implicitly multiplied both sides by a variable (when we flipped the fractions over). So we need to make sure that this x is not an extraneous solution. It isn't, so we have the solution $\boxed{x = 13/2}$.

5. (10 pts.) Find all solutions of the following inequality.

$$|-3x-4| + 8 \geq 10$$

Solution: First, solve for the absolute value expression by subtracting 8 from both sides.

$$|-3x-4| \geq 2.$$

This is an absolute value inequality of the form "absolute value greater than", so we can now rewrite as a compound inequality as follows.

$$-3x - 4 \geq 2 \quad \text{OR} \quad -3x - 4 \leq -2.$$

Solving each of these for x, we have

$$x \geq -2/3 \quad \text{OR} \quad -2 \geq x.$$

(Remember to change the direction of the inequality when dividing by -3!) In interval notation, the solution is $\boxed{(-\infty, -2] \cup [-2/3, \infty)}$.

6. (10 pts.) Find all solutions of the following equation.

$$2x^2 - 17x + 35 = 0$$

Solution: We can solve this equation by factoring. We have

$$2x^2 - 17x + 35 = (2x-7)(x-5) = 0,$$

which is true if $\boxed{x = 7/2 \text{ or } x = 5}$. (We can also use the quadratic formula.)

7. (10 pts.)

   Find all solutions of the following equation.

$$x^2 - 6x - 3 = 0$$

   Solution: We can try factoring this equation, but there are no two integers whose sum is -6 and whose product is -3. Instead we will use the quadratic formula as follows.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2} \\ &= \frac{6 \pm \sqrt{48}}{2} \\ &= \frac{6 \pm 4\sqrt{3}}{2} \\ &= 3 \pm 2\sqrt{3} \end{aligned}$$

   Thus $\boxed{x = 3 \pm 2\sqrt{3}}$.

8. (10 pts.) Find all solutions of the following equation.

$$x^2 - 9x + 18 = 0$$

   Solution: We can solve this equation by factoring. We have

$$x^2 - 9x + 18 = (x - 6)(x - 3) = 0,$$

   which is true if $\boxed{x = 6 \text{ or } x = 3}$.

9. (10 pts.) Find all solutions of the following equation.

$$\frac{x}{x-3} + 2 = \frac{4}{x-3}$$

   Solution: First, we can clear the denominators of this equation by multiplying both sides by a common denominator; in this case, multiplying by $x - 3$ will work. That gives the equation

$$x + 2(x - 3) = 4.$$

   (Remember! Multiplying by a variable may introduce extraneous solutions, so we will have to check our answers at the end.) Expanding this out and combining like terms gives $x = 10/3$. Since plugging this value in for x in the original equation does not make any denominators equal to zero, it is a solution. So $\boxed{x = 10/3}$.

10. (10 pts.) Find all solutions of the following equation.

$$x^2 - 6x + 5 = 0$$

Solution: We can solve this equation by factoring. We have

$$x^2 - 6x + 5 = (x - 5)(x - 1) = 0,$$

which is true if $\boxed{x = 5 \text{ or } x = 1}$.

Bonus. Find all solutions of the following equation.

$$2x^4 - 11x^2 + 5 = 0$$

Solution: This is a degree 4 polynomial. But note that if we make the substitution $y = x^2$, we can rewrite our equation as

$$2y^2 - 11y + 5 = 0,$$

which is quadratic. Now this equation factors as

$$(2y - 1)(y - 5) = 0,$$

and thus has two solutions: $y = 1/2$ or $y = 5$. Then $x^2 = 1/2$ or $x^2 = 5$, so that $\boxed{x = \pm\sqrt{1/2} \text{ or } x = \pm\sqrt{5}}$.