SPECTRAL ANALYSIS AND COMPUTATION OF EFFECTIVE DIFFUSIVITIES FOR TIME-DEPENDENT PERIODIC FLOWS

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Abstract. The enhanced transport of diffusing particles or tracers by turbulent fluid advection is a challenging problem with theoretical and practical importance in many fields of science and engineering, ranging from the transport of mass, heat, and pollutants in geophysical flows to turbulent combustion and stellar convection. The long time, large scale behavior of such systems is equivalent to an enhanced diffusive process with an effective diffusivity tensor $\mathbf{D}^*$. In a study of the effective or *homogenized* behavior of the advection-diffusion equation, a rigorous representation for $\mathbf{D}^*$ was given in terms of an auxilary "cell problem." Based on this result and an analytic continuation method developed for random composite media, a rigorous integral representation for $\mathbf{D}^*$ was developed for the case of a steady, random fluid velocity field $\vec{u}$, involving a spectral measure of a self-adjoint random operator acting on *vector-fields*. Later, this approach was adapted to the case of a steady, periodic fluid velocity field, providing an integral representation for $\mathbf{D}^*$ involving a spectral measure of a self-adjoint operator acting on *scalar-fields*. Here we adapt and extend both methods to the case of a *time-dependent*, periodic velocity field, providing Stieltjes integral representations for $\mathbf{D}^*$ which involve spectral measures of self-adjoint operators. We prove that the two methods are equivalent and that the correspondence is determined by a one-to-one isometry between the underlying Hilbert spaces. We develop novel Fourier methods that provide the mathematical foundation for rigorous computation of $\mathbf{D}^*$. Our numerical computations are in excellent agreement with known theoretical results.

Key words. advective diffusion, effective diffusivity, spectral measure, homogenization, turbulence, residual diffusion

AMS subject classifications. 47B15, 65C60, 35C15, 76B99 76M22 76M50 76F25 76R99

1. Introduction.

The long time, large scale mixing motion of diffusing particles or tracers being advected by turbulent fluid flows is equivalent to an enhanced diffusive process [6] with an effective diffusivity tensor $\mathbf{D}^*$. Describing the associated transport properties is a challenging problem with theoretical and practical importance in many fields of science and engineering, ranging from turbulent combustion to mass, heat, and salt transport in geophysical flows [4]. Chaotic motion of time-dependent fluid velocity fields cause instabilities in large scale ocean currents, generating geostrophic eddies [2] which dominate the kinetic energy of the ocean [3]. In the climate system, geostrophic eddies greatly enhance the meridional mixing of heat, carbon, and other climatically important tracers [2], typically more than one order of magnitude greater than the mean flow of the ocean [5]. Eddies also impact heat and salt budgets through lateral fluxes and can extend the area of high biological productivity offshore by both eddy chlorophyll advection and eddy nutrient pumping [1]. In sea ice, which couples the atmosphere to the polar oceans, the transport of vast ice floes can also be enhanced by eddie fluxes [7].

Appendix A. Title of appendix.

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