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Across-scales interactions in the ocean-atmosphere coupled system: a model-based and data-driven approach

Nicolás Bodnariuk

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1 Background and motivation

Social and economic activities, as well as cultural developments, have largely depended on Earth's climate since the origin of humankind (Rick and Sandweiss, 2020). Although awareness of this dependence dates back to ancient times, the recognition that human activities impact the environment is more recent and has only solidified after extensive debate, often influenced by political and economic interests (IPCC, 2021). Understanding the dynamics of the climate system is a central objective of the geosciences. One of the greatest challenges lies in the very nature of the object of study: climate is widely recognized as a forced, highly nonlinear, dissipative, and chaotic system operating far from equilibrium (Ghil and Lucarini, 2020). It exhibits a wide range of overlapping spatio-temporal scales, each typically associated with different physical processes (Stommel, 1963). It is not always possible to cleanly separate these scales, nor to isolate subsystems for independent study, as coupling processes lie at the heart of the climate system. In addition to this inherent complexity, the system exhibits significant natural variability (i.e. variability arising solely from internal dynamics) which is strongly modulated by relatively small changes in both natural and anthropogenic forcings (Ghil and Childress, 1987; Peixoto and Oort, 1992; Lucarini and Sarno, 2011).

Beyond the intrinsic difficulty of studying such a complex system, we also face limitations in the quality and completeness of available data. Despite significant progress in observational techniques, particularly since the advent of satellites, we still lack a comprehensive database capable of capturing the full diversity of interacting physical processes that give rise to climate (Ghil and Lucarini, 2020). Observational data are typically limited in both spatial and temporal coverage and are often heterogeneous in quality and distribution.

Fortunately, theoretical progress and the interpretation of observations have been significantly advanced by modeling efforts. The development of physical and mathematical models of the Earth system, from idealized setups to fully coupled general circulation models, has provided profound insights into system dynamics and has been integrated into many socio-economic applications. Within this context of limited observations and growing model complexity, applied mathematics has offered powerful tools to maximize the utility of available data, particularly through data assimilation techniques (Kalnay, 2002). Oceanic and atmospheric reanalyses offer historical reconstructions of past climate states, blending observations and model outputs in an optimal way (Kalnay et al., 1996; Dee et al., 2011; Balmaseda et al., 2013; Hersbach et al., 2020).

Given the system's inherent complexity, the limitations of available datasets, and the fact that we observe only one realization of climate (Ghil and Lucarini, 2020), there is a strong need for advanced methodologies to deepen our understanding. Here is where dynamical systems theory, and bifurcation theory in particular, plays a crucial role (Dijkstra and Ghil, 2005; Ghil and Lucarini, 2020). The successive bifurcation approach applied across a hierarchy of models has enabled the systematic study of how complexity arises in the system, from simple solutions to chaotic regimes. Recent years have also witnessed remarkable advances in artificial intelligence (AI), particularly machine learning techniques, which are now being applied to climate science (Reichstein, 2019; Irrgang et al., 2021). These methods hold great promise for complementing the study of successive bifurcations and advancing our understanding of the Earth system.

This project seeks to deepen our insight into the coupled ocean-atmosphere system, with a particular focus on the ocean component. A thorough analysis must consider not only the large-scale features of both the atmosphere and ocean but also smaller-scale phenomena, such as oceanic submesoscale processes, which may influence the overall dynamics, though we still lack a complete understanding of how and to what extent. Our approach builds on one of the key accomplishments of dynamical systems theory: the successive bifurcation method applied across a hierarchy of models (Schneider and Dickinson, 1974; Ghil and Robertson, 2000; Saltzman, 2001; Lucarini, 2002; Dijkstra and Ghil, 2005; Held, 2005). To construct a detailed qualitative picture of the system's behavior, we will employ classical tools from dynamical systems theory, such as continuation methods and stability analysis. Moreover, this project proposes to complement the classical approach with machine learning techniques, especially those based on cost function minimization via neural networks. The hierarchical modeling strategy, which involves analyzing systems of increasing complexity, is now supported by unprecedented computational power and new theoretical developments, including advances in algebraic topology (e.g., Templex, (Charó et al., 2022)), which enable the topological characterization of attractors that encode long-term system dynamics. These developments allow for computations that were previously infeasible, especially in high-resolution simulations. Parallel advances in algorithm design have made it possible to efficiently detect successive bifurcations in complex models. The same holds for AI, where emerging methods are beginning to rival classical approaches. These elements will be further elaborated in the *Methodology* and *Scientific Objectives and Schedule* sections of this proposal.

What follows is a brief overview of the major contributions of dynamical systems theory to physical oceanography, including studies of the double-gyre circulation, the coupled ocean-atmosphere system, and the influence of meso- and submesoscale processes on large-scale dynamics. The large-scale circulation of the ocean, such as its organization into gyres and the intrinsic variability of these basin-wide features, has been a central topic in physical oceanography since the 20$^{\text{th}}$ century. Building on the pioneering work of Veronis (1963, 1966*a*,*b*), who applied dynamical systems theory to reveal successive bifurcations in the double-gyre system, a significant body of research has since emerged. These studies have provided key insights into the dynamical regimes sustained in both quasigeostrophic (QG) (Cessi and Ierley, 1995) and shallow-water (SW) models (Jiang et al., 1995; Speich et al., 1995), particularly during the mid-1990s. Subsequent efforts aimed to elucidate the origins of temporal variability (Simonnet and Dijkstra, 2002) and to attribute dominant frequencies to underlying dynamical processes, such as interactions among mesoscale eddies (Berloff and McWilliams, 1999), transitions between steady regimes Primeau (1998, 2002), and the occurrence of global bifurcations (Meacham, 2000; Nadiga and Luce, 2001; Simonnet and Dijkstra, 2005). One of the notable achievements of this approach was the theoretical prediction that the North Atlantic Ocean could exhibit natural variability at both subannual (∼9 months) and interannual (∼7–8 years) timescales. These variations were shown to arise independently of the temporal variability in wind forcing, aligning well with observations (Da Costa and Verdiere, 2002). These results illustrate the strong potential of dynamical systems theory to yield valuable insights in physical oceanography, highlighting its capacity to explain complex dynamics.

The ocean does not stand as an isolated subsystem; on the contrary, it interacts with other components of the climate system, especially the atmosphere, through fluxes of heat, momentum, and water vapor (Fedorov and Philander, 2000). This coupling constitutes a two-way interaction: the atmosphere influences the ocean, and the ocean feeds back into the atmosphere. A typical example is El Niño, which involves a large region of the ocean in the Equatorial Pacific that interacts with the tropical atmosphere and impacts the climate worldwide through teleconnections (Philander, 1990; Vannitsem et al., 2021). Irrespective of singular examples, the coupling occurs throughout the ocean-atmosphere interface. Regarding momentum fluxes, the transfer is typically represented as a function of the 10-meter wind, but it has been proposed that this relationship should be modified to include ocean velocities, so that the relative velocity between the fluid layers defines the wind stress Duhaut and Straub (2006); Hutchinson et al. (2010). By the same token, latent and sensible heat exchanges also depend on the states of both fluid layers Kelly et al. (2010). Dynamical systems theory has also been applied to understand complex interactions, such as coupled variability modes, their origins, and dynamics (Simonnet et al., 2009; Vannitsem et al., 2021).

Theoretical developments have long sought to incorporate the influence of small-scale processes on the large-scale structure of ocean circulation. However, despite remarkable efforts, no comprehensive theory of ocean circulation has yet been established, largely due to the closure problem and the coexistence of multiple interacting spatial and temporal scales in oceanic flows Fig. 1. Among theoretical progress, quasigeostrophic turbulence theory has laid the groundwork for understanding mesoscale interactions with the mean flow (Rhines and Young, 1982; Pedlosky, 1987; Salmon, 1998; Vallis, 2017). The energy exchanges and the inverse cascade in 2D turbulence have established the foundations for understanding energy flux across scales Kraichnan (1967); McWilliams (1984); Pedlosky (1987); Vallis (2017). Nevertheless, assuming two-dimensional dynamics neglects the inherently three-dimensional structure of the flow. In particular, submesoscale dynamics plays a key role by communicating processes between the ocean interior and the surface, and bridging a gap between 2-D turbulence a full 3-D turbulence Mahadevan (2016); McWilliams (2016); Capet et al. (2008); Levy et al. (2012). At these scales, rotational effects do not constitute the leading force in the equations of motion and therefore, geostrophic balance no longer holds Mahadevan (2016); Klein and Lapeyre (2009). Both the ageostrophic motions and the vertical exchanges set a new theoretical challenge, as classical SW and QG models present significant limitations at submesoscales.

Understanding internal dynamics is fundamental to distinguishing intrinsic variability from forced variability associated with external perturbations, such as those related to greenhouse gas emissions and associated changes in the radiative balance (Hasselmann, 1976; Ghil and Robertson, 2000). In this context, unveiling the internal dynamics supported by the coupled ocean-atmosphere system through the analysis of a hierarchy of models could provide further insight to distinguish forced behavior from natural variability.

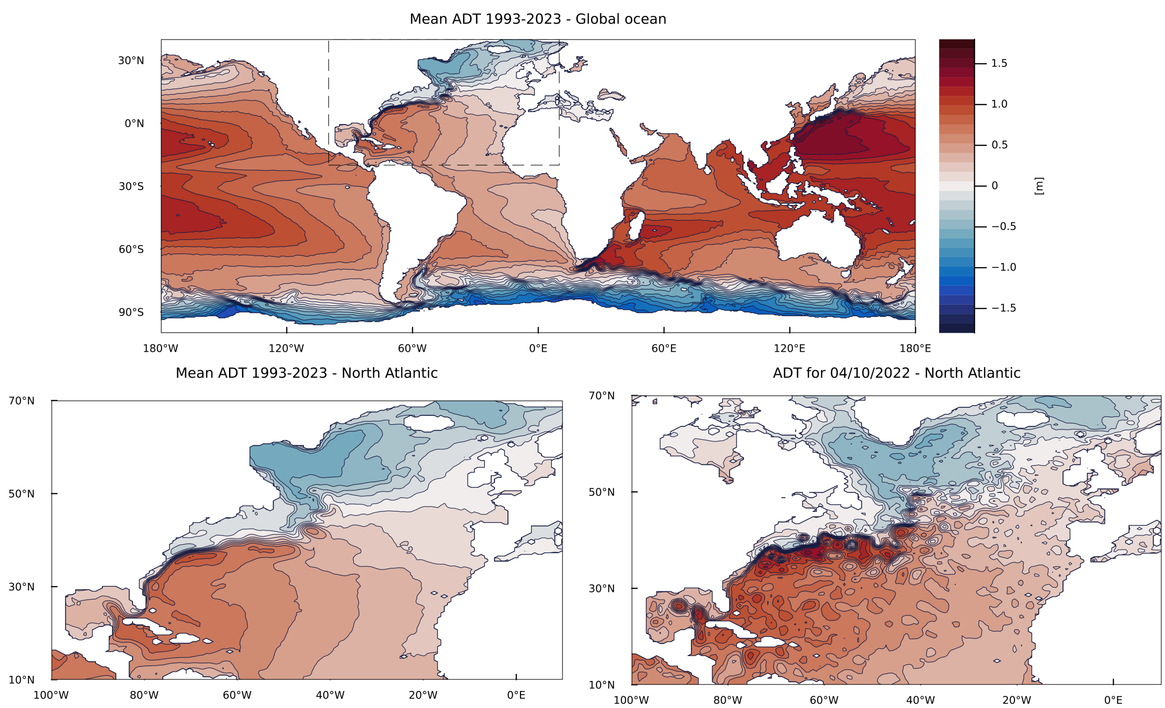

Figure 1: Mean Absolute Dynamic Topography (ADT) as derived from satellite altimetry for the period 1993–2023 over the global ocean (upper panel). The same as above but focused on the North Atlantic Ocean (lower left panel). ADT on 04/10/2022 for the North Atlantic (lower right panel). Note that, unlike the averages, the latter reveal the presence of multiple spatial and temporal scales. Ocean circulation is inherently irregular and cannot be accurately captured by smooth mean-field descriptions.

Given that ocean dynamics emerge from the interaction between processes across scales and the coupling with other components of the climate system, particularly the atmosphere, it is fundamental to investigate the interaction across spatial and temporal scales and between subsystems. Understanding these is crucial for advancing our knowledge of flow organization, especially in the context of energy transport, heat redistribution within the Earth system, and the maintenance of the mean basin-scale circulation. Within this framework, we identify three central scientific questions that will be addressed by the project:

- What is the role of ocean mesoscale and submesoscale features in shaping the large-scale circulation? In particular, how does energy cascade across scales, and to what extent do smaller scales influence the structure and variability of the large-scale flow?
- Does the coupled ocean–atmosphere system present a characteristic internal variability (e.g., mode of variability)? And can it be distinguished from externally forced variability? Clarifying this distinction is essential for understanding natural climate variability and for attributing changes to anthropogenic influences. In addition, studying the coupled system will advance past approaches, which have mainly focused on individual components.
- How well do simplified models that isolate fundamental dynamical mechanisms compare with observations in addressing these questions? Specifically, can low-order models, which emphasize core elements of geophysical fluid dynamics, provide insights that help interpret observed variability? What processes should be integrated in higher stages of the hierarchy to effectively minimize the distance with observations?

To answer these questions, we propose a combined approach based on a hierarchy of models and observational data (direct observations and reanalyses). As mentioned before, the analysis will build upon classical tools of dynamical systems theory, complemented by recent advancements in both algebraic topology and AI. It is envisaged that the new insight derived from the study be directly transferred to the modeling community, thereby providing feedback into the physical processes that are not properly represented in the current generation of climate models.

The next section details the main objectives of the project along with a tentative development timeline. This is followed by a concise presentation of the datasets and methodology. Finally, an evaluation of the project's feasibility is provided, drawing on the candidate's previous experience, proposed collaborations, and the resources available at the Laboratoire de Météorologie Dynamique (LMD, CNRS). A comprehensive list of references concludes the document.

2 Scientific Objectives and Schedule

The general objective of this project is to contribute to the understanding of the climate system by analyzing the interactions between large-scale ocean dynamics and mesoscale and submesoscale processes, as well as the coupling with the atmosphere, as guided by the central scientific questions presented in the Background and Motivation section. The work is structured around partial objectives, presented below in chronological order according to the proposed schedule (see Table 1). A total duration of 36 months is envisaged, based on the complexity of the proposed experiments and the expected time required to explore each of the ideas addressed by the project.

Work Package I - Development of the modelling framework

(a) Mesoscale–mean flow interaction through baroclinic instabilities

The candidate has already developed and tested a reduced-gravity quasi-geostrophic model in a 1.5-layer configuration for the double-gyre (Fig. 2) (Simonnet and Dijkstra, 2005) during the postdoctoral fellowship at the Laboratoire de Météorologie Dynamique (LMD), under the supervision of *Sabrina Speich* and co-supervision of *Michael Ghil*, within the framework of the ANR project ANR-23-CE56-0002 *Topological Methods for the Planet's Dynamics* (Templex), coordinated by *Denisse Sciamarella*. We aim to increase the complexity of this model by incorporating baroclinic processes. The primary motivation for this extension is to enable the representation of baroclinic instability, a fundamental dynamical mechanism responsible for the generation of mesoscale eddies through the destabilization of the background flow (Vallis, 2017; Pedlosky, 1987). Including this process is expected to provide new insight into the interactions between the mean flow and mesoscale dynamics. We are particularly interested in analyzing the qualitative changes in system behavior (i.e., the emergence of different dynamical regimes) resulting from variations in stratification. To this end, we plan to perform a successive bifurcation analysis, complemented by a topological data analysis based on Templex method, with which the candidate is already familiar. Using this approach, we will identify transitions between dynamical regimes and link them to the underlying physical processes in the ocean. These experiments will be performed in an autonomous model configuration, where the external forcing (a sinusoidal zonal wind profile) remains constant in time. This setup allows us to focus on the system's intrinsic dynamics and the internal variability arising from nonlinear interactions. The development of the baroclinic model builds directly on the existing barotropic framework. The primary structural addition involves introducing a separation between fast barotropic modes and slower baroclinic modes, enabling efficient time integration. Validation of the extended model will rely on comparisons with the literature (Simonnet et al., 2003*a*,*b*).

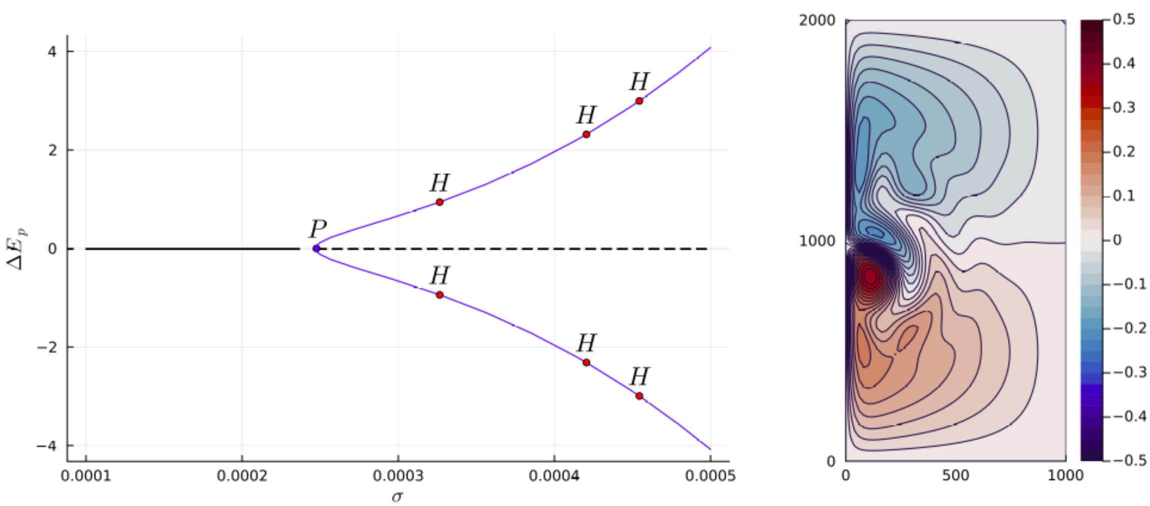

Figure 2: Bifurcation diagram of the 1.5-layer reduced-gravity quasi-geostrophic (QG) model for the double-gyre under varying parameter σ (wind stress intensity). The ordinate represents the difference in potential energy between the subtropical and subpolar gyres. The letter P denotes a Pitchfork bifurcation, while H indicates Hopf bifurcations. Dashed segments correspond to unstable branches (left panel). The streamfunction for the stable state on the lower branch at the first Hopf bifurcation is shown in the right panel. This bifurcation gives rise to the interannual variability mentioned in the text and discussed in the literature

(b) The role of the submesoscale in the ocean dynamics

At spatial scales of the order of 100 m to 10 km and temporal scales of hours to days, processes in the ocean can no longer be considered to be in geostrophic balance (McWilliams, 2016). Horizontal velocities are generally not divergence-free and therefore vertical movements connecting the surface of the ocean to its interior develop. Submesoscale features such as fronts, filaments and small-scales eddies are believed to play a fundamental role in vertical mixing and exchanges of properties (Mahadevan, 2016; Levy et al., 2012). Few studies have addressed the role of the submesoscale in shaping the large-scale ocean circulation. Based on the dynamical characteristics of processes occurring at these scales, QG models and SW models are no longer suitable. Typical models employed for the study are primitive equation models such as ROMS (Shchepetkin and McWilliams, 2005) and CROCO (Jullien et al., 2024), which allow for very high-resolution (km) runs in regional domains (e.g. channels and basins). In the context of this project, based on the hierarchy of models approach, simpler approaches may bridge the gap between the QG model proposed in the previous section and the full primitive equation. Surface QG models (QGS; (Held et al., 1995), (Lapeyre, 2017)) allow for the representation of some processes characteristic of the submesoscale: (i) the sharpening and intensification of surface fronts, and (ii) the generation of strong horizontal buoyancy gradients. However, some other important processes cannot be represented, such as (i) ageostrophic motions including vertical velocities and ageostrophic secondary circulations, and (ii) communication with the ocean interior through vertical exchanges and interior mixing processes.

In the context of gradual model complexification, it is not expected that low-order models reproduce all physical processes, but rather only those believed to be the dominant ones. It is clear that interaction between layers is a crucial element in the study of submesoscale features. For this reason, we propose coupling a QSG model (representing the upper layer) to a 2.5-layer QG model (representing the ocean interior). The proposal consists of two stages: (i) build a QSG model, programmed similarly to the 1.5-layer model we already have, and evaluate successive bifurcations in this standalone model to establish a benchmark; and then (ii) couple the Stage I model with the QSG model to study dynamical differences and interactions between the surface, the interior, and across different scales. Regarding scales, we propose working at multiple resolutions to capture the distinct roles that mesoscale and submesoscale processes may play in ocean dynamics. This coupling between surface quasigeostrophic (SQG) and interior quasigeostrophic (QG) dynamics has been previously tested in the literature and shown to provide a good compromise for representing key submesoscale processes, particularly the interaction between surface density anomalies and the balanced flow in the upper ocean layers (Lapeyre and Klein, 2006).

(c) Interactions and coupling of the ocean and atmosphere

The existing 1.5 layers QG model described above, as well as the proposed baroclinic extension, are initially run in an autonomous configuration (i.e., the external forcing is steady in time). In this phase of the project, we aim to introduce external variability into the ocean system in two successive steps of increasing complexity. First, we will consider a periodic forcing, representative of the seasonal displacement of the subtropical anticyclones. This simple configuration allows the system to be autonomized, making it suitable for the application of the dynamical systems and topological techniques previously introduced. In the second step, we move toward non-periodic and more realistic forcing scenarios. In these cases, the system is no longer autonomous, and different mathematical tools are required to study its long-term behavior. In particular, we will employ the framework of pullback attractors to analyze the system's response to non-autonomous forcing and to characterize the different regimes of variability that emerge over time Caraballo and Han (2017); Carvalho et al. (2012).

Within this hierarchy of complexity, we also aim to move beyond the assumption of a passively driven ocean. As mentioned in the *Background and Motivation section*, the atmosphere and ocean are dynamically coupled, and this interaction can be crucial to understanding feedbacks in the climate system. To capture this, we propose to couple the 2.5-layer baroclinic ocean model described in (a) with a 2-layer quasigeostrophic atmospheric model, following the approach of Vannitsem et al. (2015) who employed such coupling to study the impact of El Niño on mid-latitude dynamics. The coupling is performed through (i) the exchange of heat and momentum fluxes at the ocean-atmosphere interface, and (ii) the frictional coupling between atmospheric and oceanic boundary layers. The atmospheric model represents the mid-latitude large-scale circulation, capturing baroclinic instabilities and jet stream dynamics, which are critical for realistic representation of climate variability. In *Feasibility and Collaborations* section we propose working with *Stéphane Vannitsem* within the context of this project to undertake the proposed study, leveraging his expertise in low-order coupled ocean-atmosphere models to deepen our understanding of ocean-atmosphere feedbacks.

(d) The role of the ocean submesoscale in the coupled ocean–atmosphere system

In (b) we analyzed submesoscale dynamics and their impacts both in the upper portion of the gyre and its interior. However, the role of submesoscale processes extends beyond the internal variability of the ocean; they are intimately linked to exchanges with the atmosphere, influencing air-sea fluxes of heat, momentum, and moisture McWilliams (2016); Levy et al. (2012). Building on the analysis of energy transfer across scales within the gyre and exploring the ocean-atmosphere coupling, this project aims to evaluate the effect that small-scale oceanic features have on heat, momentum, and vapor exchanges at the air-sea interface.

While the work in (b) considered an ocean passively forced by the atmosphere to isolate internal ocean dynamics, this investigation requires a fully coupled ocean-atmosphere modeling framework. We therefore propose coupling the SQG model developed in (b), representing upper-ocean small-scale dynamics, with the 2.5-layer baroclinic ocean model from (a) for the ocean interior, and the quasi-geostrophic atmospheric model from (c), all configured for the double gyre (the idealized North Atlantic basin).

Previous project phases will have provided comprehensive benchmarks for the individual components and two-way coupled subsystems, including ocean-only and ocean-atmosphere interactions. Gradually increasing model resolution will allow the explicit representation of submesoscale processes and their feedbacks. This multi-scale coupled approach will enable us to (i) reveal the role of submesoscale features in modulating air-sea exchanges, (ii) identify characteristic regions of strong coupling, and (iii) assess the broader influence of these processes on the general circulation and climate variability.

(e) Double-gyre dynamics in the context of climate change

Recent studies have highlighted the sensitivity of submesoscale dynamics to climate change, raising critical questions about their evolving role in the climate system. In particular, warming of the upper ocean leads to increased stratification and modifies horizontal buoyancy gradients, resulting in altered mixed layer depths and available potential energy (APE) for submesoscale instabilities Boccaletti et al. (2007); Su et al. (2018); Richards et al. (2021). These background changes can reshape the intensity and spatial distribution of submesoscale activity, influencing vertical mixing, tracer transport, and the ocean's capacity to communicate with the atmosphere.

This is at the frontier of current ocean-climate science. Submesoscale processes play a pivotal role in mediating air-sea exchanges of momentum, heat, freshwater, and gases such as carbon dioxide McWilliams (2016). They also affect nutrient fluxes, biological productivity, and ecosystem structure in the upper ocean. However, these processes remain poorly represented in most global climate models, which typically do not resolve submesoscales. This leads to systematic biases in the simulation of heat budgets, carbon uptake, and the response of the coupled system to external forcing Richards et al. (2021); Seo et al. (2023).

To develop improved parameterizations of submesoscale effects in coarse-resolution models, a detailed understanding of their dynamics and feedbacks is essential. In particular, climate-change scenarios should not treat the ocean as passively driven, since changes in submesoscale activity can feed back into the atmosphere and modulate its state through altered fluxes and large-scale circulation adjustments.

In this context, the coupled QG model proposed in (c) and the three-component coupling framework described in (d) provide ideal platforms for systematically investigating these questions. By modifying the atmospheric emissivity to reflect elevated greenhouse gas concentrations, we can simulate a warming scenario and assess its impacts across scales. Approach (c) will allow us to evaluate changes in large-scale circulation patterns, such as the migration, spin-up, or spin-down of subtropical gyres, while approach (d) will provide insight into how small-scale oceanic features influence coupled feedbacks under increased stratification. Together, these tools will help identify regimes where submesoscale processes are most active and assess their broader climate implications.

Work Package II - Validating model insights with observations

The first phase of the project is expected to yield significant physical insights into mean-flow to mesoscale interactions, ocean-atmosphere coupling, and the role of the submesoscale. In the second stage, we aim to contrast model insights with observations to assess the extent of their agreement and to identify processes that are not well resolved by the models. To address this, we will focus on the analysis of reanalysis datasets and in-situ data collected during oceanographic campaigns (see the *Data* section for details). The methodology applied to these observational datasets will mirror that used for the model output, namely, a dynamical systems framework combined with topological methods. The latter enable the reconstruction of nonlinear variability modes from time series by characterizing the attractor underlying the system dynamics. In this phase, we will identify observational time series with good observability properties, suitable for phase-space reconstruction via differential embeddings Takens (1981). It is envisaged that data reduction techniques such as Empirical Orthogonal Function analysis or Multichannel Singular Spectrum Analysis (see the *Methodology* section for further details) could be employed to construct time series representative of the global double-gyre dynamics. The objectives are twofold:

- Understand the modes of variability revealed by the topology of the attractor: how the flow organizes itself, how it transitions between regimes, the likelihood and timescales of such transitions, and the global geometry of the system's dynamics.
- Compare the attractor structures derived from observations and those generated by the model, in order to detect both similarities and discrepancies in their phase-space organization. Since the attractor encodes the essential features of the system's long-term behavior, such comparisons are fundamental to determining whether the model captures the correct dynamical mechanisms.

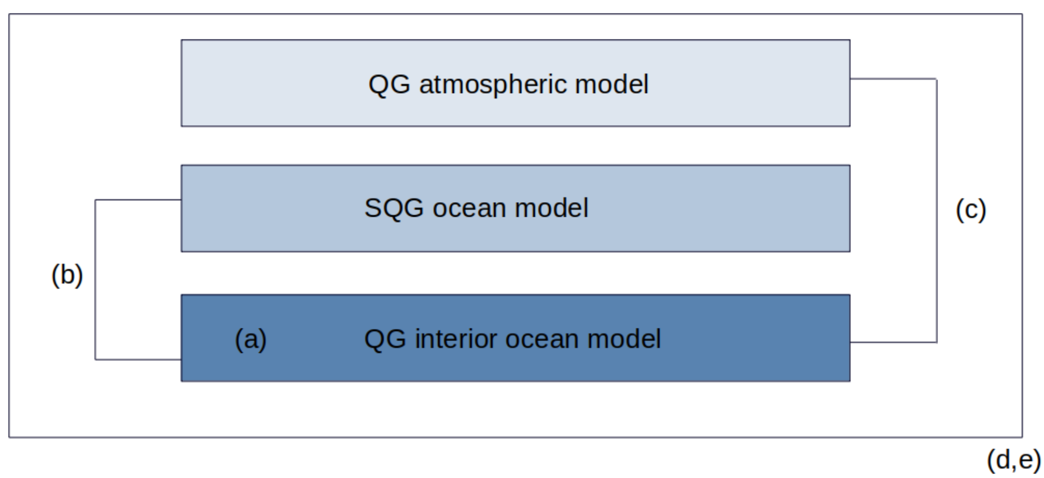

Figure 3: Schematic representation of the modelling approach. The lower block represents the QG model of the ocean interior, as described in (a). The coupled lower blocks represent the interaction between the surface (QSG) and the interior, as proposed in (b). The coupled ocean–atmosphere model is represented by the interaction between the lower and upper blocks, as suggested in (c). Finally, the interaction between all subsystems (three coupled models) is represented by the deep blue frame labelled (d,e), which corresponds to the activities proposed in the abovementioned subsections.

Relevance of the study for climate modelling

There is growing scientific evidence that small-scale oceanic processes play a fundamental role in shaping the general circulation of the ocean and, consequently, the global climate system. Notably, low-frequency ocean variability in the presence of mesoscale turbulence is no longer considered merely a response to atmospheric forcing or air–sea coupling. Instead, a portion of this variability arises spontaneously with chaotic behavior, influencing multiple oceanic fields, particularly ocean heat content (Penduff et al., 2018). These findings carry important implications for the detection and attribution of climate signals in a warming ocean, which warrant further investigation. Accurately determining which spatial and temporal scales must be explicitly resolved in climate models remains a central challenge, with major implications for the representation of low-frequency, high-impact events (Hewitt et al., 2022). Several studies have pointed to the existence of dynamically critical regions that are central to air-sea interactions and the modulation of large-scale circulation. Among these are oceanic frontal zones, boundary layers associated with western boundary currents, mesoscale and submesoscale eddies. Particular examples include the Gulf Stream system and its role in modulating atmospheric conditions over Western Europe through ocean-atmosphere heat and moisture exchanges (Jackson et al., 2015); the deep water formation locations linked to the Atlantic Meridional Overturning Circulation (AMOC); and the role of mesoscale dynamics on delaying the response of sea ice to atmospheric warming in the Southern Ocean (Rackow et al., 2022). State-of-the-art climate models still struggle to represent these processes adequately.

Mesoscale and submesoscale fronts and eddies, along with boundary layer processes in both the ocean and atmosphere, significantly influence the physical and biogeochemical exchanges of heat and carbon between the ocean and atmosphere. Oceanic frontal zones with pronounced sea surface temperature (SST) gradients are known to organize storm tracks, influencing the tropospheric circulation and variability (Nakamura and Shimpo, 2004; Nakamura et al., 2004; Brayshaw et al., 2008). These effects are driven by strong surface baroclinicity, sustained by sharp cross-frontal contrasts in oceanic heat supply. Coupled ocean-atmosphere studies have further shown how SST gradients can influence the deep atmosphere and, conversely, how atmospheric variability impacts the penetration of heat into the ocean interior (Thomas and Lee, 2005). Errors in the representation of SST fields, air-sea fluxes, or their variability can lead to persistent biases in climate models. Consequently, resolving eddy processes and ensuring adequate coupling between ocean and atmosphere components are of paramount importance. While kilometer-scale modeling can provide realistic simulations of such processes over limited spatial and temporal domains, it remains computationally unfeasible for long-term climate projections. To overcome this, two strategies are essential: (1) the development of robust parameterizations that can accurately capture the bulk effects of unresolved physical processes, and (2) the implementation of high-resolution nested simulations in dynamically important regions.

The critical need to better resolve these features has been explicitly emphasized in recent literature, including major scientific editorials in Nature (Hewitt et al., 2022). These recommendations underscore the urgency of improving the representation of key small-scale processes in climate models to improve the reliability of climate projections in a rapidly changing world. Within this broader context, simplified models, like the oned proposed in this project, offer an essential framework to guide and motivate the development of parameterizations. By isolating key mechanisms, these models make it possible to clearly identify and understand the fundamental physical processes involved. As such, they serve as critical building blocks toward the construction and refinement of more comprehensive climate models, bridging theoretical insight and numerical implementation.

Scientific publications and transferability of results

It is envisaged that each set of activities outlined on Stage I will contribute to the existing literature on the general circulation of the ocean gyres across scales, going beyond what is actually known in terms of submesoscale dynamics, energy flow, coupling with the atmosphere and climate change impacts. This stage would also provide new insights as to the (re-)interpretation of observation and would be informative as to the developments of parametrizations of small-scales featurs. Stage II would allow for a comparison of modeling results to existing data, which will provide feedback to the modelling comunnity by identifying both close agreements and discrepancies between models and observations. By the end of the project, we aim to identify poorly represented or missing processes in the current modelling framework. This will inform potential model improvements and help prioritize the incorporation of additional physical processes. Overall, this phase of the project bridges model-based and data-driven perspectives on large-scale ocean dynamics and provides concrete feedback to enhance future modelling efforts.

Future perspectives

The project, as currently designed, focuses on the study of simplified coupled models of the QG (Quasi-Geostrophic) or SQG type, which represent the interactions between the interior ocean, the surface, and the atmosphere. While these models are highly useful for understanding the fundamental elements of the physical processes occurring in reality, they present limitations due to their inherent simplifications. For example, they describe dynamics in the limit of the Rossby number approaching zero, thus representing large-scale, slow motions, but they are unable to capture ageostrophic processes. Furthermore, the layered configuration (one ocean layer and one surface layer) is not ideal for fully understanding vertical transport associated with submesoscale motions. In this regard, it is important to consider primitive equation models, which, as previously mentioned, have the potential to address these needs. We plan to employ CROCO in the future to complement the results obtained from the proposed models. A comparative study would enable us to understand the existing differences and their potential attribution to the explicit resolution of additional processes. Similarly, in terms of atmosphere coupling, collaboration with researchers in atmospheric sciences could facilitate the development of experiments coupling CROCO with WRF (Skamarock et al., 2008), aiming to continue advancing the line of research on coupled systems.

Working Schedule

Period	Activity
Months 1–12	Stage I (Part 1): Internal dynamics in the ocean - Study of mesoscale-mean flow interaction through baroclinic instabilities. - Study of submesoscale dynamics in the double-gyre and vertical connectivity in the ocean.
Months 13–24	Stage I (Part 2): Ocean-atmosphere dynamics - Study of interactions and coupling of the ocean and atmosphere. - Study of exchanges of properties across the ocean-atmosphere interface as influence by the submesoscale.
Months 25–36	Stage I (Part 3): Climate change scenario. Stage II: Model-observation comparison. - Study of ocean gyres dynamics in the context of global warming, with emphasis on the submesoscale. - Comparison of models to reanalysis and in-situ data. Evaluation of model performance and feedback to modelling community.

Table 1: Project timeline and proposed activities for each period of 12 months.

3 Data

We propose employing the following datasets in Stage II to complement the modeling framework established in Stage I. Additional datasets may be incorporated during the course of the project as deemed necessary.

Ocean reanalysis:
ORAS5, developed by the European Centre for Medium-Range Weather Forecasts (ECMWF), provides global ocean temperature, salinity, and currents from 1958 to present at approximately 0.25° horizontal resolution (Zuo et al., 2019). GLORYS12, produced by the Copernicus Marine Environment Monitoring Service, offers higher resolution ($1/12^\circ$, 9 km) ocean state estimates (Lellouche et al., 2021). Both products use advanced data assimilation methods integrating diverse observational sources to reconstruct ocean variability accurately. These reanalyses enable detailed investigations of ocean dynamics, heat content, and their role in climate variability.

Atmospheric reanalysis

We propose employing ERA5, the latest atmospheric reanalysis from the European Centre for Medium-Range Weather Forecasts (ECMWF) for this study. ERA5 provides high-resolution ($\sim$ 31 km) data from 1950 to present with hourly temporal frequency, offering detailed atmosphere-land variables under prescribed oceanic boundary conditions Hersbach et al. (2020). Although not a fully coupled atmosphere-ocean reanalysis, ERA5's fine spatial and temporal resolution and extensive coverage make it highly suitable for investigating large-scale atmospheric dynamics and surface interactions. Thus, it offers a robust and reliable foundation for analyzing atmospheric variability and climate processes in this project.

Coupled reanalysis

The Coupled Earth Reanalysis (CERA) system developed by ECMWF integrates atmosphere, ocean, and sea ice within a single data assimilation framework, allowing simultaneous assimilation of observations and improved physical consistency in representing air-sea interactions. CERA-20C covers the 20th century (1901–2010) at approximately 125 km atmospheric resolution and 1° ocean resolution, focusing on long-term climate variability Laloyaux et al. (2018). CERA-SAT spans from 1979 onward with higher resolution, enhancing the representation of regional coupled processes. While its resolution is coarser than ERA5 or ORAS5, CERA's fully coupled approach provides significant advantages for studying coupled climate phenomena.

In-situ data for submesoscale studies

To study submesoscale ocean dynamics (100 m–10 km scales), high-resolution data are essential. Satellite missions like SWOT (Surface Water and Ocean Topography) provide unprecedented sea surface height measurements at 1 km resolution, enabling detection of submesoscale features Fu et al. (2024).

Climate change projections:

Climate change projections rely on numerical simulations using global climate models that represent the Earth's atmosphere, oceans, land, and ice systems under different greenhouse gas emission scenarios. These models capture complex interactions among physical, chemical, and biological processes, enabling projections of future climate variables such as temperature, precipitation, and sea level rise. The Coupled Model Intercomparison Project Phase 6 (CMIP6) offers the latest multi-model ensemble datasets widely used for climate impact assessments Eyring and et al. (2016). We will consider their greenhouse gas concentration trajectories to suggest changes in atmospheric emissivity and develop our own experiments accordingly.

4 Methodology

The primary methods proposed in this project are succinctly outlined below, with appropriate references provided for completeness.

Dynamical system theory

Dynamical systems theory provides a mathematical framework to study how systems evolve over time under deterministic rules Guckenheimer and Holmes (1983). The system's state is represented as a point in a phase space, and its evolution is governed by ordinary differential equations. This approach allows us to characterize long-term behavior through invariant sets, attractors, and bifurcations.

Bifurcation analysis

Bifurcation analysis is used to study qualitative changes in a system's behavior as parameters vary. Within the dynamical systems framework, it identifies transitions such as the emergence of multiple fixed points, limit cycles, or chaotic regimes Kuznetsov (2004). By tracking how invariant structures change, we gain insight into the organization of phase space. This method helps uncover critical thresholds and the stability properties of solutions. It is especially valuable in nonlinear systems where small parameter shifts can lead to major dynamical shifts. In common parlance, bifurcations are referred to nowadays as "tipping points" and phase-parameter thresholds that separate acceptable from unacceptable climate system behavior as "planetary boundaries".

Continuation methods

Continuation methods provide a systematic way to trace solution branches of a dynamical system as a parameter varies. Starting from a known solution, the method incrementally follows the path of equilibria or periodic orbits using for example, the Newton-Raphson method Dijkstra (2005). It enables the detection of bifurcation points and turning points in parameter space. This approach is essential for mapping the global structure of solutions. It is particularly effective in nonlinear systems with multiple stable and unstable regimes.

Stability analysis

Stability analysis assesses the response of a dynamical system to small perturbations about a reference state Guckenheimer and Holmes (1983); Kuznetsov (2004). By linearizing the system near fixed points or periodic orbits, we examine the eigenvalues or Floquet multipliers to determine local stability. Lyapunov exponents are used to quantify sensitivity to initial conditions. This analysis helps distinguish stable, unstable, and chaotic regimes.

Machine learning approach to successive bifurcations

We propose using a physics-informed neural network (PINN) framework to detect successive bifurcations in a dynamical system. The neural network approximates system states while minimizing a cost function that encodes the governing physical equations. By varying control parameters, the trained model reveals qualitative changes in solution structure Goodfellow and Courville (2016).

Algebraic topology and the Templex approach

Algebraic topology methods characterize the attractors of dynamical systems by capturing their intrinsic topological properties. The concept of a Templex combines the cell complex representing the underlying topological structure with the dynamical flow on it, enabling a unified description of a system's long-term behavior Charó et al. (2022); Ghil and Sciamarella (2023). This framework encodes nonlinear, topological modes of variability. These modes are not necessarily periodic but capture all the relevant information about the system's evolution and invariant features (Fig. 4)

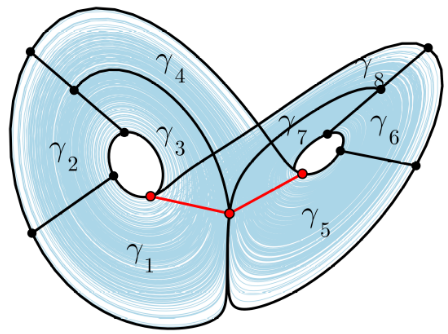

Figure 4: The blue curves represent orbits of the Lorenz attractor. Each γ-cell forms part of a cell complex that spans the attractor and serves as an essential component of a Templex. All orbits are constrained to move along the "tracks" defined by the cells. The red lines, known as joining lines, signal the presence of chaotic behavior in the dynamics: they indicate that information may enter a given outgoing cell from two distinct past states.

Quasi-geostrophic and surface quasi-geostrophic models

We propose employing quasi-geostrophic (QG) and surface quasi-geostrophic (SQG) models to study large-scale geophysical flows. These models approximate the rotating, stratified dynamics of the ocean and atmosphere under the geostrophic balance. QG captures interior potential vorticity dynamics, while SQG focuses on surface-driven motions and frontal structures. Both models retain essential nonlinear and advective processes while filtering fast waves. They provide a tractable yet physically grounded framework for analyzing mesoscale and submesoscale variability Pedlosky (1987); Lapeyre (2017).

Data analysis for spatio-temporal fields
Empirical Orthogonal Functions (EOFs) identify dominant spatial patterns and their corresponding temporal coefficients by decomposing data variance Lorenz (1956); Preisendorfer (1988). Multichannel Singular Spectrum Analysis (MSSA), as described by Ghil et al. (2002) and Vautard and Ghil (1992), extends EOFs by analyzing lagged covariance matrices to extract oscillatory modes, enabling the detection of quasi-periodic oscillations and trends in multivariate time series. These methods reduce data dimensionality and help isolate key dynamical modes. They are fundamental for interpreting complex variability in geophysical systems.

5 Feasibility and collaborations

Candidate's experience in the field

The candidate has developed a barotropic quasigeostrophic model for the double gyre circulation that is representative of the North Atlantic Ocean, and he is well acquainted with the numerical methods that enable further complexification of the model in terms of both resolution and additional processes (Bodnariuk, Simonnet, Speich, Sciamarella and Ghil, 2025). He also has a strong background in dynamical systems, is contributing to a book on the subject (Ghil et al., 2025), and is familiar with the topological techniques proposed in the project (Bodnariuk, Charo, Sciamarella and Ghil, 2025), having implemented them during his postdoctoral research at LMD (2024–present). Furthermore, he has extensive experience in the exploration of reanalysis data (Bodnariuk, Simionato and Saraceno, 2021*b*; Bodnariuk, Simionato, Osman and Saraceno, 2021; Bodnariuk, Simionato and Saraceno, 2021*a*; Bodnariuk et al., 2022) and the application of data reduction techniques, developed during his PhD at the University of Buenos Aires and his previous postdoctoral position at CONICET (Argentina). His scientific achievements earned him a permanent research position at CONICET, Argentina's national research council. However, the administrative implementation of this position has been delayed due to recent government policies that have severely defunded both the academic and scientific sectors, most severely so in the areas of climate change and its impacts. This challenging context strongly motivated his decision to participate in the Choose France for Science program, seeking to continue his climate-related research in a stable professional environment.

Proposed collaborations

– **Physical Oceanography:** *Sabrina Speich* is a leading expert in observational physical oceanography, with a particular focus on multi-scale interactions in the ocean system and its coupling with the atmosphere. *Gianluca Menheguello* focuses on the use of observations and numerical models to improve our understanding of the dynamics of the ocean and the atmosphere, and on the development of autonomous observational platforms.

– **Atmospheric Science:** *Fabio D'Andrea* works on atmospheric physics and climate dynamics. In particular, he focuses on the dynamics of extreme climate events in the context of climate change.

– **Nonlinear dynamics in the ocean and atmosphere:** *Guillaume Lapeyre* is an expert in the nonlinear dynamics of atmospheric and oceanic vortices. His research focuses on tracer transport and mixing properties, the dynamics of ocean surface layers, synoptic-scale atmospheric depressions, and ocean–atmosphere interactions.

– **Dynamical Systems and Algebraic Topology:** *Michael Ghil* (currently co-supervising my postdoctoral work at LMD) is internationally recognized for his contributions to dynamical systems theory and its applications to geophysical flows. *Denisse Sciamarella* (supervisor of the ANR project that supports my current postdoc) specializes in topological approaches to dynamical systems, particularly in geoscientific contexts. *Gisela Charo* (postdoctoral researcher in the same ANR project) also brings extensive experience in the application of algebraic topology to complex system analysis.

– **Mathematical and Computational Methods, and Artificial Intelligence:** *Eric Simonnet* has extensively studied double-gyre ocean dynamics, applying reduced models and dynamical systems theory. In recent years, he has expanded his expertise into machine learning applications for climate research. He has been actively collaborating with me as a scientific advisor during my postdoctoral stay at LMD.

– **Ocean–Atmosphere Coupling:** In the context of atmospheric coupling, we propose collaboration with *Stéphane Vannitsem* (Royal Meteorological Institute of Belgium), who has developed a simplified yet powerful coupled ocean–atmospheric model (MAOOAM) suitable for coupling experiments. We also aim to engage with meteorologists *Marisol Osman* and *Juan Ruiz* at *CIMA* (CONICET, Argentina), who bring valuable expertise in atmospheric variability, modeling, and climate dynamics.

Available resources in the proposed institution

The candidate has access to both the Spirit server (https://mesocentre.ipsl.fr/) and the National Supercomputing Center to carry out the intensive computational studies required (http://www.idris.fr/jean-zay/). In addition, he has access to the server Noise at IFAECI (Franco-Argentinian International Research Laboratory).

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