# Models with Disaster Risk

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# Barro (2006)

- This model accounts for the observed equity premium by introducing in the standard CCAPM rare disasters (i.e., low probability large aggregate negative shocks).
- Disasters:
  - economic events: the Great Depression, the Great Financial Crisis,
  - natural disasters: earthquakes,
  - epidemics of disease: Black Death, Covid-19.
- In economic terms, Barro (2006) argues that **wars** have been the most important economic disasters of the twentieth century globally.

# Wars are associated with a large drop in real GDP

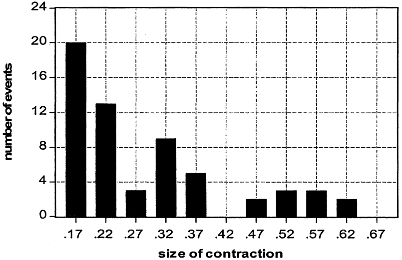
TABLE I
DECLINES OF 15 PERCENT OR MORE IN REAL PER CAPITA GDP

Part A: Twenty OECD Countries in Maddison [2003]

| Event | Country | Years | % fall in real per capita GDP |
|---|---|---|---|
| World War I | Austria | 1913–1919 | 35 |
| | Belgium | 1916–1918 | 30 |
| | Denmark | 1914–1918 | 16 |
| | Finland | 1913–1918 | 35 |
| | France | 1916–1918 | 31 |
| | Germany | 1913–1919 | 29 |
| | Netherlands | 1913–1918 | 17 |
| | Sweden | 1913–1918 | 18 |
| Great Depression | Australia | 1928–1931 | 20 |
| | Austria | 1929–1933 | 23 |
| | Canada | 1929–1933 | 33 |
| | France | 1929–1932 | 16 |
| | Germany | 1928–1932 | 18 |
| | Netherlands | 1929–1934 | 16 |
| | New Zealand | 1929–1932 | 18 |
| | United States | 1929–1933 | 31 |
| Spanish Civil War | Portugal | 1934–1936 | 15 |
| | Spain | 1935–1938 | 31 |
| World War II | Austria | 1944–1945 | 58 |
| | Belgium | 1939–1943 | 24 |
| | Denmark | 1939–1941 | 24 |
| | France | 1939–1944 | 49 |
| | Germany | 1944–1946 | 64 |
| | Greece | 1939–1945 | 64 |
| | Italy | 1940–1945 | 45 |
| | Japan | 1943–1945 | 52 |
| | Netherlands | 1939–1945 | 52 |
| | Norway | 1939–1944 | 20 |
| Aftermaths of wars | Canada | 1917–1921 | 30 |
| | Italy | 1918–1921 | 25 |
| | United Kingdom | 1918–1921 | 19 |
| | United Kingdom | 1943–1947 | 15 |
| | United States | 1944–1947 | 28 |

Figure 1: Economic Disasters and Economic Consequences (Barro, QJE 2006)

# Large disasters are *rare*




Panel A: Contractions in Table I

Figure 2: Frequency of Disasters and Size of Contraction (Barro, QJE 2006)

# In GDP terms, the impact of a pandemic is less severe

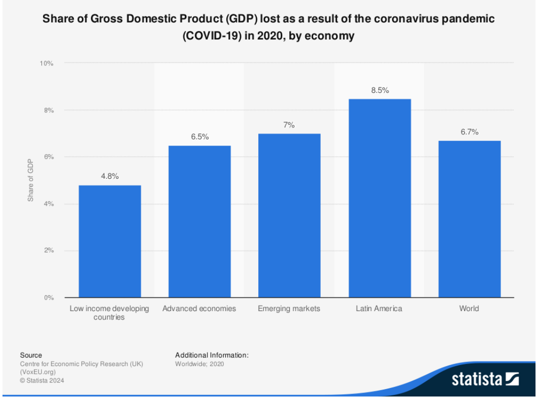



Figure 3: GDP cost of Covid-19 pandemic (Statista)

# Stocks tend to have a low payoffs when disasters hit

TABLE II
STOCK AND BILL RETURNS DURING ECONOMIC CRISES

| Event | Real stock return (% per year) | Real bill return (% per year) |
|---|---|---|
| **World War I** | | |
| Austria, 1914–1918 | — | −4.1 |
| Denmark, 1914–1918 | — | −6.9 |
| France, 1914–1918 | −5.7 | −9.3 |
| Germany, 1914–1918 | −26.4 | −15.6 |
| Netherlands, 1914–1918 | — | −5.2 |
| Sweden, 1914–1918 | −15.9* | −13.1 |
| **Great Depression** | | |
| Australia, 1928–1930 | −3.6 | 8.2 |
| Austria, 1929–1932 | −17.3* | 7.1 |
| Canada, 1929–1932 | −23.1* | 7.1 |
| Chile, 1929–1931 | −22.3* | — |
| France, 1929–1931 | −20.5 | 1.4 |
| Germany, 1928–1931 | −14.8 | 9.3 |
| Netherlands, 1929–1933 | −14.2* | 5.7 |
| New Zealand, 1929–1931 | −5.6* | 11.9 |
| United States, 1929–1932 | −16.5 | 9.3 |
| **Spanish Civil War** | | |
| Portugal, 1934–1936 | 13.4* | 3.8 |
| **World War II** | | |
| Denmark, 1939–1945 | −3.7* | −2.2 |
| France, 1943–1945 | −29.3 | −22.1 |
| Italy, 1943–1945 | −33.9 | −52.6 |
| Japan, 1939–1945 | −2.3 | −8.7 |
| Norway, 1939–1945 | 1.7* | −4.5 |
| **Post-WWII Depressions** | | |
| Argentina, 1998–2001 | −3.6 | 9.0 |
| Chile, 1981–1982 | −37.0* | 14.0 |
| Indonesia, 1997–1998 | −44.5 | 9.6 |
| Philippines, 1982–1984 | −24.3 | −5.0 |
| Thailand, 1996–1997** | −48.9 | 6.0 |
| Venezuela, 1976–1984 | −8.6* | — |

Figure 4: Equity and Bill Returns during Crises (Barro, QJE 2006)

# US GDP fluctuates around an upward trend at least since WWII

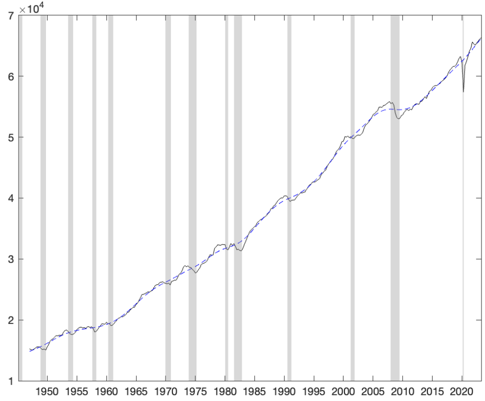



Figure 5: US quarterly per capital GDP. Seasonally adjusted, blue-dashed line is HP-filtered trend. Data are from FRED for the period 1947 to 2024.

# Models with Disaster Risk

- Barro (QJE 2006) assumes that log consumption growth follows:

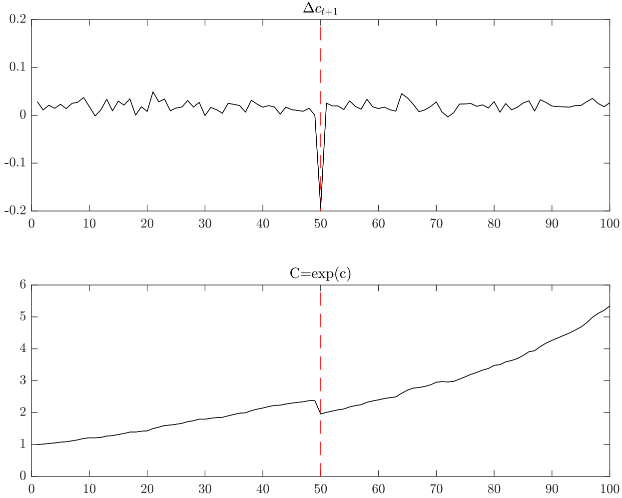
$$\begin{aligned}\Delta c_{t+1} &= \mu + \sigma\epsilon_{t+1} \quad \text{with probability } (1-p) \\ &= \mu + \sigma\epsilon_{t+1} + \log(1-b) \quad \text{with probability } p\end{aligned}$$

  where $\epsilon_{t+1} \sim$ *i.i.d.* $N(0,1)$ and $b \in (0,1)$.

- $b$ is a *rare* disaster which causes **a permanent drop in consumption** $\rightsquigarrow$ disasters are *costly*!

- Investors have standard CRRA preferences

# The disaster causes a permanent drop in consumption



# Risk-Free Rate (I/II)

- The risk-free rate follows from the standard Euler equation P = E(MX) and CRRA preferences:

$$R_t^f = \{E_t[M_{t+1}]\}^{-1} = \left\{E_t[\beta e^{-\gamma \Delta c_{t+1}}]\right\}^{-1}$$

which under the specified endowment process implies that:

$$\begin{aligned} R^f &= \left\{\beta(1-p)e^{-\gamma\mu+(1/2)\gamma^2\sigma^2} + \beta p e^{-\gamma\mu+(1/2)\gamma^2\sigma^2-\gamma\log(1-b)}\right\}^{-1} \\ &= \left\{\beta e^{-\gamma\mu+(1/2)\gamma^2\sigma^2}\left[(1-p)+p(1-b)^{-\gamma}\right]\right\}^{-1} \end{aligned}$$

- Taking logs:

$$r^f = -\log\beta + \gamma\mu - (1/2)\gamma^2\sigma^2 - \log\left[(1-p)+p(1-b)^{-\gamma}\right] \quad (1)$$

# Risk-Free Rate (II/II)

- If $p = 0$ (no disaster), then the risk-free rate is same as with CRRA and iid log consumption growth:

$$r^f = -\log \beta + \gamma\mu - (1/2)\gamma^2\sigma^2$$

- Assume $p = 1/100 > 0$, $b = 0.2$ and $\gamma = 1$ (from Barro's paper), then:

$$\log \left[(1-p) + p(1-b)^{-\gamma}\right] = 0.0025 = 25bp$$

- Intuition:
  - $r^f$ is lower because investors save more for both intertemporal substitution and risk aversion effects,
  - the higher is $b$ (disaster size), the lower is the risk-free rate.

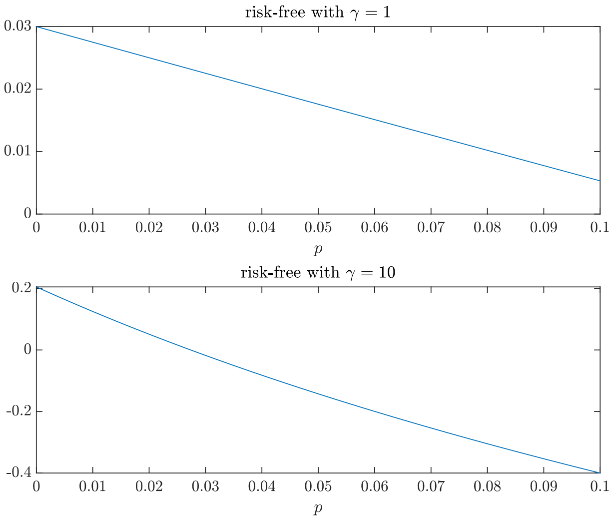



Figure 6: Risk-free rate as function of disaster probability

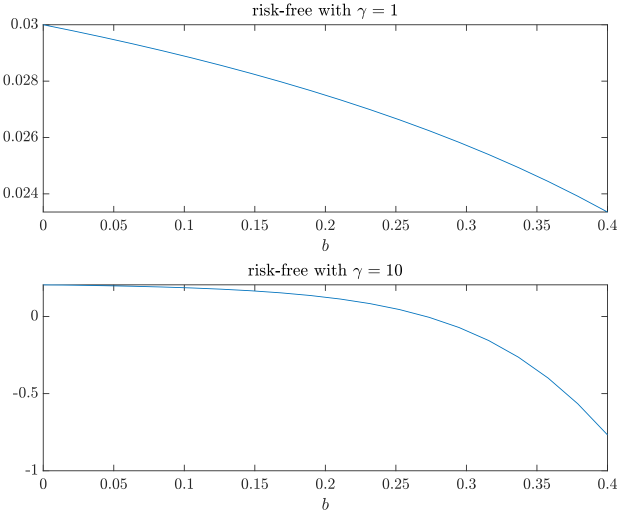



Figure 7: Risk-free rate as function of disaster size

# Price-Dividend Ratio

- Since $\Delta c$ is $i.i.d.$ (i.e., disasters are unpredictable), the $PD$ ratio is constant as in the standard CCAPM.

$$q \equiv \frac{P_t}{D_t} = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\left(\frac{P_{t+1}}{D_{t+1}}+1\right)\right] = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}(1+q)\right]$$

- After some simple algebra:

$$q = \beta(1+q)\left[e^{(1-\gamma)\mu+\frac{(1-\gamma)^2}{2}\sigma^2}\left((1-p)+p(1-b)^{1-\gamma}\right)\right] \tag{2}$$

note: we compute the PD ratio because it is useful in the derivation of the (log) equity return in the model (i.e., $\log(ER^e)$).

# Expected Equity Return

- The expected equity return is:

$$E_t(R^e_{t+1}) = E_t\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) = E_t\left(\frac{P_{t+1}/D_{t+1} + 1}{P_t/D_t}\frac{D_{t+1}}{D_t}\right)$$
$$= \frac{1+q}{q}E_t\left(\frac{D_{t+1}}{D_t}\right)$$
$$= \frac{1+q}{q}\left[e^{\mu+(1/2)\sigma^2}\left(1-p+p(1-b)\right)\right]$$

- We can derive the expected log equity return using (2) and taking the log of the equation above:

$$\log\left[E(R^e)\right] = -\log\beta + \gamma\mu - \frac{\gamma^2}{2}\sigma^2 + \gamma\sigma^2 + \log\left[\frac{1-p+p(1-b)}{1-p+p(1-b)^{1-\gamma}}\right] \tag{3}$$

# Equity Premium

- The (log) equity premium is then:

$$\log\left(\frac{ER^e}{R^f}\right) = \gamma\sigma^2 + \log\left[\frac{(1-p+p(1-b))(1-p+p(1-b)^{-\gamma})}{1-p+p(1-b)^{1-\gamma}}\right] \tag{4}$$

- The term in the square brackets comes from the default premium and is equal to zero if $p = 0$.

# Equity Premium and the Disaster Probability

- The derivative of the (log) equity premium $r^e$ with respect to $p$ is:

$$\frac{\partial r^e}{\partial p}(p) = \underbrace{-\frac{b}{1-pb}}_{\leq 0} + \underbrace{\frac{b(1-b)^{-\gamma}}{\left[1-p+p(1-b)^{-\gamma}\right]\left[1-p+p(1-b)^{1-\gamma}\right]}}_{\geq 0} \tag{5}$$

- Note that for small $p$ we have:

$$\frac{\partial r^e}{\partial p}(0) = -b + b(1-b)^{-\gamma} > 0$$

- Intuition:
  - for $p$ small, an increase in $p$ increases the risk premium,
  - for $p$ large, risk premium drops because *uncertainty drops* even though disaster is large.

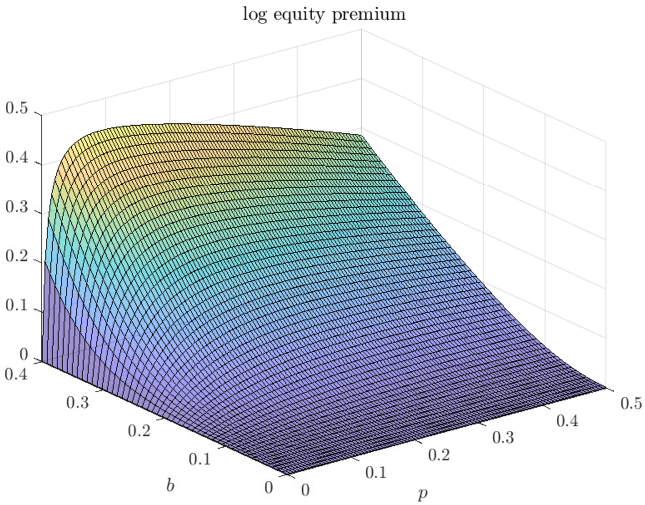



Figure 8: Equity premium as function of disaster probability and size

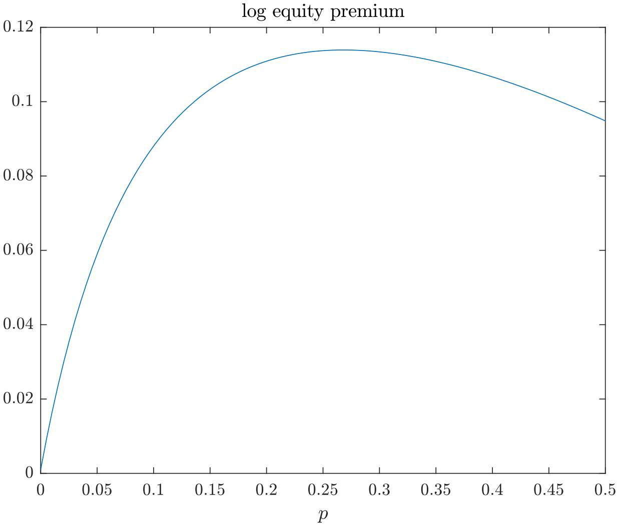



Figure 9: Equity Premium as function of disaster probability for fixed $b = 0.2$

TABLE V
Calibrated Model for Rates of Return

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| | | | Parameters | | | | |
| | No disasters | Baseline | Low $\theta$ | High $p$ | Low $q$ | Low $\gamma$ | Low $\rho$ |
| $\theta$ (coeff. of relative risk aversion) | 4 | **4** | **3** | 4 | 4 | 4 | 4 |
| $\sigma$ (s.d. of growth rate, no disasters) | 0.02 | **0.02** | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| $\rho$ (rate of time preference) | 0.03 | **0.03** | 0.03 | 0.03 | 0.03 | 0.03 | **0.02** |
| $\gamma$ (growth rate, deterministic part) | 0.025 | **0.025** | 0.025 | 0.025 | 0.025 | **0.020** | 0.025 |
| $p$ (disaster probability) | **0** | **0.017** | 0.017 | **0.025** | 0.017 | 0.017 | 0.017 |
| $q$ (bill default probability in disaster) | **0** | **0.4** | 0.4 | 0.4 | **0.3** | 0.4 | 0.4 |
| | | | Variables | | | | |
| Expected equity rate | 0.128 | 0.071 | 0.076 | 0.044 | 0.071 | 0.051 | 0.061 |
| Expected bill rate | 0.127 | 0.035 | 0.061 | −0.007 | 0.029 | 0.015 | 0.025 |
| Equity premium | 0.0016 | 0.036 | 0.016 | 0.052 | 0.042 | 0.036 | 0.036 |
| Expected equity rate, conditional | 0.128 | 0.076 | 0.081 | 0.052 | 0.076 | 0.056 | 0.066 |
| Face bill rate | 0.127 | 0.037 | 0.063 | −0.004 | 0.031 | 0.017 | 0.027 |
| Equity premium, conditional | 0.0016 | 0.039 | 0.019 | 0.056 | 0.045 | 0.039 | 0.039 |
| Price-earnings ratio | 9.7 | 19.6 | 17.8 | 37.0 | 19.6 | 27.8 | 24.4 |
| Expected growth rate | 0.025 | 0.020 | 0.020 | 0.018 | 0.020 | 0.015 | 0.020 |
| Expected growth rate, conditional | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.020 | 0.025 |
| | | Levered results (debt-equity ratio is $\lambda$ = 0.5) | | | | | |
| Expected equity rate | 0.129 | 0.089 | 0.084 | 0.071 | 0.092 | 0.069 | 0.079 |
| Equity premium | 0.0024 | 0.054 | 0.024 | 0.078 | 0.063 | 0.059 | 0.054 |
| Expected equity rate, conditional | 0.129 | 0.096 | 0.091 | 0.080 | 0.099 | 0.076 | 0.086 |
| Equity premium, conditional | 0.0024 | 0.059 | 0.028 | 0.084 | 0.068 | 0.059 | 0.059 |

Figure 10: Model Results (Barro, QJE 2006)

# Gourio (2008)

- Barro (QJE 2006) assumes that disasters are permanent.
- However, in the data, disasters are usually followed by recoveries.
- Gourio (AER 2008) shows that the effect of recoveries hinges on the intertemporal elasticity of substitution (IES):
  - when the IES is low, recoveries may increase the equity premium implied by the model,
  - when it is high, the opposite happens.

Table 1—Measuring Recoveries
*(In percent)*

| | All disasters (57 events) | | Disaster greater than 25% (27 events) | |
|---|---|---|---|---|
| Years after trough | Growth from trough | Loss from previous peak | Growth from trough | Loss from previous peak |
| 0 | 0 | −29.8 | 0 | −41.5 |
| 1 | 11.1 | −22.8 | 16.1 | −32.7 |
| 2 | 20.9 | −16.8 | 31.3 | −24.2 |
| 3 | 26.0 | −13.7 | 38.6 | −20.4 |
| 4 | 31.5 | −10.2 | 45.5 | −16.9 |
| 5 | 37.7 | −6.1 | 52.2 | −13.4 |

*Note:* The table reports the average of (a) the growth from the trough to 1, 2, 3, 4, 5 years after the trough and (b) the difference from the current level of output to the previous peak level, for 0, 1, 2, 3, 4, 5 years after the trough.

Figure 11: Disasters and Recoveries (Gourio, AER 2008)

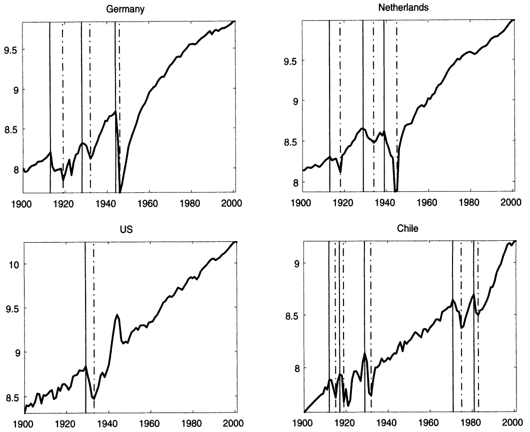



FIGURE 1. LOG GDP PER CAPITA (IN 1990 DOLLARS) FOR FOUR COUNTRIES: GERMANY, NETHERLANDS, THE US, AND CHILE

*Note:* The disaster start (end) dates are taken from Barro (2006), and are shown with a vertical full (dashed) line.

Figure 12: Disasters and Recoveries (Gourio, AER 2008)

TABLE 2

| Probability of a recovery $\pi$ | 0.00 | 0.30 | 0.60 | 0.90 | 1.00 |
|---|---|---|---|---|---|
| IES = 0.25 | 3.31 | 4.62 | 5.91 | 7.19 | 7.64 |
| IES = 0.50 | 3.31 | 3.30 | 3.03 | 2.26 | 1.68 |
| IES = 1 | 3.31 | 2.69 | 1.94 | 1.00 | 0.54 |
| IES = 2 | 3.31 | 2.42 | 1.52 | 0.63 | 0.30 |

*Notes:* Unconditional log geometric equity premium, as a function of the IES and the probability of a recovery. This table sets risk aversion at 4 and the other parameters as in Barro (2006).

Figure 13: Equity Premium and IES (Gourio, AER 2008)

# Intuition I

- To understand the result, recall the present value identity in the case of power utility:

$$\frac{P_t}{C_t} = E_t \sum_{k \geq 1} \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{1-\gamma}$$

- The fact that a recovery may arise can increase or decrease the stock price today depending on whether $\gamma > 1$ or $\gamma < 1$ (recall: IES in the case of power utility is $1/\gamma$).

# Intuition II

- Good news about the future has two effects:
  1. on the one hand, it increases future dividends, which increases the stock price today (cash-flow effect),
  2. but on the other hand, it increases interest rates, which lowers the stock price today (discount-rate effect).
- The second effect (discount-rate) is stronger when interest rates rise more for a given change in consumption, i.e., when the IES is low.

# Other relevant papers

- Some other relevant papers in the strand of the asset pricing literature incorporating disasters:
    - Lettau et al. (2014): downside risk CAPM
    - Wachter (2013): rare disasters and stock market volatility
    - Gabaix (2012): time-varying severity of disasters
    - Farhi and Gabaix (2016): rare disasters and exchange rates
    - Bernstein et al. (2019): climate risk and asset prices

# References I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