Models with habits

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The wish list

- We would like to develop a model that:
  1. explains a high market Sharpe ratio,
  2. and the high level and volatility of stock returns,
  3. with low and relatively constant interest rates,
  4. roughly i.i.d. consumption growth with small volatility.
- In addition, we would like to explain the predictability of excess returns.

Limits of power utility

- Equity premium puzzle, risk-free rate puzzle, predictability.
- Recall:

$$
\begin{aligned}
E_t(R_{t+1}) - R_t^f &= -\frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}\sigma_t(R_{t+1})\rho_t(M_{t+1}, R_{t+1}), \\
&\approx \gamma_t \sigma_t(\Delta c_{t+1})\sigma_t(R_{t+1})\rho_t(\Delta c_{t+1}, R_{t+1}).
\end{aligned}
$$

- **A possible solution comes from variation in** γ_t (i.e., a time-varying risk aversion)

Solution: Time-varying risk aversion?

- A natural explanation for the predictability of returns from dividend yields is that people get *less risk averse* as consumption and wealth increase in a boom, and *more risk averse* as consumption and wealth decrease in a recession.
- We cannot tie risk aversion to the *level* of consumption and wealth (why?).
- We must specify a model in which risk aversion depends on the level of consumption or wealth *relative* to some “trend”, “reference” value, or recent past.

Original “Habit” Literature

- Sundaresan (1989).
- Constantinides (1990).
- Abel (1990).
- ⋆ Campbell and Cochrane (1999).
- Chan and Kogan (2002).

Specifics

- The “habits” form the “trend” in consumption.
- Preferences over habits X defined using ratios or differences: C/X or $C - X$.
- $X_t = f(C_{t-1}, C_{t-2}, ...)$.
- **Internal** or **external** habits: each individual’s habit is determined by his consumption or by everyone else’s consumption.

Outline

Outline

Campbell and Cochrane's model

- CC preferences generate:
  - pro-cyclical variations of stock prices,
  - long horizon predictability,
  - counter-cyclical variation of stock market volatility,
  - counter-cyclicality of the Sharpe ratio,
  - and the short- and long-run equity premium.

Preferences

- The agent maximizes:

$$E\sum_{t=0}^{\infty}\beta^t\frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \tag{1}$$

where γ denotes the risk-aversion coefficient, X_t the external habit level and C_t consumption.

Habits

- Habits should move slowly in response to consumption, something like:

$$x_t \approx \lambda \sum_{j=0}^{\infty} \phi^j c_{t-j},$$

or equivalently: $x_t = \phi x_{t-1} + \lambda c_t$, with $\phi \in (0, 1)$.

Surplus consumption ratio

- Rather than letting the habit level follow an AR(1), CC assumes that it is related to consumption through the following AR(1) process for the (log) **surplus consumption ratio** $S_t \equiv (C_t - X_t)/C_t$:

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g). \tag{2}$$

  where:
  - Lowercase letters correspond to logs.
  - The surplus consumption ratio measures the difference between current consumption and the habit level as a fraction of current consumption.
  - $\lambda(s_t)$ is called sensitivity function.
  - g is the average growth rate of the log-normal consumption process.

Sensitivity function I

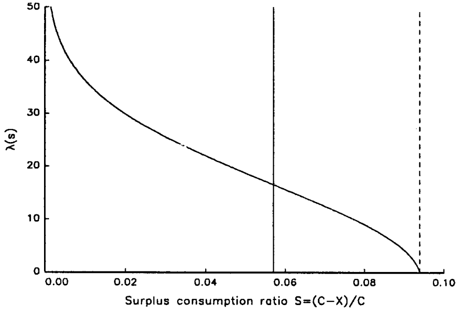
- CC suggest the following sensitivity function:

$$\lambda(s_t) = \frac{1}{\overline{S}}\sqrt{1 - 2(s_t - \overline{s})} - 1, \text{ when } s \leq s_{\max}, \text{ 0 elsewhere,}$$

  where $\overline{S}$ and $s_{\max}$ are respectively the steady-state and upper bound of the surplus-consumption ratio.

- Note that $\lambda(s_t)$ is a counter-cyclical (time-varying) volatility of the shock to the surplus consumption ratio
- How did CC come up with the function for λ? Reverse engineering!

Sensitivity function II



Sensitivity function III

- CC chose the sensitivity function $\lambda(s_t)$ so that the habit level at date $t+1$ does not actually depend on consumption level on the same date.
- This can be shown using a first order Taylor approximation of the law of motion of the habit level x_{t+1} when s_t is close to its steady-state value $\overline{s}$ and consumption growth Δc_{t+1} is close to its average g.

Sensitivity function IV

- The log surplus consumption ratio is equal to:

$$s_t = \ln(\frac{e^{c_t} - e^{x_t}}{e^{c_t}}) = \ln(1 - e^{x_t - c_t}).$$

- Let $x - c$ be the steady-state value of $x_t - c_t$.
- Then a first-order Taylor approximation of s_t around $\overline{s}$ leads to (Details):

$$s_t - \overline{s} \simeq (1 - \frac{1}{\overline{S}})(x_t - c_t - (x - c)).$$

Sensitivity function V

- Recall the law of motion of s_t:

$$s_{t+1} = (1-\phi)\overline{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g).$$

- Substitute for the Taylor approximations for s_t and get:

$$\begin{aligned}(1-\frac{1}{\overline{\overline{S}}})(x_{t+1} - c_{t+1} - (x-c)) &= \phi(1-\frac{1}{\bar{\bar{S}}})(x_t - c_t - (x-c)) + \\ &+\lambda(\overline{s})(c_{t+1} - c_t - g).\end{aligned}$$

- Note that the $\bar{s}$ terms cancel out.

Sensitivity function VI

- CC choose the sensitivity function $\lambda(s_t)$ so that the habit level x_{t+1} does not depend on c_{t+1} (they set $\lambda(\overline{s}) = -(1 - \frac{1}{\overline{S}})$).
- Thus:
$$x_{t+1} - (x - c) = \phi(x_t - c_t - (x - c)) + c_t + g,$$
leads to:
$$x_{t+1} = \phi x_t + [(1 - \phi)(x - c) + g] + (1 - \phi)c_t.$$
- As a result, habits *move slowly* in response to past consumption.

Endowment

- Assume that endowment shocks are *i.i.d* log-normally distributed:

$$\Delta c_{t+1} = g + u_{t+1}, \text{ where } u_{t+1} \sim i.i.d.\ N(0, \sigma^2).$$

Risk-free rate I

- **External habits**: the habit is assumed here to depend only on aggregate, not on individual, consumption.
- Note that $U_c(C, X) = (C - X)^{-\gamma}$.
- Thus, the inter-temporal marginal rate of substitution is here:

$$
\begin{aligned}
M_{t+1} &= \beta \frac{U_c(C_{t+1}, X_{t+1})}{U_c(C_t, X_t)} \\
&= \beta \frac{(C_{t+1} - X_{t+1})^{-\gamma}}{(C_t - X_t)^{-\gamma}} \\
&= \beta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t}\right)^{-\gamma} \\
&= \beta e^{-\gamma[s_{t+1} - s_t + \Delta c_{t+1}]} \\
&= \beta e^{-\gamma[(1-\phi)\bar{s} + (\phi-1)s_t + \lambda(s_t)(\Delta c_{t+1} - g) + \Delta c_{t+1}]} \\
&= \beta e^{-\gamma[g + (\phi-1)(s_t - \overline{s}) + (1+\lambda(s_t))(\Delta c_{t+1} - g)]}.
\end{aligned}
$$

Risk-free rate II

- Assuming that $\overline{S} = \sigma\sqrt{\frac{\gamma}{1-\phi-B/\gamma}}$ and $s_{\max} = \overline{s} + (1-\overline{S}^2)/2$ leads to a linear time-varying risk-free rate:

$$r_t = \overline{r} - B(s_t - \overline{s}), \tag{3}$$

where $\overline{r} = -\ln(\beta) + \gamma g - \frac{\gamma^2\sigma^2}{2\overline{S}^2}$ and $B = \gamma(1-\phi) - \frac{\gamma^2\sigma^2}{\overline{S}^2}$.

▸ Details

Risk-free rate III

- Depending on the value of the structural parameters, the model implies **constant, pro- or counter-cyclical interest rates**.
- Interest rates are constant when B=0: this is the baseline case considered by CC.

Interpretation of B

- **Consumption smoothing** and **precautionary savings** affect the real interest rate, and the parameter B here summarizes these two different effects:

  1. In good times, after a series of positive consumption shocks that result in a high surplus consumption ratio s, the agent wants to save more in order to smooth consumption. This leads to a decrease in the interest rate through an **inter-temporal substitution effect**.
  2. But, in good times, the representative agent is less risk-averse (the local curvature of his utility function is γ / S_t). He is less interested in saving, leading to an increase in the real interest rate through a **precautionary saving effect**.

Interpretation of B (cont'd)

- The case of $B < 0$ is thus the one in which the precautionary effect overcomes the substitution effect. As a result, (real) interest rates are low in bad times and high in good times (i.e., pro-cyclical).

Key mechanism

- Time-varying local risk-aversion coefficient (RRA):

$$\gamma_t = -\frac{CU_{CC}}{U_C} = \frac{\gamma}{S_t}.$$

- As consumption falls toward habit, people become much less willing to tolerate further falls in consumption; they become very risk averse.
- Thus a low power coefficient γ can still mean a high, and time-varying curvature.

Counter-cyclical (max) Sharpe ratio

- Start from:

$$SR_t = \left|\frac{E_t(R^{e,mv}_{t+1})}{\sigma_t(R^{e,mv}_{t+1})}\right| = \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}.$$

- Assuming log-normal SDF leads to:

$$\begin{aligned}
E_t(M_{t+1}) &= e^{E_t(\log M_{t+1})+\frac{1}{2}Var_t(\log M_{t+1})},\\
Var_t(M_{t+1}) &= E_t(M^2_{t+1}) - [E_t(M_{t+1})]^2,\\
&= e^{2E_t(\log M_{t+1})+2Var_t(\log M_{t+1})} - \\
&\quad - e^{2E_t(\log M_{t+1})+Var_t(\log M_{t+1})}.
\end{aligned}$$

Counter-cyclical (max) Sharpe ratio (cont'd)

- As a result:

$$SR_t = \sqrt{e^{Var_t(\log M_{t+1})} - 1} \simeq \sigma_t(\log M_{t+1}) = \frac{\gamma\sigma}{\overline{S}}\sqrt{1 - 2(s_t - \overline{s})}.$$

- At the steady-state, $\overline{SR} = \gamma\sigma/\overline{S}$.

- Approximation is for small values of the variance of the log SDF (i.e., take Taylor approximation of $e^{Var_t(\log M_{t+1})} - 1$ around 0).

Precautionary saving I

- This model gets around interest rate problems with *precautionary saving*.
- **The habit model allows high "risk aversion", with low "aversion to intertemporal substitution."**
- Contrast this with power utility: high "risk aversion" goes along high "aversion to intertemporal substitution."

Precautionary saving II

- $S = (C - X) / C$ is a recession indicator and it is low after several quarters of consumption declines; and high in booms.
- Investors fear stocks because they do badly in *occasional* serious recessions.

Conditional Sharpe ratio

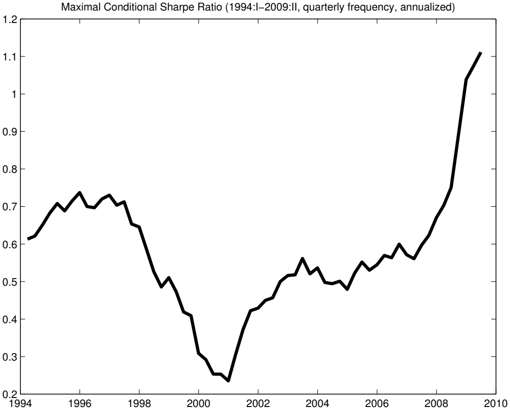


Figure 1: Conditional Sharpe ratio simulated by feeding US real consumption data to the model for the period 1994:1-2009:III.

Outline

Fixed-point method (original CC paper)

- The aggregate market is represented as a claim to the future consumption stream.
- Let P_t denote the ex-dividend price of this claim.
- Then, $E_t[M_{t+1} R_{t+1}] = 1$ implies that in equilibrium P_t satisfies:

$$\begin{aligned} E_t[M_{t+1} \frac{P_{t+1} + C_{t+1}}{P_t}] &= 1, \\ \text{or } E_t[M_{t+1} (\frac{P_{t+1}}{C_{t+1}} + 1) \frac{C_{t+1}}{C_t}] &= \frac{P_t}{C_t}. \end{aligned}$$

Fixed-point method (original CC paper)

- Conjecturing a solution for the price-consumption ratio policy function $G^0(s_t)$, then $G^1(s_t)$ is obtained on a grid of values for s_t as:

$$
\begin{aligned}
G^1(s_t) &= E_t[M_{t+1}(G^0(s_{t+1})+1)\frac{C_{t+1}}{C_t}] \\
&= \beta e^{(1-\gamma)g-\gamma(1-\phi)(\bar{s}-s_t)} \times \ldots \\
\ldots &\times \int_{-\infty}^{\infty} p(\nu) e^{(1-\gamma)\nu-\gamma\lambda(s_t)\nu} \times \ldots \\
\ldots &\times [G^0(\underbrace{(1-\phi)\bar{s}+\phi s_t+\lambda(s_t)\nu}_{s_{t+1}})+1]d\nu,
\end{aligned}
$$

  where $p(\nu)$ is the probability density function of a normal distribution with mean zero and standard deviation σ.

- ν are the shocks to log consumption growth.

- $G(.)$ is the price-dividend policy function (i.e., feed a value for the state s to G to get the corresponding equilibrium PD ratio).

Fixed-point method (original CC paper)

- More generally:

$$
\begin{aligned}
G^{k+1}(s_t) \quad &= \quad \beta e^{(1-\gamma)g-\gamma(1-\phi)(\overline{s}-s_t)} \times \ldots \\
\ldots \quad &\times \int_{-\infty}^{\infty} p(\nu) e^{(1-\gamma)\nu-\gamma\lambda(s_t)\nu} \times \ldots \\
\ldots \quad &\times [G^k((1-\phi)\overline{s}+\phi s_t+\lambda(s_t)\nu)+1]d\nu.
\end{aligned}
$$

- The procedure is repeated until G^{k+1} and G^k differ by at most 10^{-4} (i.e., convergence).

- Note that each step in the recursion requires computing the function obtained in the previous step at a set of points $(1-\phi)\overline{s}+\phi s_t+\lambda(s_t)\nu_j$, where the ν_j are required by the numerical integration routine.

Series method I

- Wachter (2005) proposes the following method.
- Start from the price-consumption recursion and iterate N times:

$$G^N(s_t) = \sum_{n=1}^{N} E_t[(\prod_{j=1}^{n} M_{t+j})\frac{C_{t+n}}{C_t}] + E_t[(\prod_{j=1}^{N} M_{t+j})\frac{C_{t+N}}{C_t}G^0(s_{t+N})].$$

- Choose an initial G^0 such that:

$$\lim_{N\to\infty} E_t[(\prod_{j=1}^{n} M_{t+j})\frac{C_{t+N}}{C_t}G^0(s_{t+N})] = 0.$$

Series method II

- Then, the price-dividend ratio (policy function) is an infinite sum of expectations:

$$G(s_t) = \lim_{N\to\infty} G^N(s_t) = \sum_{n=1}^{\infty} E_t[(\prod_{j=1}^{n} M_{t+j})\frac{C_{t+n}}{C_t}].$$

- Each term above is the time-t price of a claim to the aggregate dividend n periods from now, divided by the dividend today.
- Think of of “zero-coupon equity” with maturity n.
- This suggests another way of solving for the price dividend ratio: **computing each expectation on the right hand-side of the expression above, or at least *enough* terms so that what remains is sufficiently small**.

Series method III

- Let denote $F_n(s_t)$ the n-th term:

$$F_n(s_t) = E_t[(\prod_{j=1}^{n} M_{t+j})\frac{C_{t+n}}{C_t}],$$

and $F_n(s_t)C_t$ is then the price of a “zero-coupon equity” that matures in n periods.

Series method IV

- The one-period return of this security is (no dividends, zero coupon):

$$R_{n,t+1} = \frac{F_{n-1}(s_{t+1})C_{t+1}}{F_n(s_t)C_t},$$

and using $E_t[M_{t+1}R_{n,t+1}] = 1$ leads to:

$$F_n(s_t) = E_t[M_{t+1}\frac{C_{t+1}}{C_t}F_{n-1}(s_{t+1})].$$

Series method V

- We can write the recursion as:

$$
\begin{aligned}
G(s_t) &= \sum_{n=1}^{\infty} E_t\Big[\Big(\prod_{j=1}^{n} M_{t+j}\Big)\frac{C_{t+n}}{C_t}\Big] \\
&= \sum_{n=1}^{\infty} F_n(s_t).
\end{aligned}
$$

Series method VI

- When the equity matures, it pays the aggregate dividend (i.e., we are starting from last period and then move backward):

$$F_0(s_t) = 1.$$

- We need to solve on a grid the following recursion:

$$\begin{aligned} F_n(s_t) &= \beta e^{(1-\gamma)g-\gamma(1-\phi)(\bar{s}-s_t)} \times \ldots \\ \ldots &\times \int_{-\infty}^{\infty} p(\nu) e^{(1-\gamma)\nu-\gamma\lambda(s_t)\nu} \times \ldots \\ \ldots &\times [F_{n-1}((1-\phi)\bar{s}+\phi s_t+\lambda(s_t)\nu)] d\nu. \end{aligned}$$

- As in the fixed-point method, $F_{n-1}((1-\phi)\bar{s}+\phi s_t+\lambda(s_t)\nu)$ is found by interpolating between grid points.

Calibration I

Table 1: This table reports the assumed paramters in CC and Wachter (2005). The CC specification is simulated at a monthly frequency, while the Wachter specification is simulated at a quarterly frequency. Parameters are annualized, e.g.,$12g$, $\sqrt{12}\sigma$, ϕ^{12} for the CC values, and $4g$, 2σ, ϕ^4 for the Wachter values.

	CC value	Wachter Value
Mean Consumption Growth (%) g	1.89	2.20
Std Consumption growth (%) σ	1.50	0.86
Utility curvature γ	2.00	2.00
Parameter in risk-free rate B	0.00	0.011
Habit persistence ϕ	0.87	0.89
Discount Rate β	0.90	0.93

Calibration II

- CC choose parameters to fit the mean and volatility of consumption growth, the average risk-free rate, the Sharpe Ratio, and the persistence of the price-dividend ratio in annual data from 1947 until 1995.
- The parameter B is set to zero, and therefore the (real) risk-free rate is constant.
- Note that CC calculate the price dividend ratio using the fixed-point method and simulate the model at a monthly frequency, and then aggregate the data to an annual frequency.

Results

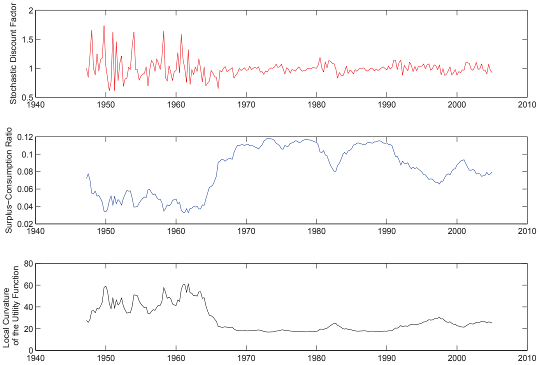


Figure 1. Reality check. Stochastic discount factor, surplus consumption ratio and local curvature of the utility function of an American investor, computed with actual US consumption data only over the 1947:II-2004:IV period using the parameters presented in the first column of Table II.

Calibration in Wachter (2005)

- Wachter (2005) chooses parameters to fit the same moments as in CC, but in quarterly data from 1952 until 2004.
- Also, the parameter B is allowed to differ from zero to match the upward-sloping yield curve for nominal Treasury Bonds.
- The model is simulated at quarterly frequency.

Outline

Bonds I

- Wachter (2006) uses the CC's framework to study bond risk premia.
- Counter-cyclical risk-free rates ($B > 0$) lead to an upward sloping real yield curve.
- To model nominal bonds, she postulates an exogenous ARMA(1,1) process for inflation.
- We cannot obtain closed-form expressions for bond prices but we can easily simulate the model.

Upward sloping yield curve: intuition (I/III)

- Recall that when $B > 0$ the inter temporal smoothing effect dominates and interest rates $\downarrow$ when the surplus consumption ratio $\uparrow$, because investors save more.
- On the other hand, when $B < 0$ the precautionary effect dominates, so that when the surplus consumption ratio $\uparrow$ investor are less risk averse, save less and interest rates $\uparrow$.

Upward sloping yield curve: intuition (II/III)

- The link between B and the slope of the yield curve can be understood looking at the investors' Euler equation:

$$E(R_{t,t+n} - R_{t,t+1}) = \frac{-1}{E(M_{t+1})} cov(R_{t,t+n} - R_{t,t+1}, M_{t+1}),$$

  where $R_{t,t+i}$ is the return on a bond between period t and $t+i$ and $R_{t,t+n} - R_{t,t+1}$ is the term premium.
- if $B > 0$, then in good times when s_t is high we have low short-term interest rates.
- Bond returns move in opposite direction to interest rates (i.e., when interest rate $\downarrow$, bond prices go $\uparrow$ and returns $\uparrow$).

Upward sloping yield curve: intuition (III/III)

- **This means that bonds have high returns in good times, and bad returns in bad times**: they are risky assets, and investors demand a risk premium to hold them!

  $$\frac{E(R_{t,t+n} - R_{t,t+1})}{\sigma(R_{t,t+n})} = -\rho(R_{t,t+n} - R_{t,t+1}, M_{t+1})\frac{\sigma(M_{t+1})}{E(M_{t+1})},$$

  where the last term is the max SR and is countercyclical.

- The price of long-term bonds is more sensitive to changes in interest rates than that of short-term bonds.

- Because long-term bonds have higher expected returns than if there were no risk premia, they must have higher yields.

Term structure I

- A simple way to compute the real yield curve is to use the **series method** presented above.
- Instead of looking for zero-coupon *equities* at different horizons n, we compute zero-coupon *bonds* (the promise of paying 1 dollar n years from now).

Zero-coupon bond prices

- The price of a zero-coupon bond of maturity n is:

$$P_n(s_t) = \beta^n E_t\left[\frac{U_c(C_{t+n,}X_{t+n})}{U_c(C_{t,}X_t)}\right] = E_t[\prod_{j=1}^{n} M_{t+j}].$$

Zero-coupon bond returns

- The one-period return of this security is:

$$R_{n,t+1} = P_{n-1}(s_{t+1}) / P_n(s_t),$$

- Using $E_t[M_{t+1} R_{n,t+1}] = 1$ leads to:

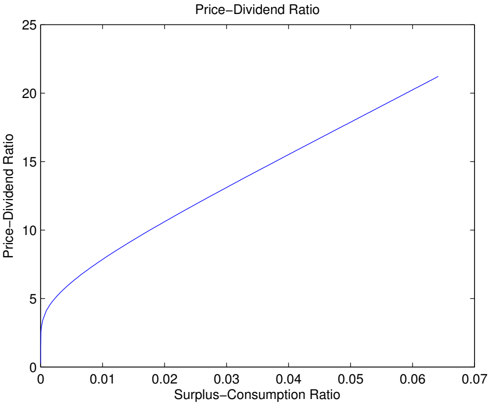
$$P_n(s_t) = E_t[M_{t+1} P_{n-1}(s_{t+1})].$$

Recursive algorithm

- Finally, when the bond matures, its value is: $P_0(s_t) = 1$.
- As a result, we need to solve on a grid the following recursion:

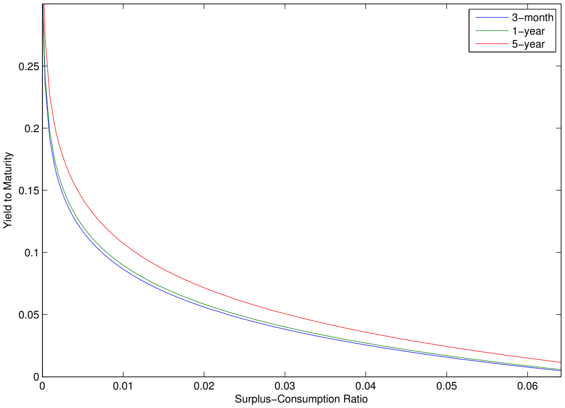
$$
\begin{aligned}
P_n(s_t) &= \beta e^{-\gamma(g+(\phi-1)(\overline{s}-s_t))} \times \ldots \\
\ldots & \times \int_{-\infty}^{\infty} p(\nu) e^{-\gamma(1+\lambda(s_t))\nu} \times \ldots \\
\ldots & \times P_{n-1}((1-\phi)\overline{s} + \phi s_t + \lambda(s_t)\nu) d\nu.
\end{aligned}
$$

Figure 2: Price dividend ratio as function of the surplus consumption ratio. Simulation results using Wachter (2006)'s parameters.



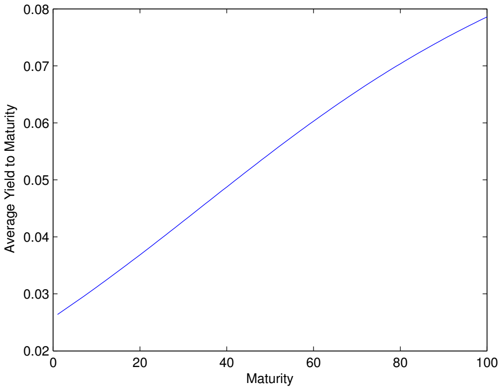
Bonds

Figure 3: Yield to maturity as function of the surplus consumption ratio. Simulation results using Wachter (2006)'s parameters.



Bonds

Figure 4: Real yield curve. Simulation results using Wachter (2006)'s parameters.

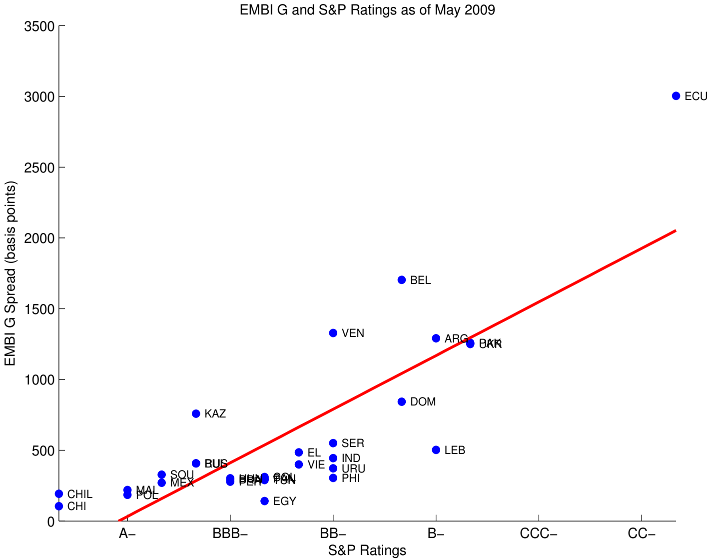


Outline

Borri & Verdelhan 2012

- We study:
  - **sovereign** debt
  - issued by **emerging and developing** countries
  - in **US dollars**.
- We focus on benchmark JP Morgan EMBI indices.
- We take the perspective of a US investor.

EMBI Spreads and Standard & Poors' Credit Ratings



Is It All About Default Probabilities?

- Think about a one-period zero-coupon sovereign bond.
- Let r^f denote the real risk-free rate, r the return on the sovereign bond, p the default probability and *rec* the recovery rate in case of defaults.
- Invest one dollar in a risk-free asset or in a sovereign bond.
- If investors are risk-neutral, expected returns are the same:

$$1 + r^f = (1 - p) \times (1 + r) + p \times rec$$

  - NB: We observe r^f and r. If we assume a value for *rec*, then we can compute p:

$$p = \frac{r - r^f}{1 + r - rec}$$

- Then expected excess returns should be zero: Not true in the data!

Risk Matters!

Here's the intuition:

- Emerging countries tend to default when they are in trouble (i.e in bad times for them).
- Take two countries: Mexico and Thailand.
- Assume that they have the same default probability.
  - Bad times in Mexico are likely to correspond to bad times in the US,
  - Less so for Thailand.
- For a given identical default probability, Mexican sovereign bonds are more risky than Thai bonds for the US investor.

Our Findings

- **Large cross section of average holding period excess returns** between countries with low and high probability of default and low and high connection to the US:
  - Use Standard and Poor's credit ratings to measure default probability;
  - Use bond market betas to measure connection to the US.
- These excess returns are **risk premia.**
  - One **single** risk factor, the BBB US corporate bond return, explains the cross sectional variation in average excess returns.
- A **model with optimal sovereign borrowing and default and external habit preferences** replicates qualitatively these results.
  - Contradicts pure "Peso" explanations of excess returns;
  - Risk premia link countries: countries default **more** frequently when **investors' risk aversion is high**.

EMBI Portfolios

- Countries are ranked on **two** dimensions:
  - their default **probabilities**,
  - their **bond market betas**.

- At the end of each month t, sort all countries into 2 groups on the basis of their $\beta^i_{EMBI,t}$ (information up to date t).

- Within each group, sort countries into 3 portfolios on the basis of their default probabilities at the end of each month t.

- Compute the log excess return $r^{e,j}_{t+1}$ for each portfolio $j = 1, 2, \ldots, 6$ by averaging:

$$r^{e,j}_{t+1} = \frac{1}{N_j} \sum_{i \in P_j} r^{e,i}_{t+1}.$$

Cross-section of Excess Returns

Portfolios	1	2	3	4	5	6
β^j_{EMBI}		Low			High	
S&P	*Low*	*Medium*	*High*	*Low*	*Medium*	*High*
			EMBI Bond Market Beta: β^j_{EMBI}			
Mean	0.09	0.13	0.10	0.39	0.47	0.64
Std. Dev.	0.16	0.20	0.20	0.33	0.31	0.38
			S&P Default Rating: dp^j			
Mean	7.15	9.60	13.10	10.05	12.25	15.22
Std. Dev.	1.52	1.00	1.03	1.68	0.96	1.47
			Excess Return: $r^{e,j}$			
Mean	**3.75**	4.13	6.92	8.44	8.78	**14.62**
s.e	[1.75]	[2.15]	[2.76]	[2.98]	[3.90]	[5.09]
Std. Dev.	7.34	9.08	11.42	11.80	15.25	20.72
SR	0.51	0.45	0.61	0.71	0.58	0.71
			Debt/GNP			
Mean	0.11	0.16	0.33	0.27	0.30	0.33
Std. Dev.	0.05	0.09	0.08	0.09	0.08	0.09

Annualized monthly excess returns. Monthly Data. Higher S&P's credit ratings is higher default probability.
Sample period is 1/1995 – 5/2011.

Spreads in Sovereign Bond Returns

- Spreads between high and low default probability countries: **470 bp** on average.

- Spreads between high and low bond market beta countries: **570 bp** on average.

Macro Implications?

- **Cost of debt**, **borrowing** and **default** choice are all endogenous.
- We need a model of optimal sovereign borrowing and default:
  - N-1 small open economies (‘borrowers’),
  - 1 large developed economy (US, ‘lender’),
  - Framework a la Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008),
  - But:
    - Lenders are **risk-averse**: external habit preferences as in Campbell and Cochrane (1999).
    - The borrowers’ endowment is composed of a transitory component **and** a time-varying mean.
    - One source of **heterogeneity** among ‘borrowers’: countries differ in their correlation with respect to the US business cycle.

Why External Habits?

- Models with power utility preferences do not produce large spreads in excess returns.
- Intuition:
  - Assume same probability of default, same yield volatility.
  - Spreads depend on $cov_t(M_{t+1}, R^{e,i}_{t+1}) - cov_t(M_{t+1}, R^{e,j}_{t+1})$.
  - Maximum spreads between high and low-correlation countries:

$$2\gamma\sigma_{\Delta c}\sigma_{r^e} \simeq 2 \times 2 \times 1.5\% \times 13\% \simeq 80bp.$$

  - Matching the data with CRRA implies a very high risk-aversion coefficient and implausible risk-free rates.
- $\Rightarrow$ External habit implies that risk aversion is time-varying and higher in 'bad times'.

The Borrowing Economy

- Timing of events in period t:
  1. household receives stochastic endowment Y_t^B,
  2. government **repays** outstanding debt B_{t-1}^t or **defaults**,
  3. if government **repays**:
     - it borrows B_t^{t+1} at the price Q_t,
     - it makes a lump-sum good transfer to the household.
  4. if government **defaults**:
     - it is excluded from international capital markets for a stochastic number of periods,
     - faces a direct output cost $\Rightarrow Y_t^{B,def}$.

Endowment: borrowers

- Two components: $Y_t^i = e^{z_t^i}\Gamma_t^i$
  - a transitory component z_t^i,
  - a time-varying mean (or permanent component) Γ_t^i.

$$
\begin{aligned}
z_t^i &= \mu_z(1-\alpha_z) + \alpha_z z_{t-1}^i + \epsilon_t^{z,i} \\
\Gamma_t^i &= G_t^i \Gamma_{t-1}^i, \\
g_t^i &= log(G_t^i) = \mu_g(1-\alpha_g) + \alpha_g g_{t-1}^i + \epsilon_t^{g,i}.
\end{aligned}
$$

- $\epsilon^{g,i}$ and $\epsilon^{z,i}$ are $i.i.d$ normal, $E(\epsilon^{g,i'}\epsilon^{z,i}) = 0$.
- Similarities across countries: $E([\epsilon^{z,i}]^2) = \sigma_z^2$ and $E([\epsilon^{g,i}]^2) = \sigma_g^2$.

Endowment: lenders

- US consumption growth is a random walk:

$$\Delta c_t = g + \epsilon_t.$$

- Emerging countries only differ according to their conditional correlation to the developed economy:

$$E(\epsilon^{z^{i'}} \epsilon) = \rho^i.$$

Investors

- Lenders are:
    - **risk averse**,
    - behave competitively,
    - supply any quantity of funds demanded at a price Q_t,
    - have **external habit preferences** over their consumption endowment described by:

$$E_t \sum_{t=0}^{\infty} (\beta)^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma}.$$

    - H_t is the external habit level and corresponds to a time-varying subsistence level or social externality.
- $\Rightarrow$ If lenders were **risk neutral**, there would be no role for covariances in sovereign bond prices.

Recursive Equilibrium

- State variables:
  - $x = [y^i, s]'$, Markovian with conditional density $f(x', x)$.
  - B is the quantity of debt coming to maturity.

- Value of the **option** to default or stay in the contract is:

$$v^o(B, x) = max\{v^c(B, B', x), v^d(x)\}$$

- Value of **default** is:

$$v^d(x) = u(y^{def}) + \beta \int_{x'} [\pi v^o(0, x') + (1 - \pi) v^d(x')] f(x', x) dx'$$

- Value of staying in the **contract** is:

$$v^c(B, x) = Max_{B'}\{u(c) + \beta \int_{x'} v^o(B', x') f(x', x) dx'\}$$

Results

- A model with **both large sovereign debt levels and large spreads**:
  - time-varying mean growth rates $\Rightarrow$ incentives to borrow (e.g., good times ahead, but want to consume now) $\Rightarrow$ large debt
  - But, sometimes, ... unexpected bad news (e.g., negative temporary shocks) $\Rightarrow$ defaults $\Rightarrow$ large spreads
- Defaults and bond prices depend not only on the borrowers', but also on the lenders' economic conditions

Country-Level Simulation Results (I/II)

	Macro Moments			
	Model			Data
Cross-country correlation:	Low	Zero	High	
$\sigma(Y)$	6.61	6.61	6.61	5.39
$\sigma(\Delta Y)$	4.60	4.60	4.60	3.60
$\rho(Y)$	0.78	0.78	0.78	0.81
$\rho(\Delta Y)$	0.15	0.15	0.15	0.45
$\sigma(C)/\sigma(Y)$	1.66	1.61	1.56	1.30
$\sigma(TB/Y)$	8.12	7.59	7.03	5.00
$\rho(TB/Y, Y)$	-0.11	-0.13	-0.15	-0.33
$\rho(C, Y)$	0.69	0.71	0.74	0.59
E(Default)	6.67	4.97	3.27	1.91
E(Debt/Y)	-30.30	-28.96	-27.50	-49.68

Country-Level Simulation Results (I/II)

	Asset Pricing Moments			
		Model		Data
Cross-country correlation:	Low	Zero	High	
$E(R^e)$	-2.20	$--$	1.15	7.00
$\sigma(R^e)$	26.21	$--$	18.36	18.07
$\rho(Y, R^e)$	-0.07	$--$	-0.06	-0.19
$\rho(TB/Y, R^e)$	-0.02	$--$	-0.01	0.14
$E(spread)$	4.69	4.92	4.50	5.44
$\sigma(spread)$	1.15	1.22	1.07	3.77

APPENDIX

Surplus Consumption Ratio

- Take first order Taylor expansion around $x - c$ and recall that $s_t = \ln(1 - e^{x_t - c_t})$.
- Note that by chain-rule:

$$\frac{\partial s_t}{\partial(x_t - c_t)} = -\frac{1}{1 - e^{x_t - c_t}} e^{x_t - c_t}.$$

- We need to evaluate the derivative at $x - c$:

$$-\frac{e^{x-c}}{1 - e^{x-c}}.$$

- Further note that:

$$1 - \frac{1}{\overline{\overline{S}}} = \frac{\overline{S} - 1}{\overline{S}} = -\frac{\frac{X}{C}}{\frac{C-X}{C}} = -\frac{e^{x-c}}{1 - e^{x-c}},$$

where $\overline{S}$ is the value of the surplus consumption ratio evaluated at $x - c$.

- Therefore:

$$s_t - \overline{s} \simeq (1 - \frac{1}{\overline{\overline{S}}})(x_t - c_t - (x - c)).$$

Back

Risk-free rate

- Recall that the risk-free rate is equal to $R_t^f = 1/E(M_{t+1})$.
- Compute first $E(M_{t+1})$:

$$E(M_{t+1}) = \beta e^{-\gamma[g+(\phi-1)(s_t-\bar{s})]+\frac{\gamma^2}{2}(1+\lambda(s_t))^2\sigma^2},$$

where we used the log-normal trick and the fact that $E(\Delta c_{t+1} - g) = 0$.

- Therefore:

$$R_t^f = [\beta e^{-\gamma[g+(\phi-1)(s_t-\bar{s})]+\frac{\gamma^2}{2}(1+\lambda(s_t))^2\sigma^2}]^{-1}.$$

- And:

$$r_t^f = \ln(R_t^f) = -\ln(\beta) + \gamma g + \gamma(\phi - 1)(s_t - \bar{s}) - \frac{\gamma^2}{2}(1 + \lambda(s_t))^2\sigma^2.$$

Back

Risk-free rate

- Now use the fact that $[1+\lambda(s_t)]^2 = \frac{1}{\overline{S}^2}(1 - 2(s_t - \bar{s}))$:

$$r_f = -\ln(\beta) + \gamma g - \frac{\gamma^2}{2}\frac{1}{\overline{S}^2}\sigma^2 - (s_t - \bar{s})[\gamma(1-\phi) - \frac{\gamma^2}{\overline{S}^2}\sigma^2].$$

- CC define $B = \gamma(1-\phi) - \frac{\gamma^2\sigma^2}{\overline{S}^2}$ and $\bar{r} = -\ln(\beta) + \gamma g - \frac{\gamma^2}{2}\frac{1}{\overline{S}^2}\sigma^2$so that:

$$r_f = \bar{r} - B(s_t - \overline{s}).$$

Back

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