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The curvature measurement of the human eyeball

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The curvature of the cornea in the ophthalmology field is essential data, especially for humans when they are being tested for wearing contact lenses, or for patients undergoing refractive surgery by either knife or laser. It is extremely difficult to measure the curvature of the eye precisely because the measurement of the cornea, using a contact tool, is not possible. In this paper, we propose a method to measure the curvature using lens aberration theory and an image processing technique. We project a circular grating, known as a Placido disc, on to the cornea. The reflective image does not have equal divisions and this distortion value contains the curvature information. ©1998 Elsevier Science Ltd.

KEYWORDS: distortion aberration, image processing, curvature, ophthalmology, Placido disc

Introduction

In ophthalmology, the doctor always uses a keratometer to diagnose the astigmatism of the eyeball by projecting a circular grating onto the patient's cornea and checking the reflective image. If the reflective image is a set of circular gratings, then the eyeball is normal; if not, the eyeball is astigmatic. The spacing of the rings also gives the skilled practitioner information about the curvature of the cornea. A flat disc with a circular grid, known as a Placido disc, was developed as an instrument over 100 years ago. The concept of using more circles to improve the resolution has been used by several authors[1]. Many of these systems have then been computerized to produce three-dimensional maps of the corneal surface. In fact, such systems are available from a number of commercial companies, as corneal mapping is now an important area with the advent of laser refractive surgery. However, all of these systems are expensive and are not suitable for use by most opticians' shops. Whereas most commercial systems use cones rather than flat plates, our proposed system incorporates a simple and low-cost design. We use a flat plate and, by adding more circles and decreasing the size of each circle, we obtain good resolution from a transportable system that is easy to manufacture and low cost[2].

We place a CCD camera in the centre of the disc behind the circular grating to record the image of the reflective circular grating from the cornea. First, we obtain the thinning image using the thinning algorithm[3], and this enables us to distinguish each order of circles. Secondly, we calculate the centre of the image. Finally, we compare the distortion image with a standard circular grating to calculate the offset value of each specific sampling point. These offset values are caused by the distortion, which is proportional to the curvature of the eyeball.

If a circular grating was reflected by a convex spherical mirror, we show that the circular image produced is of equal division using ray tracing techniques. However, in the real world, the image is seriously distorted with the space between each circle becoming increasingly closer at the outer circles. We used the lens design software GENII[4] to simulate the relationship between the object and the image and found the distortion which dominates the five different kinds of aberrations[5]. We need only to calculate the offset values of each fringe, and the curvatures of the whole surface can then be obtained.

The distortion concept

To discuss the distortion produced by a single refracting surface[6,7], we considered that the object surface is flat and normal to the axis at L, as in Fig. 1. H is an object height on the flat object surface. The image plane is conjugated to L at L' for paraxial rays. H' is the image traced by the chief ray from H through O to the image screen. The auxiliary axis from H through C penetrates the image screen at H''. If there is no distortion, H' must be located at the same point as H''. We can clearly find that when the aperture-stop was placed at the centre of curvature, all the chief rays would pass through the refracting surface normal to the centre of curvature. Therefore, we get a perfect projection image from the object. The chief ray from H coincides

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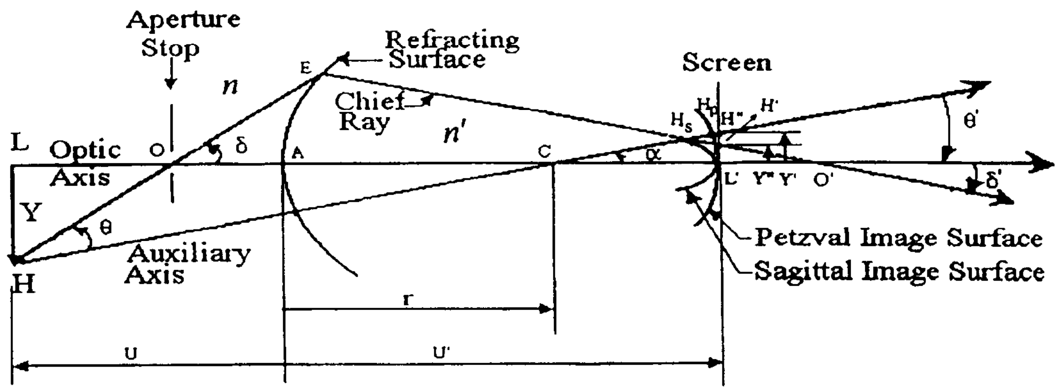


Fig. 1 Distortion produced by a spherical refracting surface

with the auxiliary axis at H' and also coincides with H''. Otherwise, distortion occurs. The amount of distortion is the displacement from H' to H''. The chief ray crosses the auxiliary axis at H_s and H_p, which is the sagital image and Petzval image of each H. The equation for distortion can be derived as follows

$$\text{distortion} = \overline{H'H''} = -(\overline{H''H_p} + \overline{H_pH_s})\frac{\sin\theta'}{\cos\delta'} \quad (1)$$

For small angles, the sine is equal to the radian value and the cosine is equal to unity, and equation (1) reduces to (2). The error can be minimized when we select a suitable E (distortion coefficient) to calculate the distortion amount

$$\text{distortion} = \overline{H'H''} = -(\overline{H''H_p} + \overline{H_pH_s})\theta' \quad (2)$$

Then, $\overline{H''H_p}$ represents the Sag of the Petzval surface and can be represented as

$$\text{Sag} = -\left(\frac{n'-n}{2nr}\right)Y'^2, \quad Y' = \overline{L'H''} \quad (3)$$

where n is the index in air, and n' is the index of the image-side medium. $\overline{H_pH_s}$ is the spherical aberration of $\overline{\text{HO}}$ based on the auxiliary[6]. The equation is as follows

$$\overline{H_pH_s} = \bar{A}\theta^2 = \bar{A}\left(\frac{u'}{u}\theta'\right)^2 \quad (4)$$

where

$$\bar{A} = \left[\frac{1}{2}\frac{n}{n'}(u')^2\left(\frac{n'-n}{n'^2}\right)\left(\frac{r-u}{r}\right)^2\left(\frac{n'+n}{u} - \frac{n}{r}\right)\right] \quad (5)$$

$$\theta' = \delta' - \alpha \quad (6)$$

$$\alpha = \frac{Y'}{CL'} \quad (7)$$

$$\theta' = \left(\frac{\overline{CO'}}{\overline{O'C'}\cdot\overline{CL'}}\right)Y' \quad (8)$$

Equations (3), (4) and (8), the distortion equations, can be derived as follows:

$$\text{distortion} = E(Y')^3 \quad (9)$$

where

$$E = \left(\frac{u'}{u}\right)^2\left(\frac{\overline{CO'}}{\overline{O'L'}\cdot\overline{CL'}}\right)^3\bar{A} - \left(\frac{\overline{CO'}}{\overline{O'L'}\cdot\overline{CL'}}\right)\left(\frac{n'-n}{2nr}\right) \quad (10)$$

Distortion increases in proportion to the cube of Y'. In our case, we measured from a reflecting surface. Therefore, we derive the distortion equation produced

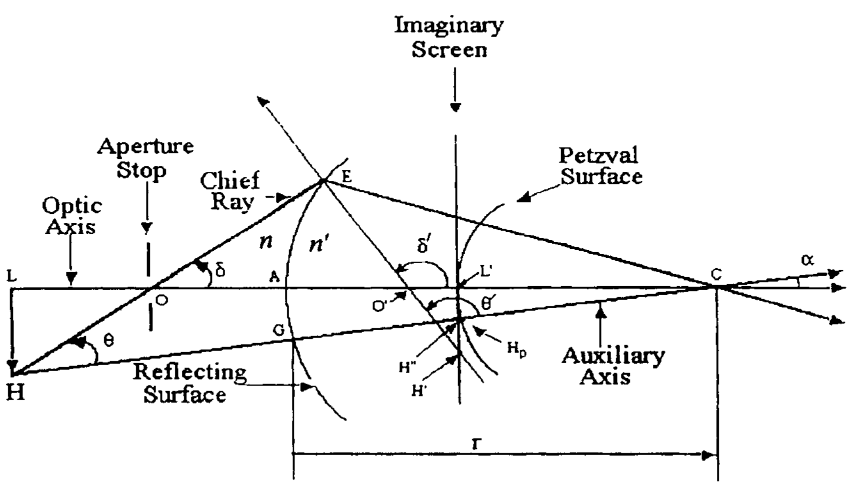


Fig. 2 Distortion produced by a spherical reflecting surface

by a spherical reflecting surface, as shown in Fig. 2. The distortion equation is the same as (9), where

$$E = \left(\frac{u'}{u}\right)^2 \left(\frac{\overline{CO'}}{\overline{O'L'} \cdot \overline{CL'}}\right)^3 \overline{A} - \left(\frac{\overline{CO'}}{\overline{O'L'} \cdot \overline{CL'}}\right)\left(\frac{1}{r}\right) \quad (11)$$

$$\overline{A} = (u')^2 (u - r)^2 \left(\frac{1}{r}\right)^3 \quad (12)$$

and where r is the curvature of the reflecting surface, and E is the distortion coefficient.

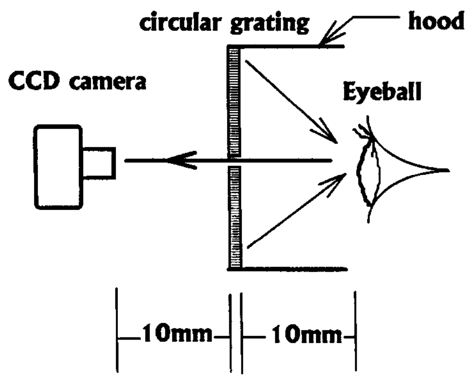


Fig. 3 The set-up of eyeball curvature testing

Image processing

The system set-up is shown in Fig. 3. The grid has 27 rings, 1 mm in diameter spaced on 2.0 mm centres. The distance from plate to cornea is 10 mm. The lens is a 50 mm/2.8 micro lens made by Schneider and located 10 mm behind the plate. We used this system to obtain a real eyeball image upon which a circular grating was projected, as shown in Fig. 4. We mainly utilized an image processing technique[8] for dealing with the fringe pattern the CCD camera records, to obtain the information we required.

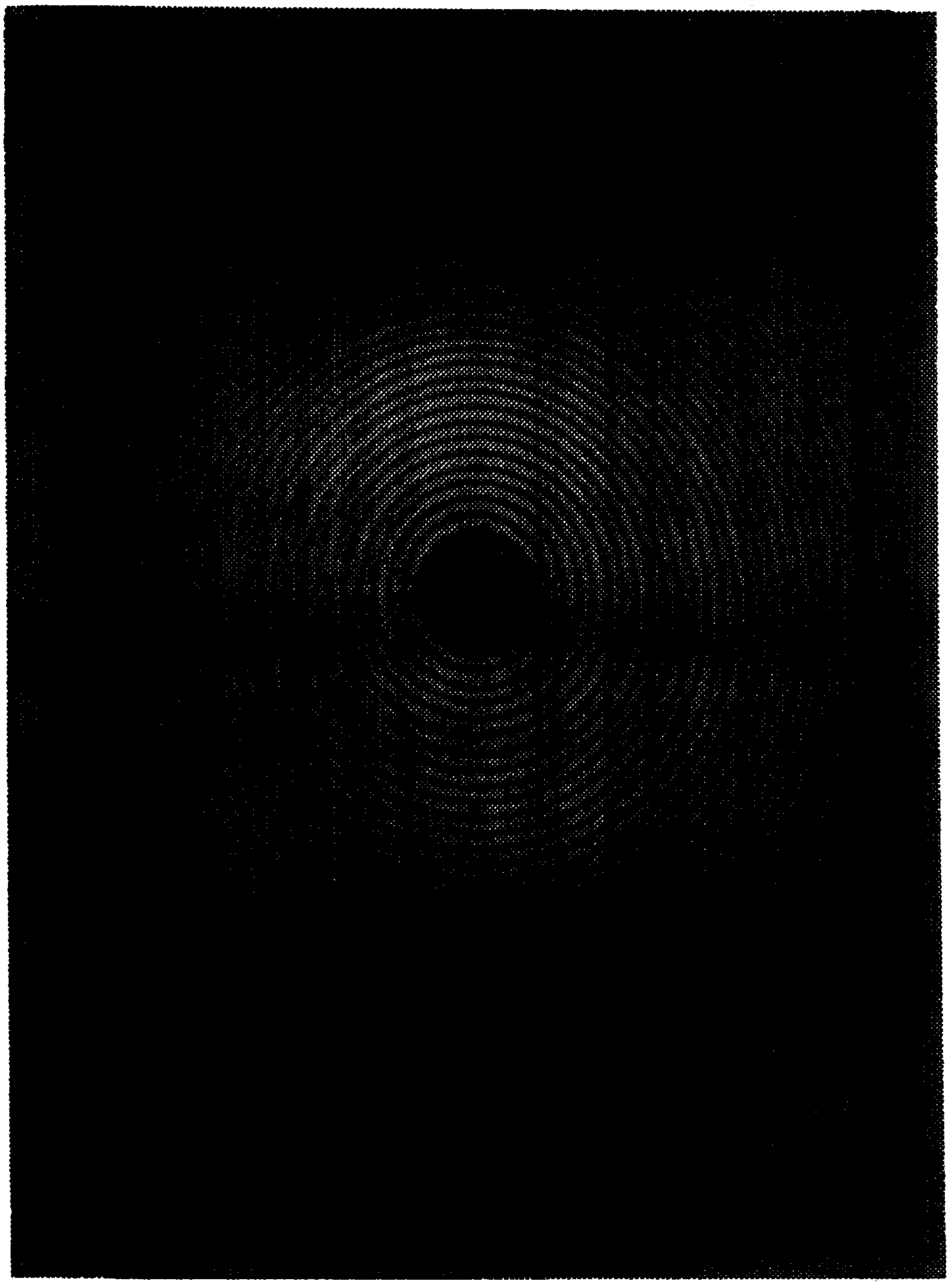


Fig. 4 The distortion circular image produced by a real cornea obtained from the proposed system

The first step is noise-suppression. This step filters the noise of the digitized image, and enhances the connectivity of the broken segments. It consists of isolated point deletion, dilation point deletion and missing point recovery. The second step is histogram equalization and histogram extension. This increases the contrast of the background and the object for the purpose of thresholding. The third step is thresholding. The purpose of thresholding is to obtain the binary image and to classify the background and the object. The final step is thinning and, in this step, we use a new thinning method to extract the skeleton of the fringe. The thinned pattern preserves the connection and the shape of the original pattern. In short, we use iterative deletions of the dark points (that is, change them to white) along the edges of a pattern until the pattern is thinned to a line drawing. In our thinning method, we modify the coding method of Carlo Arcelli and Gabriella Sanniti[9] to encode the fringe pattern. This coding method consists of forward scanning (from bottom right to top left), which will mark all pixels of the image using different numbers to distinguish the boundary and core from the object. We then utilize the new 'saving-deletion rule'[3] to save or delete these marked pixels and, therefore, the outer boundary, which does not destroy the skeleton shape, will be gradually deleted. We repeat the steps above, and the fringe skeleton is obtained. We used this thinning algorithm to get the thinned fringe and compensated the lost pixels of each fringe. We calculated the centre of gravity of the image in order to compare easily the distortion fringe image with the perfect grating. The formula of gravity (x^*, y^*) is

$$x^* = \frac{\sum_{\text{all } y}\sum_{\text{all } x} x \cdot f(x,y)}{\sum_{\text{all } y}\sum_{\text{all } x} f(x,y)} \tag{13}$$

$$y^* = \frac{\sum_{\text{all } y}\sum_{\text{all } x} y \cdot f(x,y)}{\sum_{\text{all } y}\sum_{\text{all } x} f(x,y)} \tag{14}$$

We also need to determine the order of each fringe. We selected 36 sets of lines, as in Fig. 5, which were through the centre every 10 arc degrees, and counted the coordination of the cross point of each order. The sampling points would be 36 pixels for each order.

Experiment and results

The distortion value is the distance between the image pixel and the perfect grating, which is then multiplied by the magnification factor of the CCD lens, at the same order fringe. If the circular grating included N orders, then the total sampling points would be $N \times 36$. To get the correct data, all experimental data should be adjusted by a suitable E value to match the real value. The distortion coefficient, the E value, can be calculated by the object height, image height, and the distortion percentage. We solved the E value at the different curvatures and orders using (9) to get the E value table. Table 1 shows part of the whole E value. We measured the D (distortion) value and used (9) and Table 1 to find

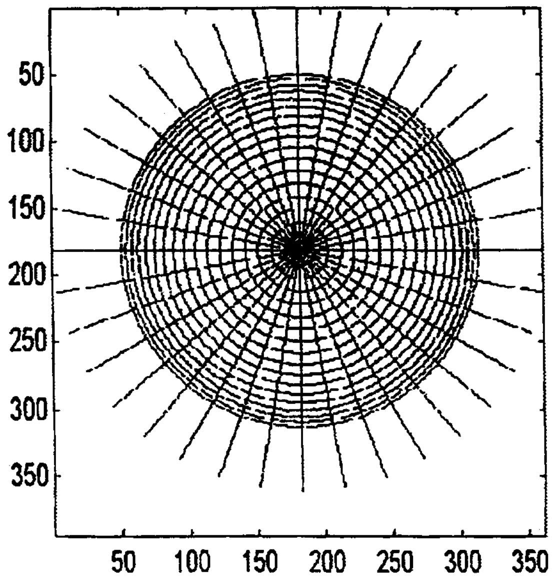


Fig. 5 The sampling method

the best matching E value to solve the Y' (image height), then used the following equations

$$M = \frac{Y'}{Y} = -\frac{u'}{u} \tag{15}$$

$$\frac{1}{u} + \frac{1}{u'} = -\frac{2}{r} \tag{16}$$

where M is the magnification factor.

The r of each sampling point is a diopter, which is the total power of the front and rear surface of a cornea. We assume the index number of the cornea to be 1.3375, the thickness of cornea to be 0.08 mm, and the curvature of the rear cornea surface to be 6.8 mm (see Ref. 10). It is easy to calculate the cornea surface curvature r for every sampling point of the eyeball. Figure 6 is a three-dimensional mesh picture of a real eyeball. We compared the cornea data with the EyeSys[11] system's output—the system is an ophthalmology machine for measuring the cornea curvature. Our prototype system uses a manual focus system. We grabbed an image of the cornea as a testing sample 15 times. After calculation, we find each testing sample has less than 5% error caused by the position error, which makes up the focus difference. The focus difference causes the different magnifications of the grating image. We take the average contour value of these 15 testing samples and plot the contour map shown in Fig. 7. The testing result of the EyeSys system

Table 1. Part of the E value for different curvatures and object heights

	E value	
	Curvature 7.5 mm	Curvature 7.7 mm
Object height 100%	0.2127	0.1943
Object height 90%	0.2035	0.1862
Object height 70%	0.1854	0.1701
Object height 50%	0.1699	0.1565

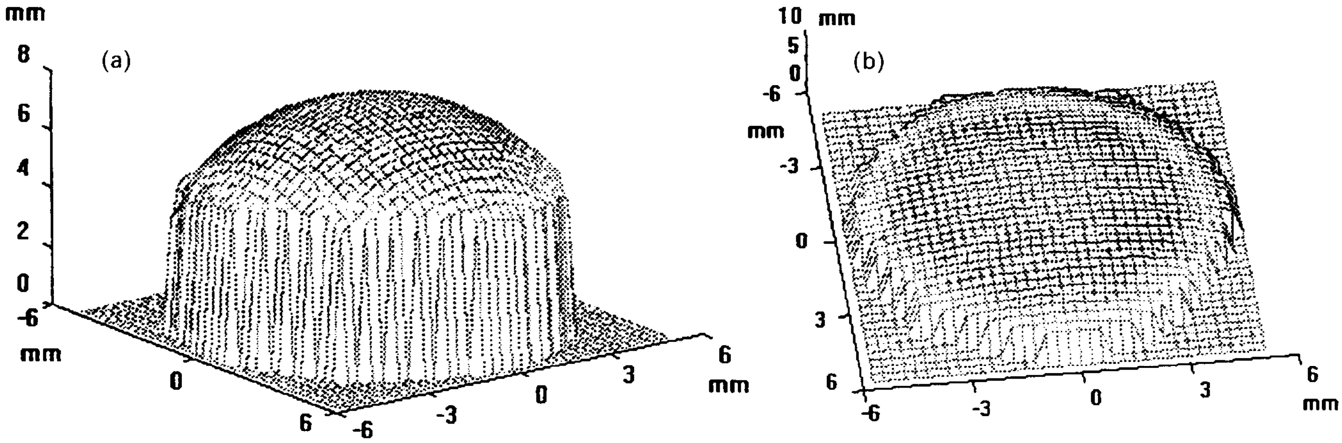
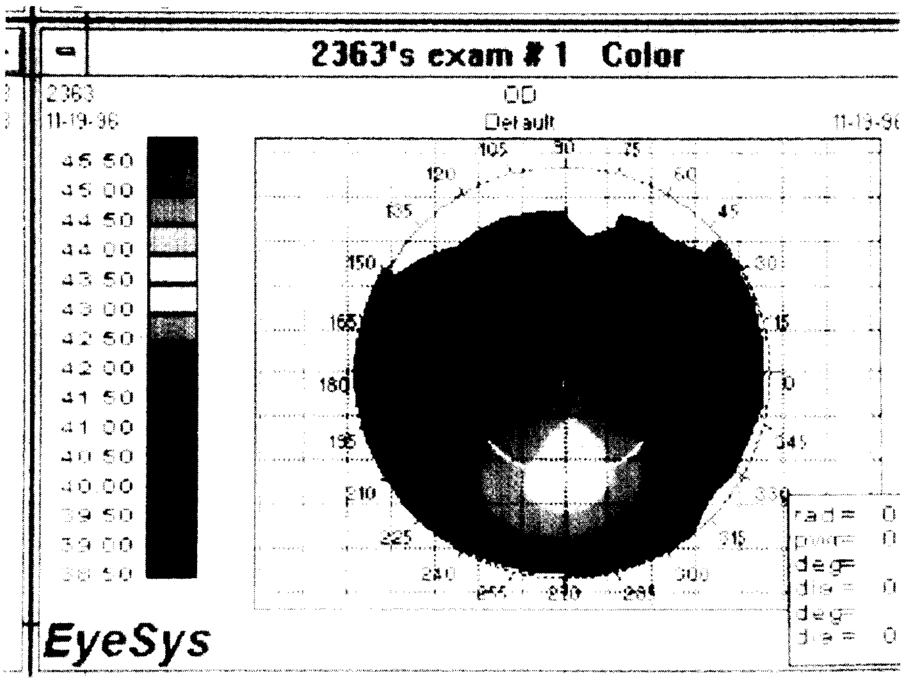


Fig. 6 The three-dimensional mesh picture of a real cornea using our proposed system: (a) side view; (b) oblique view



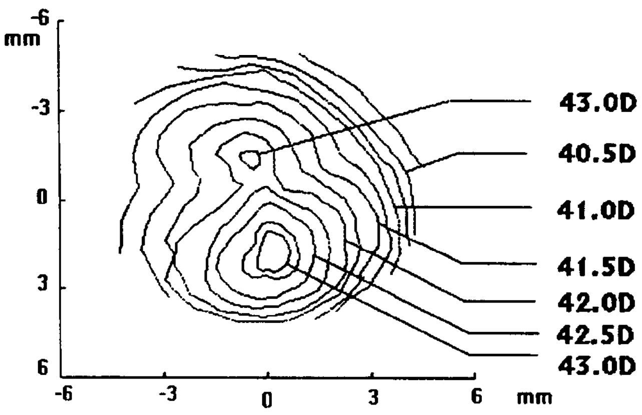


Fig. 7 The contour map of a cornea: (a) by EyeSys; (b) by the proposed system

is a contour map which is composed of many different areas. We also take the average value of each contour area to become a thin curve contour. We overlap these two thin contour maps and calculate the difference in order to obtain the total error. The two diopter contour maps obtained from two different systems are almost the same. The average error of these 15 testing samples is less than 5% compared with the data obtained from the EyeSys system.

Conclusions

Distortion is a problem for optical design. All lens design engineers try to remove this problem, especially in wide angle lens design. However, in our proposed method, distortion is a contribution of the eyeball curvature measurement system. The distortion image contains the curvature messages. Using the Placido disc is economic and it is easy to manufacture. We have demonstrated that using the Placido disc it is possible to produce good curvature data of the cornea. However, in our experiments, using an equally spaced circular grating, the outer ring is closer and we can design a differently spaced circular grating to overcome the problem of distortion.

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