DETECTION AND MODEL ANALYSIS OF CIRCULAR FEATURE FOR ROBOT VISION

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Abstract:

A novel detection and model analysis of circular feature with error compensation is addressed in this paper for robot vision. We use an efficient method of fitting ellipses to data points by minimizing the algebraic distance subjected to a constraint that a conic should be an ellipse, and solve the ellipse parameters through Lagrange multiplier method. Using closed form solution, the 3D position and orientation of a circular feature with known radius can be obtained. Since the center of projected circle does not correspond to the ellipse center, we compensate for the projection error between them. Experimental results show that our method is robust, efficient and accurate.

Keywords:

Circular feature; detection; 3D model; robot vision

1. Introduction

One of the key elements of robot vision systems is the analysis and description of 3D objects. To analyze the 3D model of objects, detection of geometry features in digital images is an important task for image analysis and perception. Circular features such as circle and ellipse are not only the basic element in nature but also very common shapes in many man-made objects, which have been commonly used in robot vision fields. Therefore estimation of 3D information from 2D image coordinates for these features is an important problem. Circular feature is a particular case of conic feature, because its perspective projection in any arbitrary orientation is always an exact ellipse, and a circle has been shown to have high center-location accuracy for its property of isotropy. Monocular vision used in navigation of a mobile robot can find 3D position and orientation of an object if the objection model is known [1], [2]. In many other vision based fields such as automatic assembly, industrial metrology and unmanned drive, it is also important to estimate the 3D location of objects.

For many advantages mentioned above, circular features have been widely used in robot vision for accurate self-location using circular landmarks and football tracking in robot soccer competition because the perspective projection of a spherical football is always a circle [3], [4]. However, the center of projected circle does not correspond to the ellipse center [5], therefore the error compensation is a key scale to the accuracy. There have been several methods proposed to solve the model analysis of circular feature [2], [6], [7], [8], but most of them are mathematically complex and have not performed the error compensation of circular feature, so the precision is not very good. Since the radii of many circular features such as football and circular landmarks used for location are known, we focused on accurate estimation of the 3D position and 3D orientation of a circular feature with known radius. We use a board machined with circular features to perform circles detection and 3D model analysis based on accurate model and rapid algorithm with error compensation. Experimental results show that our method is accurate and reliable.

2. Circular feature detection

It is well known that the accurate estimation of the basic parameters of an elliptical shape is important to the accuracy of the 3D model of circular features. The general form of a common quadratic curve can be expressed as the form of

$$F(S,U) = SU = au^2 + buv + cv^2 + du + ev + f = 0 \qquad (1)$$

where $S = [\, a \;\; b \;\; c \;\; d \;\; e \;\; f \,]$, $U = [\, u^2 \;\; uv \;\; v^2 \;\; u \;\; v \;\; 1 \,]^T$.

The previous research [7], [8], [9] focused on ellipse fitting can be divided into two basic techniques: Hough transform and least squares fitting. Based on mapping set of points to the parameter space, Hough transform have some great advantages especially of high robustness to occlusion and noise, but suffer from such great shortcomings as high computational complexity and non-uniqueness of solutions. Looking for accurate estimation of elliptical parameters, the least squares method is centered on finding the set of parameters that minimize the squares sum of error of fit between the data points and the ellipse:

$$e = \sum_{i=1}^{n}(au_i^2 + bu_iv_i + cv_i^2 + du_i + ev_i + f)^2 = \|SW\|^2 \quad (2)$$

where $W = [U_1\ U_2\ \cdots\ U_n]^T$.

In order to fit the data points to be ellipse, the constraint for the conic is well known that the discriminant $b^2 - 4ac$ is negative. Since we have the freedom to arbitrarily scale the parameters of the conic, we can impose the equality constraint $4ac - b^2 = 1$, which can be expressed in the matrix form of $S^TGS = 1$, where

$$G = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

By introducing the Lagrange multiplier λ, we can get the simultaneous equations:

$$\begin{cases} HS = \lambda GS \\ S^TGS = 1 \end{cases} \quad (4)$$

where $H = W^TW$. To solve these equations, we first need to get all possible solutions of the generalized eigenvectors then select those that are satisfied with the constraint.

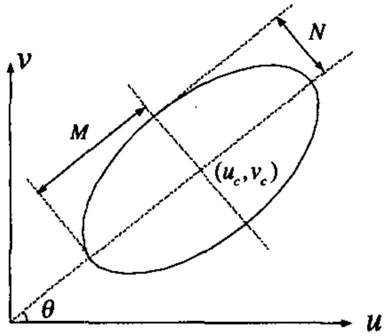

Figure1. Ellipse parameter estimation

When obtained the conic equation, we can estimate the ellipse parameters, as illustrated in Figure1. The five parameters of an ellipse such as the center point coordinates (u_c, v_c), the major axis length M, minor axis length N, the angle or orientation of the ellipse θ can be calculated using the following formula:

$$u_c = \frac{2cd - be}{b^2 - 4ac} \quad (5)$$

$$v_c = \frac{2ae - bd}{b^2 - 4ac} \quad (6)$$

$$\theta = \arctan\left[\frac{(c-a) + \sqrt{b^2 + (c-a)^2}}{b}\right] \quad (7)$$

$$M^2 = \frac{2(1-\rho)[(a+c) + \sqrt{b^2 + (c-a)^2}\,]}{b^2 - 4ac} \quad (8)$$

$$N^2 = \frac{2(1-\rho)[(a+c) - \sqrt{b^2 + (c-a)^2}\,]}{b^2 - 4ac} \quad (9)$$

where $\rho = \dfrac{bde - ae^2 - cd^2}{b^2 - 4ac}$.

3. 3D model analysis

The perspective projection of a spatial circle is commonly an ellipse, as illustrated in Figure 2. Let both the world coordinate system $\Omega_1(x_w, y_w, z_w)$ and the camera coordinate system $\Omega_2(x_c, y_c, z_c)$ be centered in the optics center o, and let z_c axis be perpendicular to the CCD plane Π_2, whose corresponding image axis u and v are parallel to x_c and y_c respectively. In projective projection, a 3D circle Γ_1 whose radius is known as r would be commonly perceived as elliptical shapes. The position and orientation of a circular feature in 3D space can be completely specified by the coordinates of its center and the surface normal vector [2].

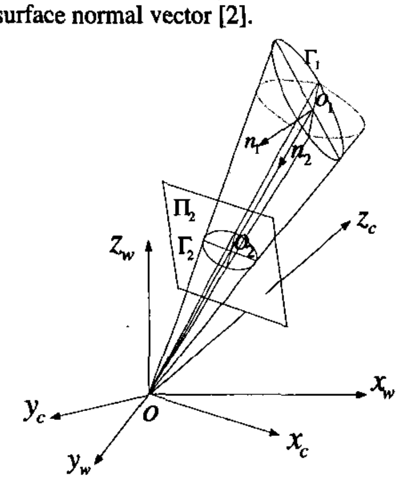

Figure 2. Perspective projection of circular feature

In the camera coordinate system, the equation of the line oo_1 can be given by:

$$\frac{x_c}{u_c} = \frac{y_c}{v_c} = \frac{z_c}{f} \quad (10)$$

where u_c and v_c are the coordinates of o_2.

$$\frac{r}{M}=\frac{z_c}{f} \tag{11}$$

so we can get the center point o_1 of the 3D circle with a given radius r in the camera coordinate system:

$$\begin{cases} x_c=\dfrac{r}{M}u_c \\ y_c=\dfrac{r}{M}v_c \\ z_c=\dfrac{r}{M}f \end{cases} \tag{12}$$

It is well known that the general form of a common quadratic curve can be expressed as the form of

$$au^2+buv+cv^2+du+ev+f=0. \tag{13}$$

In the perspective projection with pinhole camera model, the relationship between the camera coordinate (x_c, y_c, z_c) and the image coordinate (u,v) can be described as:

$$\begin{matrix} u=f\cdot x_c/z_c \\ v=f\cdot y_c/z_c \end{matrix}. \tag{14}$$

From Eq. (13) we can see that each point (u,v) of an ellipse on the image plane forms a line of the skewed cone through (u,v,f) and $(0,0,0)$. Substituting Eq. (14) into Eq. (13), we can get the cone equation in the camera coordinate system:

$$Ax_c^2+Bx_cy_c+Cy_c^2+Dx_cz_c+Ey_cz_c+Fz_c^2=0 \tag{15}$$

where $A=a\cdot f^2$, $B=b\cdot f^2$, $C=c\cdot f^2$, $D=d\cdot f$, $E=e\cdot f$, $F=f$. It can be rewritten in the following matrix form:

$$[x_c\ y_c\ z_c]P\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}=0 \tag{16}$$

where $P=\begin{bmatrix} A & \frac{1}{2}B & \frac{1}{2}D \\ \frac{1}{2}B & C & \frac{1}{2}E \\ \frac{1}{2}D & \frac{1}{2}E & F \end{bmatrix}$.

To make the direction of the elliptic cone is the new axis z_e, there must be a rotational matrix Q subjected to orthonormal transformation $Q^TQ=I$, so we can change the reference frame to a new coordinate frame $\Omega_3(x_e, y_e, z_e)$ that have the same origin as the camera frame to simplify the analysis:

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}=Q\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} \tag{17}$$

where $Q=\begin{bmatrix} q_1 & q_2 & q_3 \\ q_4 & q_5 & q_6 \\ q_7 & q_8 & q_9 \end{bmatrix}$.

So Eq. (16) becomes:

$$[x_e\ y_e\ z_e]Q^TPQ\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix}=0. \tag{18}$$

It can be rewritten as:

$$\eta_1x_e^2+\eta_2y_e^2+\eta_3z_e^2=0 \tag{19}$$

where η_i are eigenvalues of $G=Q^TPQ$, and we can get Q because it is a diagonalizing matrix for P, that's $Q^TPQ=Diag(\eta_1,\eta_2,\eta_3)$.

The surface normal vector T_k of the circle can be solved by considering a central cone intersected with a plane defined by $k_1x_e+k_2y_e+k_3z_e=k_4$, where $k_1^2+k_2^2+k_3^2=1$. Therefore, to find the coefficients of the equation of a plane for which the intersection is circular can be expressed mathematically as finding k_1, k_2, and k_3 such that the intersection of a conical surface with the above surface is a circle [10].

In this paper, the radius of the circular feature has been supposed to be known, and only the positive solution whose location is in front of the camera would be acceptable in our research.

The surface normal vector of the 3D circle relative to the coordinate frame $\Omega_3(x_e, y_e, z_e)$ can be transformed with respect to the camera frame $\Omega_2(x_c, y_c, z_c)$:

$$T_c=QT_k=\begin{bmatrix} q_1k_1+q_2k_2+q_3k_3 \\ q_4k_1+q_5k_2+q_6k_3 \\ q_7k_1+q_8k_2+q_9k_3 \end{bmatrix} \tag{20}$$

where $T_k=[k_1\ k_2\ k_3]^T$.

4. Error compensation of circular feature

The accurate center of projected circles provides exact correspondences between ellipses in 2D image plane and circles in 3D plane. The perspective projection of a spatial circle is commonly an ellipse, however, the center of projected circle does not correspond to the ellipse center, as illustrated in Figure 3. Let both the world coordinate system $\Omega_1(x_w, y_w, z_w)$ and the camera coordinate system $\Omega_2(x_c, y_c, z_c)$ be centered in the optics center o, and let z_w axis be orthogonal to the object plane Π_1, also let z_c axis be perpendicular to the CCD plane Π_2, whose corresponding image axis u and v are parallel to x_c and y_c respectively. If the direction angle of vector $\overline{oo_1}$ is α, β, γ in Ω_1, and the distance between o_1 and o_2 equal to d, then the projection of the circle Γ_1 on the $x_w o y_w$ plane is a circle Γ_3 whose radius r is known, which can be given by:

$$(x_w - z_w \cdot a_1)^2 + (y_w - z_w \cdot a_2)^2 = (z_w a_3)^2 \qquad (21)$$

where $a_1 = \dfrac{\cos\alpha}{\cos\gamma}$, $a_2 = \dfrac{\cos\beta}{\cos\gamma}$, $a_3 = \dfrac{r}{d}$.

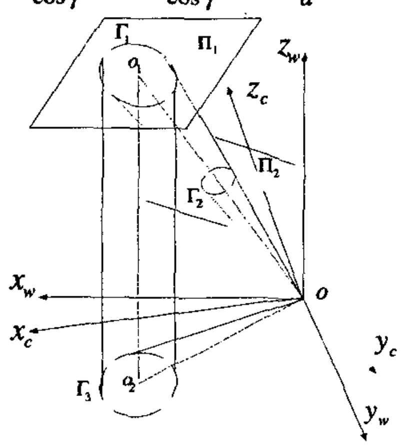

Figure 3. Projection Error between circle and ellipse

The relationship between the world coordinate system Ω_1 and the camera coordinate system Ω_2 can be given by:

$$\begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} = \begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \qquad (22)$$

where the vectors $[b_1, b_4, b_7]^T$, $[b_2, b_5, b_8]^T$, $[b_3, b_6, b_9]^T$ form an orthonormal basis. The orthogonal distance between the optics center o and the image plane Π_2 is the focal length f, so we can express Eq. (21) in the camera coordinate system:

$$\begin{aligned} &(g^2 + l^2 - p^2)x_c^2 + 2(gh + lm - pq)x_c y_c \\ &+(h^2 + m^2 - q^2)y_c^2 + 2(gk + ln - ps)x_c \\ &+2(hk + mn - qs)y_c + k^2 + n^2 - s^2 = 0 \end{aligned} \qquad (23)$$

where $g = b_1 - a_1 b_7$

$h = b_2 - a_1 b_8$

$k = (b_3 - a_1 b_9) f$

$l = b_4 - a_2 b_7$

$m = b_5 - a_2 b_8$

$n = (b_6 - a_2 b_9) f$

$p = a_3 b_7$

$q = a_3 b_8$

$s = a_3 b_9 f$.

As we know that a common quadratic curve can be expressed as the form of:

$$A x_c^2 + B x_c y_c + C y_c^2 + D x_c + E y_c + F = 0 \qquad (24)$$

and the projection of a common circle on the image plane is an ellipse, then the image of the circle Γ_1 is an ellipse Γ_2 located on the image plane Π_2. Thus the center point coordinates of the ellipse can be calculated using the following formula:

$$\begin{cases} u_c = \dfrac{c_1 c_2 - c_3 c_4}{(gq - hp)^2 + (pm - ql)^2 - (gm - lh)^2} \\ v_c = \dfrac{c_1 c_5 - c_2 c_3}{(gq - hp)^2 + (pm - ql)^2 - (gm - lh)^2} \end{cases} \qquad (25)$$

where: $c_1 = (h^2 + m^2 - q^2)$

$c_2 = (gk + ln - ps)$

$c_3 = (gh + lm - pq)$

$c_4 = (hk + mn - qs)$

$c_5 = (g^2 + l^2 - p^2)$.

In the camera coordinate system the equation of the line oo_1 can be give by:

$$\frac{b_1 x_c + b_2 y_c + b_3 z_c}{\cos\alpha} = \frac{b_4 x_c + b_5 y_c + b_6 z_c}{\cos\beta} = \frac{b_7 x_c + b_8 y_c + b_9 z_c}{\cos\gamma} \qquad (26)$$

The real coordinate of the ellipse center is the intersection of the line oo_1 and the plane $z_c = f$ is:

$$\begin{cases} \tilde{u}_c = \dfrac{d_3 d_5 - d_2 d_6}{d_1 d_5 - d_2 d_4} \\ \tilde{v}_c = \dfrac{d_1 d_6 - d_3 d_4}{d_1 d_5 - d_2 d_4} \end{cases} \qquad (27)$$

where: $d_1 = b_1 \cos\gamma - b_7 \cos\alpha$,

$d_2 = b_2 \cos\gamma - b_8 \cos\alpha$,

$d_3 = (b_9 \cos\alpha - b_3 \cos\gamma) f$,

$d_4 = b_4 \cos\gamma - b_7 \cos\beta$,

$d_5 = b_5 \cos\gamma - b_8 \cos\beta$,

$d_6 = b_9 \cos\beta - b_6 \cos\gamma$.

The observed image coordinate (u,v) should be corrected with the error compensation:

$$\begin{cases} u = u - N_x(\tilde{u}_c - u_c) \\ v = v - N_y(\tilde{v}_c - v_c) \end{cases} \quad (28)$$

After the error compensation, the 3D model parameters are computed again and it need not have further iterations, so we can get more accurate parameter, to guarantee high accuracy for robot vision.

5. Illustration of the algorithm

After the image is processed with noise elimination, the algorithm goes as follows:

1. Detect of circular feature using the fitting method. We can obtain the parameters of ellipse such as the center point coordinates, the major axis length, minor axis length, the angle of orientation.
2. Perform the 3D model analysis of circular feature. We can estimate the 3D position and 3D orientation of the circle.
3. Do the error compensation of circular feature that exists between the center of projected circle and the ellipse center.

6. Experiment results

Our robot vision system consisted of the camera TM1400, which is made in Pulnix company of United States with high resolution 1392*1040, and digital image board named Pc2-camlink, which was made in Coreco company of Canada. We use a board with 9 identical circles whose radii are 50mm to measure the validity of our 3D model, as shown in Figure 4.

To check the accuracy of our method, when our camera is calibrated we use 9 circles to measure the difference between their world coordinates of the center points and the coordinates our method estimated with:

$$\varepsilon_1 = \frac{\sum_{i=1}^{N} |(X_s - X_d)|}{N} \quad (29)$$

where X_s is the world coordinate of the circle center point while X_d is the value our method estimated, and N is the number of circles. In the same way, the difference between the surface normal vectors in the world coordinates and the surface normal vector of the circle that our method estimated can be measured with:

$$\varepsilon_2 = \frac{\sum_{i=1}^{N} |(V_s - V_d)|}{N} \quad (30)$$

where V_s is the surface normal vector of the circle in the world coordinate while V_d is the surface normal vectors our method estimated, both of their measurement units are degrees defined for orientation angle. The average error of our vision system is ε_1=1.15mm, ε_2=0.76degrees, so that the accuracy of our method is very high.

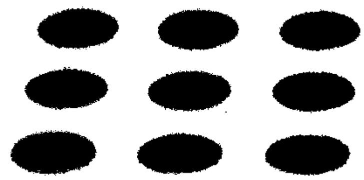

Figure 4. The image of board with circles

7. Conclusion

In this paper, a novel detection and model analysis of circular feature with error compensation is presented for robot vision. Fitting ellipses to data points subjected to a constraint, our circular detection method can uniquely and efficiently yield elliptical parameters. We decompose the location estimation of circular feature into the 3D position and 3D orientation estimation, and do the error compensation of circular feature that exists between the center of projected circle and the ellipse center. Experimental results have shown that our method is robust, efficient and accurate, so this method can be popularized to other vision based applications.

Acknowledgements

This paper is supported by the National Natural Science Foundation of People's Republic of China under grant 60175015.

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