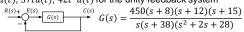
Steady-State Error - Problems

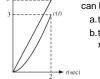
7.1 Find the steady-state errors for the following test inputs: 25u(t), 37tu(t), $42t^2u(t)$ for the unity feedback system



System Dynamics and Control

Steady-State Error - Problems

7.2 The figure shows the ramp input r(t) and the output c(t) of a system. Assuming the output's steady state can be approximated by a ramp, find a.the steady-state error;



b.the steady-state error if the input becomes r(t) = 2.5tu(t)

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System Dynamics and Control

Steady-State Error - Problems

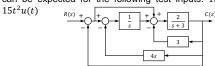
7.3 Find the steady-state errors if the input is $80t^2u(t)$ for the unity feedback system

$$G(s) = \frac{60(s+3)(s+4)(s+8)}{s^2(s+6)(s+17)}$$

System Dynamics and Control

Steady-State Error - Problems

7.4 For the system shown in the figure, what steady state error can be expected for the following test inputs: 15u(t), 15tu(t),



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Steady-State Error - Problems

7.5 For the unity feedback system shown in the figure, where
$$\overbrace{-\bigcirc_{s}^{R(s)+} \overbrace{-G(s)}^{E(s)}}^{C(s)} G(s) = \frac{500}{(s+24)(s^2+8s+14)}$$

find the steady-state error for inputs of 30u(t), 70tu(t), and $81t^2u(t)$

System Dynamics and Control

Steady-State Error - Problems

7.6 An input of $25t^3u(t)$ is applied to the input of a Type 3 unity feedback system, as shown in the figure, where

$$G(s) = \frac{210(s+4)(s+6)(s+11)(s+13)}{s^3(s+7)(s+14)(s+19)}$$

Steady-State Error - Problems

7.7 The steady-state error in velocity of a system is defined to be

$$\left. \left(\frac{dr}{dx} - \frac{dc}{dx} \right) \right|_{t \to \infty} \qquad \text{where, } r\text{:the system input} \\ c\text{:the system output}$$

Find the steady-state error in velocity for an input of $t^3u(t)$ to a unity feedback system with a forward TF of

$$G(s) = \frac{100(s+1)(s+2)}{s^2(s+3)(s+10)}$$

System Dynamics and Control

Steady-State Error - Problems

7.8 What is the steady-state error for a step input of 15 units applied to the unity feedback system, where

$$G(s) = \frac{E(s) + E(s)}{G(s)}$$

$$G(s) = \frac{1020(s+13)(s+26)(s+33)}{(s+65)(s+75)(s+91)}$$

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System Dynamics and Control 9 Steady-State Error - Problems 7.9 A system has $K_p=4$. What steady state error can be

expected for inputs of 70u(t) and 70tu(t)?

System Dynamics and Control

Steady-State Error - Problems

7.10 For the unity feedback system shown in the figure, where

$$G(s) = \frac{1}{s(s+75)}$$

- a. What is the expected percent overshoot for a unit step input?
- b. What is the settling time for a unit step input?
- c. What is the steady-state error for an input of 5u(t)?
- d. What is the steady-state error for an input of 5tu(t)?
- e. What is the steady-state error for an input of $5t^2u(t)$?

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7.11 Given the unity feedback system shown in the figure, where $G(s) = \frac{100500(s+5)(s+14)(s+23)}{s(s+27)(s+\alpha)(s+33)}$

Find the value of α to yield a $K_v = 25000$

System Dynamics and Control

Steady-State Error - Problems

7.12 For the unity feedback system of the figure, where

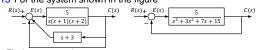
 $G(s) = \frac{K(s+2)(s+4)(s+6)}{s^2(s+5)(s+7)}$

find the value of K to yield a static error constant of 10,000

Steady-State Error - Problems

Steady-State Error - Problems

7.13 For the system shown in the figure



- b. Find the steady-state error for an input of 50u(t), 50tu(t), $50t^2u(t)$
- c. State the system type

input. Find the steady-state position error for this input if the forward TF is $G(s) = \frac{1030(s^2 + 8s + 23)(s^2 + 21s + 18)}{s^3(s+6)(s+13)}$

7.14 A Type 3 unity feedback system has $r = 10t^3$ applied to its

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Steady-State Error - Problems

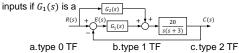
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7.15 Find the system type for the system of the figure



Steady-State Error - Problems 7.16 What are the restrictions on the feedforward TF $G_2(s)$ in the system of the figure to obtain zero steady-state error for step



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System Dynamics and Control 7.16

$$E(s) = \frac{s(s+3) - 20G_2}{s(s+3) + 20G_1}R(s)$$

$$e(\infty) = \lim_{s \to 0} sE(s) = -\lim_{s \to 0} \frac{G_2(s)}{G_1(s)}$$

$$e(\infty) = \lim_{s \to 0} sE(s) = -\lim_{s \to 0} \frac{G_2(s)}{G_1(s)}$$

to obtain zero steady-state error for step inputs, $G_2(s)$ must be lower order than $G_1(s)$. Therefore, if $G_1(s)$ is a

a.type 0 TF \rightarrow G_2 must be zero

b.type 1 TF \rightarrow G_2 must be type 0

c.type 2 TF \rightarrow G_2 must be type 1

System Dynamics and Control

Steady-State Error - Problems

7.19 For the system shown in the figure

$$G(s) = \frac{K(s+a)}{s(s+2)(s+13)}$$

Find the value of Ka so that a ramp input of slope 40 will yield an error of 0.006 in the steady state when compared to the output

 $e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$

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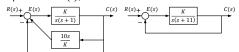
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Steady-State Error - Problems

Steady-State Error - Problems

7.20 Given the system of the figure, design the value of K so that for an input of 100tu(t), there will be a 0.01 error in the steady state



7.21 Find the value of K for the unity feedback system where

 $G(s) = \frac{K(s+3)}{s^2(s+7)}$

if the input is $10t^2u(t)$, and the desired steady-state error is 0.061 for this input

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7.22 The unity feedback system of the figure, where

$$G(s) = \frac{K(s^2 + 3s + 30)}{s^n(s + 5)}$$

is to have 1/6000 error between an input of 10tu(t) and the output in the steady state

- a. Find K and n to meet the specification
- b. What are K_p , K_v , and K_a

System Dynamics and Control

System Dynamics and Control

Steady-State Error - Problems

7.23 For the unity feedback system of the figure, where

$$G(s) = \frac{K(s^2 + 6s + 6)}{(s+5)^2(s+3)}$$

- a. Find the system type
- b. What error can be expected for an input of 12u(t)?
- c. What error can be expected for an input of 12tu(t)?

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Steady-State Error - Problems

7.27 The unity feedback system of the figure, where

$$G(s) = \frac{K(s+\alpha)}{(s+\beta)^2}$$

is to be designed to meet the following specifications: steady-state error for a unit step input = 0.1, damping ratio = 0.5, natural frequency = $\sqrt{10}$. Find K, α , and β

System Dynamics and Control

Steady-State Error - Problems

7.28 A second-order, unity feedback system is to follow a ramp input with the following specifications: the steady-state output position shall differ from the input position by 0.01 of the input velocity; the natural frequency of the closed-loop system shall be 10rad/s. Find the following

- a. The system type
- b. The exact expression for the forward-path TF
- c. The closed-loop system's damping ratio

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Steady-State Error - Problems

7.29 The unity feedback system of the figure, where $\frac{R(s) + R(s)}{r} = \frac{R(s) + R(s)}{r}$

$$G(s) = \frac{K(s+\alpha)}{S(s+\beta)}$$

is to be designed to meet the following requirements: The steady-state position error for a unit ramp input equals 1/10; the closed-loop poles will be located at $-1 \pm j1$. Find K, α , and β in order to meet the specifications

System Dynamics and Control

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Steady-State Error - Problems

7.30 Given the unity feedback control system of the figure, where

$$G(s) = \frac{K}{s^n(s+a)}$$

find the values of $n,\,K,\,{\rm and}$ a in order to meet specifications of 12% overshoot and $K_v=110$

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Steady-State Error - Problems

7.31 Given the unity feedback control system of the figure, where

$$G(s) = \frac{K}{s(s+a)}$$

find the following

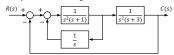
- a.K and a to yield $K_v = 1000$ and a 20% overshoot
- b.K and a to yield a 1% error in the steady state and a 10% overshoot

System Dynamics and Control

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Steady-State Error - Problems

7.32 Given the system in the figure



Find the following

- a. The closed-loop TF
- b.The system type
- c. The steady-state error for an input of 5u(t)
- d. The steady-state error for an input of 5tu(t)
- e.Discuss the validity of your answers to Parts c and d

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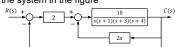
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Steady-State Error - Problems

7.33 Given the system in the figure



Find the following

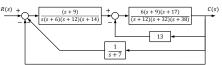
- a. The closed-loop TF
- b. The system type
- c. The steady-state error for an input of 5u(t)
- d. The steady-state error for an input of 5tu(t)
- e.Discuss the validity of your answers to Parts c and d

System Dynamics and Control

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Steady-State Error - Problems

7.34 For the system shown in the figure



Use matlab to find the following

- a. The system type
- $b.K_p, K_v, and K_a$
- c.The steady-state error for inputs of 100u(t), 100tu(t), and $100t^2u(t)$

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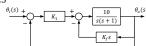
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Steady-State Error - Problems

7.35 The system of the figure is to have the following specifications: $K_v = 10, \, \zeta = 0.5$



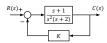
Find the values of K_1 and K_f required for the specifications of the system to be met

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Steady-State Error - Problems

7.44 Given the system shown in the figure



find the following

- a. The system type
- b. The value of K to yield 0.1% error in the steady state

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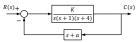
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Steady-State Error - Problems

7.50 Given the system shown in the figure



find the sensitivity of the steady-state error to parameter a. Assume a step input. Plot the sensitivity as a function of parameter a

System Dynamics and Control

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Steady-State Error - Problems

7.55 For each of the following closed-loop systems, find the steady-state error for unit step and unit ramp inputs. Use both the final value theorem and input substitution methods

alue theorem and input substitution methods
$$\mathbf{a}.\,\dot{x} = \begin{bmatrix} -5 & -4 & -2 \\ -3 & -10 & 0 \\ -1 & 1 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} r, \quad y = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} x$$

$$\mathbf{b}.\,\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -9 & 7 \\ -1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

$$\mathbf{c}.\,\dot{x} = \begin{bmatrix} -9 & -5 & -1 \\ -1 & 0 & -2 \\ -3 & -2 & -5 \end{bmatrix} x + \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & -2 & 4 \end{bmatrix} x$$

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