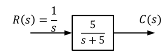
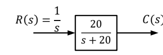
4.1 Derive the output responses for the system with step input

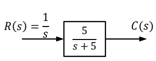
a. $\frac{9}{s^2+9s+9}$ b. $\frac{9}{s^2+2s+9}$ c. $\frac{9}{s^2+9}$ d. $\frac{9}{s^2+6s+9}$

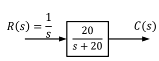
4.2 Find the output response, $c(t)$, for each of the systems shown in the figure. Also find the time constant, rise time, and settling time for each case

a.


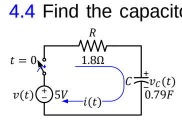
b.


4.3 Plot the step responses using matlab

a.


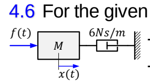
b.


4.4 Find the capacitor voltage in the network if the switch closes at $t=0$. Assume zero initial conditions. Also find the time constant, rise time, and settling time for the capacitor voltage



4.5 Plot the step response for P.4.4 using matlab. From your plots, find the time constant, rise time, and settling time

4.6 For the given system, find an equation that relates



- settling time of the velocity of the mass to M
- rise time of the velocity of the mass to M

4.7 Plot the step response for P.4.6 using matlab. From your plots, find the time constant, rise time, and settling time. Use $M = 1, 2$

4.8 For each of the TF shown below, find the locations of the poles and zeros, plot them on the s-plane, and then write an expression for the general form of the step response without solving for the inverse Laplace transform. State the nature of each response (overdamped, underdamped, and so on)

a. $T(s) = \dfrac{2}{s+2}$

b. $T(s) = \dfrac{5}{(s+3)(s+6)}$

c. $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$

d. $T(s) = \dfrac{20}{s^2+6s+144}$

e. $T(s) = \dfrac{s+2}{s^2+9}$

f. $T(s) = \dfrac{s+5}{(s+10)^2}$

4.9 Use matlab to find the poles of

$$T(s) = \frac{s^2+2s+2}{s^4+6s^3+4s^2+7s+2}$$

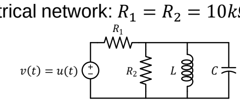
4.10 Find the transfer function and poles of the system

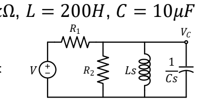
$$\dot{\boldsymbol{x}} = \begin{bmatrix} 8 & -4 & 1 \\ -3 & 2 & 0 \\ 5 & 7 & -9 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} -4 \\ -3 \\ 4 \end{bmatrix}u(t),\ y = \begin{bmatrix} 2 & 8 & -3 \end{bmatrix}\boldsymbol{x},\ \boldsymbol{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4.11 Find the TF and poles of the system using matlab

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 8 & -4 & 1 \\ -3 & 2 & 0 \\ 5 & 7 & -9 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} -4 \\ -3 \\ 4 \end{bmatrix}u(t),\ y = \begin{bmatrix} 2 & 8 & -3 \end{bmatrix}\boldsymbol{x},\ \boldsymbol{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

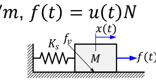
4.12 Write the general form of the capacitor voltage for the electrical network: $R_1 = R_2 = 10k\Omega$, $L = 200H$, $C = 10\mu F$





4.13 Use matlab to plot the capacitor voltage in P.4.12

4.14 Solve for $x(t)$ in the system if $f(t)$ is a unit step: $M = 1kg$, $K_s = 5N/m$, $f_v = 1Ns/m$, $f(t) = u(t)N$



4.15 The system has a unit step input. Find the output response as a function of time. Assume the system is underdamped. Notice: the result will be Eq.4.28

$$R(s) \rightarrow \boxed{\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \rightarrow C(s)$$

4.16 Derive the relationship for damping ratio as a function of percent overshoot, Eq.4.39

4.17 Calculate the exact response of each system using Laplace transform techniques

a. $T(s) = \dfrac{2}{s+2}$

b. $T(s) = \dfrac{5}{(s+3)(s+6)}$

c. $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$

d. $T(s) = \dfrac{20}{s^2+6s+144}$

e. $T(s) = \dfrac{s+2}{s^2+9}$

f. $T(s) = \dfrac{s+5}{(s+10)^2}$

4.18 Find the damping ratio and natural frequency for each second-order system and show that the value of the damping ratio conforms to the type of response (underdamped, overdamped, and so on) predicted in that problem

a. $T(s) = \dfrac{2}{s+2}$

b. $T(s) = \dfrac{5}{(s+3)(s+6)}$

c. $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$

d. $T(s) = \dfrac{20}{s^2+6s+144}$

e. $T(s) = \dfrac{s+2}{s^2+9}$

f. $T(s) = \dfrac{s+5}{(s+10)^2}$

4.19 A system has a damping ratio of $\zeta = 0.5$, a natural frequency of $\omega_n = 100 rad/s$, and a dc gain of 1. Find the response of the system to a unit step input

$\mathcal{L}\{\sin\omega t u\} = \frac{\omega}{s^2+\omega^2}$ $\mathcal{L}\{\cos\omega t u\} = \frac{s}{s^2+\omega^2}$ $\mathcal{L}\{tu(t)\} = \frac{1}{s^2}$ $\mathcal{L}\{e^{-at}f(t)\} = F(s+a)$

4.20 For each of the second-order systems that follow, find ζ, ω_n, T_s, T_p, T_r, and $\%OS$

a. $T(s) = \dfrac{16}{s^2 + 3s + 16}$ b. $T(s) = \dfrac{0.04}{s^2 + 0.02s + 0.04}$

c. $T(s) = \dfrac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$

4.21 Write a matlab program to estimate the given specifications and plot the step responses. Estimate the rise time from the plot

a. $T(s) = \dfrac{16}{s^2 + 3s + 16}$ b. $T(s) = \dfrac{0.04}{s^2 + 0.02s + 0.04}$

c. $T(s) = \dfrac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$

4.22 Use matlab's LTI Viewer and obtain settling time, peak time, rise time, and percent overshoot for each of the systems

a. $T(s) = \dfrac{16}{s^2 + 3s + 16}$ b. $T(s) = \dfrac{0.04}{s^2 + 0.02s + 0.04}$

c. $T(s) = \dfrac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$

4.23 For each pair of second-order system specifications that follow, find the location of the second-order pair of poles

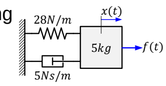
a. $\%OS = 12\%, T_s = 0.6s$ b. $\%OS = 10\%, T_p = 5s$ c. $T_s = 7s, T_p = 3s$

4.24 Find the transfer function of a second-order system that yields a 12.3% overshoot and a settling time of $1s$

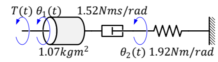
4.25 For the given system, do the following

a. Find the TF $G(s)=X(s)/F(s)$

b. Find ζ, ω_n, $\%OS$, T_s, T_p, and T_r



4.26 For the given system, a step torque is applied at $\theta_1(t)$. Find



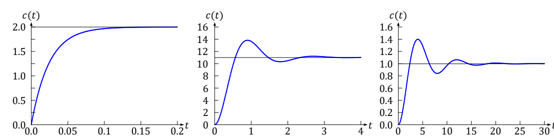
a. the TF $G(s)=\theta_2(s)/T(s)$

b. the percent overshoot, settling time, and peak time for $\theta_2(t)$

4.28 Find the percent overshoot, settling time, rise time, and peak time for

$$T(s)=\frac{14.145}{(s^2+0.842s+2.829)(s+5)}$$

4.29 For each of the unit step responses, find the TF of the system



a. b. c.

Hint: a. $c_{ss}=2.0, T_c=0.025s$ b. $c_{max}=13.8, c_{ss}=11.0, T_s=2.2s$ c. $c_{max}=1.4, c_{ss}=1.0, T_p=3.9s$

4.30 For the following response functions, determine if pole-zero cancellation can be approximated. If it can, find percent overshoot, settling time, rise time, and peak time

a. $C(s)=\frac{s+3}{s(s+2)(s^2+3s+10)}$ c. $C(s)=\frac{s+2.1}{s(s+2)(s^2+s+5)}$

b. $C(s)=\frac{s+2.5}{s(s+2)(s^2+4s+20)}$ d. $C(s)=\frac{s+2.01}{s(s+2)(s^2+5s+20)}$

4.31 Using matlab, plot the time response of the following systems and from the plot determine percent overshoot, settling time, rise time, and peak time

a. $C(s)=\frac{s+3}{s(s+2)(s^2+3s+10)}$ c. $C(s)=\frac{s+2.1}{s(s+2)(s^2+s+5)}$

b. $C(s)=\frac{s+2.5}{s(s+2)(s^2+4s+20)}$ d. $C(s)=\frac{s+2.01}{s(s+2)(s^2+5s+20)}$

4.32 Find peak time, settling time, and percent overshoot for only those responses below that can be approximated as second-order responses

a. $c(t) = 0.003500 - 0.001524e^{-4t} - 0.001976e^{-3t}\cos 22.16t - 0.0005427e^{-3t}\sin 22.16t$

b. $c(t) = 0.05100 - 0.007353e^{-8t} - 0.007647e^{-6t}\cos 8t - 0.01309e^{-6t}\sin 8t$

c. $c(t) = 0.009804 - 0.0001857e^{-5.1t} - 0.009990e^{-2t}\cos 9.796t - 0.001942e^{-2t}\sin 9.796t$

d. $c(t) = 0.007000 - 0.001667e^{-10t} - 0.008667e^{-2t}\cos 9.951t - 0.0008040e^{-2t}\sin 9.951t$

4.35 A system is represented by the state and output equations that follow. Without solving the state equation, find the poles of the system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u(t),\ y = \begin{bmatrix} 3 & 2 \end{bmatrix}\boldsymbol{x}$$

4.36 A system is represented by the state and output equations that follow

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 6 & 5 \\ 1 & 4 & 2 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}u(t),\ y = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}\boldsymbol{x}$$

Without solving the state equation, find

a. the characteristic equation

b. the poles of the system

4.37 Given the following state-space representation of a system, find $Y(s)$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\sin 3t,\ y = \begin{bmatrix} 1 & 2 \end{bmatrix}\boldsymbol{x},\ \boldsymbol{x}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

4.38 Given the following system represented in state space, solve for $Y(s)$ using the Laplace transform method for solution of the state equation

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -4 & 1 \\ 0 & 0 & -6 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}e^{-t},\ y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\boldsymbol{x},\ \boldsymbol{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4.39 Solve the following state equation and output equation for $y(t)$, where $u(t)$ is the unit step. Use the Laplace transform method

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u(t),\ y = \begin{bmatrix} 0 & 1 \end{bmatrix}\boldsymbol{x},\ \boldsymbol{x}(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

4.40 Solve for $y(t)$ for the following system represented in state space, where $u(t)$ is the unit step. Use the Laplace transform approach to solve the state equation

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -6 & 1 \\ 0 & 0 & -5 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} e^{-t},\ y = [0 \quad 1 \quad 1]\boldsymbol{x},\ \boldsymbol{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4.41 Use matlab to plot the step response of P.4.40

4.49 A human responds to a visual cue with a physical response. The TF that relates the output physical response $P(s)$ to the input visual command $V(s)$ is

$$G(s) = \frac{P(s)}{V(s)} = \frac{s+0.5}{(s+2)(s+5)}$$

Do the following

a. Evaluate the output response for a unit step input using the Laplace transform

b. Represent the transfer function in state space

c. Simulate the system and obtain a plot of the step response