2.2 Using the Laplace transform pairs of Table 2.1 and the Laplace transform theorems of Table 2.2, derive the Laplace transforms for the following time functions

a. $e^{-at}\sin\omega t u(t)$ b. $e^{-at}\cos\omega t u(t)$ c. $t^3 u(t)$

2.3 Solve the following differential equations using Laplace transform methods with zero initial conditions

a. $\dfrac{dx}{dt} + 7x = 5\cos 2t$ b. $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 8x = 5\sin 3t$

c. $\dfrac{d^2x}{dt^2} + 8\dfrac{dx}{dt} + 25x = 10u(t)$

Assume that the forcing functions are zero prior to $t = 0_-$

2.4 Solve the following differential equations using Laplace transform methods with the given initial conditions

a. $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 2x = \sin 2t \qquad x(0) = 4, \dfrac{dx}{dt}(0) = -4$

b. $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + x = 5e^{-2t} + t \qquad x(0) = 4, \dfrac{dx}{dt}(0) = 1$

c. $\dfrac{d^2x}{dt^2} + 4x = t^2 \qquad x(0) = 2, \dfrac{dx}{dt}(0) = 3$

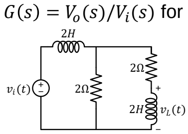
Assume that the forcing functions are zero prior to $t = 0_-$

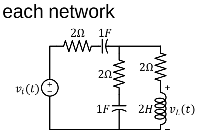
2.8 For each of TF, write the corresponding differential equation

a. $\dfrac{X(s)}{F(s)} = \dfrac{7}{s^2 + 5s + 10}$ b. $\dfrac{X(s)}{F(s)} = \dfrac{15}{(s+10)(s+11)}$

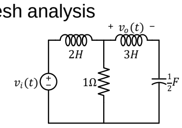
c. $\dfrac{X(s)}{F(s)} = \dfrac{s+3}{s^3 + 11s^2 + 12s + 18}$

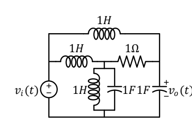
2.17 Find $G(s) = V_o(s)/V_i(s)$ for each network



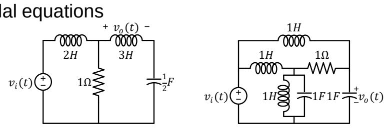


2.18 Find $G(s) = V_o(s)/V_i(s)$ for each network. Solve the problem using mesh analysis





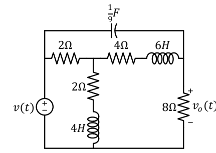
2.19 Find $G(s) = V_o(s)/V_i(s)$ for each network. Solve the problem using nodal equations



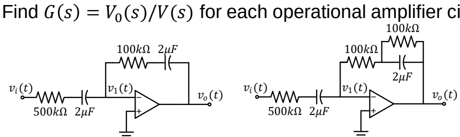
2.20

a. Write, but do not solve, the mesh and nodal eq.s for the network

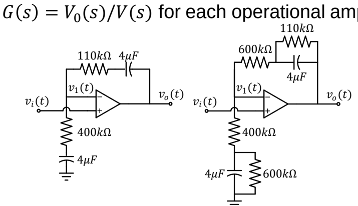
b. Use matlab, the Symbolic Math Toolbox, and the eq.s found in part a to solve for the TF $G(s) = V_0(s)/V(s)$. Use both the mesh and nodal equations to show that either set yields the same TF



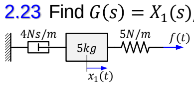
2.21 Find $G(s) = V_0(s)/V(s)$ for each operational amplifier circuit



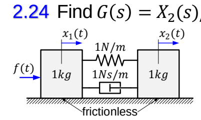
2.22 Find $G(s) = V_0(s)/V(s)$ for each operational amplifier circuit



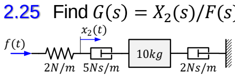
2.23 Find $G(s) = X_1(s)/F(s)$ for the translational mechanical system



2.24 Find $G(s) = X_2(s)/F(s)$ for the translational mechanical network

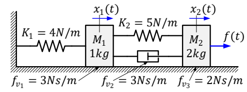


2.25 Find $G(s) = X_2(s)/F(s)$ for the translational mechanical system

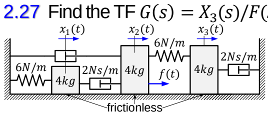


Hint: place a zero mass at $x2(t)$

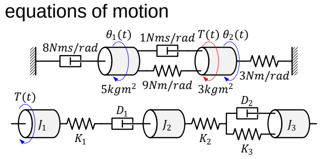
2.26 Find the TF for the system $G(s) = X_1(s)/F(s)$



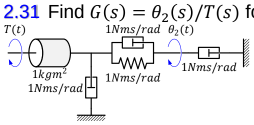
2.27 Find the TF $G(s) = X_3(s)/F(s)$ for translational mechanical system



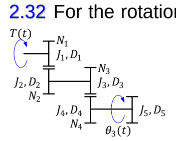
2.30 For each of the rotational mechanical systems, write, but do not solve, the equations of motion



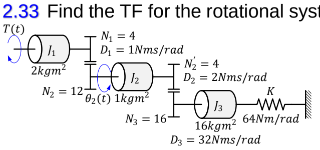
2.31 Find $G(s) = \theta_2(s)/T(s)$ for the rotational mechanical system



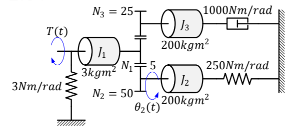
2.32 For the rotational mechanical system with gears, find the TF $G(s) = \theta_3(s)/T(s)$. The gears have inertia and bearing friction as shown



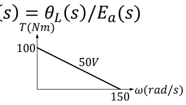
2.33 Find the TF for the rotational system, $G(s) = \theta_2(s)/T(s)$

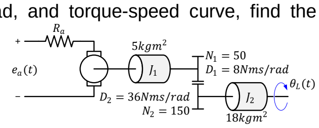


2.34 For the rotational mechanical system, find $G(s) = \theta_2(s)/T(s)$

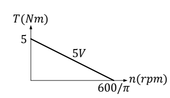


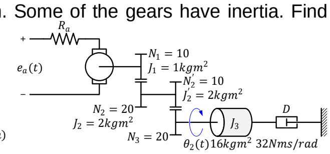
2.42 For the motor, load, and torque-speed curve, find the TF $G(s) = \theta_L(s)/E_a(s)$



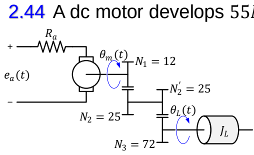


2.43 The motor whose torque-speed characteristics drives the load shown in the diagram. Some of the gears have inertia. Find the TF $G(s) = \theta_2(s)/E_a(s)$



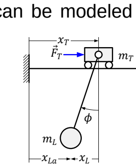


2.44 A dc motor develops $55Nm$ of torque at a speed of $600rad/s$ at $12V$. It stalls out at this voltage with $100Nm$ of torque. If the inertia, damping of the armature are $7kgm^2$ and $3Nms/rad$, respectively, find the TF $G(s) = \theta_L(s)/E_a(s)$, of this motor if it drives an inertia load of $105kgm^2$ through a gear train



2.62 The figure shows a crane hoisting a load. Although the actual system's model is highly nonlinear, if the rope is considered to be stiff with a fixed length L, the system can be modeled using the following equations

$$m_L\ddot{x}_{La} = m_L g\phi$$
$$m_T\ddot{x}_T = f_T - m_L g\phi$$
$$x_{La} = x_T - x_L$$
$$x_L = L\phi$$



where, m_L: the mass of the load

m_T: the mass of the cart

x_T, x_L: displacements as defined in the figure

f: the rope angle with respect to the vertical

$\vec{f}_T$: the force applied to the cart

2.62

a. Obtain the TF from cart velocity to rope angle $\Phi(s)/V_T(s)$

b. Assume that the cart is driven at a constant velocity V_0 and obtain an expression for the resulting $\phi(t)$. Show that under this condition, the load will sway with a frequency $\omega_0 = \sqrt{g/L}$

c. Find the TF from the applied force to the cart's position $X_T(s)/F_T(s)$

d. Show that if a constant force is applied to the cart, its velocity will increase without bound as $t \to \infty$