18.304 Final Project
Hadamard Matrices

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1 Abstract

2 Introduction

Definition 2.1. A Hadamard matrix is a square matrix whose entries are either +1 or -1 and whose rows are mutually orthogonal.

3 Construction

There are several ways to construct Hadamard matrices. For example, James Joseph Sylvester proposed the following: Let H be a Hadamard matrix of order n. Then

$$\begin{bmatrix} H & H \\ H & -H \end{bmatrix}$$

is a Hadamard matrix of order $2n$. This construction could lead to the following sequence of Hadamard matrices: $H_1 = \begin{bmatrix} 1 \end{bmatrix}$, $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $H_{2^k} = \begin{bmatrix} H_{2^{k-1}} & H_{2^{k-1}} \\ H_{2^{k-1}} & H_{2^{k-1}} \end{bmatrix}$

4 Equality of Hadamard Matrices

5 Results

Theorem 5.1. *Let H be a Hadamard matrix of order n. Then $HH^T = nI_n$.*

Proof. Since each entry in H is ± 1, we know that the length of each row vector is $\sqrt{n}$. Further, we know from the definition of Hadamard matricecs that each row is orthogonal to each other row, so if we divide H by $\sqrt{n}$ we obtain an orthognal matrix $Q = \frac{1}{\sqrt{n}}H$. We then see that

$$\begin{aligned} QQ^T &= I_n \\ \left(\frac{1}{\sqrt{n}}H\right)\left(\frac{1}{\sqrt{n}}H^T\right) &= I_n \\ HH^T &= nI_n \end{aligned}$$

$\square$

Theorem 5.2. *If H is an $n \times n$ Hadamard matrix, then $n = 1$ or $n = 2$ or $n \equiv 0 \mod 4$.*

Proof. $\square$

Theorem 5.3. *There exists an $n \times n$ matrix with entries ± 1 whose determinant is greater than $\sqrt{n!}$*

Proof. $\square$

6 Applications

7 Current Research

8 Conclusion