Name and section: ______________________________

ID number: ______________________________

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1. Evaluate by **hand** and **approximate** the following definite integrals using the **composite Trapezoidal** rule $I_{\text{trap}}(h)$ for $N = 2, 4, 8, 16, 32$ and 64, where $h = (b-a)/N$:

   (a) $I_f = \displaystyle\int_0^1 (3x+1)\, dx,$

   (b) $I_f = \displaystyle\int_0^1 xe^{-x^2}\, dx,$

   (c) $I_f = \displaystyle\int_0^{2\pi} (\cos(x)+1)\, dx.$

   To do so, write an m-file `trap.m` in MATLAB (or in any other programming language), the first line of which should be

   ```
   function y = trap(f,a,b,N)
   ```

   Include a copy of your code. For each of the cases above make a table with columns:

   - column 1: N
   - column 2: h
   - column 3: $I_{\text{trap}}(h)$ (this is the approximation, i.e., the output of your function)
   - column 4: $|\text{error}|$ (this is the **absolute** error)
   - column 5: $|\text{error}|/h^2$

   Are the numbers in the last column converging, and if so, what does this mean? Specifically, comment on the behavior of the error for (a) and (b). If your code is correct, you will notice that for (c) the last column is not converging, and that the approximation is very accurate. Can you explain why?

2. Consider Simpson's rule that we discussed in class:

$$I_{\text{Simp}} = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right], \quad \text{with} \quad h = \frac{b-a}{2}. \tag{1}$$

   This approximation satisfies:

$$E(f) = -\frac{f^{(4)}(\zeta)}{90}h^5, \tag{2}$$

   which implies that Simpson's rule is **exact** if $f(x)$ is a polynomial of degree $<= 3$, i.e., $p_n(x)$ for $0 \le n \le 3$.

(a) **Derive** Simpson's rule, i.e., Eq. (1).

(b) Consider $f(x) = x^3$. Then, **use** I_{Simp} to approximate $I_f = \displaystyle\int_a^b f(x)\,dx$. Is the answer exact? Justify your answer.