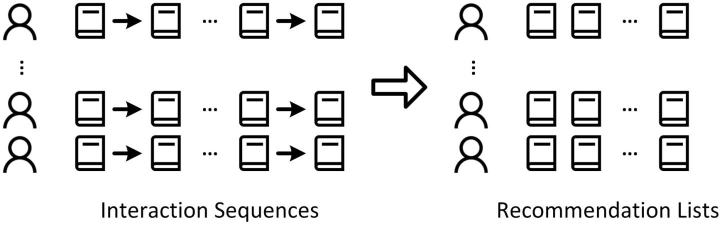
Factorizing Personalized Markov Chains

Liulan Zhong (revised by Weike Pan)

College of Computer Science and Software Engineering
Shenzhen University

Reference: Factorizing personalized Markov chains for next-basket recommendation (WWW 2010) by Steffen Rendle, Christoph Freudenthaler and Lars Schmidt-Thieme

Problem Definition



Next-Item Recommendation

- Input: $(u, \mathcal{S}_u)$, i.e., a sequence of items for each user u.
- Goal: Rank the items in $\mathcal{I} \backslash \mathcal{I}_u$ and use the top-k items with the highest preference values to construct a recommendation list for each user u.

Notations (1/2)

Table: Some notations and explanations.

n	number of users
m	number of items
u	user ID, $u \in \{1, 2, \ldots, n\}$
i	item ID, $i \in \{1, 2, \ldots, m\}$
$\mathcal{U}$	the whole set of users
$\mathcal{I}$	the whole set of items
$\mathcal{P}$	the whole set of observed (u, i) pairs
$\mathcal{I}_u$	a set of items that have been interacted by user u
γ	learning rate
$\alpha_u, \alpha_v, \alpha_p, \alpha_q$	tradeoff parameters of regularization terms
T	iteration number

Notations (2/2)

Table: Some notations and explanations (cont.).

$d \in \mathbb{R}$	number of latent dimensions
$U_{u\cdot} \in \mathbb{R}^{1\times d}$	user-specific latent feature vector w.r.t. user u
$V_{i\cdot}, P_{i\cdot}, Q_{i\cdot} \in \mathbb{R}^{1\times d}$	item-specific latent feature vector w.r.t. item i
$\hat{r}_{ui}$	predicted preference of user u to item i
$\mathcal{S}_u$	a sequence of items, $\mathcal{S}_u = \{i_u^1, i_u^2, \ldots, i_u^{\lvert\mathcal{S}_u\rvert}\}$
i_u^t	the tth item in $\mathcal{S}_u$
$\hat{r}_{ui_u^t}$	predicted preference of user u to item i_u^t

Prediction Rule (1/2)

In the training phase, the predicted preference of user u to item $i_u^t \in \mathcal{I}_u$ [Rendle et al., 2010],

$$\hat{r}_{ui_u^t} = U_{u\cdot} V_{i_u^t\cdot}^T + P_{i_u^{t-1}\cdot} Q_{i_u^t\cdot}^T, \tag{1}$$

where $U_{u\cdot}$ is the latent feature vector of user u, $V_{i_u^t\cdot}$ is the latent feature vector of item i_u^t, and $P_{i_u^{t-1}\cdot}$ and $Q_{i_u^t\cdot}$ are the auxiliary latent feature vectors of item i_u^{t-1} and item i_u^t, respectively.

Notice that the first term $U_{u\cdot} V_{i_u^t\cdot}^T$ is used to capture the user's long-term preference, while the second term $P_{i_u^{t-1}\cdot} Q_{i_u^t\cdot}^T$ is designed to capture the user's short-term sequential preference.

Prediction Rule (2/2)

- **In the training phase**, once we have sampled a (user, item) pair (u, i_u^t), we further randomly pick up an item $j \in \mathcal{I} \backslash \mathcal{I}_u$ and have

$$\hat{r}_{uj}^{i_u^t} = U_{u\cdot} V_{j\cdot}^T + P_{i_u^{t-1}\cdot} Q_{j\cdot}^T. \tag{2}$$

- **In the test phase**, for an item $j \in \mathcal{I} \backslash \mathcal{I}_u$, we have

$$\hat{r}_{uj} = U_{u\cdot} V_{j\cdot}^T + P_{i_u^{|\mathcal{S}_u|}\cdot} Q_{j\cdot}^T. \tag{3}$$

Objective Function

The objective function to be minixized is as follows,

$$\min_{\Theta} \sum_{u \in \mathcal{U}} \sum_{i_u^t \in \mathcal{S}_u, t \neq 1} \sum_{j \in \mathcal{I} \backslash \mathcal{I}_u} f_{u i_u^t j}, \tag{4}$$

where $\Theta = \{U_{u\cdot}, V_{i\cdot}, P_{i\cdot}, Q_{i\cdot}, i = 1, 2, \ldots, m; u = 1, 2, \ldots, n\}$ and $f_{u, i_u^t, j} = -\ln \sigma(\hat{r}_{u i_u^t} - \hat{r}_{uj}^{i_u^t}) + \frac{\alpha_u}{2} ||U_{u\cdot}||^2 + \frac{\alpha_v}{2} ||V_{i_u^t \cdot}||^2 + \frac{\alpha_v}{2} ||V_{j\cdot}||^2 + \frac{\alpha_p}{2} ||P_{i_u^{t-1} \cdot}||^2 + \frac{\alpha_q}{2} ||Q_{i_u^t \cdot}||^2 + \frac{\alpha_q}{2} ||Q_{j\cdot}||^2$ is a tentative objective function for a randomly sampled triple (u, i_u^t, j).

Notice that in the original paper [Rendle et al., 2010], the negative item j is randomly selected from $\mathcal{I} \backslash \{i_u^t\}$, instead of from $\mathcal{I} \backslash \mathcal{I}_u$.

Gradients (1/2)

The gradient of each parameter $\theta \in \Theta$, i.e., $\nabla\theta = \frac{\partial(f_{u,i_u^t,j})}{\partial\theta}$:

$$\nabla U_{u\cdot} = \alpha_u U_{u\cdot} - e_{ui}(V_{i_u^t\cdot} - V_{j\cdot}), \tag{5}$$

$$\nabla V_{i_u^t\cdot} = \alpha_v V_{i_u^t\cdot} - e_{ui} U_{u\cdot}, \tag{6}$$

$$\nabla V_{j\cdot} = \alpha_v V_{j\cdot} + e_{ui} U_{u\cdot}, \tag{7}$$

Gradients (2/2)

$$\nabla Q_{i_u^t \cdot} = \alpha_q Q_{i_u^t \cdot} - e_{ui} P_{i_u^{t-1}}, \tag{8}$$

$$\nabla Q_{j \cdot} = \alpha_q Q_{j \cdot} + e_{ui} P_{i_u^{t-1}}, \tag{9}$$

$$\nabla P_{i_u^{t-1}} = \alpha_p P_{i_u^{t-1}} - e_{ui} (Q_{i_u^t \cdot} - Q_{j \cdot}), \tag{10}$$

where $e_{ui} = \sigma(\hat{r}_{uj}^{i_u^t} - \hat{r}_{ui_u^t})$.

Update Rule

We have the update rule for each parameter,

$$\theta = \theta - \gamma \nabla \theta, \tag{11}$$

where $\gamma > 0$ is the learning rate,
$\theta \in \Theta, \Theta = \{U_{u\cdot}, V_{i\cdot}, P_{i\cdot}, Q_{i\cdot}, i = 1, 2, \ldots, m; u = 1, 2, \ldots, n\}$.

Algorithm

Algorithm 1 The algorithm of FPMC.

Initialize the model parameters
for $t = 1, \cdots, T$ **do**
  for each $(u, i_u^t) \in \mathcal{P} \backslash \{(u, i_u^1)\}$ in a ramdon order **do**
    Calculate $\hat{r}_{ui_u^t}$ via Eq.(1)
    Calculate $\hat{r}_{uj}^{i_u^t}$ via Eq.(2)
    Calculate $e_{ui} = \sigma(\hat{r}_{uj}^{i_u^t} - \hat{r}_{ui_u^t})$
    Calculate the gradients via Eqs.(5-10)
    Update the model parameters via Eq.(11)
  end for
end for

Dataset

We adopt the commonly used dataset in the experiments, i.e., MovieLens 100K. We treat all the observed behaviors as positive feedback and preprocess the dataset as follows.

- We remove the records of the users who rate fewer than five times.
- We remove the records of the items that are rated fewer than five times.
- We sort all the records according to the timestamps and split each user's sequence into three parts, i.e., the item(s) at the last step for test, the item(s) at the penultimate step for validation, and the remaining items for training.

Baseline

- Bayesian personalized ranking (BPR) [Rendle et al., 2009]

Parameter Configurations

- We fix the number of dimensions $d = 20$, the learning rate $\gamma = 0.01$, and adopt stochastic gradient descent (SGD) algorithm to train both the factorization-based methods.
- We choose the tradeoff parameter of the regularization terms $\alpha_u = \alpha_v = \alpha_p = \alpha_q$ from $\{0.1, 0.01, 0.001\}$ and the iteration number T from $\{100, 500, 1000\}$ via the NDCG@20 performance on the validation data.
- We use the same sampling strategy, i.e., randomly selecting one negative sample each time, for fair comparison.
- For each validation data, we select the optimal parameters according to the averaged performance of NDCG@20 of three runs. With the optimal parameter values, the final results on the test data are also the averaged values of three runs.

Evaluation Metrics

- Precision@20
- Recall@20
- NDCG@20

Results

Method	Pre@20	Rec@20	NDCG@20
BPR	**0.0282**±0.0004	0.1974±0.0049	0.1032±0.0012
FPMC	0.0273±0.0003	**0.2292**±0.0071	**0.1147**±0.0020

Conclusion

- The sequence modeling approach in FPMC is effective.

Rendle, S., Freudenthaler, C., Gantner, Z., and Schmidt-Thieme, L. (2009).
BPR: Bayesian personalized ranking from implicit feedback.
In *Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence*, UAI '09, pages 452–461.

Rendle, S., Freudenthaler, C., and Schmidt-Thieme, L. (2010).
Factorizing personalized markov chains for next-basket recommendation.
In *Proceedings of the 19th International Conference on World Wide Web*, WWW '10, pages 811–820.