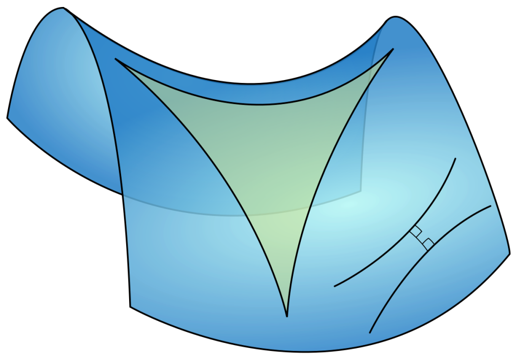
The books I found about our current understanding of the nature of the universe were either dumbed-down popular science or graduate level texts. This document attempts to bridge this gap. It assumes comfortability with high school level math, particularly differentiation and geometry with planes and lines. Much information is verbatim from sources listed at the end of the document.

1. NON-FLAT SURFACES

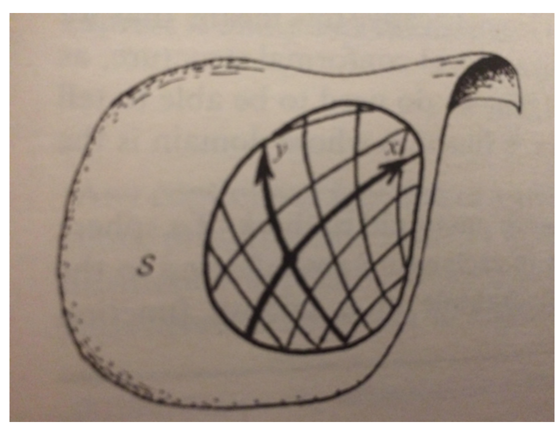
Let us imagine a non-flat surface S that looks like:

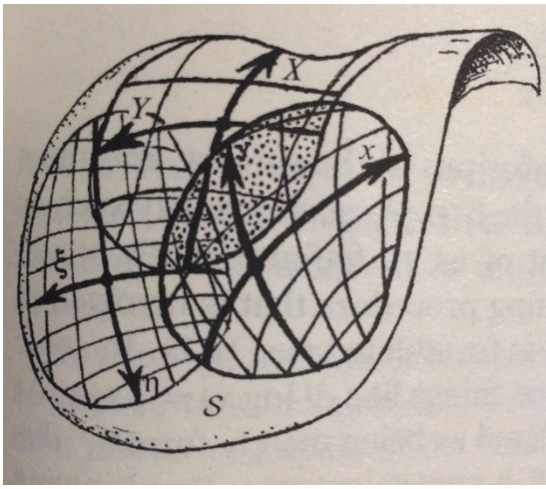


We can make this saddle-shape by taking a plain old x-y axis and "bending" it in a third direction. Something you notice is that straight lines on this non-flat surface don't look straight at all! In fact, the 3 inner angles of the green triangle in the middle sum to less than 180°! This space has an intrinsic curvature, which means the two-dimensional x-y plane is itself curved in the third dimension.

Consider a smooth, real-valued function Φ defined on surface S. That is, Φ is a smooth map from S to the space of real numbers. Φ = f(x,y). It is said that Φ is a scalar field on S. This means that on each point on S, Φ spits out a value; there is a field of values (scalars) ascribed on S.

So that we can do calculus on this non-flat surface, we can think about S being covered with many rectangular sheets called coordinate patches. One coordinate patch can look like the image on right.





To cover S, we may have to glue together several coordinate patches. $\Phi = f(x,y)$ on one patch and $\Phi = F(X,Y)$ on another. There are overlap regions where $F(X,Y)=f(x,y)$. These functions are called transition functions.

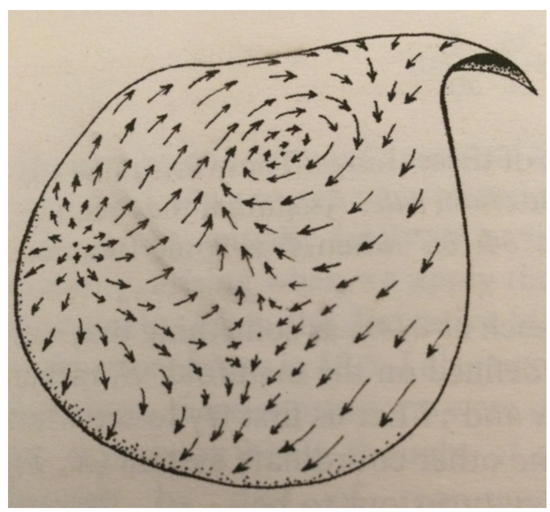
$X = X(x,y)$, $Y=Y(x,y)$, $x=x(X,Y)$, $y=y(X,Y)$

Naively, smooth surfaces are locally flat. That is, at the small scales we can think of the curved surface S as being composed of many non-curved, rectangular coordinate patches.

However, this understanding of a surface using coordinate patches is limited to coordinate choice (i.e. x, y or X, Y, etc.). There is, in fact, a notion of a derivative of a function independent of coordinate choice. This is $d\Phi = \partial f/\partial x\ (dx) + \partial f/\partial y\ (dy)$. $d\Phi$ is called a one-form or co-vector.

Φ is applied to a particular point p on S while f is applied to particular coordinate values (x,y). Because f 'knows' what the coordinates x and y are whereas Φ doesn't.

Now consider $\zeta = \partial f/\partial x$. This ζ is a differential operator called a vector field. It has a very specific way of transforming as we pass from patch to patch since $\partial f/\partial x = \zeta = \partial X/\partial x * \partial f/\partial X + \partial Y/\partial x * \partial f/\partial Y$. We can understand ζ just as a field of little arrows drawn on S like wind flow charts on TV, like this:



2. TENSORS

Vectors are objects with both a magnitude and direction. Given some coordinate system, we can specify a vector by simply giving a list of numbers. That is in fact what we're doing when we talk about v^i. Tensors are a generalization of vectors with more than 1 index. As a simple example, we can generate a tensor by taking the outer product of two vectors:

$$M^{ij} = A^i B^j$$

$$\mathbf{u} \otimes \mathbf{v} = \mathbf{u}\mathbf{v}^{\mathrm{T}} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} u_1 v_1 & u_1 v_2 & u_1 v_3 \\ u_2 v_1 & u_2 v_2 & u_2 v_3 \\ u_3 v_1 & u_3 v_2 & u_3 v_3 \\ u_4 v_1 & u_4 v_2 & u_4 v_3 \end{bmatrix}.$$

Where in Euclidian space i and j can take on 3 different values. M^{ij} represents a table of 9 numbers, each indexed by an ordered pair. The number of distinct indices is known as the order of a tensor, so M^{ij} is second order, while an ordinary vector is a first order tensor.

Tip: tensors are not matrices. Be aware that tensors don't multiply in the same way as matrices. They're just tables of numbers.

3. METRIC TENSORS

A meter stick has the very useful property that it is a meter no matter which direction it's oriented; $\text{length}^2 = x^2 + y^2 + z^2$. In hyper-surfaces, this length formula is no longer the case because of varying curvature on S. It turns out that to create our meter-stick for hyper-surfaces, we need to add some coefficients to take into account different scales & that lines no longer cross to form right angles.

In layman terms, the metric tensor endows S with the notion of distance. Consider a two-dimensional manifold with metric and put a "coordinate grid" on it, i.e. assign to each point a set of two numbers, (x,y). Then, the metric can be viewed as a 2 x 2 matrix with $2^2 = 4$ entries.

The metric is symmetric, which means that the positive distance between point a and b is the positive distance between b and a. So, $g_{ab} = g_{ba}$. Now, consider two points that are nearby, such that the difference in coordinates between the two is (dx,dy). We can denote this in shorthand notation as dl^μ where μ is either x or y, and $dl^\mu = dx$ and $dl^\nu = dy$. Then we define the square of the distance between the two points, called ds, as

$$ds^2 = g_{xx}dx^2 + g_{yy}dy^2 + 2g_{xy}dxdy = \sum_{\mu,\nu\in\{x,y\}} g_{\mu\nu} dl^\mu dl^\nu$$

The metric tensor is a geometric tool that allows to take a dot product no matter how complicated the geometry. Recall that the dot product operator allows us to compute the length of 2 vectors $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\ |\mathbf{b}|\ \cos\theta$, where θ is the angle suspended between vectors **a** & **b**. So, $\mathbf{a} \cdot \mathbf{b} = g_{ij}\ \mathbf{a}^i\ \mathbf{b}^j$. The metric tensor is a generalization of the dot product in Euclidian space for hypersurfaces, allowing us to compute length. For this reason, the generalized metric tensor is the foundation of non-Euclidian geometry.

Tip: Through integrating the tensor, we get the length of curves on S. These curves can be called geodesics, or the smallest distance line between two points on a curve. Also, the metric tensor is the derivative of the s function, where s represents distance.

This discussion of metric tensors matters because Einstein claims space-time is a three-dimensional physical reality that's bent or curved in a fourth, physical dimension. Since we cannot intuitively visualize this fourth dimension as three-dimensional creatures, techniques like integrating the metric tensor on a surface allows us to compute shortest distance paths when supplied the curvature and torsion of that surface.

4. NEWTON'S LAWS OF MOTION

Newton's first law is that any particle which is not subjected to forces moves along a straight line at constant speed. Newton's second law is $F = ma$. This means when a force acts on a mass, the mass (m) accelerates (i.e. speeds up). Newton's third law is that for every force there is a reaction force that is equal in size but opposite in direction (hence, sign).

Newton wants to be able to describe physical systems. For this, we need to define reference frames. A reference frame is the frame (point, plane, etc.) from which a measurement is taken. If you are measuring the path of the Sun in the sky, your bedroom on Earth is the frame of reference. Accordingly, in a region of space remote from all other matter and empty save for a single particle, a reference frame can be defined relative to which the particle will always have a uniform motion. Such a frame will be referred to as an inertial frame.

If S is any inertial frame and S' is another frame whose axes are always parallel to those of S but whose origin moves with a velocity u relative to S, then S' also is inertial. If v, v' are the velocities of the single particle relative to S, S' respectively then:

$$v' = v - u \tag{4.1}$$

If it is found by observation that a particle does not have a uniform motion relative to the frame, the lack of uniformity is attributed to the action of a force which is exerted upon the particle by some agency. For example, when a beam of charged particles is deflected when a bar magnet is brought into the vicinity, this phenomenon is understood to be due to the magnetic forces acting on the beam particles. If v is the particle's velocity relative to the frame at any instant t, its acceleration $a = dv/dt$ will be non-zero if the particle's motion is not uniform and this quantity is a measure of the applied force f. So, f is proportional to a.

5. SPECIAL RELATIVITY

The special principle of relativity asserts that all physical laws are covariant with respect to a transformation between inertial frames. This implies that all observers moving uniformly relative to one another and employing inertial frames will be in agreement concerning the statement of physical laws. So, the speed of light, as confirmed with experimental data, is constant.

This presents a problem. Consider two inertial frames S, S′. Suppose that an observer employing S measures the velocity of a light pulse and finds it to be c. Let c′ be the velocity of this same pulse measured by an observer from the frame S′. Then by (4.1),

$$c' = c - u \tag{5.1}$$

Clearly, c′ and c will be different and so it seems that either Maxwell's equations (presented in Appendix A) or the special principle of relativity must be abandoned for EM phenomena. Attempts were made (e.g. by Ritz) to modify Maxwell's equations, but certain consequences of the modified equations could not be confirmed experimentally. Since the special principle was always found to be valid, the only remaining alternative was to reject (4.1) and to replace it by another in conformity with the experimental result that the speed of light is the same in all inertial frames.

This can only be done at the expense of a radical revision of our intuitive ideas concerning the nature of space and time.

6. SPACETIME

The argument of this section will be founded on three postulates:

Postulate 1: A particle free to move under no forces has constant velocity in any inertial frame.
Postulate 2: The speed of light relative to any inertial frame is c in all directions.
Postulate 3: The geometry of space is Euclidian in any inertial frame.

Let the inertial frame S comprise rectangular Cartesian axes Oxyz. Let all points on S have its own atomic clock with time 0:00 synchronized with the master-clock at the origin O. The position at any event on S can now be specified by the four coordinate (x,y,z,t).

Let O′x′y′z′ be rectangular Cartesian axes determining the frame S′ and suppose their clocks, also are synchronized with a master clock at O′. Any event in S′ is (x′,y′,z′,t′), the s space coordinates being measured by scales and the time coordinate by the clock at rest in S′. If (x,y,z,t) and (x′,y′,z′,t′) relate to the same event, let's find the equations relating their coordinates.

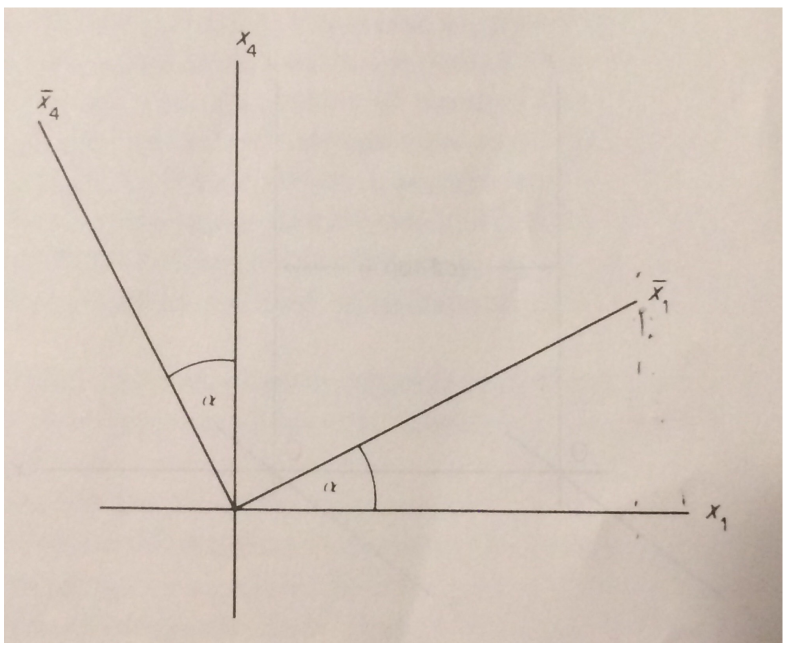
Let us imagine a particle moving uniformly in S with velocity (v_x,v_y,v_z,) that has space coordinates (x,y,z) such that

$$x = x_0 + v_x t$$
$$y = y_0 + v_y t$$
$$z = z_0 + v_z t$$

The trick here is to use a mathematical device invented by Minkowski. We will replace the time coordinate t of any event observed in S by a purely imaginary coordinate $x_4 = ict$ where $i^2 = -1$.

7. LORENTZ TRANSFORMATION

Let's imagine that we have S and S′ such that if we rotate our **x**-axes in S we get the **x′**-axes of S′. Let us say this rotation is by an angle α parallel to the x_1-x_4-plane.



The origin and the x_2, x_3 axes are unaffected by the rotation so,

$$x'_1 = x_1 \cos \alpha + x_4 \sin \alpha$$
$$x'_2 = x_2$$
$$x'_3 = x_3$$
$$x'_4 = -x_1 \sin \alpha + x_4 \cos \alpha \qquad (7.1)$$

To interpret these equations, let's consider a plane which is stationary relative to the S′′ frame with its y′′ and z′′ axes parallel to y′ and z′ on S′. Its equation relative to S is

x = -ict tan α. y and z are the same …

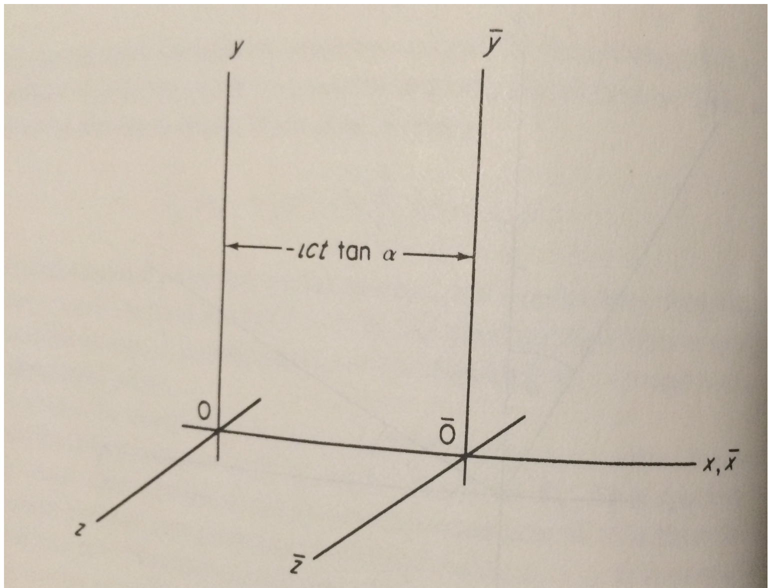


FIG. 2

If u is the speed of translation of S′ relative to S,

u = -ic tan α

We have tan α = iu/c

$$\cos \alpha = \frac{1}{\sqrt[2]{\frac{i - u^2}{c^2}}}$$

$$\cos \alpha = \frac{\frac{iu}{c}}{\sqrt[2]{\frac{i - u^2}{c^2}}}$$

Substituting these in (7.1),

$$x' = \beta(x - ut) \qquad y' = y$$
$$t' = \beta\left(t - \frac{ux}{c^2}\right) \qquad z' = z \qquad (7.2)$$

where $\beta = \sqrt[2]{\frac{i - u^2}{c^2}}$

If u is small by comparison with c as is generally the case, these equations may evidently be approximated by the equations

$$x' = x - ut \qquad y' = y \qquad (7.3)$$
$$t' = t \qquad z' = z$$

This set of equations, called the special Galilean transformation equations, is, of course, the set which was assumed to relate space and time measurements in the two frames in classical physical theory. However, $t' = t$ was rarely stated explicitly, since it was taken as self-evident that time measurements were absolute, i.e. quite independent of the observer. It appears from (7.3) that this view of the nature of time can no longer be maintained and that in fact time and space measurements are related, as is shown by the dependence of t' on t and x. The revolutionary idea is also suggested by the manner in which the special Lorentz transformation has been derived, viz. a rotation of axes in a manifold which has both space-like and time-like characteristics. However, this does not imply that space and time are now to be regarded as basically similar physical quantities, for it has only been possible to place the time coordinate on the same footing as the space coordinate in $\mathbf{E}^4$ by multiplying the former by i. Since x_4 must always be imaginary whereas x_1, x_2, x_3 are real, the fundamentally different nature of space and time measurements is still maintained in the new theory.

8. FITZGERLAND CONTRACTION

Consider a rigid rod stationary in S′ and lying along the $\mathbf{x'}$-axis. Let x'_1 and x'_2 at the two ends of the bar so that its length as measured in S′ is given by

$$l' = x'2 - x'1 \qquad (8.1)$$

In the frame S, the bar is moving with speed u and, to measure its length, it is necessary to observe the positions of its two ends at the same instant t. Suppose chalk marks are mode on the **x**-axis at x_1 and x_2 lying at opposite the two ends, at the instant t. The making of these marks constitutes a pair of events with space-time coordinates (x_1,t), (x_2,t) in S. In S′ this pair of events must have coordinates (x'_1, t'_1), (x'_2, t'_2). Equations (7.2) now require that

$$x'_1 = \beta\,(x_1 - ut) \qquad (8.2)$$
$$x'_2 = \beta\,(x_2 - ut)$$

But $x_2 - x_1 = l$ is the length of the bar as measured in S and it follows by subtraction of (8.2) that

$$l = l' \sqrt[2]{\frac{1-u^2}{c^2}}$$

The length of the bar accordingly suffers contraction when it is moved longitudinally relative to an inertial frame. This is the Fitzgerald contraction.

This contraction is not to be thought of as the physical reaction of the rod to its motion and as belonging to the same category of physical effects as the contraction of a metal rod when it is cooled. It is due to a changed relationship between the rod and the instruments measuring its length. l' is a measurement carried out by scales which are stationary relative to the bar, whereas l is the result of a measuring operation with scales which are moving with respect to the bar. Also, the first operation can be carried out without the assistance of a clock, but the second operation involves is simultaneous observation of the two ends of the bar and hence clocks must be employed. In classical physics, it was assumed that these two measurement procedures would yield the same result, since it was supposed that a rigid bar possessed intrinsically an attribute called its length and that this could in no way be affected by the procedure employed to measure it. It is no understood that length, like every other physical quantity, is defined by the procedure employed for its measurement and that it possesses no meaning apart from being the result of this procedure. From this point of view, it is not surprising that, when the procedure must be altered to suit the circumstances, the result will also be changed.

9. TIME DILATION

Consider again the two events when chalk marks are made on the **x**-axis. Applying equations (7.3) to the space-time coordinates of the events in the two frames, the following equations are obtained:

$$t'_1 = \beta\ (t-ux_1/c^2)$$
$$t'_2 = \beta\ (t-ux_2/c^2)$$

These equations show that $t'_1 \neq t'_2$; i.e. although the events are simultaneous in S, they are not simultaneous in S′. The concept of simultaneity is accordingly also a relative one and has no absolute meaning as was previous thought.

The registration by the clock moving with O′ of the times t'_1, t'_2 constitutes two events having coordinates $(0, 0, 0, t'_1)$, $(0,0,0,t'_2)$ respectively in S′. Employing the inverse transformation to (7.3), it follows that the times t_1 and t_2 of these events as measured in S are given by

$$t_1 = \beta\ t'_1$$
$$t_2 = \beta\ t'_2$$

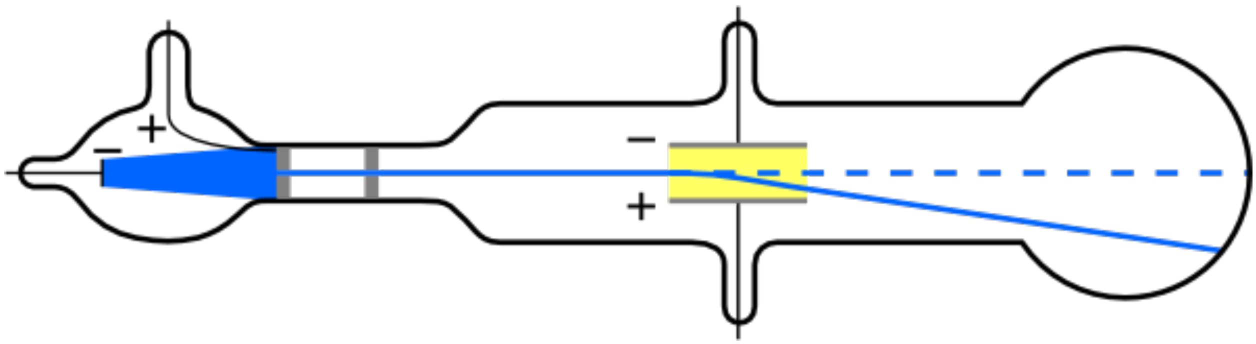
This equation shows that the clock moving with O′ will appear from S to have its rate reduced by a factor of $\sqrt[2]{\frac{1-u^2}{c^2}}$. This is the time dilation effect.

This result implies that all physical processes will evolve more slowly when observed from a frame relatively to which they are moving.

It may also be deduced that, if a human passenger were to be launched from the earth in a rocket which attained a speed approaching that of light and after proceeding to a great distance returned to the earth with the same high speed, suitable observation made from the earth would indicate that all physical processes occurring within the rocket, including the metabolic and physiological processes taking place inside the passenger's body, would suffer a retardation. Since all physical processes would be affected equally, the passenger would be unaware of this effect. Nonetheless, upon return to the earth he would find that his estimate of the duration of the flight was less than the terrestrial estimate.

10. THOMSON'S EXPERIMENT

JJ Thomson ran an electric current through H_2 gas. Then he fed the gas from the left side of the device shown below.



Since the gas is ionized, the negatively charged electrons in the hydrogen gas separated from the positively charged protons. As the gas travelled to the right, the particles encounter charged plates shown as the line with the minus and plus sign. These plates produce an electromagnetic field around them, where the negatively charged plate attracts the protons and the positively charged plate attracts the electrons.

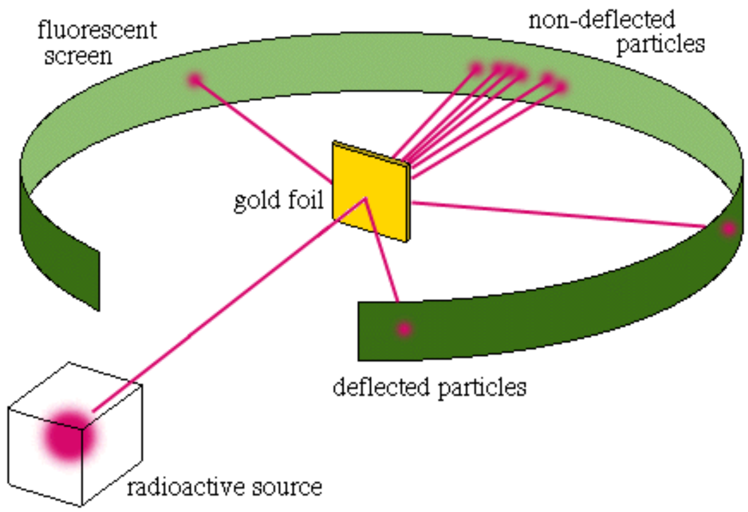
Thomson found that as the ionized particles of hydrogen passed through the device, the positively charged plates deflected the electrons a lot, but the negatively charged plates barely affected the protons! The result of the experiment was that the mass of protons was way larger than the mass of electrons.

$$m_+ >>> m_-$$

In fact, the protons are so dense that a teaspoon of nuclear matter would weigh 500,000,000 tonnes! It is understandable, then, that the number of protons in an atom determines its element on the periodic table.

11. GOLD FOIL EXPERIMENT

Ernest Rutherford fired radioactive particles from a source (bottom-left box in the image below) through a gold foil. All around the foil, he placed a fluorescent screen which were able to pick up the presence of these radioactive particles.



Most of the particles went right through the gold foil and ended up being picked up on the fluorescent screen right behind it. However, sometimes, these particles actually encounter something solid in the foil & so rebounds.

Therefore, if you picture the atom to be the size of your bedroom, the nucleus is the size of a grain of sand! In the analogy, the electrons are whizzing around the walls. This explains why most of the radioactive particles flow straight through the foil: atoms are mostly empty space!

12. ELECTRON ORBIT

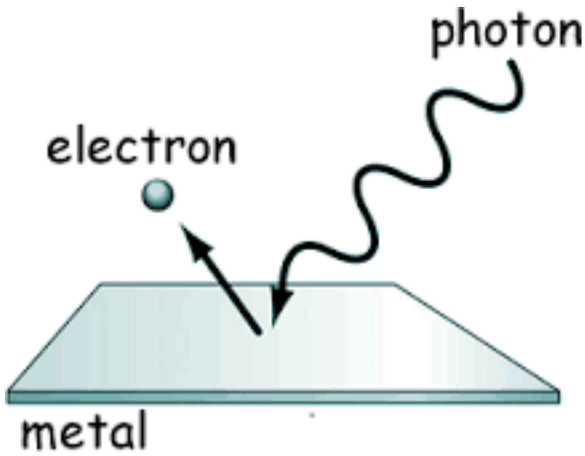
TV and radio work by accelerating electrons up and down transmitters.

As electrons orbit around the nucleus, this means electrons are accelerating. This is because they're constantly changing direction. Imagine you're speeding up in one direction, but when you turn around you must stop and begin speeding up (i.e. accelerating again). Likewise, electrons are also undergoing constant acceleration because of always changing directions.

However, accelerating charges emit radiation. If it's emitting radio waves, it's losing energy, so the electron would spiral inward toward the nucleus to annihilate the atom! Why doesn't this happen? Spoiler alert: because the electron acts as both a wave and particle and so it's forced to stay in discrete energy levels.

13. LIGHT IS A WAVE

When a photon (light particle) comes into contact with an atom from a metal, it hits the electron and transfers energy to that electron so it can escape from the metal's atom. This only occurs if the frequency f of that photon is greater than some frequency f_0. That is, if $hf >$ binding energy keeping the electron in the atom.



$E_{photon} = hf$. Light is a packet of energy. So, light is made of particles and is a wave.

This is called the photoelectric effect.

14. ELECTRONS ARE WAVES

Electrons are also waves! We know

$E=mc^2$ (1)

$p=mv$ (2)

$E=hf$ (3)

So,

$m = E/c^2$

$p = (Ec)/c^2 = E/c$ for a particle travelling at light speed

$p = hf/c = h/\lambda$ λ is wavelength

$\lambda = h/p = h/mv$ (4)

A Nobel Prize was won by de Broglie for this relation (4), showing the wave nature of the electron. This is because momentum is a property of a wave so an electron, can, in fact, be described as a wave using (4)!

15. A WAVE FUNCTION IS A SOLUTION TO ITS SCHRODINGER'S EQUATION

Misinterpretation of the ideas of quantum mechanics has spawned some of the worst quackery pseudo-science and unfounded mystical story-telling of any scientific theory. It's easy to see why. Quantum experiments do seem to demand weird explanations of reality.

One part of this explanation is that the wave function is not a wave in anything physical, but an abstract distribution of probabilities. In the absence of measurement, the unobserved universe is only a suite of possibilities of the various states it could take were a measurement be made. Upon measurement, fundamental randomness determines the path of say, a particle that would emerge from its wave function.

Some were opposed to this way of thinking. Einstein believed that quantum mechanics was a statistical approximation to an underlying deterministic theory. Schrodinger, also, was opposed to a statistical or probabilistic approach. Still, this approach encounters much experimental success and is a pillar of the Standard Model. This becomes clear as we continue.

Newton's Second Law describes a physical system in classical mechanics. Let us try to describe a quantum system. Let us take a single particle in 3 dimensions (x,y,z). Let its wave function be ψ. Since experimental evidence strongly suggests a probabilistic approach[1], ψ^2 = probability and so ψ is the probability amplitude…

I picture a wave function Ψ as a trigonometric function of the form $\Psi = A\cos kx + C$. This implies $\Psi' = \cos kx$ and since $k=p/\hbar$, $\Psi = \cos(p/\hbar)x$. This implies we know x and p precisely and violate Heiseinberg's Uncertainty Principle. Also, probability of finding an electron at a point is Ψ^2. So $\Psi'^2 = \cos^2 kx \neq$ constant. We need to put our trigonometric function Ψ in another form. Schrodinger's insight was to express the phase of a plane wave as a complex phase factor. This is because $\cos(x) + i\sin(x) = e^{i\theta}$. This form is called polar coordinate form.

Now let us return to our single particle in 3 dimensions (x,y,z). We make a few assumptions. First, the total energy of the particle is

$$E = T + V = \frac{p^2}{2m} + V$$

Second, Einstein's light quanta hypothesis of 1905, which asserts that the energy E of a photon is proportional to the frequency f of the corresponding electromagnetic wave.

$$E = hf = \frac{h}{2\pi}(2\pi f) = \hbar\omega$$

where the frequency f of the quanta of radiation (photons) are related by Planck's constant h, and where $w = 2\pi f$ is the angular frequency of the wave.

Third, we use de Broglie's hypothesis:

$$p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$$

We can express momentum (p) and constant k as vectors.

[1] Demonstration of Correctness : http://www.applet-magic.com/micromacro.htm

Schrödinger's great insight, late in 1925, was to express the wave function in complex phase form, which is complex polar form with angles:

$$\Psi(\mathbf{x},t) = Ae^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} = Ae^{i\mathbf{k}\cdot\mathbf{x}}e^{-i\omega t} = \psi(\mathbf{x})\phi(t)$$

where

$$\psi(\mathbf{x}) = Ae^{i\mathbf{k}\cdot\mathbf{x}}$$
$$\phi(t) = e^{-i\omega t}$$

And to realize that since

$$\frac{\partial}{\partial t}\Psi = -i\omega\Psi$$

then

$$E\Psi = \hbar\omega\Psi = i\hbar\frac{\partial}{\partial t}\Psi$$

And similarly since,

$$\frac{\partial}{\partial x}\Psi = ik_x\Psi$$

then

$$p_x\Psi = \hbar k_x\Psi = -i\hbar\frac{\partial}{\partial x}\Psi$$

and hence

$$p_x^2\Psi = -\hbar^2\frac{\partial^2}{\partial x^2}\Psi$$

And by inserting these expressions for the energy and momentum into the classical formula we started with we get Schrödinger's famed equation for a single particle in the 3-dimensional case in the presence of a potential V:

$$p^2\Psi = (p_x^2 + p_y^2 + p_z^2)\Psi = -\hbar^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Psi = -\hbar^2\nabla^2\Psi$$

The wave function to the system is the solution to its Schrodinger’s equation.

16. HEISENBERG'S UNCERTAINTY PRINCIPLE

Veritasium fired photons from a green laser on one side of the room through a green slit and onto a screen on the other side of the room. This slit can be made narrower or wider. Naturally, we would expect that as the slit narrows, the spot on the screen behind it also narrows. This does happen until a certain point.

But, when we make the slit very small, something very weird happens. As you keep making the slit even narrower, the green spot on the wall starts to spread out! Veritasium is making the slit narrower and yet the spot on the wall is getting wider!

As we narrow delta x, we are getting more precise about the location of these green photons. At a certain point, we reach some limit. What happens after this limit, is that the momentum increases. This means that if before photons were going perfectly straight now they must veer off to the left or right to ensure we don't break this uncertainty relation. The more we decrease the size of the slit, the higher the momentum of the photons go up![2]

17. MOTIVATION FOR DIRAC EQUATION

Schrodinger's Equation breathed life into the then-emerging model of quantum mechanics. He describes how matter waves, represented as wave functions, change over time and allowed physicists to predict the evolution of quantum systems, like the photons in the double split experiment.

There is a problem though: it is incompatible with Einstein's Special Relativity.

First, although relativity tells us space & time are intrinsically connected, Schrodinger's Equation tracks time with 1 clock (typically of the observer). t is the variable for time, and not $t_1, t_2, ..., t_n$.

Second, relativity tells us passage of time depends on velocity, SE only works for slow objects. So, it gives wrong answers for objects like electrons.

Third, it describes particles as simple wave functions – distribution of possible positions and momenta that have no internal property - yet we now know that many particles have an internal property called spin.

Spin results in a sort of quantum angular momentum. Like electron spins allows them to align with magnetic fields. The axis of spin can point in different directions like up or down. Pauli realized that to explain electron energy levels in atoms, those electrons must obey the Pauli Exclusion Principle. It applies to all particles called fermions. It suggests that we should find only 1 electron per atomic orbital, if we count each orbital as a

[2] derivation: applet-magic.com/Uncertainty.htm

quantum state. But, we found 2 electrons per shell, so Pauli realized there must exist a hidden quantum spin. Pauli, here, introduced a new degree of freedom internal to electrons, one that could take one of two values. Let's call them up or down. Spin allows two electrons to occupy the same energy level without violating the Pauli Exclusion Principle.

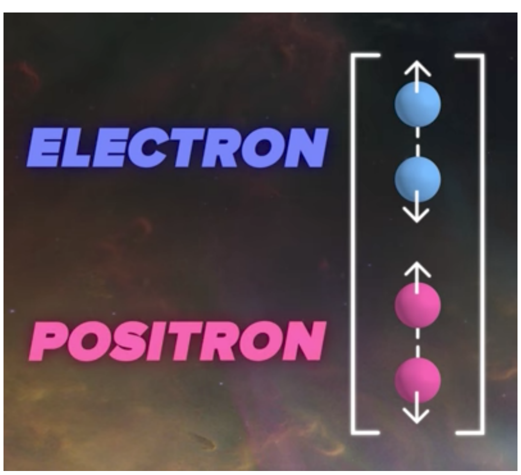
It's okay to ignore spin in the old Schrodinger's equation and get approximate answers. But when a magnetic field is present, spin direction becomes very important. So, for fast moving electrons and for electrons in EM fields, the SE gives the wrong answer. Since SE does not take into account relativistic effects, it is invariant under a Galilean transformation but not a Lorentz transformation. To include relativity, the physical picture must be altered in a radical way.

18. DIRAC EQUATION

Inducing Lorentz invariance on the Schrodinger's Equation consumed the mind of brilliant British physicist Paul Dirac. Dirac started with the full form of $E=mc^2$, starting with relativity. he then used quantum mechanical expressions for energy and momentum. the result was a huge mess, but Dirac stumbled upon an idea that cause the horrendous mathematics to collapse into an incredibly simple and beautiful equation. That simplification required Dirac to expand the internal workings of the electron even further. Instead of having a 2-component spinor (up and down) like in Pauli's theory, he needed 4 components.

$$i\,\hbar\,\gamma^{\mu}\partial_{\mu} = m\,c\,\psi$$

The equation describes the space-time evolution of this 4-component particle wave function. It contains the marks of both QM in the Plank's constant and relativity through c (speed of light). It accurately predicts the motion of electrons in any speed, including in an EM field. It was a major victory but opened up more questions than it answered. The first question is what on earth are the 2 extra degrees of freedom in the 4-component electron?



Only a few years after Dirac wrote down his equation in 1928, the positron (the antimatter electron) was spotted in cosmic rays by Carl Anderson. These 2 extra components correspond to the electron's spins of their antimatter counterpart. In fact, the electron and positron can't exist without each other; they're 2 sides to the same coin…positive and negative energy solutions to the same vibration of the electron field. They are both vibrations on the same field.

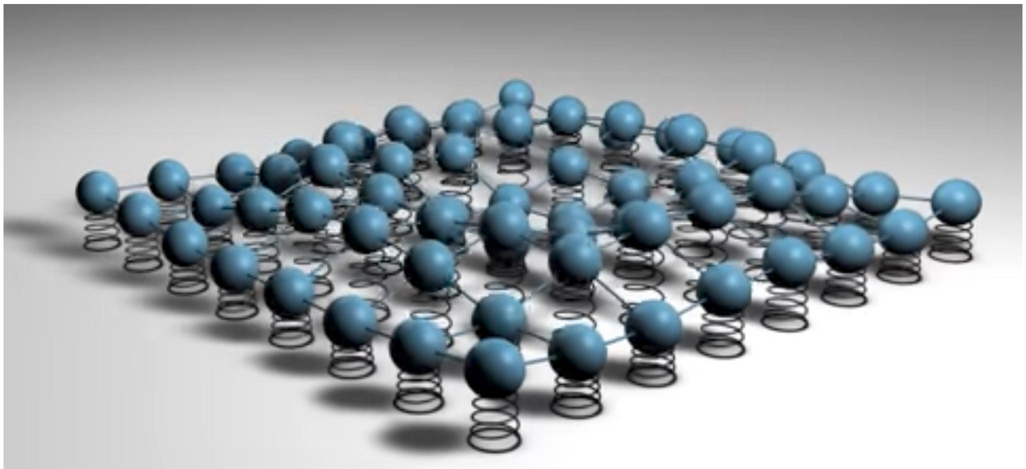
19. QUANTUM FIELD THEORY

Quantum field theory (QFT) is the best description of the fundamental workings of reality. It describes all elementary particles as vibrational modes in the fundamental fields that exist in all points of space and time throughout the universe. Quantum electrodynamics (QED) provides this description for 1 such field - the EM field. The pillars of QED are the description of the behaviour of the EM field and the behaviour of the description of the electron (via the Dirac equation).

Let's start with looking at vibrations as displacement. Imagine a guitar string. The string vibrates with a certain frequency when plucked. It also vibrates with an amplitude depending on how hard you pluck it. A larger amplitude or frequency means the vibration carries more energy. Any point on the vibrating string is displaced from its relaxed or equilibrium position; that distance changes over time as the string moves back and forth. Guitar strings are 1D. For 2D, imagine a drum skin. Everywhere on the surface of a vibrating drum skin, there's a displacement of the flat equilibrium space in the up-down direction.

The 3D analogy is harder to imagine. Every point in 3-space has a vibration in an additional dimension.

We saw how Dirac found an equation that worked to describe the behaviour of an electron in space-time. However, understanding the behaviour of light and its interaction with matter required a different approach. The Schrodinger's equation worked badly for many-particle systems since it tracks the state of every individual particle. But this is very inefficient; imagine doing your finances by keeping a tag on each dollar you spend. No one cares which dollar is which, you only care about how many dollars end up in whose bank account. Dirac's solution was that instead of quantizing electrons' physical properties like x, p, etc., Dirac quantized the EM field itself. He imagined each point as a simple harmonic oscillator (like a spring). The oscillation at each point can be complicated but can be built up from a number of minimum amplitude quantum oscillations, or photons.



Dirac described a space of quantum states as an infinite array of springs. His math kept track of the number of particles (i.e. quantum oscillations) in each spring. Dirac called this method the second quantization, which is the process of counting the changing number of quantum oscillations or particles per state. This implies the first quantization would be Schrodinger's book-keeping of the physical state of each individual particle. The second quantization allows for calculations in radiation and the creation/destruction of particles. It's been carefully tested! It can predict how spin changes electron energy (hyperfine splitting), or spins interacting with vacuum fluctuations (lamb shift).

Particles are vibrations in their own field! Fields are fundamental. Particles and their antimatter counterparts are just ways in which that field vibrate.

Calculations of QFT are about counting the number of ways a quantum phenomenon can occur…So a huge part of QFT is about taming infinities in equations since there are infinite ways in which anything can occur.

20. FEYNMAN'S PATH INTEGRAL

Our goal was to be able to mathematically describe quantum systems, like photons in the slit experiment. So, our goal would be to calculate the probability of a photon taking a path from point A to B. Feynman's approach is to slice the space between A and B into infinitely many slices. The particle can go from A to B with no restrictions on paths. This includes the particle starting at A, going to the edge of the universe, and then going to B.

Feynman's idea was to the use an older theorem called Principle of Least Action. Let S be action.

$$S = \int_{t_i}^{t_f} L dt = \int_{t_i}^{t_f} L\gamma d\tau$$

L is the Lagrangian. L= Kinetic Energy – Potential Energy.

Dirac is using the insight that objects always take the path of least action. Each path of least action contributes a probability amplitude to the entire A→B journey.

The principle of least action is a function of the particle's path through spacetime so works with special relativity (since space and time are treated symmetrically).

This converts to a QFT, where instead of summing over all paths particles can take, you sum over all possible histories of quantum fields.

Feynman diagrams tame infinities by describing antimatter as regular matter travelling backward in time.

21. ELECTRON SCATTERING

Vibrations in the all-permeating EM field are called photons (light). Electrons are excitations of vibrations in a different field (electron field). Fields are connected, as seen in processes like electron scattering, or Møller scattering.

Electron scattering occurs when 2 electrons repel each other. Since all infinite number of intermediate events that lead to the same result actually do happen, we need to sum all the possible histories of the quantum field… How do we deal with the infinities of quantum theory? These calculation problems currently plague quantum mechanics. There are, though, methods to "get around" this.

22. PERTURBATION THEORY

If the correct equation is unsolvable, just find a similar equation that you can solve and make small modifications to (i.e. perturb it). This approach is called perturbation theory.

How 2 electrons repel each other in QFT is that 1 electron excites a photon and that photon delivers a bit of its momentum to the 2nd electron. This photon is called a virtual photon since its existence is ambiguous.

Electrons are always interacting with virtual photons! This impedes its motion & increases its mass! This is called self-energy.

Q. If you try to calculate the self-energy to the electron mass using QED, you get infinite extra mass because you're adding up all possible (arbitrarily large) photon energies. How do we find the limit for the energies of these photons? The answer probably lies in some theory of quantum gravity which we do not yet have, but we can estimate!).

A. Renormalization. The trick is that every measurement we make for the mass of the electron already includes self-energy. So, set this mass in the equation based on the measurement. We do pay a price for renormalization: renormalization can only make predictions of other properties relative to your lab measurement.

APPENDIX: PHILOSOPHICAL AFTER-THOUGHTS

Albert Camus found a contradiction between the human struggle to find meaning in life and the inherent meaninglessness of the universe. To Camus, we are Sisyphus - the King of Greek mythology condemned to repeat the same task of pushing a boulder on a mountain only to see it roll down again, and again, and again for the rest of his life. Indeed, Sisyphus' never-ending and seemingly pointless task resembles the scientific quest to understand the universe: the more questions we answer, even more questions arise. As we seemingly get to know more about the universe, prior paradigms are overturned and tossed in the loony bin. Aristotle, Archimedes, Newton, Maxwell, Einstein, and Schrodinger are spokes on the wheel of knowledge, where one is on top for a few centuries until he is let go in favour of another, perhaps contradictory, theory. It's hard to accept that our efforts will be largely pointless and quickly forgotten in due time.

However, Camus says one must imagine Sisyphus happy. He argues we have to recognize the absurd, meaninglessness background of our existence and accept it. Once we do, it frees is to find our own subjective meaning and purposes.

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