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ANFIS izvještaj

Oblik pravila koja se koriste Ako **x je A_i** i **y je B_i** onda je $z_i = p_i x + q_i y + r_i$

Sigmoidalna funkcija pripadnosti $\mu = \frac{1}{1+e^{b_i(x-a_i)}}$

Funkcija pripadnosti za skup A $\mu_a = \frac{1}{1+e^{b1_i(x-a1_i)}}$

Funkcija pripadnosti za skup B $\mu_b = \frac{1}{1+e^{b2_i(y-a2_i)}}$

Funkcija pogreške $E_k = \frac{1}{2}(y_k - o_k)^2$

Izlaz $o_k = \frac{\sum_{i=1}^{m} w_i z_i}{\sum_{i=1}^{m} w_i}$

Težina $w_i = \mu_a \mu_b = \frac{1}{1+e^{b1_i(x-a1_i)}} \quad \frac{1}{1+e^{b2_i(y-a2_i)}}$

kao t-norma se koristi algebarski produkt

Ažuriranje parametra ψ $\psi(t+1) = \psi(t) - \eta \frac{\partial E_k}{\partial \psi}$

Ažuriranje parametra p_i

$p_i(t+1) = p_i(t) - \eta \frac{\partial E_k}{\partial p_i}$

p_i utječe na z_i, z_i utječe na o_k, o_k utječe na E_k pa je stoga lanac jednak $p_i \Rightarrow z_i \Rightarrow o_k$

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial p_i}$$

$$\frac{\partial E_k}{\partial o_k} = \frac{\partial}{\partial o_k} \frac{1}{2}(y_k - o_k)^2 = -\frac{1}{2} * 2(y_k - o_k) = -(y_k - o_k)$$

$$\frac{\partial o_k}{\partial z_i} = \frac{\partial}{\partial z_i} \frac{\sum_{j=1}^{m} w_j z_j}{\sum_{j=1}^{m} w_j} = \frac{w_i}{\sum_{j=1}^{m} w_j}$$

$$\frac{\partial z_i}{\partial p_i} = \frac{\partial}{\partial p_i}(p_i x + q_i y + r_i) = x_i$$

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial p_i} = -(y_k - o_k) \frac{w_i}{\sum_{j=1}^{m} w_j} x_i$$

$$p_i(t+1) = p_i(t) - \eta \frac{\partial E_k}{\partial p_i} = p_i(t) - \eta(-(y_k - o_k) \frac{w_i}{\sum_{j=1}^{m} w_j} x_i) = \mathbf{p_i(t) + \eta(y_k - o_k)} \frac{w_i}{\sum_{j=1}^{m} w_j} \mathbf{x_i}$$

Ažuriranje parametra q_i

$$q_i(t+1) = q_i(t) - \eta \frac{\partial E_k}{\partial q_i}$$

q_i utječe na z_i, z_i utječe na o_k, o_k utječe na E_k pa je stoga lanac jednak $q_i \Rightarrow z_i \Rightarrow o_k$

$$\frac{\partial E_k}{\partial q_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial q_i}$$

$$\frac{\partial E_k}{\partial o_k} = \frac{\partial}{\partial o_k} \frac{1}{2} (y_k - o_k)^2 = -\frac{1}{2} * 2 (y_k - o_k) = -(y_k - o_k)$$

$$\frac{\partial o_k}{\partial z_i} = \frac{\partial}{\partial z_i} \frac{\sum_{j=1}^{m} w_j z_j}{\sum_{j=1}^{m} w_j} = \frac{w_i}{\sum_{j=1}^{m} w_j}$$

$$\frac{\partial z_i}{\partial q_i} = \frac{\partial}{\partial q_i} (p_i x + q_i y + r_i) = y_i$$

$$\frac{\partial E_k}{\partial q_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial q_i} = -(y_k - o_k) \frac{w_i}{\sum_{j=1}^{m} w_j} y_i$$

$$q_i(t+1) = q_i(t) - \eta \frac{\partial E_k}{\partial q_i} = q_i(t) - \eta(-(y_k - o_k) \frac{w_i}{\sum_{j=1}^{m} w_j} y_i) = \mathbf{q_i(t) + \eta(y_k - o_k)} \frac{w_i}{\sum_{j=1}^{m} w_j} \mathbf{y_i}$$

Ažuriranje parametra r_i

$$r_i(t+1) = r_i(t) - \eta \frac{\partial E_k}{\partial r_i}$$

r_i utječe na z_i, z_i utječe na o_k, o_k utječe na E_k pa je stoga lanac jednak $r_i \Rightarrow z_i \Rightarrow o_k$

$$\frac{\partial E_k}{\partial r_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial r_i}$$

$$\frac{\partial E_k}{\partial o_k} = \frac{\partial}{\partial o_k} \frac{1}{2} (y_k - o_k)^2 = - \frac{1}{2} * 2 (y_k - o_k) = - (y_k - o_k)$$

$$\frac{\partial o_k}{\partial z_i} = \frac{\partial}{\partial z_i} \frac{\sum_{j=1}^{m} w_j z_j}{\sum_{j=1}^{m} w_j} = \frac{w_i}{\sum_{j=1}^{m} w_j}$$

$$\frac{\partial z_i}{\partial r_i} = \frac{\partial}{\partial r_i} (p_i x + q_i y + r_i) = 1$$

$$\frac{\partial E_k}{\partial r_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial r_i} = - (y_k - o_k) \frac{w_i}{\sum_{j=1}^{m} w_j}$$

$$r_i(t + 1) = r_i(t) - \eta \frac{\partial E_k}{\partial r_i} = r_i(t) - \eta(- (y_k - o_k) \frac{w_i}{\sum_{j=1}^{m} w_j}) = \mathbf{r_i(t) + \eta(y_k - o_k)} \frac{w_i}{\sum_{j=1}^{m} w_j}$$

Ažuriranje parametra $a1_i$

$$a1_i(t + 1) = a1_i(t) - \eta \frac{\partial E_k}{\partial a1_i}$$

$a1_i$ utječe na w_i, w_i utječe na o_k, o_k utječe na E_k pa je stoga lanac jednak $a1_i \Rightarrow w_i \Rightarrow o_k$

$$\frac{\partial E_k}{\partial a1_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial a1_i}$$

$$\frac{\partial E_k}{\partial o_k} = \frac{\partial}{\partial o_k} \frac{1}{2} (y_k - o_k)^2 = - \frac{1}{2} * 2 (y_k - o_k) = - (y_k - o_k)$$

$$\frac{\partial o_k}{\partial w_i} = \frac{\partial}{\partial w_i} \frac{\sum_{j=1}^{m} w_j z_j}{\sum_{j=1}^{m} w_j} = \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{(\sum_{j=1}^{m} w_j)^2}$$

$$\frac{\partial w_i}{\partial a1_i} = \frac{\partial}{\partial a1_i} \mu_a \mu_b = \frac{\partial}{\partial a1_i} \left(\frac{1}{1+e^{b1_i(x - a1_i)}} \frac{1}{1+e^{b2_i(y - a2_i)}} \right) = b1_i \mu_a (1 - \mu_a) \mu_b$$

$$\frac{\partial E_k}{\partial a1_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial a1_i} = -(y_k - o_k) \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} b1_i \mu_a (1 - \mu_a) \mu_b$$

$$a1_i(t+1) = a1_i(t) - \eta \frac{\partial E_k}{\partial a1_i} = a1_i(t) - \eta(-(y_k - o_k)) \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} b1_i \mu_a (1 - \mu_a) \mu_b$$

$$= \mathbf{a1_i(t) + \eta(y_k - o_k)} \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} \mathbf{b1_i \mu_a (1 - \mu_a) \mu_b}$$

Ažuriranje parametra $b1_i$

$$b1_i(t+1) = b1_i(t) - \eta \frac{\partial E_k}{\partial b1_i}$$

$b1_i$ utječe na w_i, w_i utječe na o_k, o_k utječe na E_k pa je stoga lanac jednak $b1_i \Rightarrow w_i \Rightarrow o_k$

$$\frac{\partial E_k}{\partial b1_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial b1_i}$$

$$\frac{\partial E_k}{\partial o_k} = \frac{\partial}{\partial o_k} \frac{1}{2} (y_k - o_k)^2 = -\frac{1}{2} * 2 (y_k - o_k) = -(y_k - o_k)$$

$$\frac{\partial o_k}{\partial w_i} = \frac{\partial}{\partial w_i} \frac{\sum_{j=1}^{m} w_j z_j}{\sum_{j=1}^{m} w_j} = \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2}$$

$$\frac{\partial w_i}{\partial b1_i} = \frac{\partial}{\partial b1_i} \mu_a \mu_b = \frac{\partial}{\partial b1_i} \left(\frac{1}{1+e^{b1_i(x-a1_i)}} \frac{1}{1+e^{b2_i(y-a2_i)}} \right) = -(x - a1_i)\mu_a (1 - \mu_a)\mu_b$$

$$\frac{\partial E_k}{\partial b1_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial b1_i} = -(y_k - o_k) \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} -(x - a1_i)\mu_a (1 - \mu_a)\mu_b$$

$$b1_i(t+1) = b1_i(t) - \eta \frac{\partial E_k}{\partial b1_i} = b1_i(t) - \eta(-(y_k - o_k)) \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} -(x - a1_i)\mu_a (1 - \mu_a)\mu_b$$

$$= \mathbf{b1_i(t) - \eta(y_k - o_k)} \quad \frac{\sum_{j=1}^{m} w_j(z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} \quad \mathbf{(x - a1_i)\mu_a(1 - \mu_a)\mu_b}$$

Ažuriranje parametra $a2_i$

$$a2_i(t+1) = a2_i(t) - \eta \quad \frac{\partial E_k}{\partial a2_i}$$

$a2_i$ utječe na w_i, w_i utječe na o_k, o_k utječe na E_k pa je stoga lanac jednak $a2_i \Rightarrow w_i \Rightarrow o_k$

$$\frac{\partial E_k}{\partial a2_i} = \frac{\partial E_k}{\partial o_k} \quad \frac{\partial o_k}{\partial w_i} \quad \frac{\partial w_i}{\partial a2_i}$$

$$\frac{\partial E_k}{\partial o_k} = \frac{\partial}{\partial o_k} \quad \frac{1}{2} \quad (y_k - o_k)^2 = - \quad \frac{1}{2} \quad * 2\,(y_k - o_k) = -\,(y_k - o_k)$$

$$\frac{\partial o_k}{\partial w_i} = \frac{\partial}{\partial w_i} \quad \frac{\sum_{j=1}^{m} w_j z_j}{\sum_{j=1}^{m} w_j} = \frac{\sum_{j=1}^{m} w_j(z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2}$$

$$\frac{\partial w_i}{\partial a2_i} = \frac{\partial}{\partial a2_i} \quad \mu_a\mu_b = \frac{\partial}{\partial a2_i} \quad \left(\frac{1}{1+e^{b1_i(x-a1_i)}} \quad \frac{1}{1+e^{b2_i(y-a2_i)}} \right) = b2_i\,\mu_a(1 - \mu_b)\mu_b$$

$$\frac{\partial E_k}{\partial a2_i} = \frac{\partial E_k}{\partial o_k} \quad \frac{\partial o_k}{\partial w_i} \quad \frac{\partial w_i}{\partial a2_i} = -\,(y_k - o_k) \quad \frac{\sum_{j=1}^{m} w_j(z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} \quad b2_i\,\mu_a(1 - \mu_b)\mu_b$$

$$a2_i(t+1) = a2_i(t) - \eta \quad \frac{\partial E_k}{\partial a1_i} = a2_i(t) - \eta(-\,(y_k - o_k)) \quad \frac{\sum_{j=1}^{m} w_j(z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} \quad b2_i\,\mu_a(1 - \mu_b)\mu_b$$

$$= \mathbf{a2_i(t) + \eta(y_k - o_k)} \quad \frac{\sum_{j=1}^{m} w_j(z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} \quad \mathbf{b2_i\,\mu_a(1 - \mu_b)\mu_b}$$

Ažuriranje parametra $b2_i$

$$b2_i(t+1) = b2_i(t) - \eta \quad \frac{\partial E_k}{\partial b2_i}$$

$b2_i$ utječe na w_i, w_i utječe na o_k, o_k utječe na E_k pa je stoga lanac jednak $b2_i \Rightarrow w_i \Rightarrow o_k$

$$\frac{\partial E_k}{\partial b2_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial b2_i}$$

$$\frac{\partial E_k}{\partial o_k} = \frac{\partial}{\partial o_k} \frac{1}{2} (y_k - o_k)^2 = - \frac{1}{2} * 2 (y_k - o_k) = - (y_k - o_k)$$

$$\frac{\partial o_k}{\partial w_i} = \frac{\partial}{\partial w_i} \frac{\sum_{j=1}^{m} w_j z_j}{\sum_{j=1}^{m} w_j} = \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2}$$

$$\frac{\partial w_i}{\partial b2_i} = \frac{\partial}{\partial b2_i} \mu_a \mu_b = \frac{\partial}{\partial b2_i} \left(\frac{1}{1+e^{b1_i(x-a1_i)}} \frac{1}{1+e^{b2_i(y-a2_i)}} \right) = -(x - a2_i)\mu_a (1 - \mu_b)\mu_b$$

$$\frac{\partial E_k}{\partial b2_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial b2_i} = - (y_k - o_k) \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} -(x - a2_i)\mu_a (1 - \mu_b)\mu_b$$

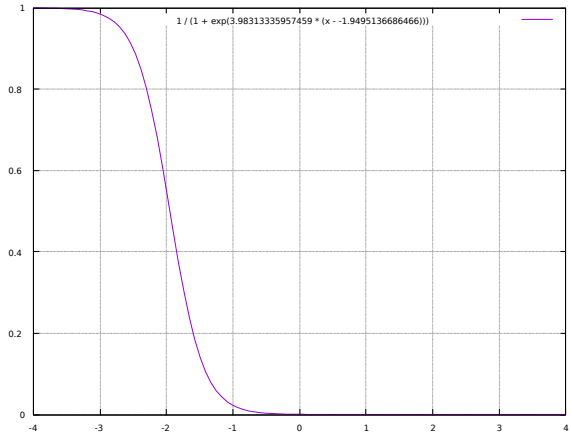
$$b2_i(t + 1) = b2_i(t) - \eta \frac{\partial E_k}{\partial b2_i} = b2_i(t) - \eta(- (y_k - o_k)) \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} -(x - a2_i)\mu_a (1 - \mu_b)\mu_b$$

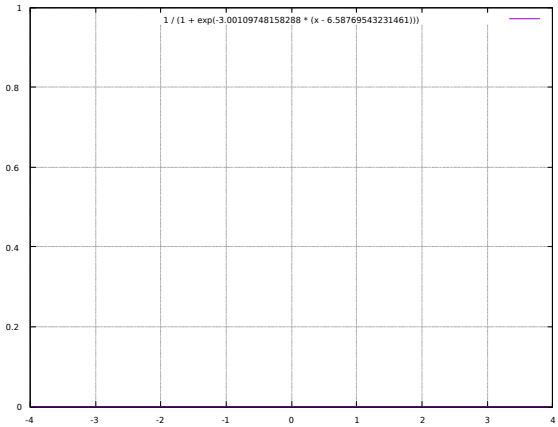
$$= \mathbf{b2_i(t) - \eta(y_k - o_k)} \frac{\sum_{j=1}^{m} w_j (z_i - z_j)}{\left(\sum_{j=1}^{m} w_j\right)^2} \mathbf{(x - a2_i)\mu_a (1 - \mu_b)\mu_b}$$

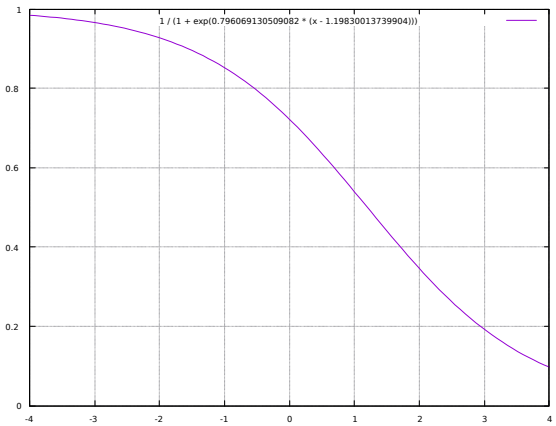
Razlika između stohastičkog i pravog gradijenta je u tome što se kod pravog radi sumiranje po svim uzorcima, a ne pojedinačno te su stoga gornje formule validne i za pravi gradijent uz sumiranje po svim uzorcima.

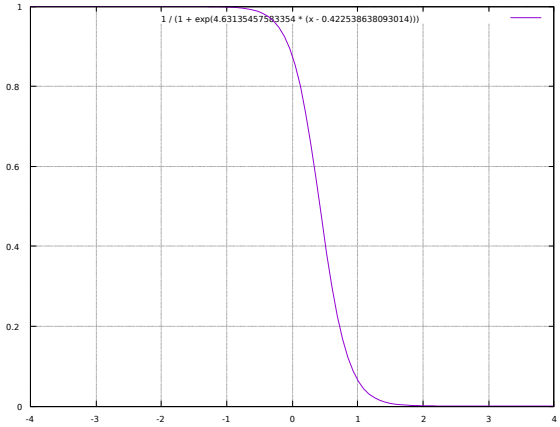
- Stohastički gradijent brže knovergira od običnog budući da je moguće koristiti za red veće vrijednosti, npr. za $\eta = 0.005$ obični gradijent stagnira dok stohastički knovergira jako brzo

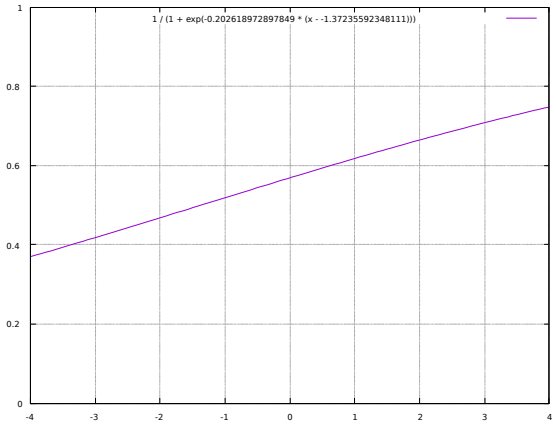
Funkcije pripadnosti za m = 8

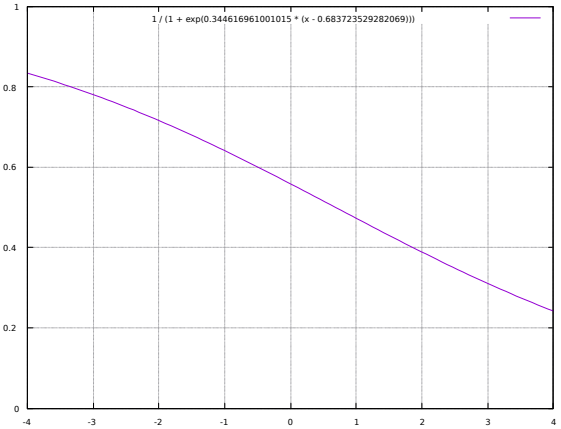


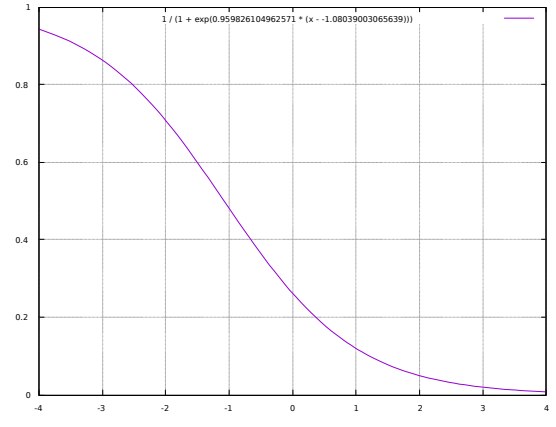


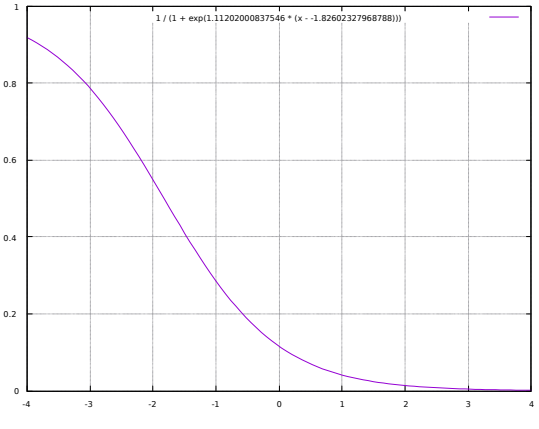


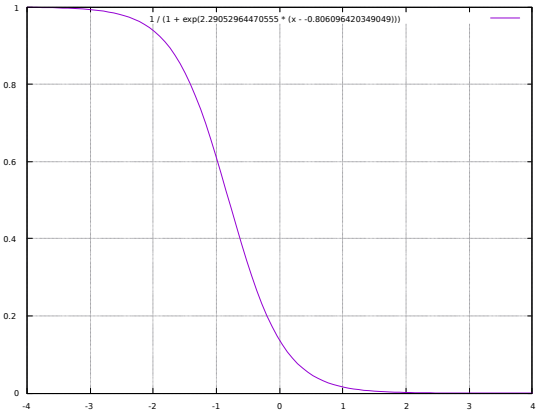


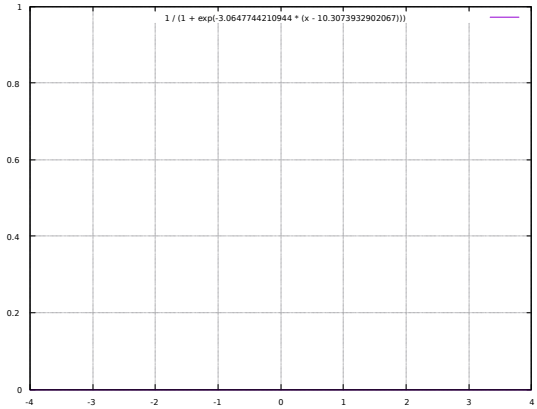


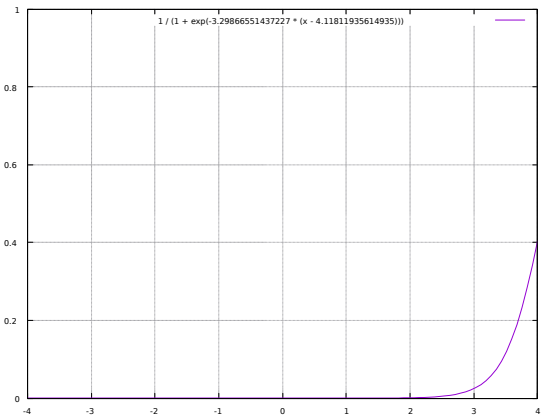


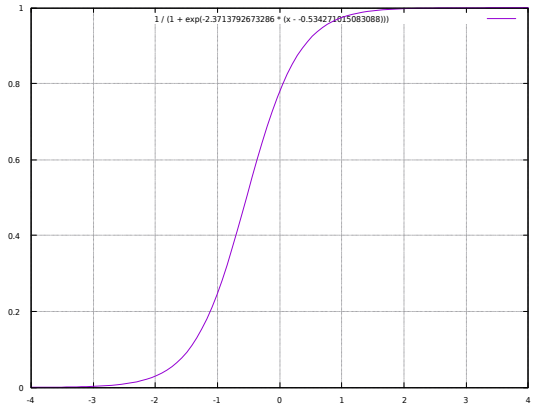


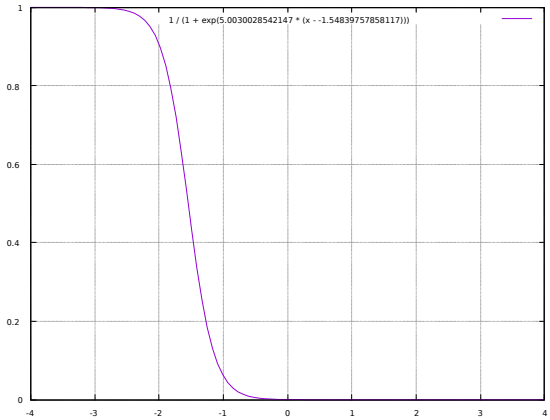


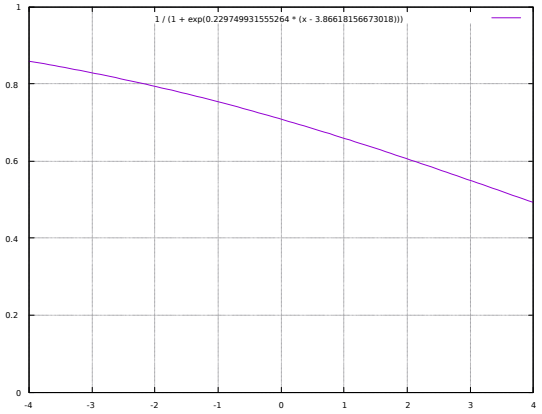

1 / (1 + exp(2.29052964470555 * (x - -0.806096420349049)))

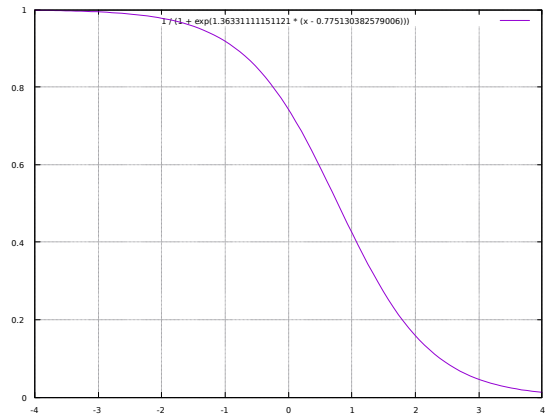

1 / (1 + exp(-3.0647744210944 * (x - 10.3073932902067)))

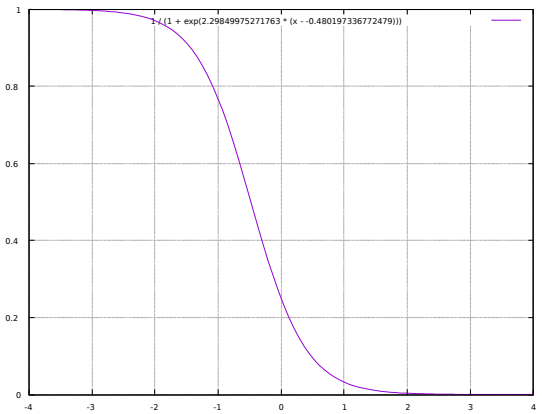

1 / (1 + exp(-3.29866551437227 * (x - 4.11811935614935)))


1 / (1 + exp(-2.3713792673286 * (x - -0.534271015083088)))


1 / (1 + exp(5.0030028542147 * (x - -1.54839757858117)))

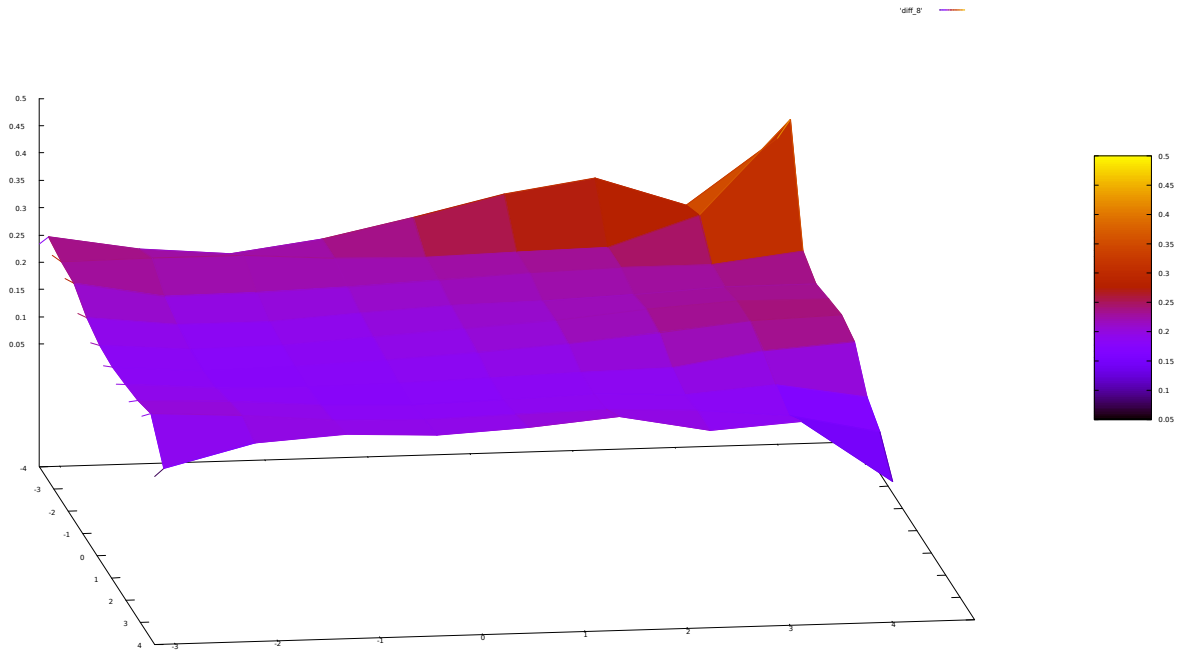

1 / (1 + exp(0.229749931555264 * (x - 3.866181566673018)))


1 / (1 + exp(1.36331111151121 * (x - 0.775130382579006)))

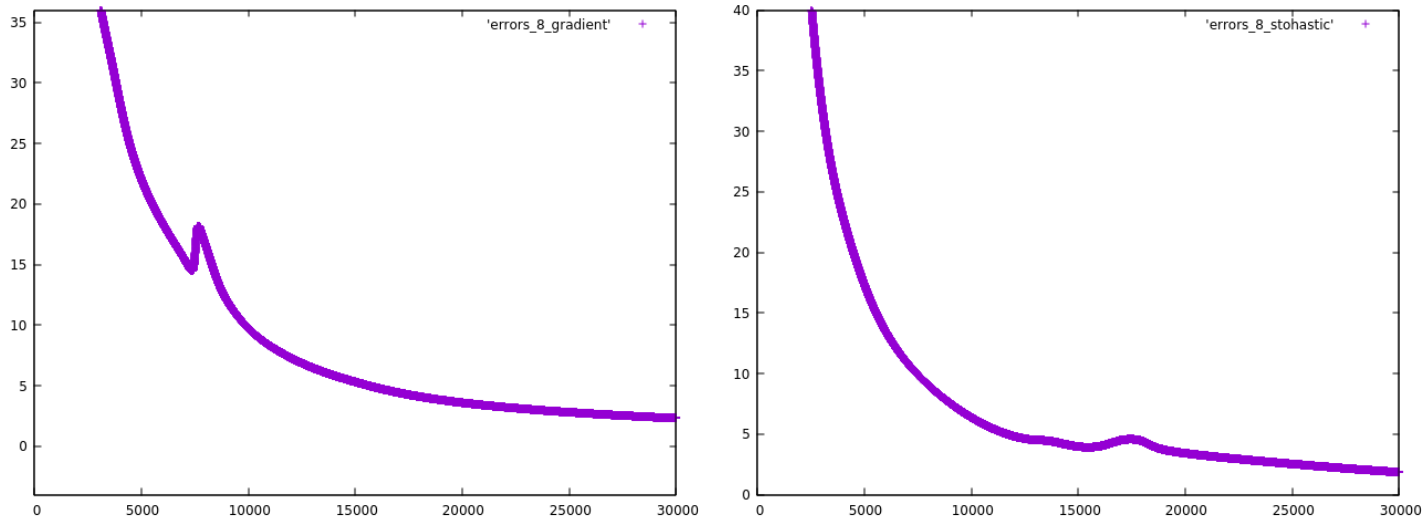

1 / (1 + exp(2.29849975271763 * (x - -0.480197336772479)))

Na gornjim grafovima su prikazane funkcije pripadnosti za neizrazite skupove: lijevi stupac predstavlja neizrazite skupove A_i, a desni neizrazite skupove B_i

Razlika između stvarne vrijednosti i naučene



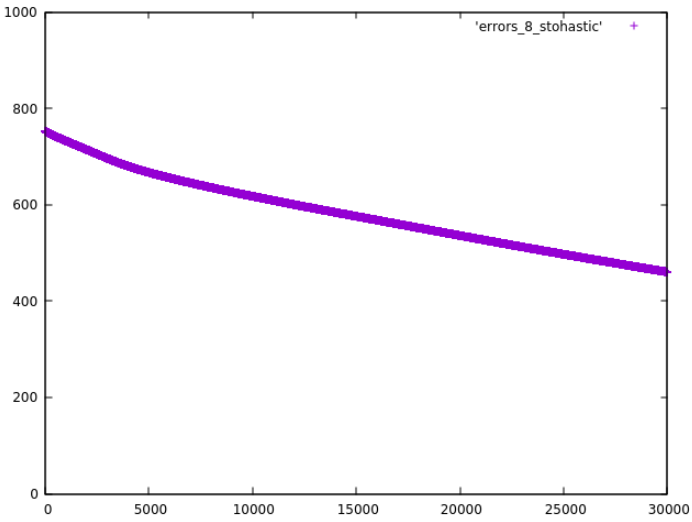
Usporedba pogrešaka po epohama

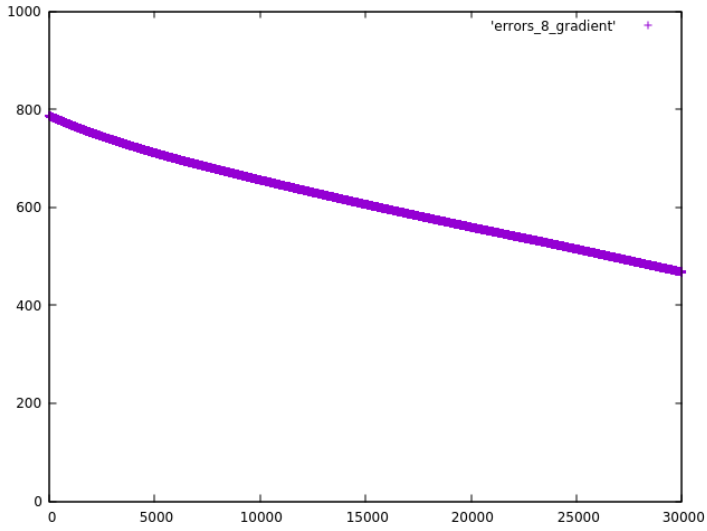


Lijevi graf prikazuje pogreške po epohama za gradijent, a desni za stohastički

Vidljivo je da stohastički gradijent brže konvergira od običnog

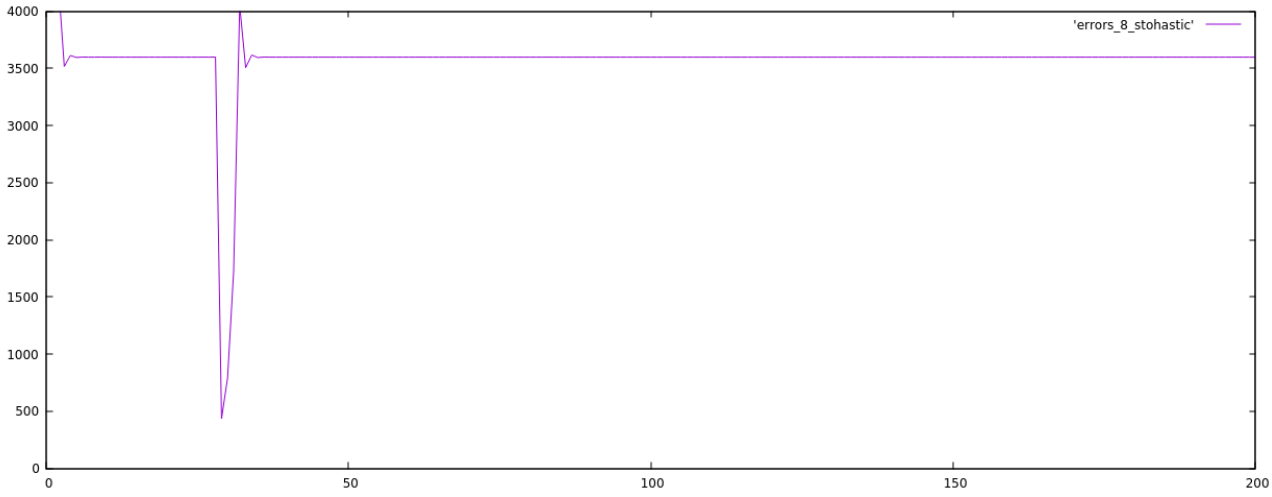
Usporedba različitih stopa učenja

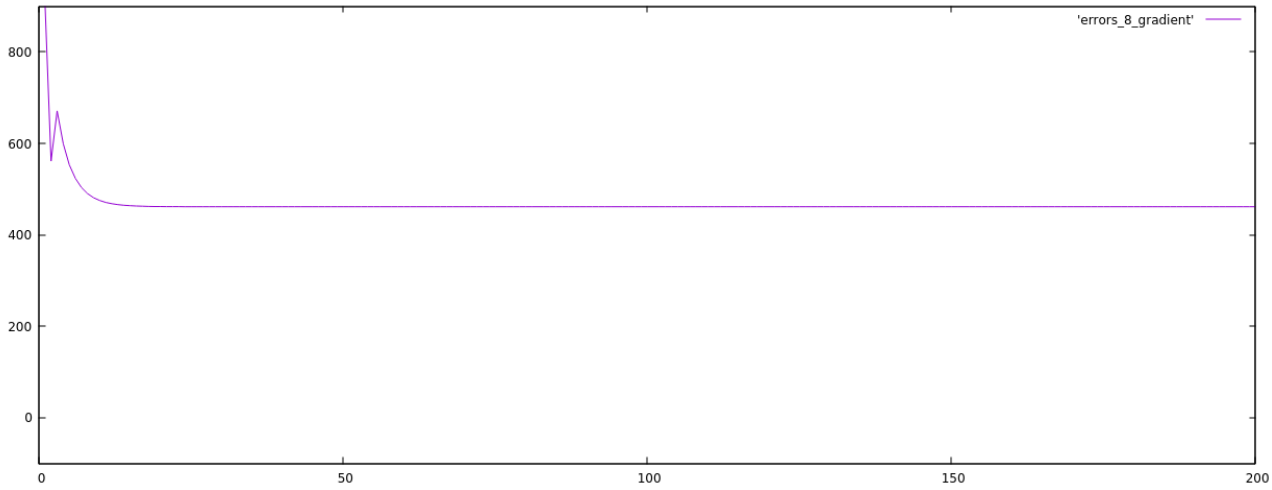




Premala stopa učenja: lijevo stohastički, desno obični

Kako je stopa učenja premala, pomaci u gradijentima su mali pa je konvergencija bitno usporena





Prevelika stopa učenja: gore stohastički, dolje obični

Kako je stopa učenja prevelika, dolazi do stagnacije nakon nekoliko epoha, ukoliko bi se stopa učenja dodatno povećala onda bi došlo do divergencije rezultata i greška bi krenula prema beskonačnosti

Lokalni optimumi

Prilikom testiranja algoritma dogodila su se zapinjanja u lokalnim optimumima što je za očekivati s obzirom da se koristi algoritam gradijnetnog spusta za mijenjanje vrijednosti parametara.