Modelling Call-Time Choice as Effect using Scoped Free Monads

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1 Introduction

2 Preliminaries

2.1 Coq

• Introduce the necessary Coq concepts to understand the paper

2.2 Haskell

• Introduce the necessary Haskell concepts to understand the paper

2.2.1 Monad and MonadPlus

2.3 Curry

• Introduce the necessary Curry concepts to understand the paper

2.3.1 Non-strictness

2.3.2 Sharing

2.3.3 Non-determinism

2.4 Modelling Curry Programs using Monadic Code Transformation

- Why is the naive MonadPlus approach not sufficient to model Curry semantic?
- Motivate usage of monadic data types
- · Introduce explicit sharing

Modelling Curry programs in a language like Haskell requires a transformation of non-deterministic code into a semantically equivalent, deterministic program. First, we have a look at the direct representation of non-determinism used in the KiCS2 implementation as described by Braßel et al. [2011].

Non-determinism in Curry is not limited to flat non-determinism but can occur within components of data structures and anywhere in a computation. This means that expressing non-determinism via Haskell's list monad is not sufficient to model Curry's non-determinism. Instead, existing data types receive additional constructors that represent

TODO: Example

failure and the choice between two values. For example, the extended list data type looks as follows.

```
data List a = Nil | Cons a (List a) | Choice (List a) (List a) | Fail
```

Since this transformation adds new constructors, all functions need to cover these cases, too. The new rules return Fail if the function's argument is a failed computation and distribute function calls to both branches if the argument is a choice.

One issue with this approach is that call-time choice is not implemented yet. If a choice is duplicated during evaluation, this information cannot be recovered later. Therefore, each Choice constructor has an additional ID argument that identifies the same choices. Since each choice needs a fresh ID, functions use an additional IDSupply argument when choices are created.

The evaluation of a non-deterministic value is implemented by transforming the value into a search tree which can be traversed with different search strategies. In the process, each choice ID's decision is stored and then repeated if the same ID is encountered again.

While this approach is useful when the host language supports laziness and sharing, another approach is necessary to model these effects when they are not built into the language.

Fischer et al. [2009] introduce a monadic representation of non-determinism that supports sharing and non-strict evaluation. Out of simplicity, the implementation idea is presented in Haskell, similar to the approach of the original authors, using the example of permutation sort. The algorithm consists of three components. Firstly, a function insert that inserts an element non-deterministically at every possible position within a list.

The second part is the function perm that inserts the head of a given list into the permutations of the list's tail.

Finally, the function sort generates permutations and then tests whether they are sorted.

```
return ys
```

The function isSorted compares each element in a list to the next one to determine whether the list is sorted. When we test this implementation, we can see that the runtime increases significantly when adding even a few elements.

```
λ> sort [9, 8..1] :: [[Int]]
[[1,2,3,4,5,6,7,8,9]]
(0.69 secs)
λ> sort [10, 9..1] :: [[Int]]
[[1,2,3,4,5,6,7,8,9,10]]
(6.67 secs)
λ> sort [11, 10..1] :: [[Int]]
[[1,2,3,4,5,6,7,8,9,10,11]]
(77.54 secs)
```

The reason for the factorial runtime is that the implementations is needlessly strict. A list of length n has n! permutations, all of which are generated when running sort. This matches our observation above, since adding a tenth element increases the runtime by a factor of 10 and an eleventh element multiplies the runtime of the ten-element list by eleven.

If we consider the implementation of isSorted, we can see that, as soon as the comparison of two elements yields False, the function returns False and does not evaluate the remainder of the list.

However, since we use bind to pass permutations from perm to isSorted, each permutation is fully evaluated before it is determined whether the permutation is sorted. This leads to the complete evaluation of every permutation, which results in an inefficient program.

Similarly, when we consider the Curry example head (1 : head [] : []), the strictness of our MonadPlus approach shows again. The corresponding Haskell expression is as follows.

```
hd [] >>= \x -> \x d (1 : x : [])
```

Here hd:: MonadPlus a => [a] -> m a is the lifted head function. Evaluating the expression in Haskell yields mzero, that is, no result, while Curry returns 1. The reason is the definition of the bind operator. For example, the monad instance for lists defines bind as xs >>= f = concatMap f xs. In the expression above, this means that the pattern matching within concatMap evaluates hd [] to mzero and thus returns mzero.

The strictness observed in both examples is the motivation for an alternative approach. The problem with the above implementations is that non-deterministic arguments of constructors need to be evaluated completely before the computation can continue. There-

fore, we would like to be able to use unevaluated, non-deterministic computations as arguments of constructors.

As mentioned before, we can implement this idea by adapting all data types so that they may contain non-deterministic components.

```
data List m a = Nil | Cons (m a) (m (List m a))
```

The list data type now has an additional argument m of type * -> * that represents a non-determinism monad. Instead of fixed constructors like Choice, the monad m determines the structure and evaluation strategy of the non-determinism effect. Two smart constructors cons and nil make handling the new list type more convenient.

```
nil :: Monad m => m (List m a)
nil = return Nil

cons :: Monad m => m a -> m (List m a) -> m (List m a)
cons x y = return (Cons x y)
```

Adapting the permutation sort functions to the lifted data type requires us to replace [] with List m However, this is not sufficient because the list itself can be the result of a non-deterministic computation. Therefore, an additional m is wrapped around every occurrence of List.

Whenever pattern matching occurred in the original definition, we now use bind to extract a List value. Since this only evaluates flat non-determinism and not non-determinism that occurs in the components, non-strictness is upheld as much as possible.

All functions now take arguments of the same type they return. Thus, the definition of sort does not need bind in order to pass permutations to isSorted.

```
sort' :: MonadPlus m => m (List m Int) -> m (List m Int)
sort' xs = let ys = perm' xs in
  isSorted' ys >>= \sorted -> guard sorted >> ys
```

We are now able to take advantage of isSorted's non-strict definition. The implementation generates permutations only if there is a chance that the permutation is sorted, that is, only recursive calls of perm that are demanded by isSorted are executed.

We reconsider the Curry example head (1: head []: []). Since the List data type now takes monad values as arguments, we can write the example using the smart constructors and a lifted head function as follows.

```
λ> hd' (cons (return 1) (cons (hd' nil) nil))
1
```

Because we do not need to use bind to get the result of hd' nil, the expression is not evaluated due to non-strictness and the result is equal to Curry's output.

Data types with non-deterministic components solve the problem of non-strictness because each component can be evaluated individually, instead of forcing the evaluation of the whole term. Unfortunately, this leads to a problem. When unevaluated components are shared via Haskell's built-in sharing, computations, rather than results, are being shared. This means that the results can be different each time the computation is evaluated, which contradicts the intuition of sharing.

The solution to this problem is an explicit sharing combinator share :: m a -> m (m a) that allows sharing the results of a computation in a non-strict way. Here, m is a MonadPlus instance, similar to the monad used in the definition of the data type, that supports sharing. Thus, share takes a computation and then returns a computation that returns the result, that is, the shared value. The reason for this nesting of monad layers is that, in short, the share combinator performs some actions that can be immediately executed by bind (the outer monad layer), while the inner monad layer should only be evaluated when needed. This is explained in more detail later. With the explicit sharing operator we can adapt perm' to share the generated permutations in order to achieve non-strictness in combination with sharing.

The share operator must satisfy certain laws, which we discuss in section 4.4. The implementation of share is subject of the next chapter.

3 Call-Time Choice modelled in Haskell

Based on the ideas presented in the last chapter, we now want to model call-time choice, that is, non-strictness, sharing and non-determinism, in Haskell. We still use MonadPlus to parameterize our programs. However, instead of, for example, using the list instance to make non-determinism visible, we define an effect functor that can express many different effects, including non-determinism and sharing. This approach, as introduced by Wu et al. [2014], will also be the base of the Coq implementation shown in chapter 4.

TODO: Definition effect

For the implementation of call-time choice, we want to be able to express different effects within our programs. However, not every program contains effects. There are also pure programs that have no side-effects besides the computation of a value. A data type that represents such programs could look as follows.

```
data Void a = Return a
```

Here, Void means the absence of effects.

If we consider programs that contain effects, also called impure, like, non-determinism, a data type that represents such values could look like the following.

This data type also has a constructor to model pure values but in addition, there are constructors that represent failed computations and the non-deterministic choice between two values. We could go on and list data types that model many more effects but the question is: Is it possible to create a data type that, if appropriately instantiated, behaves like the original effect functor? This would allow us to represent programs with many different effects using one compact data type.

Answering this question requires abstracting the concrete form of effect functors into a general program data type. As we saw in the examples above, we need a way to represent pure values in a program. Therefore, the first constructor of our new program data type should be Return a for the type a, that is, the result type of the program. To model effects like non-determinism, the program is parameterized over effect functors of type * -> * that represent, for example, Fail and Choice. We call this argument sig because the signature of a program tells us which effects can occur. So far, programs are defined as data Prog sig a = Return a. In order do make use of the sig component, we need to add a constructor for impure operations. The ND data type shows us that effect functors can be defined recursively. Thus, the constructor for impure programs should be recursive, too, to be able to represent this structure.

```
data Prog sig a = Return a | Op (sig (Prog sig a))
```

With this definition of Prog, we are able to represent the original functors by instantiating sig appropriately. For Void, we already have the Return constructor. Therefore, the data type we can use with Prog does not need a constructor anymore, that is, data Void' a.

The type Prog Void' now resembles the original type Void since the Op constructor would require a value of type Void', which we cannot construct. ¹ Only Return can be used to define values, similar to the original data type.

Similar to Void', we can define a data type Choice that represents Choice in combination with Prog.

```
data ND p = Fail | Choice p p
```

Again, we can omit the Return constructor because it is already part of the Prog data type. For the same reason, the type variable a has been replaced with the variable p, since ND does not have values as arguments but rather programs that return values.

Since Op applies sig recursively, this yields the following type, which is equivalent to the original data type.

We have found a way to model effect functors as instances of the data type Prog, which essentially models a tree with leafs, represented by the Return constructor, and branches that have the form defined by sig.

TODO: Tree structure visualization?

3.1 Free Monads

- · What are free monads?
- Why do we use free monads?

The data type Prog is better known as the free monad. We saw in the previous chapter that Free can be used to model other data types. In addition, Free is a monad that can turn any functor into a monad.

¹It is possible to use undefined to create an impure value of type Prog Void' a. Since this is not possible in Coq, we do not consider this in the Haskell implementation.

We consider, for example, the type Free One where data One a = One. Here a is a phantom type that we need because Free expects a functor. The monad instance for Free is as follows.

```
data Free f a = Pure a | Impure (f (Free f a))
instance (Functor f, Applicative (Free f)) => Monad (Free f) where
  return = Pure
  Pure x >>= g = g x
  Impure fx >>= g = Impure (fmap (>>= g) fx)
```

Since One has only a single, non-recursive constructor One, the only possible impure value is Impure One, whereas the usual Return constructor remains. If bind encounters the value One, the function g is distributed deeper into the term structure using fmap. Since fmap One = One, it becomes apparent that the monad constructed by Free One is the Maybe monad.

Since we want to model different effects in our program, the free monad makes writing programs easier by allowing monadic definitions without defining a separate monad instance for each effect.

TODO: Visualization of fmap tree

TODO: Find more reasons for using free monads

3.2 Modelling Effects

- Explanation of the Prog/sig infrastructure
- ND and state effect implementation

In the previous sections the free monad and its ability to represent effect functors was discussed. The goal of this section is to explore the infrastructure that allows us to combine multiple effects, write effectful programs and compute the result of such programs.

3.2.1 Combining Effects

Firstly, we would like to combine multiple effects. For this purpose, we use the technique introduced by Swierstra [2008] to define a data type that combines the effect functors sig1 and sig2. The infix notation simplifies combining multiple effects via nested applications of :+:.

```
data (sig1 :+: sig2) a = Inl (sig1 a) | Inr (sig2 a)
```

For example, the type ND:+: One is a functor that we can use with Prog to define programs that contain non-determinism and partiality as follows.

```
progNDOne :: Prog (ND :+: One) Int
progNDOne = Op (Inl (Choice (Op (Inr One)) (Return 42)))
```

In the example progNDOne we define a program that represents the non-deterministic choice between a program whose value is absent and a program that returns 42. The

complexity of nesting constructors of Prog and :+: correctly increases quickly for bigger terms. Therefore, we define a type class that allows us to define such expressions more conveniently. The class is parameterized over two functors, one of which is a subtype – regarding :+: – of the other.

```
class (Functor sub, Functor sup) => sub :<: sup where
inj :: sub a -> sup a
```

We need a few instances of the class :<: to make it useful. The simplest case is sig :<: sig where want to inject a value of type sig a into the same type. Since we do not need to modify the value in any way, id is used to define in j.

```
instance Functor sig => sig :<: sig where
inj = id</pre>
```

The next instance covers the case sig1 :<: (sig1 :+: sig2). Since we already know that sig1 is part of the sum type, we only need to apply the correct constructor of :+:, that is, Inl because sig1 is the left argument.

```
instance (Functor sig1, Functor sig2) => sig1 :<: (sig1 :+: sig2) where
inj = Inl</pre>
```

The last instance assumes that we can inject sig into sig2 and describes how we can inject sig into sig1 :+: sig2. In this case, we can use inj to receive a value of type sig2 a. All that remains is a situation similar to the previous instance, where we only need to use the matching constructor to complete the injection.

```
instance (Functor sig1, sig :<: sig2) => sig :<: (sig1 :+: sig2) where
inj = Inr . inj</pre>
```

These instances allow us to write a polymorphic definition of the function inject which injects constructors depending on the given type of the program.

```
inject :: sig1 :<: sig2 => sig1 (Prog sig2 a) -> Prog sig2 a
inject = Op . inj
```

inject can then be used as demonstrated in the following example.

```
λ> inject One :: Prog (One :+: ND) a
Op (Inl One)
λ> inject One :: Prog (ND :+: One) a
Op (Inr One)
```

The implementation of the function inject assumes that we can inject sig1 into sig2. This is because sig2 is the signature of the returned program and sig1 is the type of the effect constructor that we want to inject. This restriction is justified because, for example, non-deterministic syntax should only appear in a program where ND is part of the signature. With this part of the infrastructure in place, we can redefine the example progNDOne without using Inl and Inr explicitly.

```
progNDOne' :: Prog (ND :+: One) Int
progNDOne' = inject (Choice (inject One) (Return 42))
```

Deriving the appropriate instance of :<: when using inject is, however, not always unambiguous. The last two instances overlap in situations where sig = sig1. For example, the example inject One :: Prog (One :+: One) a yields different values with respect to the chosen constructor of :<:, depending on the instance.

```
λ> nothing
Op (Inl One) -- second instance
λ> nothing
Op (Inr One) -- third instance
```

This is because the type constraint of inject, in this case One :<: (One :+: One), matches both the second and third instance. Haskell does not accept overlapping instances by default, which is why we prioritize one instance via pragmas. In practice, the different term structure due to Inl and Inr does not influence the evaluation as long as we do not explicitly match for the constructors. This is ensured by an additional function prj of the type class :<:, which is discussed in the next section.

3.2.2 Simplified Pattern Matching

3.3 Sharing

- How can we implement simple sharing as an effect?
- What about deep/nested sharing?
- Examples (exRecList, ...)

4 Call-Time Choice modelled in Coq

The goal of this chapter is to transfer the Haskell implementation of call-time choice to Coq. We begin with the data structure Prog, that is, the free monad, which allowed us to model programs with effects of type sig and results of type a.

```
data Prog sig a = Return a | Op (sig (Prog sig a))
```

The definition in Coq looks very similar to the Haskell version, aside from renaming and the explicit constructor types.

```
Inductive Free F A :=
| pure : A -> Free F A
| impure : F (Free F A) -> Free F A.
```

However, the definition is rejected by Coq upon loading the file with the following error message.

```
Non-strictly positive occurrence of "Free" in "F (Free F A) -> Free F A".
```

The reason for this error is explained in the next section.

4.1 Non-strictly Positive Occurrence

- What does non-strictly positive occurrence mean?
- · Motivation for usage of containers

In section 2.1, we learned that Coq distinguishes between non-recursive definitions and functions that use recursion. The reason for this is that Coq checks functions for termination, which is an important part of Coq's proof logic. To understand why functions must always terminate in Coq, we consider the following function.

```
Fail Fixpoint loop (x : unit) : A := loop x.
```

The function receives an argument x and calls itself with the same argument. Since this function obviously never terminates, the result type A is arbitrary. In particular, we could instantiate A with False, the false proposition. The value loop tt: False could be used to prove anything, according to the principle of explosion. For this reason, Coq requires all recursive functions to terminate provably.

Returning to the original data type, what is link between Free and termination? It is well known that recursion can be implemented in languages without explicit recursion

syntax by means of constructs like the Y combinator or the data type Mu for type-level recursion.

```
Fail Inductive Mu A := mu : (Mu A -> A) -> Mu A.
```

Mu is not accepted by Coq for the same reason as Free: non-strictly positive occurrence of the respective data type. The problematic property of non-strictly positive data types is that the type occurs on the left-hand side of a constructor argument's function type. This would allow general recursion and thus, as described above, make Coq's logic inconsistent.

In case of Free, the non-strictly positive occurrence is not as apparent as before because the constructors do not have functional arguments. However, F is being applied to Free F A. If F has a functional argument with appropriate types, the resulting type becomes non-strictly positive, as shown below.

In the type of impureC contains a non-strictly positive occurrence of ContF R A. Consequently, Coq rejects Free because it is not guaranteed that no instance violates the strict positivity requirement. Representing the Free data type therefore requires a way to restrict the definition to strictly positive data types. One approach to achieve this goal is described in the next section.

4.2 Containers

- How do containers work?
- How do we translate effect functors into containers?

Containers are an abstraction of data types that store values, with the property that only strictly positive data types can be modelled as a container. This will allow us to define a version of Free that works with containers of type constructors instead of the type constructors itself. First, however, we will have a more detailed look at containers.

The first component of a container is the type Shape. A shape determines how the data type is structured, regardless of the stored values. For example, the shape of a list is the same as the shape of Peano numbers: a number that represents the length of the list, or rather the number of Cons/Succ applications. A pair, on the other hand, has only a single shape.

The second component of a container is a function Pos: Shape -> Type that gives each shape a type that represents the positions within the shape. In the example of pairs, the shape has two positions, the first and second component. Each element of a list is a

position within the shape. Therefore, the position type for lists with length n is natural numbers smaller than n. Peano numbers do not have elements and therefore, the position type for each shape is empty.

Containers can be extended by a function that maps all valid positions to values. Since the position type depends on a concrete shape, the definition in Coq is quantified universally over values of type Shape.

```
Inductive Ext Shape (Pos : Shape -> Type) A :=
  ext : forall s, (Pos s -> A) -> Ext Shape Pos A.
```

The extension of a container models the concrete data type.

4.3 Modelling Effects

- In which ways is the Coq implementation simplified, compared to Haskell?
- How does the adapted Prog/sig infrastructure work?
- How do we translate recursive functions?

4.4 Sharing

· Laws of sharing

5 Curry Programs modelled in Coq

• Can we use the Coq model of call-time choice to prove properties about actual Curry programs?

Conclusion

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