Image Processing HW02

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Asagida islenecek goruntunun matrixi bulunmaktadr.

$$f = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 6 \\ 3 & 4 & 2 & 7 \\ 1 & 5 & 11 & 3 \end{bmatrix}$$

Mean Filter ise asagidaki gibidir.

$$m = 1/9 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Yatay ve dikeyde ise sirasiyla su sekildedir.

$$mX = 1/3 \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} mY = 1/3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Formul halinde yazarsak sirasiyla yatay ve dikey mean filterin formulleri asagisaki gibidir.

$$f(x, y) = 1/3(f(x-1, y) + f(x, y) + f(x+1, y)) \tag{1}$$

$$f(x, y) = 1/3(f(x, y-1) + f(x, y) + f(x, y+1)) \tag{2}$$

1.1 Filtrenin yatay duzlemde uygulanmasi

Goruntuye matrixine yatayda mirror padding uygularsak asagidaki hale gelecektir.

$$f = \begin{bmatrix} 1 & 1 & 2 & 3 & 4 & 4 \\ 5 & 5 & 6 & 7 & 6 & 6 \\ 3 & 3 & 4 & 2 & 7 & 7 \\ 1 & 1 & 5 & 11 & 3 & 3 \end{bmatrix}$$

Bu goruntuye yatay mean filteri uygulayalim.

1.1.1 Satir 1

$$\begin{aligned} f(1,0) = 1/3(f(0,0) + f(1,0) + f(2,0)) \\ 1/3(1+1+2) = 4/3 \end{aligned} \tag{3}$$

$$\begin{aligned} f(2,0) = 1/3(f(1,0) + f(2,0) + f(3,0)) \\ 1/3(1+2+3) = 6/3 = 2 \end{aligned} \tag{4}$$

$$\begin{aligned} f(3,0) = 1/3(f(2,0) + f(3,0) + f(4,0)) \\ 1/3(2+3+4) = 9/3 = 3 \end{aligned} \tag{5}$$

$$\begin{aligned} f(4,0) = 1/3(f(3,0) + f(4,0) + f(5,0)) \\ 1/3(3+4+4) = 11/3 \end{aligned} \tag{6}$$

1.1.2 Satir 2

$$\begin{aligned} f(1,1) = 1/3(f(0,1) + f(1,1) + f(2,1)) \\ 1/3(5+5+6) = 16/3 \end{aligned} \tag{7}$$

$$\begin{aligned} f(2,1) = 1/3(f(1,1) + f(2,1) + f(3,1)) \\ 1/3(5+6+7) = 6 \end{aligned} \tag{8}$$

$$\begin{aligned} f(3,1) = 1/3(f(2,1) + f(3,1) + f(4,1)) \\ 1/3(6+7+6) = 19/3 \end{aligned} \tag{9}$$

$$\begin{aligned} f(4,1) = 1/3(f(3,1) + f(4,1) + f(5,1)) \\ 1/3(7+6+6) = 19/3 \end{aligned} \tag{10}$$

1.1.3 Satir 3

$$\begin{aligned} f(1,2) &= 1/3(f(0,2)+f(1,2)+f(2,2)) \\ &1/3(3+3+4) = 10/3 \end{aligned} \tag{11}$$

$$\begin{aligned} f(2,2) &= 1/3(f(1,2)+f(2,2)+f(3,2)) \\ &1/3(3+4+2) = 3 \end{aligned} \tag{12}$$

$$\begin{aligned} f(3,2) &= 1/3(f(2,2)+f(3,2)+f(4,2)) \\ &1/3(4+2+7) = 13/3 \end{aligned} \tag{13}$$

$$\begin{aligned} f(4,2) &= 1/3(f(3,2)+f(4,2)+f(5,2)) \\ &1/3(2+7+7) = 16/3 \end{aligned} \tag{14}$$

1.1.4 Satir 4

$$\begin{aligned} f(1,3) &= 1/3(f(0,3)+f(1,3)+f(2,3)) \\ &1/3(1+1+5) = 7/3 \end{aligned} \tag{15}$$

$$\begin{aligned} f(2,3) &= 1/3(f(1,3)+f(2,3)+f(3,3)) \\ &1/3(1+5+11) = 17/3 \end{aligned} \tag{16}$$

$$\begin{aligned} f(3,3) &= 1/3(f(2,3)+f(3,3)+f(4,3)) \\ &1/3(5+11+3) = 19/3 \end{aligned} \tag{17}$$

$$\begin{aligned} f(4,3) &= 1/3(f(3,3)+f(4,3)+f(5,3)) \\ &1/3(11+3+3) = 17/3 \end{aligned} \tag{18}$$

Mirror padding alinmis halini yazarsak

$$f = \begin{bmatrix} 1 & 4/3 & 2 & 3 & 11/3 & 4 \\ 5 & 16/3 & 6 & 19/3 & 19/3 & 6 \\ 3 & 10/3 & 3 & 13/3 & 16/3 & 7 \\ 1 & 7/3 & 17/3 & 19/3 & 17/3 & 3 \end{bmatrix}$$

Sol ve sag taraftaki padding kismini kaldirirsak yatayda mean filter uyulanmis haline ulasiriz

$$f = \begin{bmatrix} 4/3 & 2 & 3 & 11/3 \\ 16/3 & 6 & 19/3 & 19/3 \\ 10/3 & 3 & 13/3 & 16/3 \\ 7/3 & 17/3 & 19/3 & 17/3 \end{bmatrix}$$

1.2 Filtrenin dikey duzlemde uygulanmasi

Goruntuye matrixine dikeyde mirror padding uygularsak asagidaki hale gelecektir.

$$f = \begin{bmatrix} 4/3 & 2 & 3 & 11/3 \\ 4/3 & 2 & 3 & 11/3 \\ 16/3 & 6 & 19/3 & 19/3 \\ 10/3 & 3 & 13/3 & 16/3 \\ 7/3 & 17/3 & 19/3 & 17/3 \\ 7/3 & 17/3 & 19/3 & 17/3 \end{bmatrix}$$

Bu goruntuye dikeyde mean filteri uygulayalim.

1.2.1 Sutun 1

$$\begin{aligned} f(0,1) = 1/3(f(0,0) + f(0,1) + f(0,2)) \\ 1/3(4/3 + 4/3 + 16/3) = 8/3 \end{aligned} \tag{19}$$

$$\begin{aligned} f(0,2) = 1/3(f(0,1) + f(0,2) + f(0,3)) \\ 1/3(4/3 + 16/3 + 10/3) = 10/3 \end{aligned} \tag{20}$$

$$\begin{aligned} f(0,3) = 1/3(f(0,2) + f(0,3) + f(0,4)) \\ 1/3(16/3 + 10/3 + 7/3) = 11/3 \end{aligned} \tag{21}$$

$$\begin{aligned} f(0,4) = 1/3(f(0,3) + f(0,4) + f(0,5)) \\ 1/3(10/3 + 7/3 + 7/3) = 8/3 \end{aligned} \tag{22}$$

1.2.2 Sutun 2

$$\begin{aligned} f(1,1) &= 1/3(f(1,0)+f(1,1)+f(1,2)) \\ &1/3(2+2+6) = 10/3 \end{aligned} \tag{23}$$

$$\begin{aligned} f(1,2) &= 1/3(f(1,1)+f(1,2)+f(1,3)) \\ &1/3(2+6+3) = 11/3 \end{aligned} \tag{24}$$

$$\begin{aligned} f(1,3) &= 1/3(f(1,2)+f(1,3)+f(1,4)) \\ &1/3(6+3+17/3) = 5 \end{aligned} \tag{25}$$

$$\begin{aligned} f(1,4) &= 1/3(f(1,3)+f(1,4)+f(1,5)) \\ &1/3(3+17/3+17/3) = 43/9 \end{aligned} \tag{26}$$

1.2.3 Sutun 3

$$\begin{aligned} f(2,1) &= 1/3(f(2,0)+f(2,1)+f(2,2)) \\ &1/3(3+3+19/3) = 37/9 \end{aligned} \tag{27}$$

$$\begin{aligned} f(2,2) &= 1/3(f(2,1)+f(2,2)+f(2,3)) \\ &1/3(3+19/3+13/3) = 41/9 \end{aligned} \tag{28}$$

$$\begin{aligned} f(2,3) &= 1/3(f(2,2)+f(2,3)+f(2,4)) \\ &1/3(19/3+13/3+19/3) = 51/9 \end{aligned} \tag{29}$$

$$\begin{aligned} f(2,4) &= 1/3(f(2,3)+f(2,4)+f(2,5)) \\ &1/3(19/3+13/3+19/3) = 51/9 \end{aligned} \tag{30}$$

1.2.4 Sutun 4

$$
\begin{aligned}
f(3,1) &= 1/3(f(3,0) + f(3,1) + f(3,2)) \\
&1/3(11/3 + 11/3 + 19/3) = 41/9
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
f(3,2) &= 1/3(f(3,1) + f(3,2) + f(3,3)) \\
&1/3(11/3 + 19/316/3) = 46/9
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
f(3,3) &= 1/3(f(3,2) + f(3,3) + f(3,4)) \\
&1/3(19/3 + 16/3 + 17/3) = 52/9
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
f(3,4) &= 1/3(f(3,3) + f(3,4) + f(3,5)) \\
&1/3(16/3 + 17/3 + 17/3) = 50/9
\end{aligned}
\tag{34}
$$

Yatayda mean filter uyulanmis hali :

$$
f = \begin{bmatrix} 8/3 & 10/3 & 37/9 & 41/9 \\ 10/3 & 11/3 & 41/9 & 46/9 \\ 11/3 & 5 & 51/9 & 52/9 \\ 8/3 & 43/9 & 51/9 & 50/9 \end{bmatrix}
$$

1.3 Efficent way Suggestion

$$f' = (f * mX) * mY$$

seklinde uygulamak yerine

$$f' = f * (mX * mY)$$

once filtrelire kendi aralarinda katlama islemine tabi tuttuktan sonra goruntuye uygularsak daha hizli bir sonuc elde ederiz.

2

$$\partial f/\partial x' = \partial f/\partial x * \partial x/\partial x' + \partial f/\partial y * \partial y/\partial x'$$

Chain rule dan gelen yukaridaki formulun ikinci turevini alirsak

$$
\begin{aligned}
\partial^2 f/\partial x'^2 = \partial/\partial x' \left(\partial f/\partial x'\right) = \\
\partial^2 f/\partial^2 x * \left(\partial x/\partial x'\right)^2 + \partial f/\partial x * \partial^2 x/\partial x'^2 \\
+2 * \partial^2 f/\partial x \partial y * \partial x/\partial x' * \partial y/\partial x' \\
+\partial^2 f/\partial^2 y * \left(\partial y/\partial x'\right)^2 + \partial f/\partial y * \partial^2 y/\partial x'^2
\end{aligned}
\tag{35}
$$

cikar.

$$\partial f/\partial y' = \partial f/\partial x * \partial x/\partial y' + \partial f/\partial y * \partial y/\partial y'$$

gene ikinci turevini alirsak

$$
\begin{aligned}
\partial^2 f/\partial y'^2 = \partial/\partial y' \left(\partial f/\partial y'\right) = \\
\partial^2 f/\partial^2 x * \left(\partial x/\partial y'\right)^2 + \partial f/\partial x * \partial^2 x/\partial y'^2 \\
+2 * \partial^2 f/\partial x \partial y * \partial x/\partial y' * \partial y/\partial y' \\
+\partial^2 f/\partial^2 y * \left(\partial y/\partial y'\right)^2 + \partial f/\partial y * \partial^2 y/\partial y'^2
\end{aligned}
\tag{36}
$$

cikar. Bu iki ikinci turevde asagidaki formulleri ve ikinci turevleri icin 0 i yerine koyarsak

$$
\begin{aligned}
\partial y/\partial x' &= sin\theta \\
\partial x/\partial x' &= cos\theta \\
\partial y/\partial y' &= cos\theta \\
\partial x/\partial y' &= -sin\theta
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
\partial^2 f/\partial x'^2 = \partial/\partial x' \left(\partial f/\partial x'\right) = \\
\partial^2 f/\partial^2 x * \left(cos\theta\right)^2 + 2\partial f/\partial x \partial y * cos\theta sin\theta \\
+\partial^2 f/\partial^2 y * \left(sin\theta\right)^2
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
\partial^2 f/\partial y'^2 = \partial/\partial y' \left(\partial f/\partial y'\right) = \\
\partial^2 f/\partial^2 x * \left(-sin\theta\right)^2 - 2\partial f/\partial x \partial y * sin\theta cos\theta \\
+\partial^2 f/\partial^2 y * \left(cos\theta\right)^2
\end{aligned}
\tag{39}
$$

Iki denklemi toplarsak ..

$$
\begin{aligned}
\partial^2 f/\partial x'^2 + \partial^2 f/\partial y'^2 = \\
\partial^2 f/\partial^2 x \left((cos\theta)^2 + (-sin\theta)^2\right) \\
+2\partial f/\partial x \partial y * cos\theta sin\theta - 2\partial f/\partial x \partial y * sin\theta cos\theta \\
\partial^2 f/\partial^2 y \left((sin\theta)^2 + (cos\theta)^2\right)
\end{aligned}
\tag{40}
$$

Sadelestirirsek

$$+2\partial f/\partial x \partial y * cos\theta sin\theta - 2\partial f/\partial x \partial y * sin\theta cos\theta$$

birbirini gotureceginden ve

$$cos\theta^2 + sin\theta^2 = 1$$

oldugundan

$$\partial^2 f/\partial x'^2 + \partial^2 f/\partial y'^2 = \partial^2 f/\partial^2 x + \partial^2 f/\partial^2 y \tag{41}$$

gelecektir. Bylece Laplacian operatorunun rotasyondan etkilenmedigini ispatlamis olduk.

3

3.1 b

H maskesinin formulunu matrix olarak yazarsak convolution maska ulasmis oluyoruz. O da asagidaki gibidir.

$$h = 1/89 \begin{bmatrix} 0 & -17 & 0 \\ 2 & 3 & 2 \\ 0 & 99 & 0 \end{bmatrix}$$

3.2 a

h maske ve f1 , f2 goruntu olmak uzere

$$h(f1 + f2) = h(f1) + h(f2)$$

oldugunu gosterebilirsek, h maskesinin lineer oldugunu ispatlamis oluyoruz. f1 ve f2 asagidaki gibi olsun, h maskesini ise b sikkindan biliyoruz.

$$f1 = \begin{bmatrix} 100 & 100 & 100 \\ 100 & 100 & 100 \\ 100 & 100 & 100 \end{bmatrix}$$

$$f2 = \begin{bmatrix} 150 & 150 & 150 \\ 150 & 150 & 150 \\ 150 & 150 & 150 \end{bmatrix}$$

h maskesini uygular ve f1' + f2' islemni uygularsak asagidaki sonuc cikyor.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 \\ 1.5 & 1.5 & 1.5 \end{bmatrix} = \begin{bmatrix} 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 \end{bmatrix}$$

Ayni sonucun ciktigini gormek icin H(f1 + f2) islemini de uygulayalim.

$$\begin{bmatrix} 100 & 100 & 100 \\ 100 & 100 & 100 \\ 100 & 100 & 100 \end{bmatrix} + \begin{bmatrix} 150 & 150 & 150 \\ 150 & 150 & 150 \\ 150 & 150 & 150 \end{bmatrix} = \begin{bmatrix} 250 & 250 & 250 \\ 250 & 250 & 250 \\ 250 & 250 & 250 \end{bmatrix}$$

Toplama isleminden cikan sonuca h maskesini uygularsak gorcegiz ki ayni sonuc cikacak.

$$f * H = \begin{bmatrix} 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 \\ 2.5 & 2.5 & 2.5 \end{bmatrix}$$

Goruldugu gibi ayni sonucu aldik bu yuzden H fonksiyonu lineedir.