Homework 2 (v1)

1) Given the following image $f:\mathbb{Z}^2 \rightarrow [0,255] \cap \mathbb{Z}$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 6 \\ 3 & 4 & 2 & 7 \\ 1 & 5 & 11 & 3 \end{bmatrix}$$

apply a mean filter to it using the 1D masks of length 3. Use mirror padding at the edges. Provide the output images after each 1D convolution (first vertically then horizontally). Can you suggest a computationally more efficient way of computing the mean filter's output? If yes, explain it in detail and provide its complexity.

2) The Laplacian operator is defined as: $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

where f represents the input image. Your task is to prove that the Laplacian operator is not influenced by rotations.

In other words, say we rotate x,y using the rotation matrix:

$$x = x' \cos(\theta) - y' \sin(\theta)$$
$$y = x' \sin(\theta) + y' \cos(\theta)$$

where x' and y' represent the rotated versions of respectively x and y.

You are requested to prove that $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}$

Hint: $\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x'}$

3) You feel lucky today, and you decide to develop a new filter h. Given a pixel $p = (x, y) \in \mathbb{Z}^2$ in an image f, your filter is defined as:

$$h(x,y) = 3f(x,y) + 2f(x-1,y) + 2f(x+1,y) - 17f(x,y-1) + 99f(x,y+1)$$

a) Is h linear? Prove your answer.
b) Provide the convolution mask corresponding to h.

4) Implement (in Java/C/C++/Matlab or Python) the morphological convex hull operator as shown in class and apply it on the attached binary image.

Good luck.

Q1: 30 points
Q2: 30 points
Q3: 40 points