CS 601 Spring 2023: Problem Set 3.

Problem 1. (10 points) Give a polynomial time algorithm to decide if a boolean formula in 2CNF (each clause contains two literals) is satisfiable. Prove that your algorithm is correct and analyze its running time.

Problem 2. (15 points) Suppose you have a set $\{x_1, \ldots x_n\}$ of n items, where item x_i has weight w_i and value v_i. You must pack these items in your bag which you will check with your airline. The airline has a weight limit W which your bag cannot exceed. Unfortunately, the total weight of all n items exceeds the limit. So, you can only take a subset of the items and the total weight of the subset must be within the weight limit. You decide to maximize the total value of the items in your subset.

Formulate this problem as a decision problem (i.e., membership of a string in a language) and prove that it is NP-complete.

Problem 3. (15 points) For a 3CNF formula ϕ with n variables and c clauses, show that you can construct in polynomial time an NFA with $O(cn)$ states that accepts all non-satisfying assignments, represented as boolean strings of length n. Conclude that if $P \neq NP$, then NFAs cannot be minimized in polynomial time.

Problem 4. (20 points) The complete bipartite graph $K_{n,n}$ consists of a set $V = V_1 \cup V_2$ of vertices, $|V_1| = |V_2| = n$, and an edge for each pair of vertices (u, v): $u \in V_1, v \in V_2$.

Define the Balanced Complete Bipartite (BCB) decision problem as follows:

Instance: An undirected graph G = (V, E) and a positive integer k.

Question: Does G contain a balanced-complete bipartite subgraph $K_{k',k'}$ with $k' \geq k$ vertices?

(a) Show that BCB is in NP.

(b) Show that CLIQUE $\leq_P$ BCB

(c) Formulate the optimization version (BCB-o) of the BCB decision problem and show a polynomial-time Turing-reduction from BCB-o to BCB.

(d) Formulate the search version (BCB-s) of the BCB decision problem, and show a polynomial-time Turing-reduction from BCB-s to BCB-o.