GoldbAuM

RISK training:

Fixed Income: Bond Pricing and Interest Rate Derivatives

DAY 2: 29-NOVEMBER-2018

TITLE: Callable Bonds and Bermudan Swaptions

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Agenda

- Introduction
- Pricing Callable Bonds - Volatility Model
- Hedging Callable Bonds
- Factors affecting the Pricing of Bermudan Swaptions
- Choosing the right model for Bermudan Swaptions
- Calibration choices
- Library

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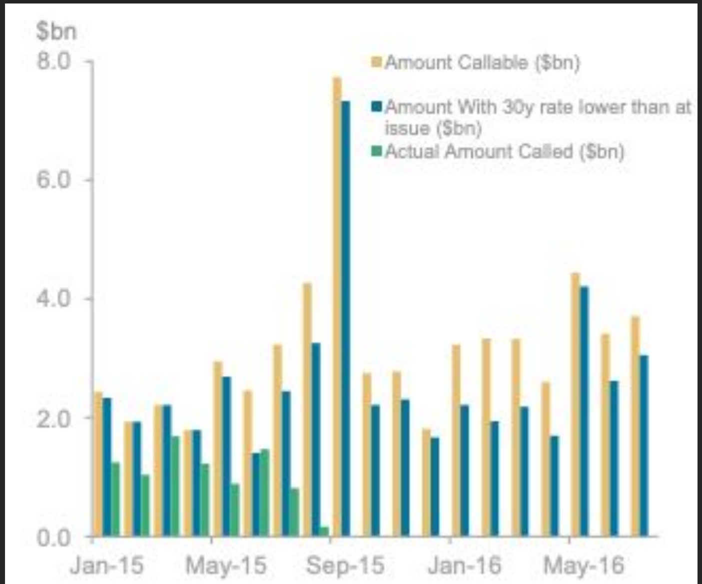
Introduction

Why “Callable Bonds” followed by “Bermudan Swaptions”?



Amount of Issuance Callable in Each Month (in billion€)

Vega	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y
1Y	-7	-5	-1	4	10	20	-8	-15	256	0
2Y	26	20	10	-2	-15	-43	16	554	0	0
3Y	-8	-7	-1	-31	-31	-30	359	0	0	0
4Y	-34	-33	-47	36	48	236	0	0	0	0
5Y	-5	16	28	-40	61	0	0	0	0	0
6Y	5	29	3	49	0	0	0	0	0	0
7Y	54	-207	-5	0	0	0	0	0	0	0
8Y	-73	178	0	0	0	0	0	0	0	0
9Y	32	0	0	0	0	0	0	0	0	0
10Y	0	0	0	0	0	0	0	0	0	0

Bucket Vega Distribution to cancel (hedge) a 10-Year Callable Swap

Source: Morgzn Stanley Research

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Pricing Callable Bonds - Volatility Model

What is a Callable Bond?

Bond Structure

>> cf. *Overview of Bond Market*

Embedded Option

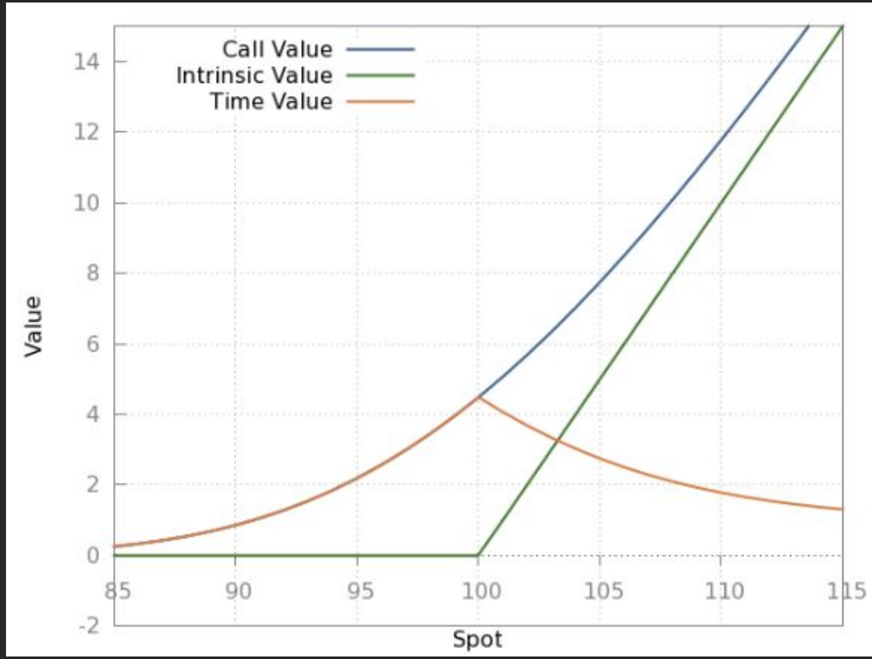
>> Call Provision (First Call Date)

Valuation

>> Dependence to Future Interest Rates only

Difference with Convertibles

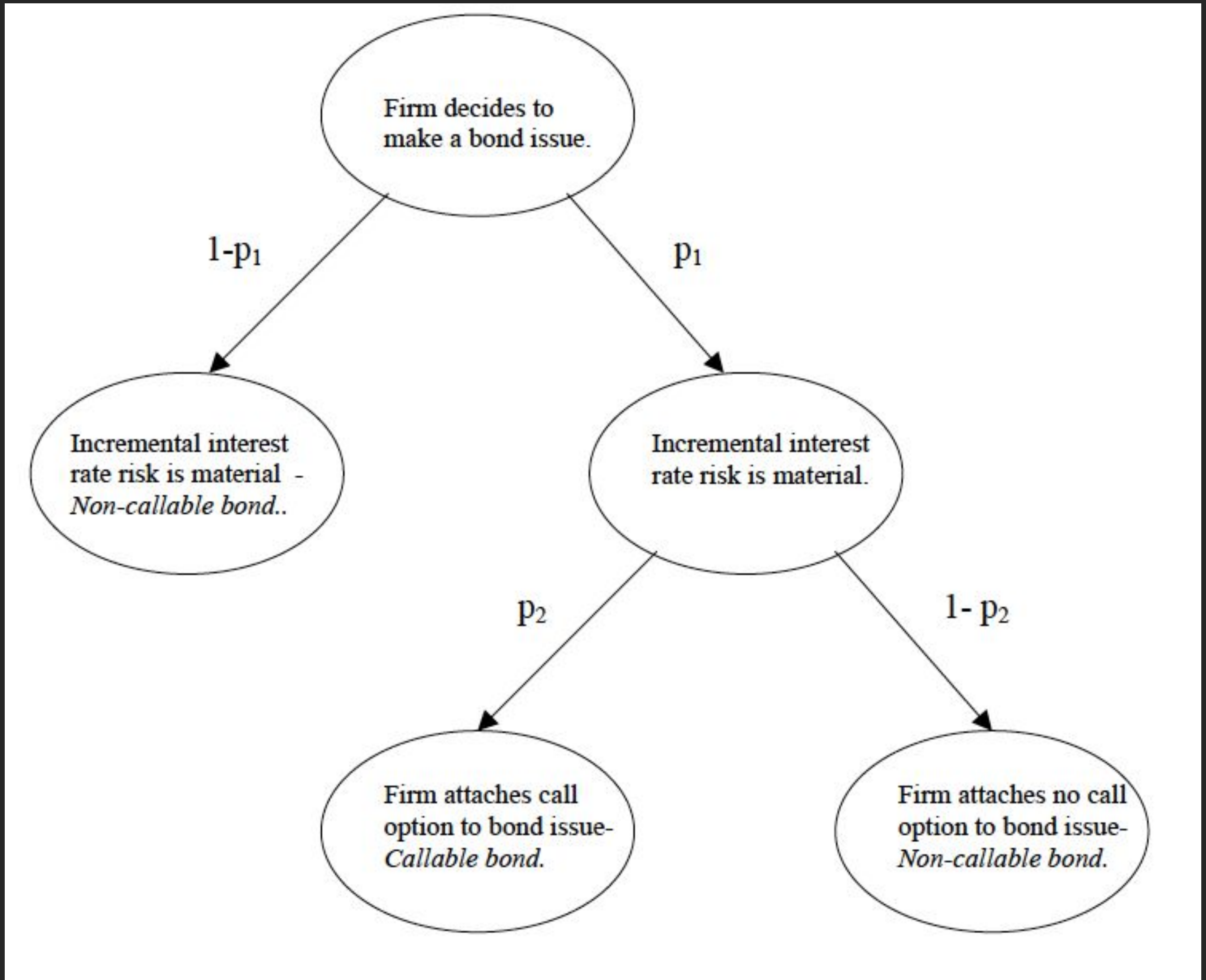
>> cf. *Convertible Bond Pricing and Modelling*



European Call Payoffs (Value, Intrinsic, Time)

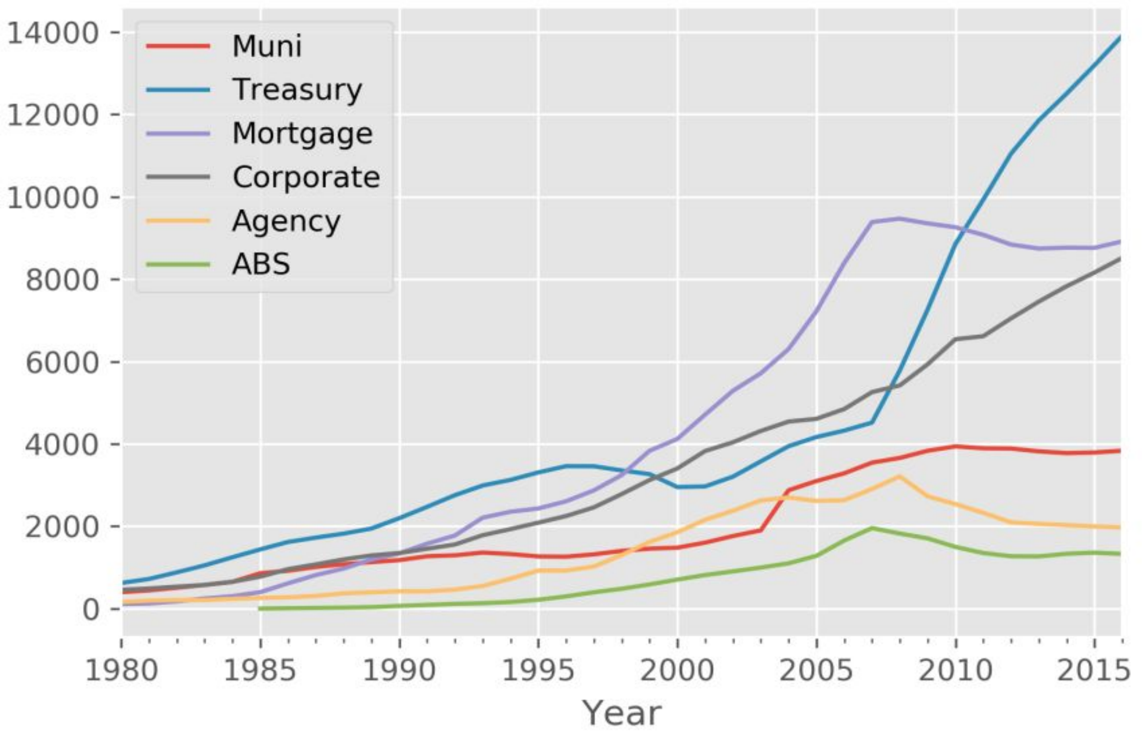
Pricing Callable Bonds - Volatility Model

Why issuing Callable Bonds?

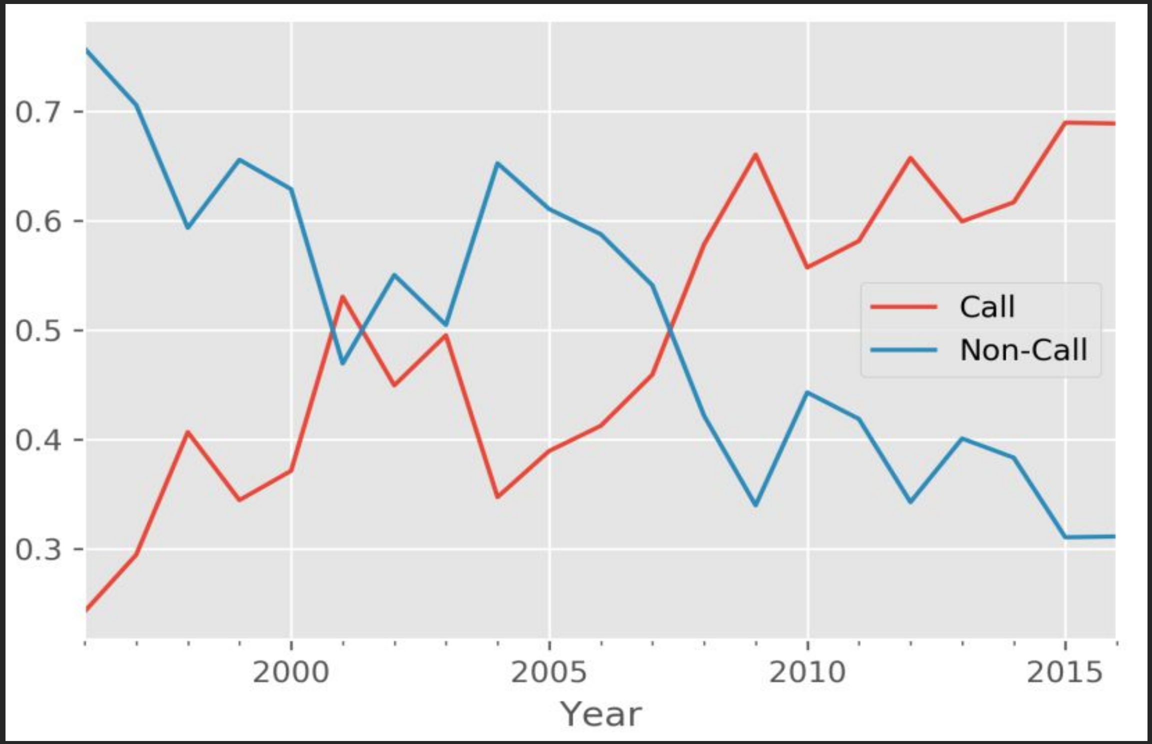


Pricing Callable Bonds - Volatility Model

Where is this market going?



Total Global Bond Issuance (in Billions€)



Share of callable and non-callable (bullet) corporate issuance (in %)

Source: Citi Research, JP Morgan Research, Wharton Financial Institutions Center,

Pricing Callable Bonds - Volatility Model

Complicated & Difficult Bond Pricing Process?

⇒ NO!

As simple as the basic principles of option-free bond modeling i.e. you just need to understand the assumptions of your model:
>> Establish the rules for when the call will be exercised
>> Determine the benchmark interest rates
>> Calibrate your interest rate volatility model

⇒ THE REAL DIFFICULTY IS TO NEVER FORGET THAT ANY FINANCIAL MODEL IS BASED ON ANOTHER MODEL

Pricing Callable Bonds - Volatility Model

Quick overview of Option-Free Bond Pricing

Look out for

>> Benchmark Interest Rates & Relative Value Analysis

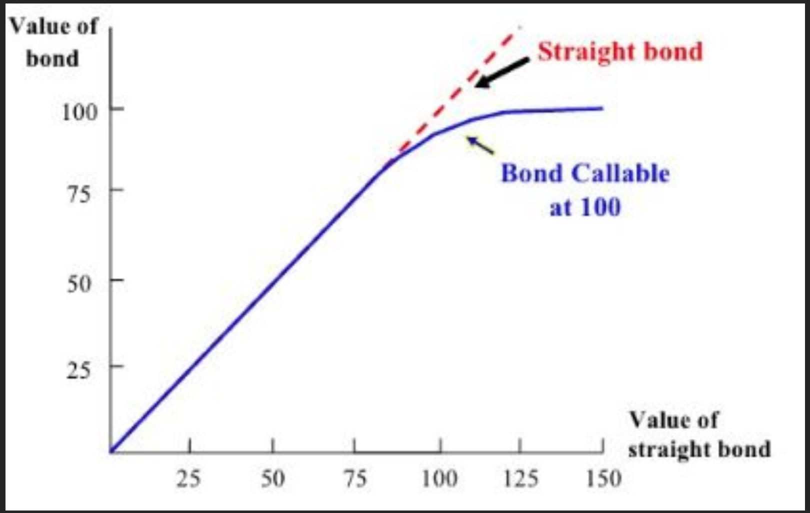
>> Interpretation of Spread Measures

>> Specific Bond Sector with a Given Credit Rating Benchmark

>> Issuer-Specific Benchmark

>> Option-Adjusted Spreads (OAS), the Benchmark, and Relative Value

⇒ PV USING BOTH SPOT AND FORWARD RATES



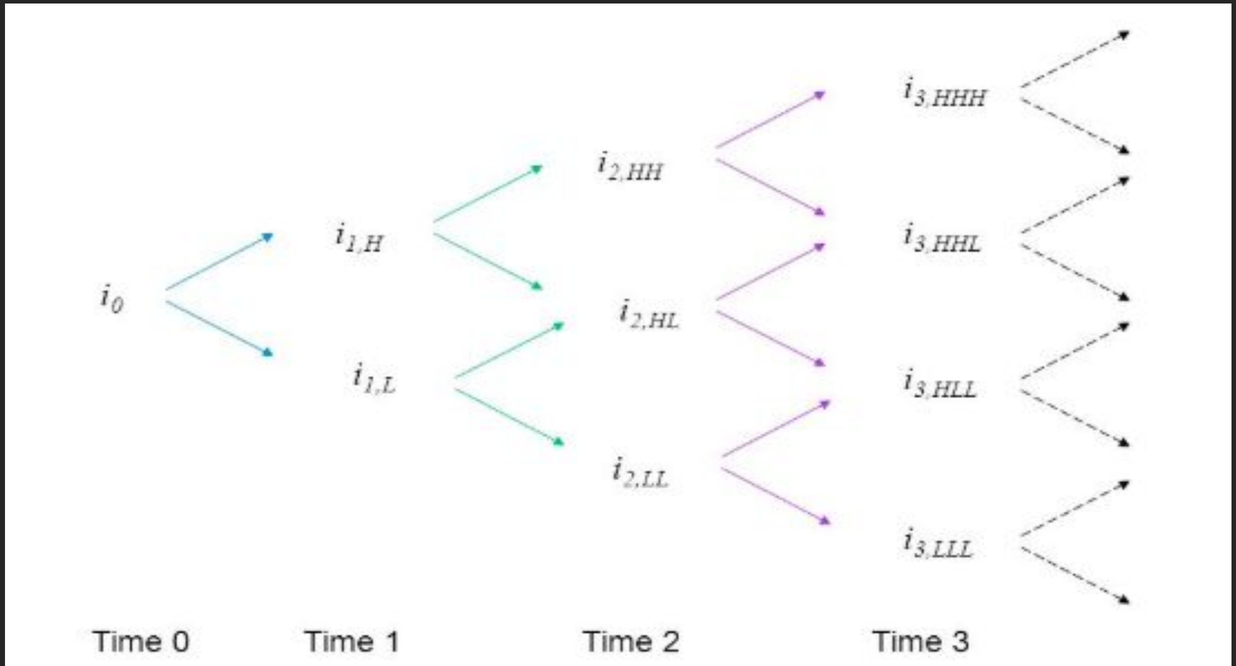
Pricing Callable Bonds - Volatility Model

The Binomial Interest Rate Model (“Tree”)

>> Inputs: On-the-run Yield Curve

>> Methodology:

- **Step 1:** Determining the Value at a Node with constant Volatility
- **Step 2:** Constructing the Tree
- **Step 3:** Option-Free Bond Pricing
- **Step 4:** Callable Bond Pricing
- **Step 5:** Resulting Option Pricing



Pricing Callable Bonds - Volatility Model

Volatility and the Arbitrage-Free Value

US High Yield Corporates OAS (in bp)

Source: ICE, BANK OF AMERICA

Pricing Callable Bonds - Volatility Model

Usual Suspects in Interest Rate & Volatility Models

IR Single Factors Models:

>> 1973: The Merton model

>> 1977: The Vasicek model

>> 1980: The Brennan-Schwartz model, The Rendleman–Bartter model

>> 1983: The Marsh Rosenfeld model

>> 1985: The Cox, Ingersoll and Ross (CIR) model

>> 1993: The Hull-White model

>> 1987: The Lognormal model

Volatility Models:

>> cf. *Interest Rate Derivative Volatility Modelling*

>> EWMA: Exponential Weighted Moving Averages

>> GARCH: Generalized Autoregressive Conditional Heteroscedasticity

Pricing Callable Bonds - Volatility Model

The Black-Scholes-Merton Model (BSM)

Option Pricing Theory

>> Know your rates

>> Know your volatility

>> BUT this time they are a function of time

>> NEW: Efficient market hypothesis

>> Asset price follows the lognormal random process i.e. volatility is constant

>> No income (coupon) is paid during the life of the options

Pricing Callable Bonds - Volatility Model

Your Tool Box: Any accurate and efficient numerical method?

⇒ **No, not really.**

Choice of several methods - each having some advantages and pitfalls:

>> Tree

>> Lattice

cf. *Factors affecting the pricing of Bermudan Swaptions*

>> Monte Carlo

Pros: Computational

Cons: Fat tails ignorance

>> Finite Difference

Pros: Closed-Form Solution

Cons: Only for Simple Structures

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- **Hedging Callable Bonds**
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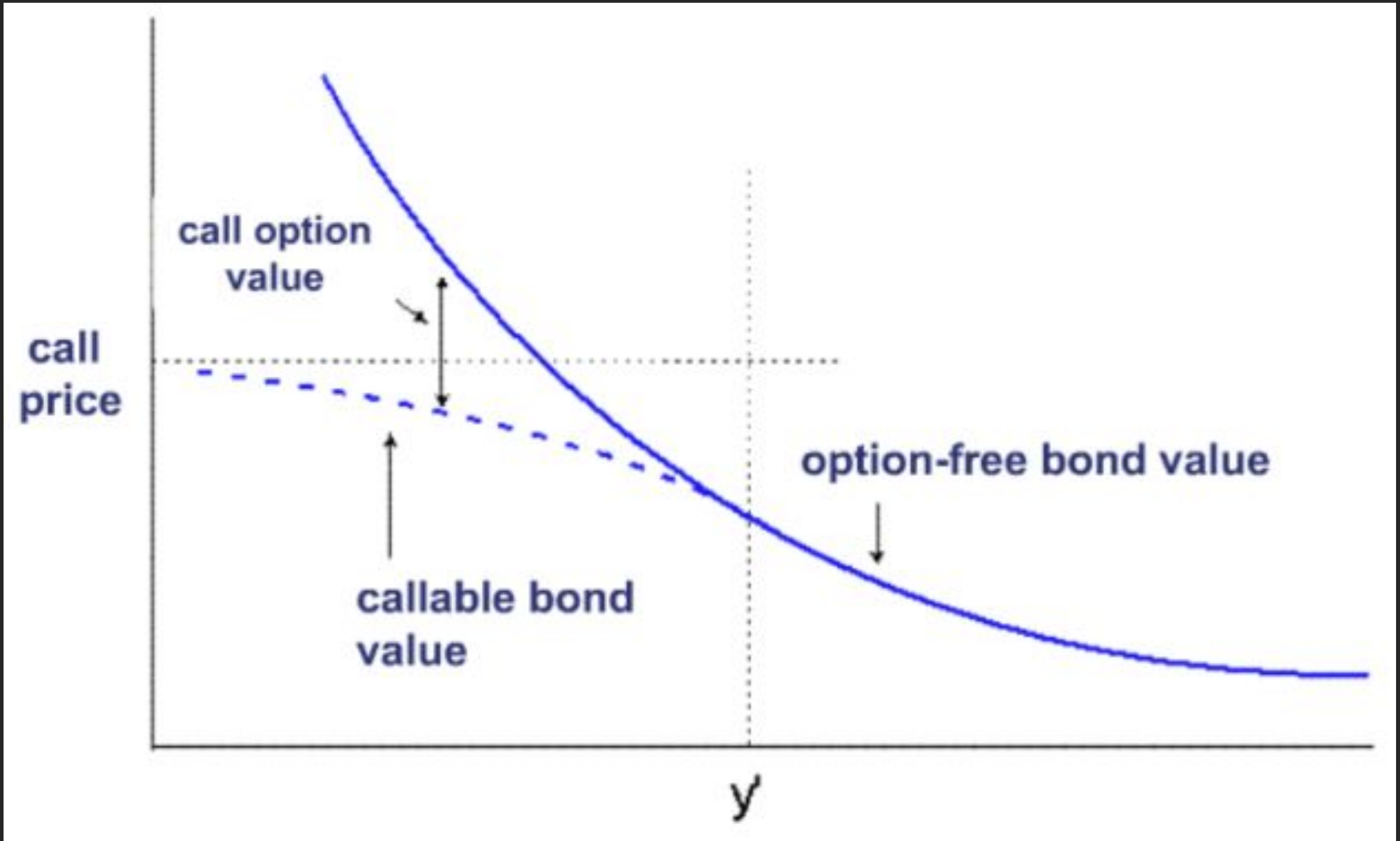
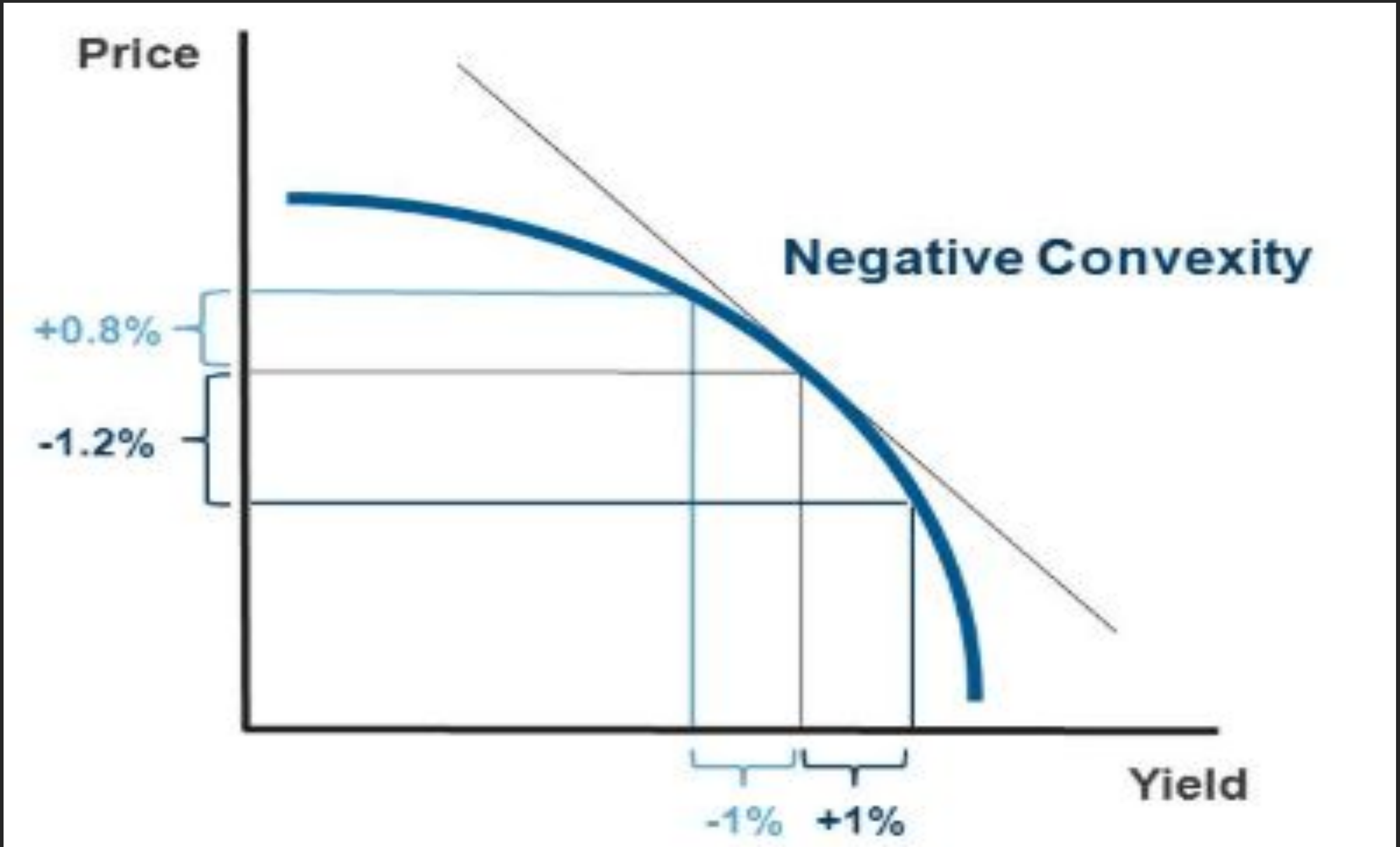
Hedging Callable Bonds

A (Quick) Word on Effective Duration



Hedging Callable Bonds

Another (Quick) Word on Convexity



Hedging Callable Bonds

Replicating Synthetically a Long Position in a Callable Bond

>> **Example 1:**

First Leg: Buy a Bullet Bond of same maturity, and

Second Leg: Sell a Call option on that same Bullet Bond

>> **Example 2:**

First Leg: Pay 1Y Swap, and

Second Leg: Buy 1Yx3Y Payer Swaption

>> **cf. *Introduction***

>> **cf. *Factors affecting the Pricing of Bermudan Swaptions***

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Factors affecting the pricing of Bermudan Swaptions

What is a Bermudan Swaption?

$$S_{T1}(\alpha_1 DF_1 + \alpha_2 DF_2) = \alpha_1 L_1 DF_1 + \alpha_2 L_2 DF_2$$

$$\Rightarrow S_{T1} = w_1 L_1 + w_2 L_2, w_i = \frac{\alpha_i DF_i}{\sum \alpha_i DF_i}$$

$$\sigma^2_{S_{T1}} S^2_{T1} T \approx w_1^2 \sigma^2_{L_1}(T) L_1^2 T + w_2^2 \sigma^2_{L_2}(T) L_2^2 T + 2\rho w_1 w_2 L_1 L_2 \sigma_{L_1}(T) \sigma_{L_2}(T) T$$

Swaption with Maturity T, Swap Tenor 1 year, Semi-annual

Factors affecting the pricing of Bermudan Swaptions

What is a Bermudan Swaption?

>> European Swaption:
Right but not the obligation to engage in an interest-rate swap to be exercised only on a fixed date as set in the contract i.e. swaption only exercised at expiry (maturity) date.

>> American Swaption:
Right but not the obligation to engage in a swap to be exercised on or before a fixed date as set in the contract i.e. swaption exercised at any time before and including the expiry date

>> Bermudan Options:
“exotic” combination of previous two option types - only exercised on predetermined dates, for example: often on one day each month

Factors affecting the pricing of Bermudan Swaptions

What is a Bermudan Swaption?

$$Payoff(T) = \max(0, V_{swap}(T))$$

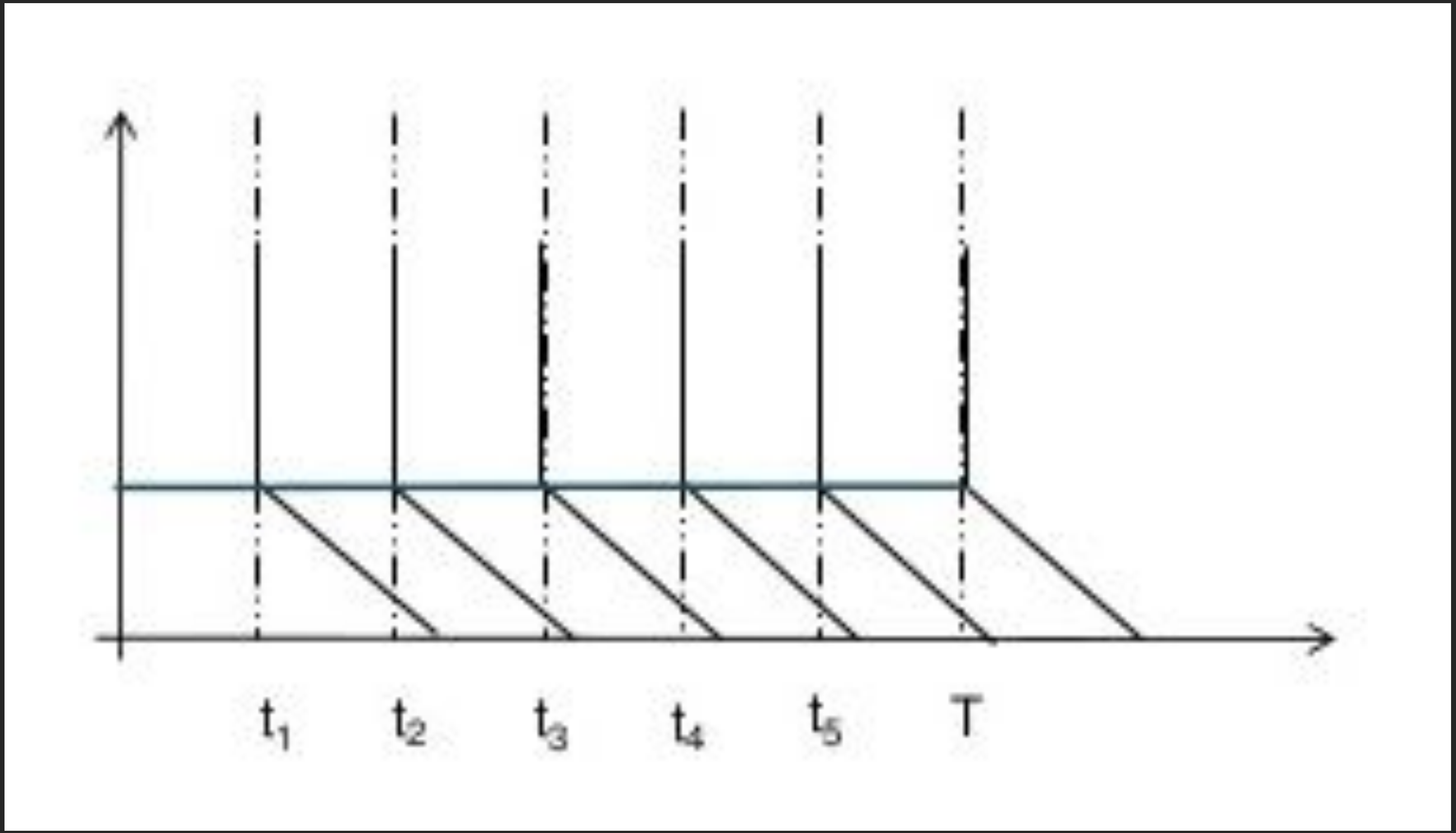
Payoff at Maturity T

$$Payoff(T_i) = max\left(V_{swap}(T_i), I(T_i)\right)$$

Payoff at any exercise date before Maturity T

Factors affecting the pricing of Bermudan Swaptions

Payoff Example of a Bermudan “Put” Swaption



Bermudan Receiver with K Strike, 6 Years Maturity

Factors affecting the pricing of Bermudan Swaptions

Interest Rate Models

The Heath-Jarrow Morton Model (HJM)

>> Too complex

The Hull-White Model (HWM)

>> Inaccurate for Computing Sensitivities

The Quadratic Gaussian Model (QGM)

>> Not that complex but computing can be tedious

Factors affecting the pricing of Bermudan Swaptions

The Binomial Tree (CRR Model)

Pros:

>> Simple Mathematics

>> Computerized

>> Adaptive

Cons:

>> Discrete

>> Heavily manual

>> Must use a constant volatility

Factors affecting the pricing of Bermudan Swaptions

The Libor Market Model (LMM)

$$P(t,T_i) = \prod_{j=1}^{i} \frac{1}{(L_j(t)(T_j - T_{j-1}) + 1)}.$$

LIBOR rates in terms of a bond price

$$A(T_i) = N\left(\sum_{k=i+1}^{\beta} P(T_i,T_k)\delta_k(L_k(T_i) - K)\right)^+$$

Bermudan Payer Swaption Payoff (not been exercised beforehand)

$$i > j, t < T_{j-1}: \quad dL_i(t) = \sigma_i(t)L_i(t)\sum_{k=i+1}^{j} \frac{\delta_j \rho_{ik}\sigma_k(t)L_k(t)}{1+\delta_j L_k(t)}dt + \sigma_i(t)L_i(t)dW^i(t).$$

Dynamics in the LIBOR market model

$$\sup_{\tau \in T} \mathbb{E}^{\beta}\left[\frac{A(\tau)}{P(\tau,T_\beta)} \middle| L_{\alpha+1}(0) = L_{\alpha+1}, \ldots, L_\beta(0) = L_\beta\right]$$

Numeraire Pricing

Factors affecting the pricing of Bermudan Swaptions

The Multi-Factor Gaussian Markov Model

$$r(t) = a(t) + x_1(t) + x_2(t)$$

Short-Term Rates dynamics

$$dx_i(t) = -\kappa_i(t)x_i(t)dt + \sigma_i(t)dW_i(t), \; i = 1,2,$$
$$dW_1(t)dW_2(t) = \rho(t)dt,$$

Correlated Ornstein-Uhlenbeck processes under Q

$$f(t,T) = -\partial \ln P(t,T) / \partial T$$

Correlation between forward rates (instantaneous)

Factors affecting the pricing of Bermudan Swaptions

LGM

$$dX(t) = \alpha(t)dW$$

Rates dynamics

$$N(t,X) = \left(H(t)X + 0.5H^2(t)\zeta(t)\right)/D(t)$$

Numeraire

$$B(t,X;T) = D(T)exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right)$$

Zero-Coupon Bond Price

Factors affecting the pricing of Bermudan Swaptions

How many factors?

Different Models Impact:

>> Assumptions on Structure of Bermudan Swaption (Payoff etc.)

>> Assumptions on Pricing versus Hedging

>> Assumptions on Volatility structures of Forward Rates

>> Assumptions on Correlation structures of Forward Rates

Key Takeaways:

⇒ Depends on the particular application

⇒ If pricing then lower order models are acceptable

⇒ Common market practice: Continuously refitting your model

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Choosing the right model for Bermudan Swaptions

Selection Criteria: Models on Models

No closed-form solution:

>> **Select Interest Rate Term Structure Model:** cf. previous list of models

Evaluate Implementation Cost vs Accuracy/Robustness

Based on errors you feel comfortable with

>> **Numerical Solution to price the Bermudan Swaption:**

Approximate underlying stochastic process: cf. your tool box

Choosing the right model for Bermudan Swaptions

Selection Criteria: Building Blocks

Available Information:

>> Yield Curve(s): Discount Function (t) & Interpolation Methodology

>> Cap/Floor Prices: Conversion to caplet volatilities (Model dependent process)

>> Vanilla Swaption Prices

⇒ **Watch out: Separately Traded Markets**

Non-available information:

>> Cap/Swaption Prices: Implied covariance matrix vs. realized movements

>> Term Structure of local Volatility

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Calibration choices

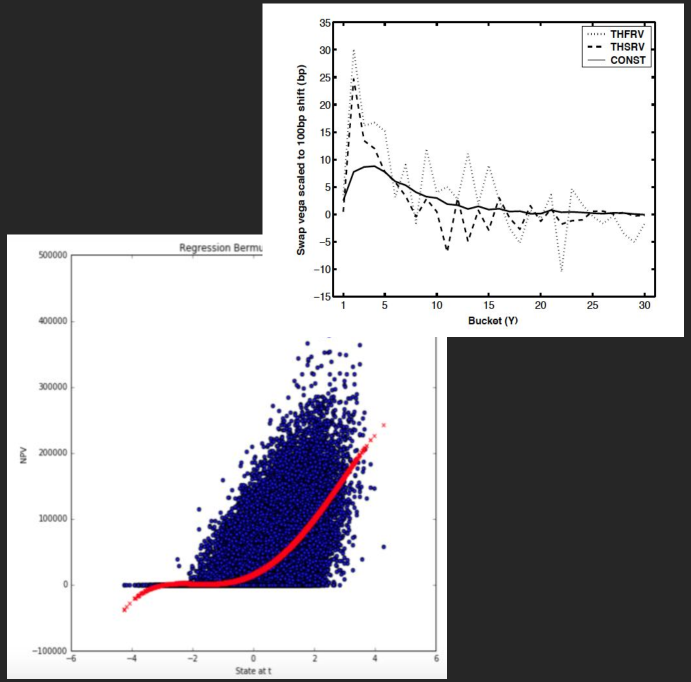
Constant Learning Process

⇒ It’s your job

>> You iteratively try to adjust the model’s structural parameters until model data = training data

Expiry (Y)	1	2	3	...	28	29	30
Tenor (Y)	30	29	28	...	3	2	1
Swaption Volatility	15.0%	15.2%	15.4%	...	20.4%	20.6%	20.8%

⇔



Calibration choices

Methodology

Calibrating the Model is to

>> Choose a numerical method with respect to minor additional assumptions

>> Choose concerned market data which is dependent on availability & quality of highly traded instruments

>> Understand why implied parameters can be higher or lower than realized

>> Analyse performance in practice: it depends on pricing or hedging approach but correct errors, reduce risk, improve calculation efficiency

Calibration choices

Example: LGM

Step 1 : Match Today's Curve

>> LGM automatically fits today's discount curve

Step 2 : Select a group of market swaptions

>> Watch out for liquidity pockets

Step 3 : Solve parameters by minimizing the relative error between the market prices and the model prices

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Library

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Any Questions?

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Thank you

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