# Incentivizing Optimal Forecasting Groups in Prediction Markets

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**Abstract**

## I. Introduction

Wisdom of crowds describes a phenomenon in which a group of individuals perform a task or make a decision with an acumen greater than any single individual. Oftentimes, the disparity between a single individual performing a task and the collective is stark. Possibly the most quoted and most widely referenced single instance of wisdom of the crowds is attributed to Sir Francis Galton's observations of a crowd estimating the weight of an ox.[1]. He noted that the crowd's average prediction was within 1 percent of the oxen's true weight. Since then, wisdom of the crowds has garnered interest from various industries and academic fields as more individuals try to ascertain what makes a crowd intelligent and how crowd intelligence be utilized to perform tasks that would be difficult for any single individual. The most prevalent example of crowds being used to accomplish tasks comes in the form of markets, where groups of individuals contribute their private information and perspective for personal gain, and in so doing, help to attain an accurate estimation of a quantity. This quantity could be the value of a particular company's stock or the likelihood of some event happening in the future.

A prediction market acts as a vehicle for obtaining likelihood estimates of a particular event or set of events occurring.[2] Participants in the market pay for a contract that is tied to a particular probability distribution over the set of events in question. The market determines both how to aggregate information elicited from market participants as well as how to reward individual contracts according to some payoff function. There is strong evidence backing up the accuracy of different prediction markets [2].

There is also substantial research focused on crowd dynamics and what characteristics make for an optimal crowd in a setting where the objective is to estimate a given quantity [3]. The purpose of this paper is to determine whether or not we can build a prediction market that changes its incentive scheme to ensure that the participating population closely matches the optimal composition given the pool of potential participants. We will first provide a brief background on scoring rules for prediction markets. Then, we will take a more in-depth look at some current literature regarding composition of forecasting groups [3]. This background will allow us to frame the motivating question for this research. After proposing the incentive structure that is meant to ensure optimal group composition, we will then evaluate the benefits and shortcomings of the scheme and suggest both improvements and directions for future work.

## II. Background

### i. Scoring Rules

In a prediction market setting, participants are asked to report a vector of probabilities denoting the likelihoods of different events occurring. The reward for a particular vector of probabilities is a function of the vector and the set of events that eventually occurred. This function is known as a scoring rule and is typically denoted as:

$$S(\vec{p}, \omega) \tag{1}$$

[1] His original paper, *Vox Populi*, was published in *Nature* in March of 1907. doi:10.1038/075450a0

[2] See [1] for more details.

where $\vec{p}$ is the vector of probabilities bid by the market participant and $\omega$ is the actual event that occurred. Note that in predicting a single event, the vector $\vec{p}$ has length 1.

There are certain properties of scoring rules that elicit different behavior from rational agents. A scoring rule is called proper if the agent cannot improve their expected payoff by misreporting their belief or what they believe to be the true probabilities over events. A scoring rule is strictly proper if the set of probability vectors that maximize an agent's expected value has a cardinality of 1, where the single element of the set is their true belief. Therefore, for both types of scoring rules, there is no incentive to report some $\hat{p} \neq \vec{p}$ and for a strictly proper scoring rule, the agent can maximize their expected payment only by reporting $\vec{p}$.

The expected value of an agent predicting a the probability of an event is given by:

$$G(\vec{p}) = pS(p,1) + (1-p)S(p,0) \tag{2}$$

We know that for a (strictly) proper scoring rule, the function $G(\vec{p})$ is (strictly) convex. Gneiting and Raftery [4] generalize the conditions that must be satisfied by the function $G(\vec{p})$ in order for a scoring rule to be proper or strictly proper.

## ii. Optimal Forecasting Groups

The following model and results stem from the work by Lamberson and Page (LP) [3] as they characterized what makes an optimal forecasting group under their assumptions. This section summarizes the details of their model and results that are relevant to the work in this paper.

### ii.1 Basic Framework

In the model, agents have a goal of estimating some quantity $V$. Each of the $M$ agents is characterized by a random variable $s_i$, which is their prediction of the quantity $V$. The random variable $\epsilon_i = s_i - V$ is the error in prediction for the $i_{th}$ agent. The aggregate estimate of the crowd of agents is $G(\vec{s}) = \frac{1}{M}\sum_i s_i$. In [3], LP focuses solely on the simple average as a way of aggregating individual estimates. Finally, the metric for accuracy in LP is group squared error, $(G(\vec{s}) - V)^2)$. LP then uses the *bias-variance-covariance* decomposition to expand the expected value of the group squared error:

$$E[(G(\vec{s}) - V)^2] = \overline{bias}(\vec{s})^2 + \frac{1}{M}\overline{var}(\vec{s}) + (1-\frac{1}{M})\overline{cov}(\vec{s}) \tag{3}$$

where the average bias of signals is given by:

$$\overline{bias}(\vec{s}) = \frac{1}{M}\sum_{i=1}^{M}(E[\epsilon_{s_i}]) \tag{4}$$

the average error variance is given by:

$$\overline{var}(\vec{s}) = \frac{1}{M}\sum_{i=1}^{M} var[\epsilon_{s_i}] \tag{5}$$

and the average error covariance is given by:

$$\overline{cov}(\vec{s}) = \frac{1}{M(M-1)}\sum_{i=1}^{M}\sum_{j\neq i} cov(\epsilon_{s_i}, \epsilon_{s_j}) \tag{6}$$

### ii.2 Group Composition

Now, let there be two types of forecasters in the crowd characterized by prediction random variables $a$ and $b$. Furthermore, LP assumes the agents to be unbiased such that:

$$E[\epsilon_a] = 0 \rightarrow E[\epsilon_a^2] = var(\epsilon_a) - (E[\epsilon_a])^2 = var(\epsilon_a) \tag{7}$$

$$E[\epsilon_b] = 0 \rightarrow E[\epsilon_b^2] = var(\epsilon_b) - (E[\epsilon_b])^2 = var(\epsilon_b) \tag{8}$$

LP goes on to define $cov(\epsilon_a)$ as the covariance in prediction errors between two individual agents of type $a$ (i.e. intra-group covariance). Likewise, $cov(\epsilon_b)$ denotes the covariance in prediction errors between two agents of type $b$. Finally, $cov(\epsilon_a, \epsilon_b)$ (inter-group covariance) is the covariance in prediction errors between the two types. In this setting, covariance indicates how similarly prediction errors for agents will vary together. For example, errors in prediction may covary if the agents draw upon similar resources or information in making their prediction or if they have similar perspectives on the problem. The final relevant assumption that LP makes is the enforcement of a condition, *type coherence*.

*Type coherence* is satisfied if the quantity:

$$TC(a,b) = cov(\epsilon_a) + cov(\epsilon_b) - 2cov(\epsilon_a, \epsilon_b) \quad (9)$$

is strictly positive: $TC(a,b) > 0$. This assumption is fairly reasonable as it states for the results of their analysis to hold, the types' errors must covary more within the two groups than they do between the two groups. In other words, the errors in prediction amongst members of groups should be more related than the errors across groups.

#### ii.3 Relevant Results from LP

From equation 3, LP gives the following for the expected squared error in average prediction for a group with types $a$ and $b$:

$$E[(G(s) - V)^2] = \frac{Avar(\epsilon_a) + Bvar(\epsilon_b) + 2\binom{A}{2}cov(\epsilon_a) + 2\binom{B}{2}cov(\epsilon_b) + 2ABcov(\epsilon_a, \epsilon_b)}{M^2}$$

where $M = A + B$. Here $A$ indicates the quantity of type $a$ in the crowd and $B$ indicates the quantity of type $b$ in the crowd. Recall that the agents are still assumed to be unbiased.

LP then treats the above equation as a function of $A$ and finds that the expected group error is minimized when the proportion of type $a$ agents in the crowd is:

$$\frac{A}{M} = \frac{[var(\epsilon_b) - var(\epsilon_a)] - [cov(\epsilon_b) - cov(\epsilon_a)]}{2M \cdot TC(a,b)} + \frac{cov(\epsilon_b) - cov(\epsilon_a, \epsilon_b)}{TC(a,b)}$$

From this equation, we can see that as $M \to \infty$, the optimal fraction of type $a$ individuals approaches $\frac{cov(\epsilon_b) - cov(\epsilon_a, \epsilon_b)}{TC(a,b)}$. This second equation shows that the proportion of $a$ type should increase as the variance (i.e. inaccuracy) of type $a$ errors decreases with respect to the variance of type $b$ errors. Furthermore, when we take the limit as $M \to \infty$, accuracy becomes less of a determining factor in the proportions of the two types. As the intra-group covariance of one group increases (the errors of that group covary more), the proportion of the opposing group needed to minimize the expected group error increases. Hence, when as the group gets larger, it favors the type with more internal disagreement in prediction errors.

This concludes the summary of the results of LP, which serve as a stepping stone for the continued work proposed in this paper.

**Table 1:** *Example table*

| Name | | |
|---|---|---|
| First name | Last Name | Grade |
| John | Doe | 7.5 |
| Richard | Miles | 2 |

## III. Incentive Scheme for Maintaining Optimal Group Composition

## IV. Results

## V. Discussion

### i. Subsection One

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### ii. Subsection Two

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