

## 7: More on magnetism; review

In this chapter we tie up some loose ends and take stock.

### 7.1 Paramagnetism and ferromagnetism

*See Mandl 3.1*

So far we have considered the ‘model magnet’ without really explaining why such a simple model is a good caricature of the real physics. This needs a knowledge of quantum mechanics that you have acquired in the quantum mechanics course.

The magnetic behaviour of solids is due to the behaviour of the electrons and/or nucleus. The simplest case is where the magnetic behaviour is due to a single ion of the solid molecule that has a single electron which acts as a ‘spin  $1/2$ ’. Quantum mechanics tells us that when we apply a magnetic field (which implies a ‘quantisation axis’) then the dipole moment is quantised parallel to the field as  $\pm m$ . This is the basis of the ‘model magnet’ which is a model of paramagnetism, i.e., magnetic ordering in response to an external field.

In the weakly interacting system, we assumed that energy was only stored in interaction between dipoles and the external field. In reality, the dipoles interact amongst themselves. The ‘model magnet’ is a good model when the molecules of the solid are large so that the single ions carrying the dipole moment are well separated (See Mandl 3.1).

Interactions between dipoles have two origins. Firstly, we have the classical electromagnetic interaction where each dipole produces a field that its neighbors sit in. This tends to align dipoles. Secondly, we have a quantum mechanical interaction which occurs when the wavefunctions of neighboring atoms overlap. This produces what is known as an ‘exchange interaction’, which generally tends to align dipoles and is stronger than the classical interaction. If the interaction between dipoles is strong enough, it can lead to the phenomenon of ferromagnetism, where the dipoles tend to align without the aid of an external field.

### 7.2 Simple model of ferromagnetism

As a very basic model of ferromagnetic interactions, let us restrict ourselves to a one-dimensional array of dipoles.

Here there is no external field, and the only energy comes from the ferromagnetic interactions between particles: if two neighboring dipoles are in the same direction, this is favorable, but if two neighboring dipoles are in opposite directions, this costs energy.

Thus, each ‘domain wall’ where neighboring dipoles are of opposite directions costs energy  $J$ , say.

$$\uparrow\uparrow \dots \uparrow\uparrow\downarrow\downarrow \dots \downarrow\downarrow\uparrow\uparrow \dots \uparrow\uparrow$$

Example of two domain walls (three domains) in a one-dimensional array of  $N$  dipoles.

To estimate the dependence of the number of such domain walls on temperature, we can use a minimization of free energy argument.

Let us consider  $n$  domain walls. The energy cost is  $nJ$ .

The number of ways of arranging the  $n$  domain walls on the lattice is the number of ways of choosing  $n$  from the  $N - 1$  possible places the domain walls can be. (Do you see why it is  $N - 1$ ? but actually  $N - 1 \approx N$  for large  $N$ .) Thus:

$$S(n) = k \ln \binom{N-1}{n} \approx -kN[x \ln x + (1-x) \ln(1-x)]$$

where  $x = \frac{n}{N}$ .

Thus, the free energy as a function of  $x = \frac{n}{N}$  is:

$$F(x) = E(x) - TS(x) = N \{Jx + kT[x \ln x + (1-x) \ln(1-x)]\}$$

Minimising with respect to  $x$  yields:

$$\begin{aligned} J + kT[\ln x - \ln(1-x)] &= 0 \\ \Rightarrow \frac{x}{1-x} &= \exp\left(\frac{-J}{kT}\right) \\ \Rightarrow \bar{n} &= N \frac{\exp\left(\frac{-J}{kT}\right)}{1 + \exp\left(\frac{-J}{kT}\right)} \end{aligned}$$

The average number of domains  $n + 1$  tends to  $N/2$  as  $T \rightarrow \infty$  and to 1 as  $T \rightarrow 0$  (in this limit  $n$  is not large so strictly the analysis isn't valid, but the results give some insight).

As usual, there is a competition between energy, here favoring ferromagnetic order, and entropy: at very low  $T$  ( $kT \ll J$ ), energy wins, and we have very large ferromagnetically ordered domains; at high  $T$  ( $kT \gg J$ ), entropy dominates and the domains are very small.

### 7.3 Pause for breath

We have now covered all the fundamental concepts of statistical mechanics. Before proceeding it is worthwhile to try and recap on the basic ideas. You should go through the key points (13 of them) of chapters 3–7. Here we try and pull some ideas together.

- First we met microstates and macrostates. One thing to remember is that for a system of  $N$  ‘particles’ the number of microstates is typically exponentially large in  $N$  e.g.  $2^N$  for the model magnet. Also to each macrostate there corresponds there typically corresponds a large number of microstates, exponentially large in  $N$ .
- For an isolated system all microstates are equally likely and the system evolves to the macrostate with the largest weight. This corresponds to the second law i.e. maximising the entropy.
- For a system in equilibrium with a heat bath the entropy of the composite (system + heat bath) is maximised and this gives the Boltzmann distribution for the system. From the point of view of the system, its free energy is minimised.
- The concept of equilibrium when energy exchange is present gives a statistical mechanics definition of temperature.
- Always, contact is made with thermodynamics through the fact that macroscopic variables are sharply defined for large  $N$  and correspond to thermodynamic functions of state.
- Calculations are most easily done for weakly interacting systems using the partition function technique. This is the usual approach. It implies each particle (of the system of weakly interacting particles) has a single particle Boltzmann distribution.
- Minimisation of the free energy is an alternative way of obtaining macroscopic variables that gives good intuition about energy–entropy competition and can be used for interacting systems (see e.g. chapter 7.2).

We have also seen some equivalences. First, a single particle in a large isolated system takes on a Boltzmann distribution since the rest of the particles act as a heat bath. Second, for a large system in equilibrium with a heat bath the energy fluctuations are vanishingly small therefore the system becomes basically equivalent to a large system in isolation.