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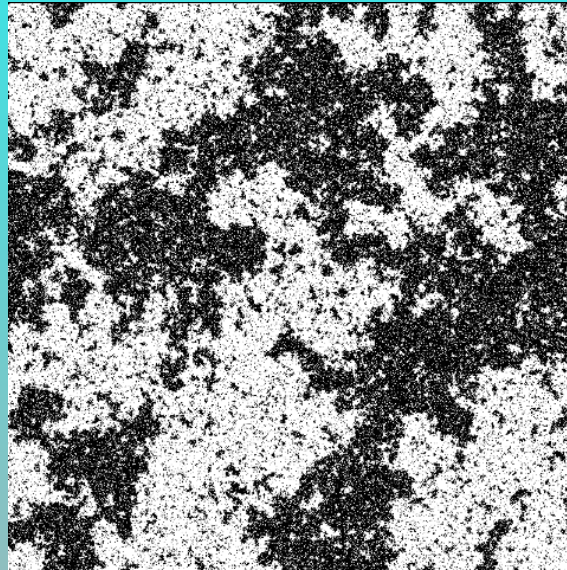
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# PHYS20040: Statistical Mechanics

Nigel Wilding



# Welcome!

# Delivery and format



Statistical Mechanics

- Detailed e-notes (see Blackboard) can be viewed on a variety of devices. Pdf available.
- I will give 'traditional' lectures (Tues, Wed, Fri) in which I use slides to summarise and explain the lecture content. Questions welcome (within reason...)
- Try to read ahead in the notes, then come to lectures, listen to my explanations and then reread the notes.
- Rewriting the notes or slides to express your own understanding, or annotating a pdf copy can help wire the material into your own way of thinking.
- There are group problem classes (Tues, Thurs) where you can try problem sheets and seek help. I will go over some problems with the class.





# 1. Introduction

# What is Statistical Mechanics?



Statistical Mechanics

Statistical mechanics, together with **classical thermodynamics**, form two branches of **thermal physics**

Each branch represents a distinct approach to thermal physics:

**Macroscopic Approach** (Classical Thermodynamics- see Properties of Matter)

- deals with **macroscopic** variables  $P, V, T, C_p, C_v$ , i.e. variables that do not refer to any microscopic details
- input is phenomenological laws e.g. an equation of state,  $P(V)$
- output is general relations between macroscopic variables eg.  $C_p = C_v + R$
- advantage is the generality of the approach

**Microscopic Approach** (Statistical Mechanics)

- starts from a **microscopic** description and seeks to explain macroscopic properties
- input is a microscopic model of a given system eg. interaction potentials between molecules
- output is predictions for macroscopic properties and behaviour
- predictions can be compared to experiment thus allowing refinement of the microscopic model

# What is Statistical Mechanics?



Statistical Mechanics

- Provides powerful concepts and tools that help us understand the properties of complex systems with very many constituents.
- Used in research of systems as diverse as earthquakes, traffic jams, neural networks, superconductivity, economics, and many more.
- **Aim of this course:** Show how key concepts that you have met in thermodynamics, such as the Boltzmann factor, entropy, and the second law, can be formulated and find expression in statistical terms.
- Develop and illustrate fundamental concepts and methods via two prototype systems:
  - Gases (classical and quantum)
  - Classical magnets
- For more advanced **Soft Matter** systems requiring a stat mech description (eg. polymers, liquid crystals, glasses, surfactants, active matter) see M-level unit: **Complex and Disordered Matter**

# The microscopic approach



- In principle, can imagine solving Newton's equation for the atomic and molecular motions in a system of interest to determine its behaviour.
- But typical systems contain of order a mol, ie.  $10^{23}$  particles - more than all the grains of sand on all the beaches in the world, or stars in the visible universe!
- So exact approach is impractical and therefore we instead appeal to concepts from statistics and seek to make probabilistic statements about a systems behaviour.
- This works well because as the number of particles becomes very large, things get simpler...



*Exercise: Revise your probability and statistics notes from first year laboratory, particularly on combinatorics, probability distributions, and summary statistics; read the section on probability in the lecture notes (end of sec 1.3)*



# Simplicity at large $N$

Toss a fair coin  $N$  times. Call this a “trial”.

What is the probability  $p_n$  of getting  $n$  heads from a trial?

Denote by  $p$  the probability that a head results from a single toss; then  $q = 1 - p$  is probability for a tail.

This is binomial statistics. Recall:

$$\begin{aligned} p_n &\equiv \text{number of distinct ways of obtaining } n \text{ heads} \\ &\times \text{probability of any specific way of getting } n \text{ heads} \\ &= \binom{N}{n} p^n q^{N-n} \end{aligned}$$

The distribution  $p_n$  has mean  $\bar{n} \equiv \sum_{n=0}^N n p_n = Np$

and variance  $\overline{\Delta n^2} \equiv \sum_{n=0}^N (n - \bar{n})^2 p_n = Npq$







# Simplicity at large $N$

- For a fair coin  $p = q = \frac{1}{2}$
- Define  $f = n/N$ , the fraction of  $N$  tosses resulting in a head
- Mean of  $f$

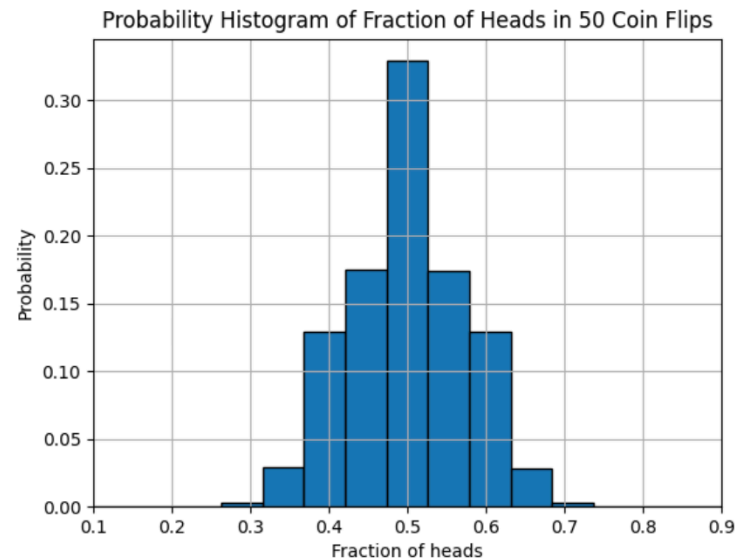
$$\bar{f} = \frac{\bar{n}}{N} = p = \frac{1}{2}$$

and standard deviation

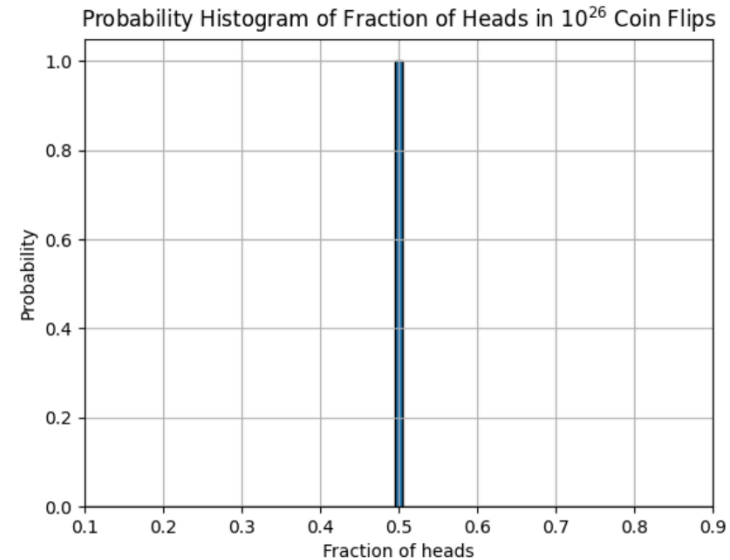
$$(\overline{\Delta f^2})^{1/2} \equiv \frac{(\overline{\Delta n^2})^{1/2}}{N} = \left( \frac{pq}{N} \right)^{1/2} = \frac{1}{2N^{1/2}}$$

- The typical deviation of  $f$  from its mean value is thus vanishingly small for large  $N$

# Simplicity at large $N$



(a) For  $N = 50$  tosses, we can be reasonably sure that  $f$  will be close to 0.5



(b) For  $N = 10^{26}$  tosses We can be absolutely sure that  $f$  will be indistinguishable from 0.5

(see python code in notes)