PHYS20040: Statistical Mechanics

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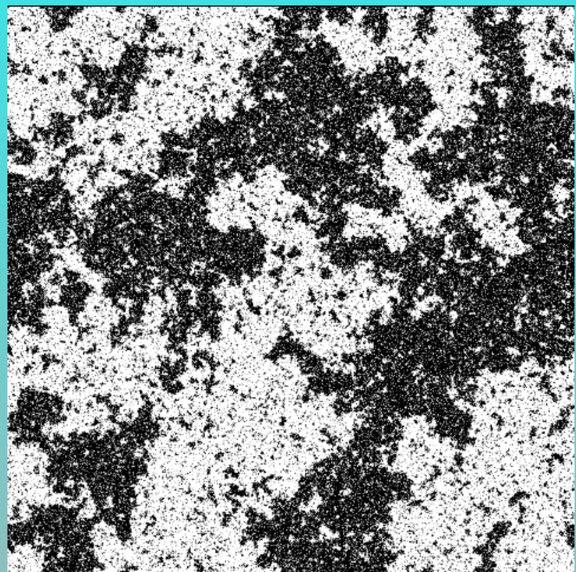


Welcome!

Delivery and format

- Detailed e-notes (see Blackboard) can be viewed on a variety of devices. Pdf available.

- I will give 'traditional' lectures (Tues, Wed, Fri) in which I use slides to summarise and explain the lecture content. Questions welcome (within reason…)

- Try to read ahead in the notes, then come to lectures, listen to my explanations and then reread the notes.

- Rewriting the notes or slides to express your own understanding, or annotating a pdf copy can help wire the material into your own way of thinking.

- There are group problem classes (Tues, Thurs) where you can try problem sheets and seek help. I will go over some problems with the class. **No classes week 19**.

1. Introduction

What is Statistical Mechanics?

Statistical mechanics, together with **classical thermodynamics**, form two branches of **thermal physics**

Each branch represents a distinct approach to thermal physics:

Macroscopic Approach (Classical Thermodynamics- see Properties of Matter)

- deals with **macroscopic** variables P, V, T, C_P, C_V ,i.e. variables that do not refer to any microscopic details
- input is phenomenological laws e.g. an equation of state, $P(V)$
- output is general relations between macroscopic variables eg. $C_P = C_V + R$
- advantage is the generality of the approach

Microscopic Approach (Statistical Mechanics)

- starts from a **microscopic** description and seeks to explain macroscopic properties
- input is a microscopic model of a given system eg. interaction potentials between molecules
- output is predictions for macroscopic properties and behaviour
- predictions can be compared to experiment thus allowing refinement of the microscopic model

What is Statistical Mechanics?

- Provides powerful concepts and tools that help us understand the properties of complex systems with very many constituents.

- Used in research of systems as diverse as earthquakes, traffic jams, neural networks, superconductivity, economics, and many more.

- **Aim of this course:** Show how key concepts that you have met in thermodynamics, such as the Boltzmann factor, entropy, and the second law, can be formulated and find expression in statistical terms.

- Develop and illustrate fundamental concepts and methods via two prototype systems:
  - Gases (classical and quantum)
  - Classical magnets

- For more advanced **Soft Matter** systems requiring a stat mech description (eg. polymers, liquid crystals, glasses, surfactants, active matter) see M-level unit: **Complex and Disordered Matter**

The microscopic approach

- In principle, can imagine solving Newton's equation for the atomic and molecular motions in a system of interest to determine its behaviour.

- But typical systems contain of order a mol, ie. 10^{23} particles - more than all the grains of sand on all the beaches in the world, or stars in the visible universe!

- So exact approach is impractical and therefore we instead appeal to concepts from statistics and seek to make probabilistic statements about a systems behaviour.

- This works well because as the number of particles becomes very large, things get simpler…

Exercise: Revise your probability and statistics notes from first year laboratory, particularly on combinatorics, probability distributions, and summary statistics; read the section on probability in the lecture notes (end of sec 1.3)

Simplicity at large N

Toss a fair coin N times. Call this a “trial”.

What is the probability p_n of getting n heads from a trial?

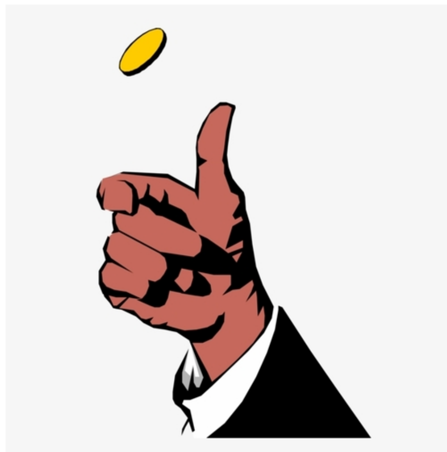
Denote by p the probability that a head results from a single toss; then $q = 1 - p$ is probability for a tail.

This is binomial statistics. Recall:

$$\begin{aligned} p_n &\equiv \text{number of distinct ways of obtaining } n \text{ heads} \\ &\times \text{probability of any specific way of getting } n \text{ heads} \\ &= \binom{N}{n} p^n q^{N-m} \end{aligned}$$

The distribution p_n has mean $\overline{n} \equiv \sum_{n=0}^{N} n p_n = Np$

and variance $\overline{\Delta n^2} \equiv \sum_{n=0}^{N} (n - \overline{n})^2 p_n = Npq$



Simplicity at large N

- For a fair coin $p = q = \dfrac{1}{2}$
- Define $f = n/N$, the fraction of N tosses resulting in a head
- Mean of f

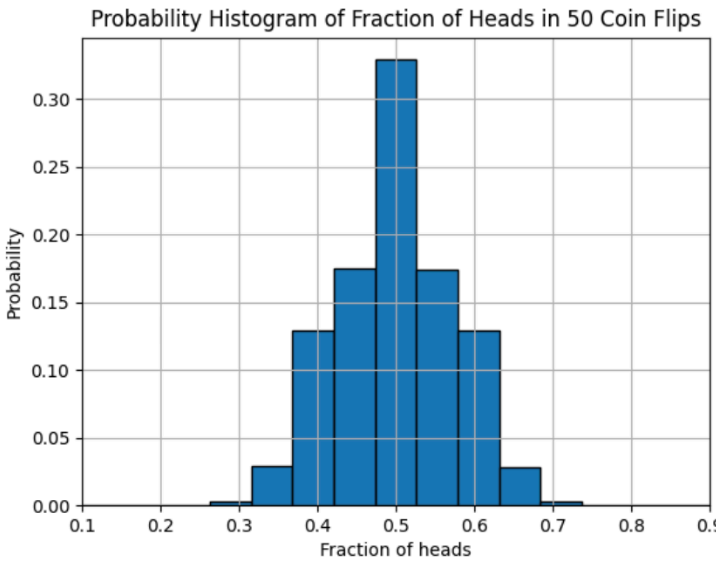
$$\bar{f} = \frac{\bar{n}}{N} = p = \frac{1}{2}$$

and standard deviation

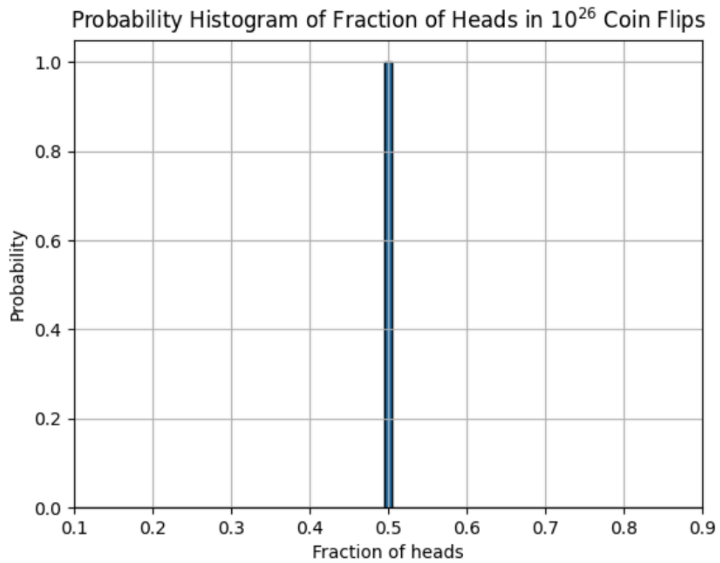
$$(\overline{\Delta f^2})^{1/2} \equiv \frac{(\overline{\Delta n^2})^{1/2}}{N} = \left(\frac{pq}{N}\right)^{1/2} = \frac{1}{2N^{1/2}}$$

- The typical deviation of f from its mean value is thus vanishingly small for large N

Simplicity at large N



(a) For $N = 50$ tosses, we can be reasonably sure that f will be close to 0.5



(b) For $N = 10^{26}$ tosses We can be absolutely sure that f will be indistinguishable from 0.5

(see python code in notes)