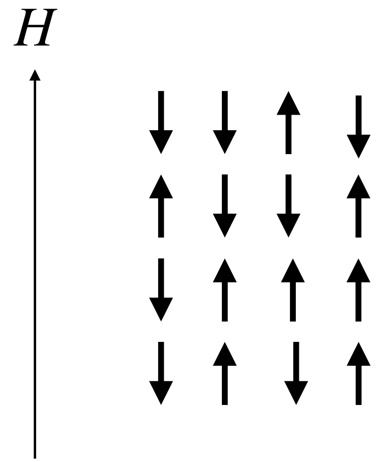
7. More on Magnetism

Paramagnetism and Ferromagnetism

- The dipoles in our simple model magnet possess energy solely through their interaction with the magnetic field H
- The two state (up/down) nature of the dipoles arises from quantum effects (spin 1/2 nature of electrons in atomic orbits)
- What the model neglects is **interactions** between dipoles: Energy is lowered if neighbouring dipoles have the same orientation (QM again).



- High T: entropy dominates and the aligning interaction is not enough to produce a spontaneous magnetisation: the system is **paramagnetic.**

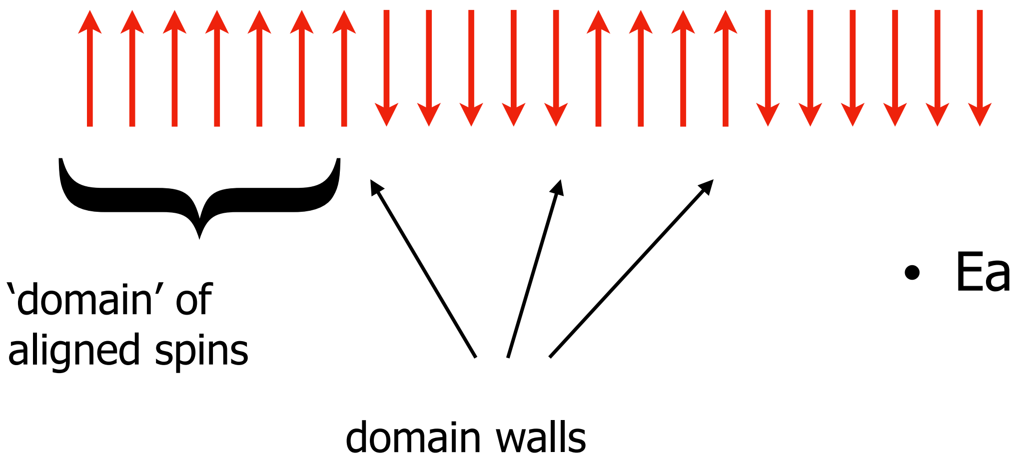
- Low T: energy dominates and the aligning interaction engenders a spontaneous magnetisation even at $H=0$: the system is **Ferromagnetic.**

Simple model of Ferromagnetism

Statistical Mechanics

- For simplicity work with a 1d chain of interacting dipoles ('spins')
- Assume energetically favourable for neighbouring spins to be aligned. Unfavourable to be antialigned.
- No magnetic field - interested in the effects of interaction energy and entropy (ie. alignment disorder)

  - Consider a chain with n 'domain' walls.



- Each 'domain wall' costs energy J, say

Simple model of Ferromagnetism

- We seek the equilibrium value of the number of domain walls n
- Need to minimise the free energy $F(n) = E - TS(n)$ with respect to n

$$E(n) = nJ, \qquad S(n) = k \ln \binom{N-1}{n}$$
$$\approx -kN[x \ln x + (1-x)\ln(1-x)]$$

where $x = n/N$

Thus

$$F(x) = E(x) - TS(x)$$
$$= N\{Jx + kT[x \ln x + (1-x)\ln(1-x)]\}$$

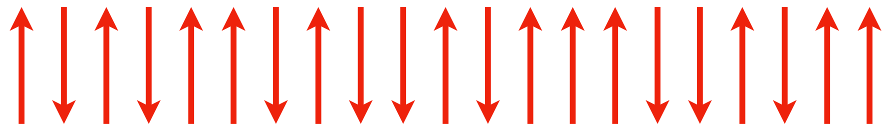
Simple model of Ferromagnetism

Minimising with respect to x yields:

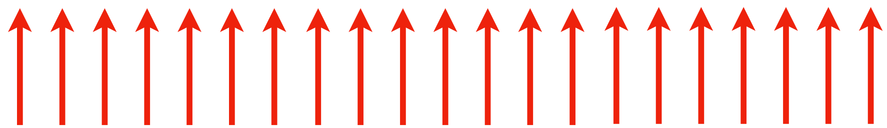
$$J + kT[\ln x - \ln(1 - x)] = 0$$

$$\Rightarrow \frac{x}{1 - x} = \exp\left(\frac{-J}{kT}\right)$$

$$\Rightarrow \bar{n} = N\frac{\exp\left(\frac{-J}{kT}\right)}{1 + \exp\left(\frac{-J}{kT}\right)}$$



High T: large number of small domains



Low T: small number of large domains

- The average number of domains $n + 1$ tends to $N/2$ as $T \to \infty$ and to 1 as $T \to 0$.