Combinatorial mathematics: Homework #0

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Problem 1

1.3 附录 B 中的 B6 是 1973 至 1978 年美国在意外事故中的死亡人数.

年份	1 月	2 月	3 月	4 月	5 月	6 月	7 月	8 月	9 月	10 月	11 月	12 月
1973	9007	8106	8928	9137	10017	10826	11317	10744	9713	9938	9161	8927
1974	7750	6981	8038	8422	8714	9512	10120	9823	8743	9192	8710	8680
1975	8162	7306	8124	7870	9387	9556	10093	9620	8285	8433	8160	8034
1976	7717	7461	7776	7925	8634	8945	10078	9179	8037	8488	7874	8647
1977	7792	6957	7726	8106	8890	9299	10625	9302	8314	8850	8265	8796
1978	7836	6892	7791	8129	9115	9434	10484	9827	9110	9070	8633	9240

利用至少两种方法对该时间序列进行分解. 要求如下:

(1) 画出数据图, 给出数据的周期 T;

(2) 给出趋势项、周期项和随机项的计算公式;

(3) 画出趋势项、周期项和随机项的数据图;

(4) 对 1979 年的意外死亡人数做出预测.

Solution

首先介绍一个引理。

Lemma 1 (Size Of Left Coset)**.** *Let H be a finite subgroup of a group G. Then each left coset of H in G has the same number of elements as H.*

Theorem 1 (Lagrange's Theorem)**.** *Let G be a finite group, and let H be a subgroup of G. Then the order of H divides the order of G.*

Proof. Let z be some element of $xH \cap yH$. Then $z = xa$ for some $a \in H$, and $z = yb$ for some $b \in H$. If h is any element of H then $ah \in H$ and $a^{-1}h \in H$, since H is a subgroup of G. But $zh = x(ah)$ and $xh = z(a^{-1}h)$ for all $h \in H$. Therefore $zH \subset xH$ and $xH \subset zH$, and thus $xH = zH$. Similarly $yH = zH$, and thus $xH = yH$, as required. □

```
death.month<- array(0, c(72))
death.detrended<-death.people-death.trended
ymat <- matrix(c(death.detrended), byrow=TRUE, ncol=12)
death.avg <- apply(ymat, 2, mean, na.rm=TRUE)
for (i in 1:72) {
death.month[i]<-death.avg[(i-1)%%12+1]
}
```

+	0	1	u	u+1	u^2	u^2+1	u^2+u	u^2+u+1
0	0	1	u	u+1	u^2	u^2+1	u^2+u	u^2+u+1
1	1	0	u+1	u	u^2+1	u^2	u^2+u+1	u^2+u
u	u	u+1	0	1	u^2+u	u^2+u+1	u^2	u^2+1
u+1	u+1	u	1	0	u^2+u+1	u^2+u	u^2+1	u^2
u^2	u^2	u^2+1	u^2+u	u^2+u+1	0	1	u	u+1
u^2+1	u^2+1	u^2	u^2+u+1	u^2+u	1	0	u+1	u
u^2+u	u^2+u	u^2+u+1	u^2	u^2+1	u	u+1	0	1
u^2+u+1	u^2+u+1	u^2+u	u^2+1	u^2	u+1	u	1	0

对于房价的预测而言，url, id, Cid, DOM, followers 是没有用的信息。对上述量进行删除处理。

```
df = df.drop(['url', 'id', 'Cid', 'DOM', 'followers'], axis = 1)
```

Problem 2

5.38. Show that if X and Y are regular, then so is the product space $X \times Y$. Conclude that $\mathbb{R}^n$ is regular.

Solution

1. For any point $(x, y) \in X \times Y$, it's the product of two closed sets in X and Y, so it's closed.

2. For any point $(x, y) \in X \times Y$ and any closed set $C = U \times V$, where U is closed in X and V is closed in Y and $x \notin U$ and $y \notin V$. X and Y are regular, so for x and U, we have there exists open sets A_1 and A_2 such that $x \in A_1$ and $U \subset A_2$ and $A_1 \cap A_2 = \varnothing$. Similarly, for y and V, we have there exists open sets B_1 and B_2 such that $y \in B_1$ and $V \subset B_2$ and $B_1 \cap B_2 = \varnothing$. Then we know that $(x, y) \in A_1 \times B_1$ and $C \subset A_2 \times B_2$, where $A_1 \times B_1$ and $A_2 \times B_2$ are open, and $(A_1 \times B_1) \cap (A_2 \times B_2) = \varnothing$.

$\mathbb{R}$ is regular, so use this conclusion n times, we know that $\mathbb{R}^n$ is regular.