Design characteristics of structural steels based on statistical analysis of metallurgical products

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Abstract

The results of experimental research of material and geometrical characteristics of Czech steel are given in the present paper. The yield strength, tensile strength and ductility were evaluated statistically. For hot-rolled steel IPE profiles, the geometrical characteristics of cross-section dimensions were evaluated statistically too. It has been proved that both material and geometrical characteristics of plates and hot-rolled IPE profiles are satisfactory. The problems are discussed also in connection with application of stochastic computations. For this purpose, correlation coefficients among the quantities measured were also investigated.
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1. Introduction

The objective information on real behaviour of a structure system can be reliably obtained and verified by material tests on real samples only. In the field of structure imperfections, maximum attention is being paid to investigation and analysis of yield strength, and of material tensile strength and ductility. Greater attention to material characteristics was paid in the initial time period of applying the calculation methods according to limit states. More extended systematic research into

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material characteristics in the 1960s and 1970s [1] brought valuable information both on quality and characteristics of Czechoslovak steels.

In recent time, introduction of new technologies and vacillating endeavour of Czech producers to assimilate the quality to international market are characteristic for production. In our contribution, we will present the results of material tests of the steels S235 and S355 produced in the Czech Republic during several recent years.

2. Mechanical characteristic of material

Traditionally, material yield strength, tensile strength and ductility have been studied among the mechanical characteristics of the structure steels. For dimensioning the structures, the yield strength is the most important above all.

In Fig. 1, the basic quantities derived from the histogram of yield strength values important for determination of design tensile strength of structural steels are presented in a simplified form:

- The characteristic value X_k with 95% statistical guarantee of maintaining the value mentioned, i.e. with 5% risk of not maintaining it. However, the nominal value X_n, is being declared and verified in frame of metallurgical process; this value should be consistent but usually it equals the characteristic value only approximately.
- The design value X_d with statistical guarantee of 99.9% maintaining this value, i.e. with nearly 0.1% risk of not maintaining it (for current steel structures).
- The material safety factor γ_{M0} is the relation of nominal value to design one.

In a short time, the unified European standard EN 1993-1-1 will be valid for the designing of steel structures; through national annexes, as expected, that the standard will recommend an unusually strict material safety factor $\gamma_{M0} = 1.0$, derived from the West European evaluation programme for hot-rolled IPE beams. In an endeavour to contribute to the European methodology of steel structures' design according to limit states, we will present the statistic characteristics of several materials and geometrical characteristics of Czech steels products.
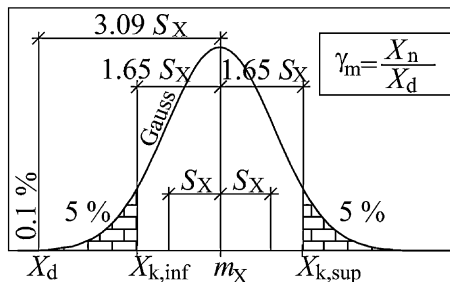

Fig. 1. The characteristic and the design value.

2.1. Experimental yield strength results obtained from plates made of S235 and S355 steels before 1999

Based on the result of plated steels with thicknesses from 9.5 to 30 mm made of S235 and S355 steel grades before 1999, the mean value $m_{fy} = 295.69$ MPa and the standard deviation $S_{fy} = 26.76$ MPa were determined [2]. The yield strength distribution of steel S235 presents a small and practically negligible skewness, so that it can be substituted, with advantage, by normal Gaussian distribution.

On the contrary, the distribution of yield strength values in plates with thicknesses from 9.5 to 30 mm made of steel S355 is asymmetrical, evidently due to the exclusion of values not satisfying the requirements of material standards for that tensile strength class, and therefore they are reclassified to lower tensile strength classes. The yield strength average value of S355 steel is $m_{fy} = 401.65$ MPa, the standard deviation, $S_{fy} = 36.80$ MPa, and skewness, $a_{fy} = 0.45$, see Fig. 7, [2].

It follows from the yield strength experimental results for plates made of steels S235 that the real characteristic yield strength value is approximately 251 MPa, so that the probability of not maintaining the guaranteed nominal value of 235 MPa is less than 5%. For steels of grade S355, the situation is just the opposite. The real characteristic value is, for the set evaluated, 346 MPa, so that the probability of not maintaining the assumed yield strength nominal value, 355 MPa, is about 8.6%, this being worse than the supposed 5% [2].

2.2. Experimental results of yield strength in plated steels S235 and S355—year 2001

The set of measured yield strength and tensile strength values is presented in Figs. 2 and 3. The measurements more or less distant in comparison with the other values are presented in the upper right corner. This contributes to a higher skewness value for tensile strength, see Table 1. However, the yield strength of these values was real. For those reasons besides others, we maintained the data within the chosen ones.

The yield strength characteristic values are given in Table 2. The design value determined according to [3] as 0.1 percentile was defined (with neglecting the cross-section area variability). According to our experience, the statistic output file is approximated, in the optimal way, by the Hermite four-parameter distribution, which takes into account even the skewness and kurtosis of random yield strength [4], see Fig. 2. The real yield strength value based on the Hermite distribution is 252.8 MPa, so the probability of not maintaining the guaranteed value, 235 MPa, is remarkably lower than 5%. The design value, 212.9 MPa, approximately corresponds to the standard value, 213.6 MPa.

The yield strength characteristic value for steel S355 determined from the Hermite probability distribution is approximately equal to the value of 357.2 MPa, see Table 4, hence it is satisfactory because 357.2 > 355 MPa. The design value, 334.0 355 MPa/1.1 = 322.7 MPa, is satisfactory too. It is evident from Tables 1–4 that the characteristic value determined as 5% quantile is not sensitive to the distribution type.
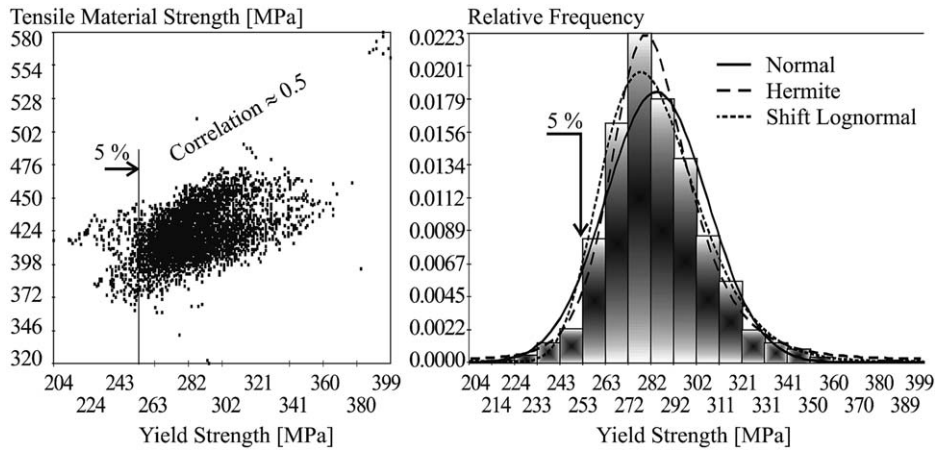
Fig. 2. Statistic characteristics of material characteristics of the S235 steel.

2.3. Experimental results of mechanical characteristics of hot-rolled IPE profiles made of S235 steel—year 2000

Hot-rolled beams are often used for practical designing of steel structures. Mechanical characteristics of these beams are evaluated, based on samples taken from one-third of a flange. In Figs. 4 and 5, there are histograms given of material yield strength, tensile strength of the IPE 160–IPE 220 profiles made of S235 steel, evaluated according to the experimental results of measurements.
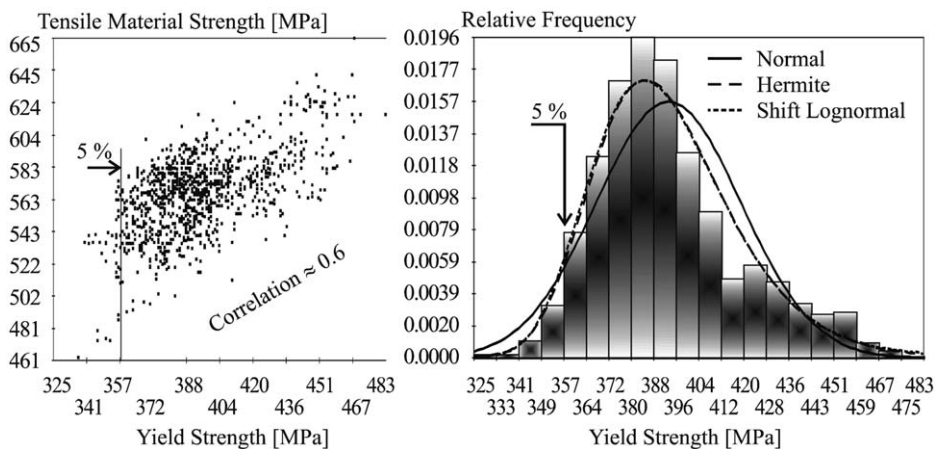

Fig. 3. Statistic characteristics of the S355 steel material characteristics.

Table 1
Mechanical characteristics of steel S235—valid observations: 5493

Quantity	Mean value	Standard deviation	Skewness	Kurtosis	Minimum value	Maximum value
Yield strength	284.5 MPa	21.5 MPa	0.60239	4.5991	204.0 MPa	399.0 MPa
Tensile strength	422.0 MPa	19.5 MPa	0.78929	8.0388	320.0 MPa	580.0 MPa
Ductility	37.9%	3.1%	−0.39691	3.8418	21.2%	49.8%

Table 2
Characteristic and design values of steel S235—valid observations: 5493

Quantile	5%			0.1%		
Distribution function	Normal	Hermite	Shift lognorm	Normal	Hermite	Shift lognorm
Value	249.1 MPa	252.8 MPa	253.0 MPa	218.0 MPa	212.9 MPa	234.0 MPa
EC3		235.0 MPa			235.0/1.1 = 213.6 MPa	

Table 3
Mechanical characteristics of steel S355—valid observations: 1089

Quantity	Mean value	Standard deviation	Skewness	Kurtosis	Minimum value	Maximum value
Yield strength	393.5 MPa	25.4 MPa	0.72471	3.3134	325.0 MPa	483.0 MPa
Tensile strength	566.1 MPa	25.1 MPa	−0.24562	4.4857	461.0 MPa	665.0 MPa
Ductility	30.963%	2.9128%	0.51372	4.6580	22.00	44.80%

Table 4
Characteristic and design values of steel S355—valid observations: 1089

Quantile	5%			0.1%		
Distribution function	Normal	Hermite	Shift lognorm	Normal	Hermite	Shift lognorm
Value	351.7 MPa	357.2 MPa	357.3 MPa	315.0 MPa	334.0 MPa	337.0 MPa
EC3		355.0 MPa			355.0/1.1 = 322.7 MPa	

In Fig. 4, the relative frequency histogram of yield strength was approximated by the Gaussian, Hermite and shift lognormal distributions of probabilities [4].

The results of the statistical analysis are presented in Table 5. According to Table 5, it is evident that yield strength shows only very small standard deviation, which also favourably influences both characteristic and design values. The characteristic value determined as 5% quantile is satisfactory as it is higher than 235 MPa in all cases. The design strength R_d in Table 6 was determined, in agreement with
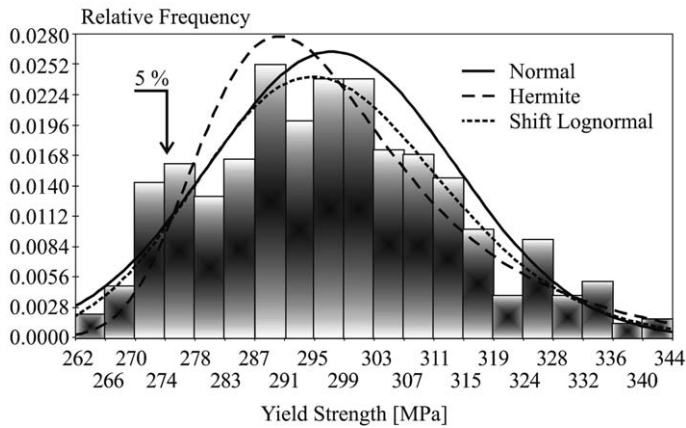
Fig. 4. Statistic characteristics of S235 steel for rolled IPE profiles.

[3], as 0.1 percentile from the function $R = f_y \cdot A$ (f_y—yield strength, see Fig. 4, A— cross-section, see Fig. 7) by the Monte-Carlo method for 200 000 simulation runs. The design value 0.1% quantile was determined as the 200th minimal value in random choice of quantity R. The design value, 255.0 MPa, determined by this way allows to consider, for hot-rolled IPE profiles also, the introduction of material safety factor $\gamma_{M0} = 1.0$. The histogram of the tensile material strength is presented in Fig. 5.

The correlation matrix is presented in the relation (1). It is evident from correlation matrix (1) that a certain positive correlation exists between the yield strength
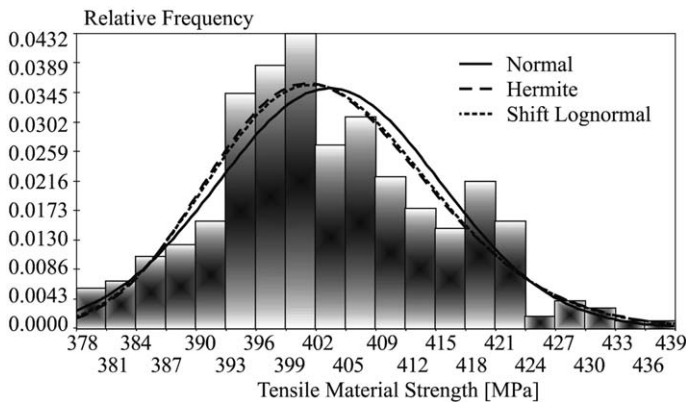

Fig. 5. Statistic tensile material strength characteristics of S235 steel for rolled IPE profiles.

Table 5
Mechanical characteristics of steel S235—valid observations: 562

Quantity	Mean value	Standard deviation	Skewness	Kurtosis	Minimum value	Maximum value
Yield strength	297.3 MPa	16.8 MPa	0.32462	2.5415	262.0 MPa	344.0 MPa
Tensile strength	403.8 MPa	11.3 MPa	0.32600	2.8447	378.0 MPa	439.0 MPa
Ductility	37.8%	2.9%	−0.75266	5.1796	27.1%	46.2%

and tensile strength values. Further on, the inverse proportion (i.e. negative correlation) can be observed between yield strength and ductility values, and between tensile strength and ductility, as well.

$$K \approx \begin{matrix} \text{Yield strength} & \text{Tensile strength} & \text{Tensility} \\ \begin{bmatrix} 1 & 0.57 & -0.32 \\ 0.57 & 1 & -0.18 \\ -0.32 & -0.18 & 1 \end{bmatrix} \end{matrix}. \quad (1)$$

The characteristics of samples presented in Tables 5 and 6 are normally taken from one-third of flange thickness. According to the experimental investigation of hot-rolled profiles IPE and HE [5], yield strength in flanges is lower than that in web due to non-uniform cross-section cooling down. The maximum deviation of beams HE200 was 26%.

When studying the web, this aspect cannot be neglected, taking into consideration the significance of flanges for the profile resistance. In connection with behaviour of various types of cross-section under tension, pressure, bending, etc., it is necessary to realize that it is not sufficient to evaluate the cross-section material characteristics by taking the samples from one cross-section point only. If we follow the load-carrying capacity of a tensile stressed beam, higher yield strength values of the profile web can contribute to increase of beam load-carrying capacity remarkably. It can be contrary in the case of simple bending, when the highest stress values are observed in extreme cross-section fibres.

By taking the samples only from one point on the flange, valuable information on yield strength variability along cross-section is neglected. After cutting both flange and web into a certain number of segments, we would obtain different yield

Table 6
Characteristic and design values of steel S235—valid observations: 562

Quantile	5%			0.1%
Distribution function	Normal	Hermite	Shift lognorm	200th minimal value in 200 000 simulation runs
Value	269.6 MPa	275.2 MPa	271.2 MPa	255.2 MPa
EC3		235.0 MPa		$f_y/\gamma_{M0} = 235.0/1.1 = 213.6$ MPa
				$(f_y/\gamma_{M0} = 235.0/1.0 = 235.0$ MPa$)$

strength values between neighbouring segments. The yield strength values of neighbouring segments would be probably very similar, i.e. they would be strongly correlated. With increasing distance among segments, the correlation coefficients would necessarily have to decrease. According to theoretical studies [6,7], the yield strength variability along the cross-section influences the variability of load-carrying capacity of short beams to a certain extent. When neglecting this variability, the load-carrying capacity scattering is higher than in the case of it being taken into consideration.

Similar results were also obtained for steel frames when studying the influence of yield strength variability on variability of the frame load-carrying capacity [8]. In all cases, the load-carrying capacity has been computed by non-linear FEM methods. The differences among the load-carrying capacity design values determined according to EC1 as 0.1% quantile can reach nearly up to 10%.

In the case of yield strength, it is therefore required to know the yield stress distribution along cross-section in more detail. The only right way to obtain information on yield strength variability in a hot-rolled profile is to carry out material tests by taking the samples from different profile points [6]. A suitable auto-correlation function and correlation length of a random field could be recommended as the output information.

2.4. Taking into account the material characteristics variability in stochastic models

In the document JCSS [9], the use of biparametric lognormal distributions with diameter m_{fy} is recommended for yield strength; it is given by the relation

$$m_{fy} = f_{y, nom} - \exp(-u \cdot v) - C, \tag{2}$$

where $f_{y,nom}$ denotes the nominal (characteristic) value of the steel considered, u is the normalized random quantity with normal distribution, for which a value from -1.5 to -2 is supposed, v denotes the variation coefficient, and C is the tensile strength reduction obtained on samples in static strength testing machine. The value of 20 MPa is given for the reduction C which generally depends on the loading rate. According to the available experience for S235 steel, this relation is satisfactory, when supposing $u = 2$ and $C = 20$ MPa. For variation coefficient $v = 0.08$, the average yield strength comes out as follows:

$$m_{fy} = 235 \exp(+2 \times 0.08) - 20 = 256 \text{ MPa}. \tag{3}$$

This value corresponds well with the above experimental results (see Table 1), 284.5 MPa $-$ 20 MPa $=$ 264 MPa, and is in good agreement even with the results [10,11].

3. Geometrical characteristic of cross-section dimension

Further, the experimental results evaluated were cross-section geometrical characteristics. Statistic quantities h, b_1, b_2, t_1, t_{21}, t_{22} were statistically evaluated from experimental measurements, see Fig. 6. In production, maintaining dimensions and
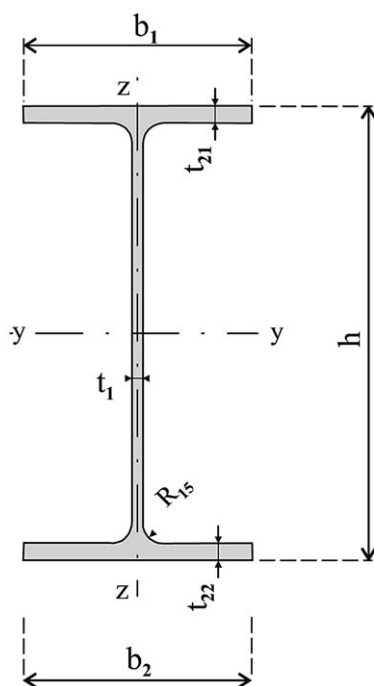
Fig. 6. Cross-section IPE.

mass of members are the geometrical characteristics verified above all. Limit deviations of dimensions and of geometrical forms are given in the Tolerance Standard EN 10034:1993. The limit deviation of real mass from theoretical one is $\pm 4\%$ for individual members.

When studying the cross-section geometry, it is appropriate to concentrate on the relative cross-section area above all, which is defined by ratio of actual cross-section area to nominal area. Steel hot-rolled IPE 160–IPE 240 beams were measured and evaluated. The histogram of relative cross-section area is given in Fig. 7.

It is evident from the relative frequency histogram in Fig. 7 that only in 2% of the members the limit deviation of cross-section area is less than 4%. When drawing a straight line through the values measured, it is evident that the relative area of larger cross-sections is in average lower than that of smaller cross-sections.

When studying the cross-section bending resistance, it is necessary to evaluate the plastic cross-section module statistically, see Fig. 8. Here it is evident, as well, that the average value of relative area is lower in larger cross-sections.

Statistic characteristics are clearly given in Table 7. The percentage of cases is given in the column marked "<1" where the measured value was lower than the nominal one. With an exception of the upper flange thickness t_{21}, the value of more than half of realization values for all the other quantities was higher than the nominal one. It was found for the flange that approximately in 96% of cases, the
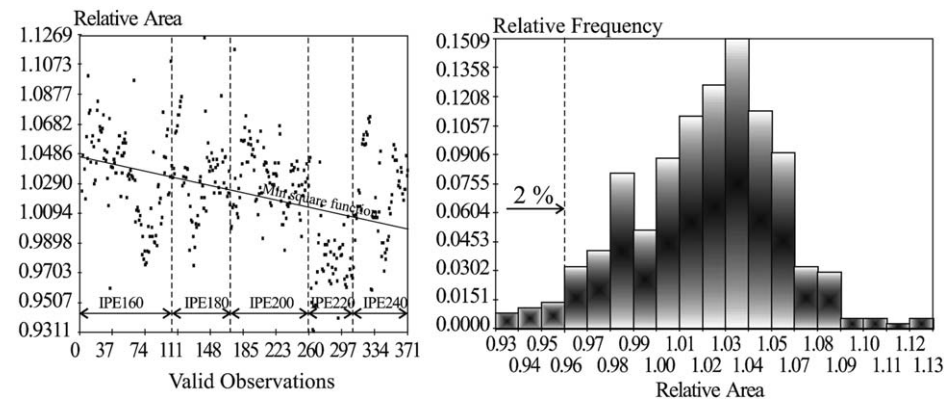
Fig. 7. Graphical representation of variability of relative IPE cross-section area.

thickness was higher than the nominal one. The maximum relative thickness found was 1.3 (exceeding the nominal value by 30%). A large representation of distant values higher than 1 is evident also in case of the value of kurtosis 7.473, see Table 7.

Flanges are hot-rolled approximately regularly both in positive and negative tolerances. The average values t_{21}, t_{22} are a little lower than 1. Statistic characteristics of the relative cross-section module are, however, satisfactory, i.e. the cross-section is not bending weakened. The deviation from nominal value was higher than 4% in 2% of real cross-section area only, see Fig. 7.

When applying the measurement results, valuable information on correlation coefficients of quantities measured can be found in stochastic models. The correlation matrix evaluated based on 371 measurements is presented in relation (4). The matrix draws attention to certain connections among geometric characteristics measured.

In matrix (4), strong correlations between width of the upper and the lower flange ($K[2, 3] = 0.6227$) and between thickness of the upper and the lower flange ($K[2, 3] = 0.7634$) are seen. Very strong correlations were found also between thickness of the upper and the lower flange and the area A ($K[5, 7] = 0.8103$, $K[6, 7] = 0.8451$) and the modulus W_{pl} ($K[5, 8] = 0.5886$, $K[6, 8] = 0.8918$).

$$
\begin{array}{cccccccc}
h & b_1 & b_2 & t_1 & t_{21} & t_{22} & A & W_{\mathrm{pl}} \\
\end{array}
$$

$$
K \approx \begin{bmatrix}
1 & -0.0068 & 0.0534 & 0.0399 & -0.0686 & -0.0989 & 0.0901 & 0.1092 \\
-0.0068 & 1 & \mathbf{0.6227} & -0.2142 & -0.2681 & -0.1456 & -0.1196 & -0.0694 \\
0.0534 & \mathbf{0.6227} & 1 & -0.2132 & -0.1596 & -0.0423 & -0.0296 & 0.0394 \\
0.0399 & -0.2142 & -0.2132 & 1 & 0.2368 & 0.2451 & \mathbf{0.6482} & 0.4712 \\
0.0686 & -0.2681 & -0.1596 & 0.2368 & 1 & \mathbf{0.7634} & \mathbf{0.8103} & \mathbf{0.8586} \\
-0.0989 & -0.1456 & 0.0423 & 0.2451 & \mathbf{0.7634} & 1 & \mathbf{0.8451} & \mathbf{0.8918} \\
0.0901 & -0.1196 & -0.0296 & \mathbf{0.6482} & \mathbf{0.8103} & \mathbf{0.8451} & 1 & \mathbf{0.9717} \\
0.1092 & -0.0694 & 0.0394 & 0.4712 & \mathbf{0.8586} & \mathbf{0.8918} & \mathbf{0.9717} & 1
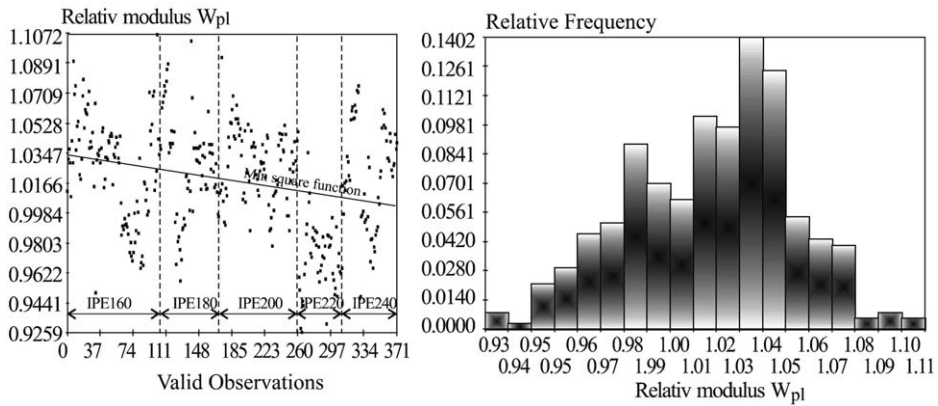\end{bmatrix}. \quad (4)
$$

Fig. 8. Graphical representation of variability of relative IPE cross-section module.

4. Sensitivity analysis

For the quantities measured, we are usually interested in how they influence the structure response dispersion, e.g. the influence of load-carrying capacity. By means of the sensitivity analysis, the quantities influencing the structure response to a high degree [12] can be identified. Therefore, the quality control activities can be focused on ensuring the satisfactory stability in statistical parameters or on decreasing their random variability and ensuring the reliability [13].

The load-carrying capacity of tensile beam can be defined as $N = A \cdot f_y$; that of bent beam, $M = W_{pl} \cdot f_y$. The area of prismatic cross-section $A = 2 \cdot b \cdot t_2 + (h - 2 \cdot t_2) \cdot t_1$; the plastic cross-section module $W_{pl} = b \cdot t_2 \cdot (h - t_2) + (h/2 - t_2)^2 \cdot t_1$. The sensitivity analysis will be evaluated in the form of Spearman rank–ordinal correlation coefficients between input random quantities f_y, t_1, t_2, h, b (represented by histograms, see Fig. 4 and Table 7) and by the load-carrying capacities N and M. Between the quantities t_{21} and t_{22}, there is, according to (4), a very

Table 7
Relative statistic characteristics of IPE profiles—valid observations: 371

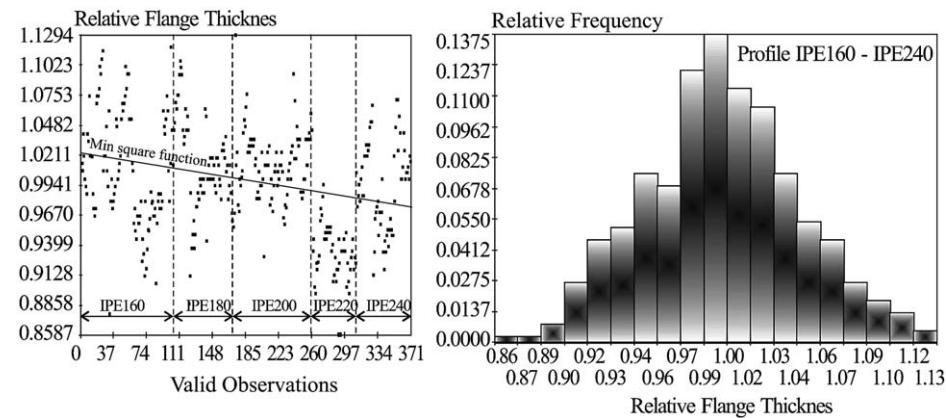| Quantity	m	S	Minimum value	Maximum value	<1 (%)	Skewness	Kurtosis
h	1.001	0.00443	0.989	1.013	34	−0.4063	3.015
b_1	1.012	0.01026	0.975	1.049	8	−0.3939	4.239
b_2	1.015	0.00961	0.975	1.037	5	−0.5448	3.887
t_1	1.055	0.04182	0.949	1.300	4	1.0545	7.473
t_{21}	0.988	0.04357	0.880	1.094	55	−0.2991	2.663
t_{22}	0.998	0.04803	0.858	1.129	47	0.3303	2.766
A	1.025	0.03245	0.931	1.127	21	−0.2152	3.076
W_{pl}	1.019	0.03347	0.926	1.107	31	−0.2583	2.650

Fig. 9. Graphical representation of the lower flange thickness variability.

strong correlation, therefore the histogram of the quantity t_{22} only was considered for simplicity sake, see Fig. 9. The sensitivity analysis was evaluated for the profile IPE180 by applying the Monte-Carlo numerical simulation method (1000 simulation runs).

It is evident from Fig. 10 that the most sensitive variability of load-carrying capacity responds to the yield strength and to the flange thickness variability as well. These quantities should be controlled with increased accuracy in the manufacturing process.

5. Conclusion

It has been found from the experimental results by statistic elaboration that for hot-rolled steel IPE beams, the value of $\gamma_{M0} = 1.0$ can be assumed with reliability.
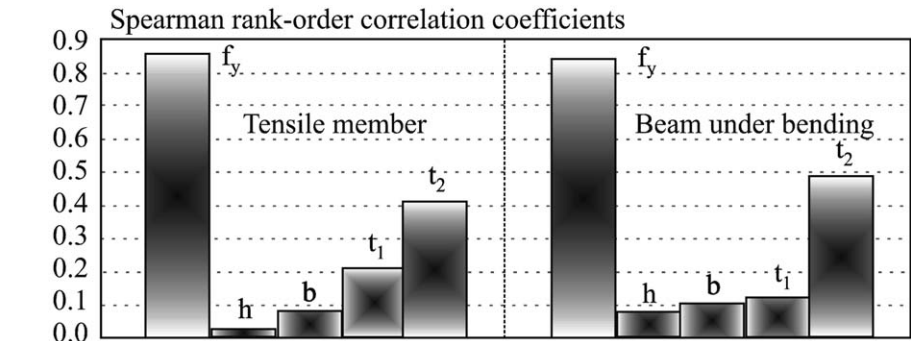

Fig. 10. Sensitivity analysis between load-carrying capacity and input quantities.

It followed from the results of sensitivity analysis that the design strength showed the maximum sensitivity to yield strength variability, as well as to that of flange thickness. These quantities should be checked with increasing accuracy in the manufacturing process. In comparison with the data from previous time periods [1,2] the production quality is evidently increasing.

As far as the geometrical characteristics of hot-rolled IPE 160–IPE 240 profiles are concerned, it has been found that the average relative cross-section area is higher than the nominal value. It was evaluated that in 96% of cases, the web thickness value was higher than the nominal one (by 30% maximum). For flanges, approximately one-half of measured results was lower and the other half was higher than the nominal value. In 2% of real cross-section area only, the deviation from nominal value was higher than 4%, which is satisfactory.

For some evaluated cases, it will therefore be possible to consider the material safety factor γ_M in agreement with the new European Standard for designing the steel structures EN 1993-1-1, with the value $\gamma_{M0} = 1.0$; nevertheless, its general application to the other assortments of metallurgical products (sheets, plates, high-strength steels) has not been verified and justified to the extent required until now. In this connection, attention needs to be paid to stability phenomena both of beams (buckling [7], lateral beam buckling [6]), and of frame systems too [8].

Acknowledgements

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