Structural Safety 53 (2015) 26–35

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Statistical analysis of the material properties of selected structural carbon steels

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ARTICLE INFO

Article history:
Received 12 August 2014
Received in revised form 3 December 2014
Accepted 28 December 2014
Available online 19 January 2015

Keywords:
Structural carbon steel
Stress–strain curves
Strain hardening
Yield plateau
Statistical analysis
Regression analysis
Elastic–plastic buckling
Safe design

ABSTRACT

Modern design procedures for steel structures increasingly employ more realistic representations of the stress–strain behaviour of steel rather than a simple ideal elastic–plastic. In particular, for buckling failure modes in the plastic range, stresses in excess of the yield stress are always involved, together with a finite post-yield stiffness. Moreover, the 'plastic plateau' in buckling curves for stocky structural members cannot be predicted computationally without a significant strain hardening representation. If a good match is to be sought between experiments and computational predictions in the elastic–plastic zone, strain hardening must be included. Most studies have either used individual laboratory measured stress–strain curves or educated guesswork to achieve such a match, but it is not at all clear that such calculations can reliably be used for safe design since the same hardening properties may not exist in the next constructed structure, or even within a different batch of the same steel grade.

A statistical exploration is presented here to assess the reliable magnitudes of post-yield properties in common structural grade steels. For simplicity, only two critically important parameters are sought: the length of the yield plateau and the initial strain hardening tangent modulus. These two are selected because they both affect the elastic–plastic buckling of stockier structural elements. The statistical analyses exploit proprietary data acquired over many years of third-party auditing at the Karlsruhe Institute of Technology to explore possible regressed relationships between the post-yield properties. Safe lower bounds for the selected properties are determined.

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1. Introduction

Early design concepts for structural members treated the behaviour as linear-elastic and limited the maximum stress to an 'allowable stress' related to a yield stress. Since an axially compressed stocky column is a structural form in which the mean axial stress can clearly exceed the yield stress before failure, early treatments of inelastic buckling such as those of Engesser [1,2] and Considère [3] used a fully nonlinear stress–strain curve. Their work was later extended into extensive buckling strength predictions for simple columns by Chwalla [4]. But in the same period, Jezek [5] was able to produce predictions for the strength of members under both axial load and bending provided the stress–strain curve was treated as ideally elastic–plastic. This difference indicates the simplicity that was then needed to address more complicated situations. With the development of the plastic theory of structural collapse [6,7], coupled with application to mild steel structures whose stress–strain relationship possesses a distinct yield plateau, it was highly desirable to continue with this ideal elastic–plastic model.

From that point onwards, the stress–strain relation for most metals was usually characterised by only two parameters (Young's modulus E and a notional yield stress σ_y) and it became internationally entrenched in both investigations of structural behaviour and design calculations. Unfortunately, this two parameter model presents a problem for precise computational predictions of the strength both of individual members and of complete structures because it implies that finite length columns cannot attain the squash load, that the full plastic moment in bending cannot be exceeded and that other configurations involving compression elements of finite slenderness cannot strictly ever achieve full plasticity as they would theoretically require infinite ductility to do so (Fig. 1). By contrast, all experiments show that the true resistance systematically exceeds the fully plastic value in moderately stocky elements and structures, and this is usually only possible due to

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http://dx.doi.org/10.1016/j.strusafe.2014.12.002
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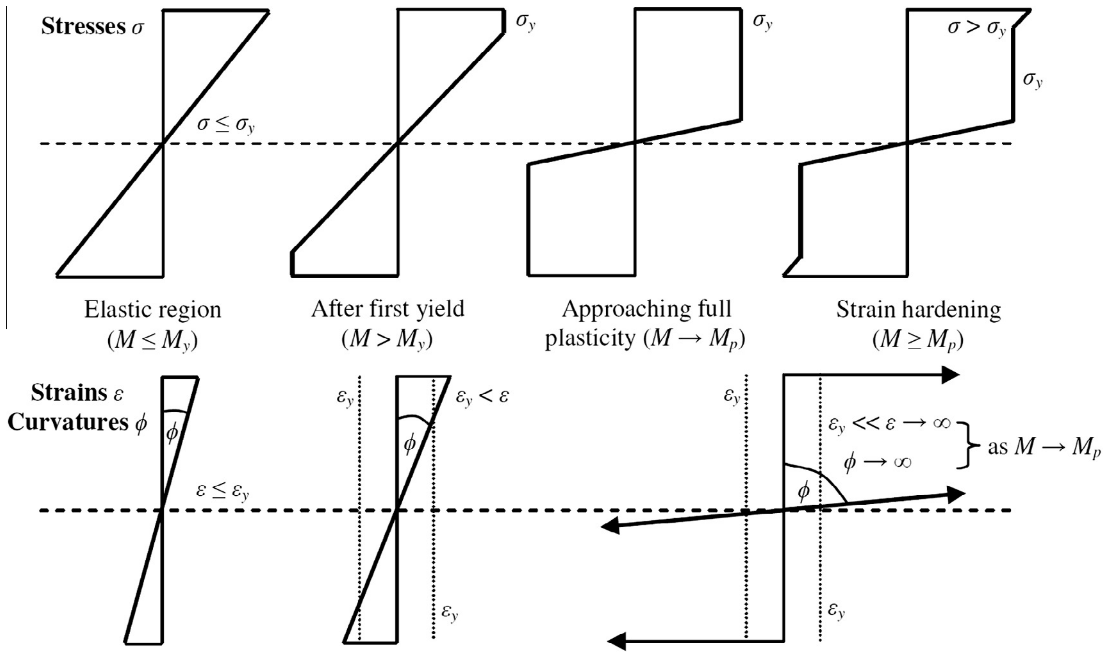

Fig. 1. Development of the stress and strain distributions under bending in a structural member: the full plastic moment M_p can only be attained with strain hardening.

strain hardening in the metal. Almost all current international design rules permit moderately stocky structures to attain a fully plastic state, but this is justified by empirical deductions from tests that are used to determine a limiting slenderness above which the full plastic resistance can no longer be attained.

The advent of computer evaluations of member strengths permitted much more sophisticated models of material behaviour to be used, but all computations relating to specific applications appear to have been based on an individual measure of the stress–strain curve obtained in the specific test series. It was tacitly assumed that what was measured in a particular series of laboratory tests would be relevant to all geometries and all international production of the same grade of steel. Unfortunately, current international standards for structural steel production do not define parameters other than the 0.2% proof stress, the ultimate tensile strength (UTS) and the elongation to rupture [8–14], so reliable and safe values for the strain hardening behaviour and yield plateau length are not commercially documented for any structural steel. If computer models are to be used to produce general recommendations for all structural elements, this situation poses a considerable challenge. When assessing the strength of stockier members and structures, it is difficult to be certain that any value derived from a single test series will produce safe estimates of the strength of all similar members. Thus it is not possible to produce safe and economical design calculations for all structures without a statistical review of existing measured steel stress–strain data.

In recent decades, experiments have become so expensive that computational modelling is more and more widely applied, and it is very difficult to justify the cost of experiments for every new investigation, especially for larger structural systems or studies where many variable parameters are involved. It is thus increasingly common that a heavy reliance is placed on computational predictions, but the safety of these calculations in the post-elastic range very much depends on the assumed ductility and strain-hardening properties. For carbon steels, the post-yield properties must include the length of the yield plateau. It is therefore critically important that more wide-ranging investigations of these post-yield properties are soundly grounded in statistical treatments. The range of structural forms, geometries, load cases and boundary conditions that require a reliable post-yield plastic characterisation is very wide and far beyond all currently available test evidence.

Uncertainties concerning the material strength are either treated in structural engineering limit state design through the concept of a 'characteristic' value, which is notionally statistically based and has a prescribed fixed probability of not being attained in a hypothetically unlimited series of tests [15,16], or an alternatively defined 'nominal' value [11,17] that has some other basis in experimental data. In either case, it is then multiplied by a 'partial factor' or 'resistance factor' that depends on the failure mode to obtain a 'design' value of the structure's strength which is then used to achieve a desired margin of safety or reliability. Thereafter the entire design process is usually deterministic. Initiatives to develop fully probabilistic structural design methods do exist [18–20] and coefficients of variation on loads, geometry and material properties have been incorporated into AISI S100 [11] and AISC 360-10 [17] LRFD provisions amongst others, but there is currently insufficient data to establish the necessary statistical bounds on all required parameters. Moreover the design process would be very complex and too laborious for all but monumental structures and failure investigations. For example, the experimental JCSS Probabilistic Model Code [20] proposes to treat material properties as random variables subject to the laws of probability but currently considers only the yield and ultimate strengths, the elastic modulus, Poisson's ratio and ultimate strain, with post-yield properties such as strain hardening and yield plateau length omitted due to a substantial lack of data [21].

A detailed study of over 40,000 mill test certificates of rolled wide flange (W), welded wide flange (WWF) and hollow structural (HSS) beam section samples mainly from ASTM A992 steels, representative of those most commonly produced for the US and Canadian markets [22], was performed by Schmidt and Bartlett [23,24]. These authors presented statistical relationships between the material properties (yield and ultimate strengths, modulus of elasticity) and geometric properties (flange/web thicknesses, web depths, diameter to thickness ratios) of these sections, and offered mean values and coefficients of variations on the most important material parameters as well as calibrating resistance factors for the Canadian limit state design provisions (currently CSA S16-14 [25]). The data set was even large enough to allow these to be reliably related to the known steel chemistry of the different samples. A parallel initiative was undertaken in the context of the AISC 360-17 [17] LFRD rules by Dexter et al. [26] who analysed a database of over 20,000 mill test certificates for various rolled shapes in a similar fashion. It is highly unfortunate that the post-yield material properties of strain hardening and yield plateau length could not be considered in these studies, as such information is generally not available on a mill test certificate.

In this context, it seems likely that many more academic institutions and materials testing laboratories worldwide possess treasure troves of data in the form of measured stress–strain curves from tensile tests which they have not yet exploited and disseminated fully. This paper presents a mathematical characterisation of the complete stress–strain curve for carbon steels to permit an accurate extraction of the relevant post-yield material properties, applied to a large experimental data set from the Karlsruhe Institute of Technology. The resulting extracted material properties are subjected to statistical analysis to establish confidence bounds for and explore the relationships between the different post-yield properties for particular grades of carbon steel. It is hoped that this paper will inspire others who have access to similar and much larger data sets to derive and publish corresponding measures so that a more complete statistical evaluation of these properties can be obtained for the benefit of the international engineering community.

2. Characterisations of stress–strain curves for carbon steels

It is well known that the locking of dislocations in structural carbon steels produces a very linear path up to a distinct proportionality limit, followed by a region of plastic flow at an approximately constant stress before rising smoothly up to a peak stress due to strain hardening, after which there is a reduction in applied stress due to local necking as the specimen becomes locally unstable and fracture occurs (Fig. 2) [27]. The presence of a yield plateau and slope discontinuities mean that a continuous characterisation is not possible. Instead, such stress–strain curves are usually characterised by a simplified piecewise-linear function in both computational and analytical studies that include strain hardening. The simplest and most widely used formulation is probably some variation on the following (Fig. 3):

$$\sigma(\varepsilon)=\begin{cases} E\varepsilon & \varepsilon\leqslant\varepsilon_y \quad \text{Linear elastic} \\ \sigma_y & \varepsilon_y<\varepsilon<(1+n)\varepsilon_y \quad \text{Yield plateau} \\ \sigma_y+E_h(\varepsilon-(1+n)\varepsilon_y) & (1+n)\varepsilon_y<\varepsilon<\varepsilon_u \quad \text{Linear strain hardening} \end{cases} \tag{1}$$

Here, σ_y is the yield stress and $\varepsilon_y = \sigma_y/E$ is the first yield strain. The 'length' of the yield plateau is defined as a multiple n of the first yield strain ε_y, such that the linear strain hardening region begins at a total strain of $(1 + n)\varepsilon_y$. Strain hardening is assumed to be linear, since in classical structural engineering the acceptable strains are generally not very large, so a single hardening modulus is defined as $E_h = hE$ where E_h is extracted from measured stress–strain curves as the initial strain hardening tangent modulus and h is its relation to the elastic value. For this simplified model, which does not attempt to model the true behaviour at large strains, the indicated total strain ε_u at the ultimate tensile stress σ_u is not an independent parameter but is deduced as:

$$\varepsilon_u=(1+n)\varepsilon_y+\left(\frac{\sigma_u-\sigma_y}{E_h}\right) \tag{2}$$

The piecewise-linear characterisation of Eq. (1) is not entirely satisfactory because it assumes a constant strain hardening stiffness up to the ultimate tensile stress, whereas real carbon steels exhibit a progressive loss in stiffness (Fig. 2). Eq. (2) thus produces an unconservative significant under-estimate of the true ultimate tensile strain, but it affects very few structural resistance calculations.

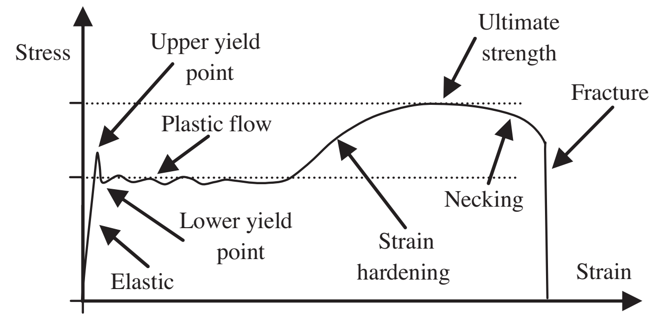

Fig. 2. Simplistic characterisation of a typical engineering stress–strain curve of low carbon steel (not to scale).

Neal's classic 1977 text [7] suggested that typical yield plateau lengths and strain hardening ratios are of the order of $h = 4\%$ and $n = 9$ respectively for structural steels. Approximations of student textbook figures showing 'typical' or 'idealised' stress–strain curves for structural steels suggest values ranging from $h \approx 2\%$ and $n \approx 6$ [28] to $h \approx 0.9\%$ and $n \approx 40$ [29]. For the purposes of computer-aided structural design, Rotter and Gresnigt [30] recommend a strain hardening modulus corresponding of a very conservative $h \approx 0.3\%$ together with a yield plateau length in the range of $9 < n < 14$, while EN 1993-1-5 [31] permits a linear strain hardening modulus of $E_h = E/100$ or $h = 1\%$ in computational limit state design irrespective of the steel grade and with no yield plateau. In research publications, Boeraeve et al. [32] analysed several stress–strain curves of S360 to S460 grade steels for the purposes of numerical validation of experimental results and found $h \approx 1.7\%$ and $n \approx 12$. The material models in their finite element analyses were, however, based only on a single stress–strain curve for each experiment. Gardner et al. [33] recommended $h = 2\%$ for cold-formed sections, while the numerical parametric study of tubular members in bending by Sadowski and Rotter [34] simply assumed $h = 2.5\%$ and $n = 0$ for a generic S250 grade carbon steel.

The inconsistency in assumed post-yield material properties is considerable and may be dangerously misleading. Indeed, very few of the above sources gave any type of rigorous justification for their particular choice. Lastly, the above assessments are concerned only with the material specification, but other factors such as residual stresses caused by the forming process, thermal effects and the strain history of the material add further uncertainty to the stress–strain relationship [35]. These considerations should be treated separately and lie outside the scope of this study.

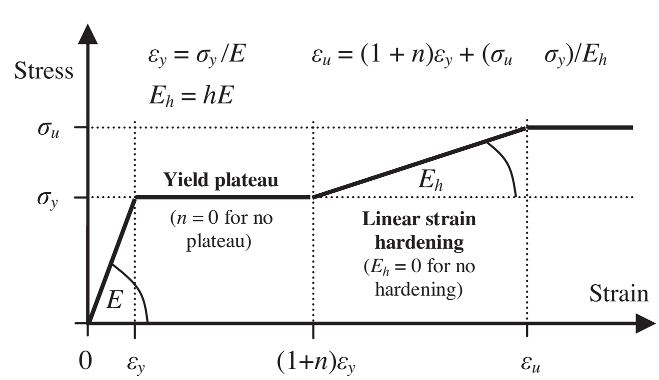

Fig. 3. Idealisation including strain hardening for design and computational purposes.

3. Experimental procedure

The purpose of this paper is to find statistically justifiable relationships and confidence intervals for the poorly-documented post-yield material parameters h and n in terms of the well-documented parameters σ_y and σ_u for implementation in a piecewise-linear material characterisation of structural carbon steels (Fig. 3). These crucial post-yield material properties are very rarely specified either by manufacturers or standards for structural steels. The study was performed on a database of stress–strain curves accumulated between 2010 and 2013 from commercial third-party auditing inspections carried out at the Research Centre for Steel, Timber and Masonry (*Versuchsanstalt für Stahl, Holz und Steine*) at the Karlsruhe Institute of Technology, Germany.

The stress–strain curves were obtained from standard tensile tests performed strictly according to the procedures of ISO 6892-1 [36] governing the coupon size and shape. The speed of loading had been controlled according to the provisions of 'Method B' which prescribes a nominal stress rate of between 6 and 60 MPa s^{-1} for materials with elastic moduli greater than 150 GPa. The strain rates were not controlled or directly recorded, but a numerical analysis of the full set of stress–strain curves suggests a well-defined mean strain rate of around of $2.5 \times 10^{-4}\ s^{-1}$ across all steel grades. The values in Fig. 4 are grouped according to steel grade and the number of stress strain curves for each grade is shown. These strain rates lie well within the order of magnitude accepted for 'quasi-static' testing [37–39] and within permissible codified bounds (e.g. ASTM A370-14 [40]) and suggest that the material properties extracted from the full set of curves are directly commensurable. However, the results of the individual tensile tests within the database can vary within the tolerance limits of these standards due to different testing machines with varying control systems and even the influence of individual operators. It should be stressed that the data was not collected with any research purpose in mind and, after being anonymised, was released for the present analysis on a strictly 'as is' basis with no possibility of obtaining further measurements.

The raw measured stress–strain curves, all in terms of engineering stresses and strains, were subjected to a careful preliminary screening. A curve was accepted into the final data set if it exhibited the typical characteristics of carbon steel (e.g., Fig. 2), including a clearly-defined proportional limit point, a yield plateau at approximately constant stress, followed by strain hardening up to an ultimate stress with fracture occurring after the peak was attained (Fig. 3). The documentation accompanying each curve was also checked to verify that precise details of the steel grade, component origin and dimensions of the specimen were known.

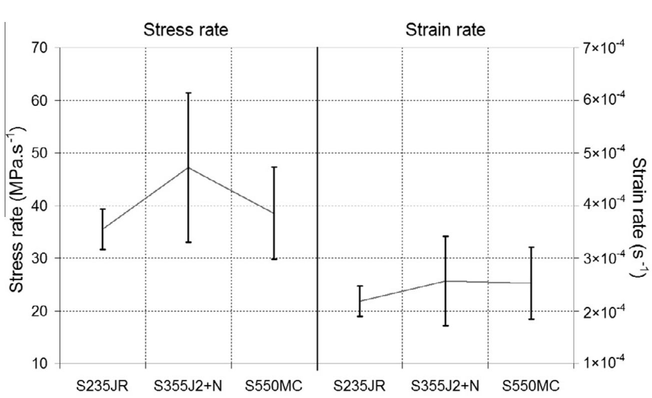

Fig. 4. Numerically-extracted mean stress and strain rates for the final accepted data set with error bars denoting the 95% confidence interval.

Once a curve passed each of these stages, it was included in the final data set with no further 'outliers' being removed. This process produced a data set of 174 stress–strain curves, summarised in Table 1 according to steel grade and specimen source. The S235JR grade also includes hollow sections designated as S235JRH as there is no difference in the source material. The JR and J2 designations refer to the Charpy impact test with 27 J at 20 °C and −20 °C respectively, the +N designation refers to the normalisation process, MC refers to thermomechanical rolling (M) and cold forming (C) while the three digit prefix refers to the nominal yield stress in MPa according to EN 10027-1 [41].

4. Processing of measured stress–strain curves

Every measured 'raw' stress–strain curve that passed the preliminary screening was carefully processed prior to the final algebraic characterisation. Many of the stress–strain curves exhibited a significant nonlinearity in the initial 'elastic' region which may be attributed either to elastic deformations in the test rig or to non-uniform deformations within a specimen that was imperfectly straight or to initially imperfect clamping conditions. Since the focus of this study is the accurate extraction of post-yield material properties, it was decided to discard entirely the initial 'elastic' region up to the commencement of the yield plateau which was assumed to begin at a reference strain ε_0 as measured by the test apparatus. Since stress–strain curves of mild carbon steel fortuitously exhibit a reasonably well-defined yield point (Fig. 2), it was possible to accurately choose the value of this reference strain ε_0 on the basis of a careful visual inspection of each curve. For the purposes of algebraic characterisation, however, the start of the yield plateau must correspond to the strain at first yield $\varepsilon_y = \sigma_y/E_{nom}$, where E_{nom} is the nominal elastic modulus conservatively taken to be 205 GPa. Thus the values of ε at each data point were transformed as $\varepsilon \rightarrow \varepsilon - \varepsilon_0 + \varepsilon_y$.

For strains greater than ε_y, the curve was next prepared for a least-squares fitting procedure to a pre-determined two-part continuous characterisation (Eq. (3)); Fig. 5). The yield plateau was characterised as a constant stress σ_y, found initially as the mean value for the range $\varepsilon_y \leqslant \varepsilon < \varepsilon_n$. The curved strain hardening region was assumed to begin at a strain ε_n and was found to be accurately represented by a 7th order polynomial. It is not suggested here that stress–strain curves should be systematically represented by such higher-order polynomials: this fit was simply a device to obtain an accurate estimate of the initial hardening tangent modulus E_h for use in the piecewise-linear material model (Fig. 3). A similar characterisation was previously used by the authors to statistically analyse tensile tests performed on spiral welded carbon steel tubes [35].

$$\sigma(\varepsilon) = \begin{cases} \text{ignored} & \varepsilon < \varepsilon_y \\ \sigma_y & \varepsilon_y \leqslant \varepsilon < \varepsilon_n \\ \sigma_y + \sum_{k=1}^{7} a_k(\varepsilon - \varepsilon_n)^k & \varepsilon \geqslant \varepsilon_n \end{cases} \tag{3}$$

The approximate boundary between the yield plateau and the strain hardening region was analysed visually to give an initial estimate for ε_n (Fig. 5a). The strain hardening region ($\varepsilon > \varepsilon_n$) was then isolated and subjected to a preliminary 7th order polynomial fit to obtain a set of trial coefficients $a_1 - a_7$. These were then used to determine a better value for ε_n, and a final fit was produced. The least-squares procedure applied to this data minimises an objective function that exhibits multiple local minima, so a careful choice of initial parameters is important to achieve an globally optimal representation of the measured stress–strain curve.

Table 1
Summary of data set according to EN steel grade, approximate ASTM grade equivalent, origin specimen and number of stress–strain curves in the final set.

EN steel grade designation	ASTM approx. equivalent grade	○ tube	□ tube	U-section	Sheet	Wedge	Other†	Observations
S235JR	A283C	70	7	26	17			120
S355J2+N	A527-50	4			25‡		2	31
S550MC	X80XLK				8	15		23
							Total	174

† Miscellaneous metal specimens not originating from any particular structural section.
‡ 10 specimens taken longitudinally and 15 specimens taken transversely to the rolling direction of the metal sheet.

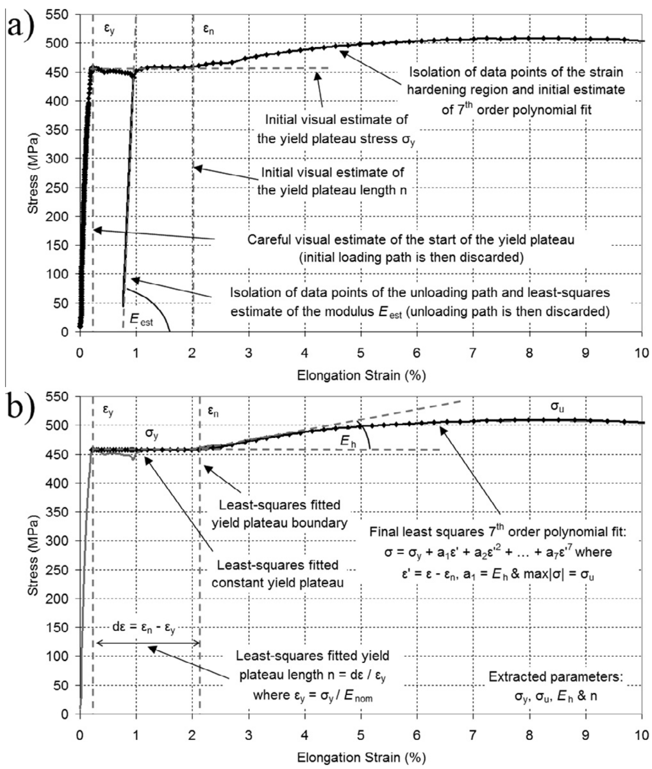

Fig. 5. Illustration of the least-squares fitting procedure to extract selected material properties from measured stress–strain curves: (a) pre-processing of the 'raw' curve and estimation of input values to the fitting procedure; (b) post-processing of the fitted curve and extraction of desired material properties.

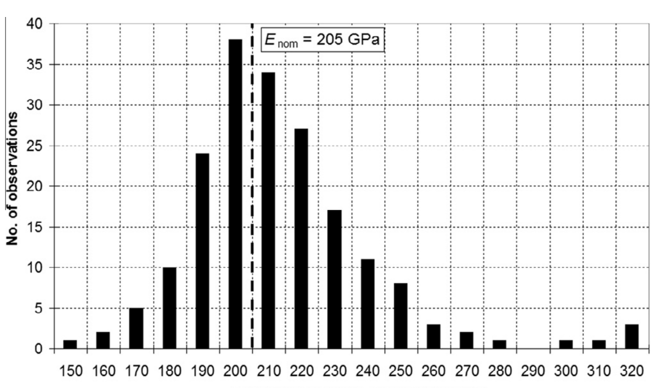

Fig. 6. Histogram of estimated moduli of elasticity E_{est} for the full data set.

The final curve fitting was performed using the nonlinear SOLVER function in Microsoft Excel® 2010, where the sum of the squared residuals between the fitted and measured values of σ at each value of ε was minimised by varying ε_n, σ_y and the coefficients $a_1 - a_7$. All the resulting fits to Eq. (2) exhibited a coefficient of determination $r^2 > 0.95$, providing an accurate characterisation of the plateau and hardening parts of the measured curve (Fig. 5b). The desired material properties were then deduced as: σ_y, $E_h = a_1$, $\sigma_u = \max|\sigma|$ and $n = (\varepsilon_n - \varepsilon_y)/\varepsilon_y$. The initial strain hardening modulus E_h was then identified dimensionlessly as a proportion of the nominal elastic modulus $h = E_h/E_{nom}$. The total strain ε_u at σ_u was not considered in this analysis because it is not an independent variable within the piecewise characterisation (Eq. (3)).

Most curves included an unloading–reloading path from the yield plateau which may be used to obtain an accurate estimate of Young's modulus E_{est} using a least-squares linear fit (Fig. 5a). The estimated values of E_{est} are shown in a histogram (Fig. 6) for all the steel grades considered in this study. There is a considerable scatter around the assumed nominal elastic modulus of E_{nom} = 205 GPa which follows an approximately log-normal distribution. The minimum and maximum values were found to be 149 GPa and 317 GPa, respectively with a mean of 208.1 GPa and a coefficient of variation (CV) of 13.2%, defined as σ/μ where μ and σ are the mean and standard deviation, respectively.

Traditional engineering practice has always accepted the elastic modulus as a material constant, with the text by Petersen [42] claiming a CV of only 1–3%, the study of Schmidt and Bartlett [23] suggesting values between 1.9% and 4.5% and Dexter et al. [26] suggesting 2.4% to 3.4%. Material properties with a CV this low can effectively be treated as deterministic, even in probabilistic design [43]. However, these reported CV values appear to be very low compared to the measurements presented in this study (CV = 13.2%), and indeed other sources report higher CVs at 6% [44] and 10.5% [45] although comparisons with data reported in older literature should be treated with care because steel fabrication properties have evolved significantly in the past five decades [22]. A detailed modern summary and review of various values is offered in Hess et al. [46]. It should be recognised that there are great technical difficulties in reliably measuring the elastic modulus because it is dependent on the strain path, the chemical composition, the orientation of the crystal lattice within the specimen and the heat treatment, whilst the stiffness of the measuring rig and minor errors in loading alignment may also introduce experimental scatter [46–48].

5. Descriptive statistics

The complete data set was first explored through simple descriptive statistics which included calculations of the mean, characteristic, minimum, maximum, nominal, standard deviations, standard errors (SE; standard deviation/$\sqrt{(\text{no. of observations})}$), coefficients of variation (CV), skew coefficients (Fisher–Pearson standardised moment coefficient adjusted for sample size [49]) and (excess) kurtosis for each of the four independent variables σ_y, σ_u, h and n for each available steel grade (Table 2), all performed using Minitab v. 16.2 [50]. The 'characteristic' material property refers to the 5th percentile (for unfavourable low values e.g., σ_y, σ_u and h) or the 95th percentile (for unfavourable high values e.g., n) as estimated from the available data set assuming a normal distribution (more accurate for larger sample sizes), while

Table 2
Summary statistics for the full data set.

	S235JR (obs = 120)				S355J2+N (obs = 31)				S550MC (obs = 23)			
	σ_y	σ_u	h	n	σ_y	σ_u	h	n	σ_y	σ_u	h	n
Mean	410.1	465.9	1.08	13.0	405.7	569.7	2.36	6.6	630.8	678.1	0.87	9.5
Characteristic	316.2	384.2	0.32	6.5	353.0	538.9	1.06	3.1	576	623	0.40	1.6
Min.	278	331	0.04	3.2	350	536	0.97	2.3	575	622	0.39	1.1
Max.	578	621	2.44	30.9	602	670	3.09	12.5	705	738	1.61	14.4
Nominal	235	360	n/a	n/a	355	470	n/a	n/a	550	600	n/a	n/a
St. dev.	53.1	51.5	0.45	5.54	69.1	29.1	0.53	2.39	33.7	37.3	0.28	3.47
SE	4.9	4.7	0.04	0.51	12.4	5.2	0.10	0.43	7.0	7.8	0.06	0.72
CV	12.96	10.97	40.79	39.77	17.04	5.1	22.62	36.38	5.3	5.5	31.45	36.45
Skew	0.27	0.39	0.43	0.61	0.70	0.22	−0.08	0.15	0.54	0.11	0.61	−0.63
Kurtosis	1.16	0.79	0.95	0.03	0.60	−0.34	−0.19	−1.11	0.08	−1.34	1.14	0.18

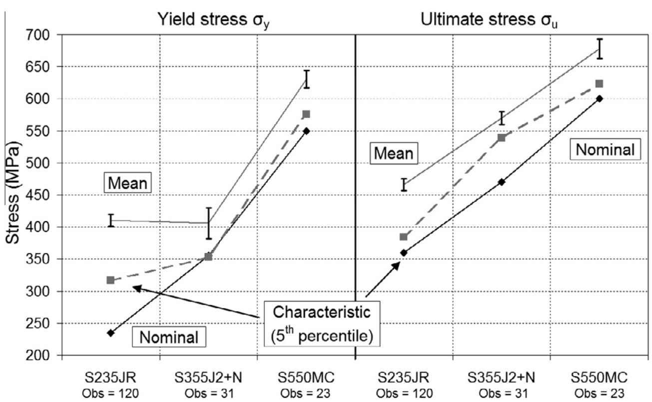

Fig. 7. Line plots of mean, characteristic and nominal (minimum codified) yield and ultimate stresses σ_y and σ_u for each steel grade with error bars denoting the 95% confidence interval.

'nominal' refers to the minimum specified material property as given by the relevant technical delivery standard. The standard error (SE), which is the standard deviation of the estimate of the population mean accounting for the volume of data in the sample, may be multiplied by ±1.96 to calculate approximate 95% Confidence Intervals (CIs) around the sample mean [51,52]. There is a 95% probability that the true population mean for each variable lies within these error bars. The skew is an indirect measure of the extent to which the given sample distribution lacks symmetry about the mean, with positive and negative skews indicating that the data is concentrated below and above the mean respectively. The kurtosis is a measure of the peakedness of the distribution with a high positive value indicating a sharp peak and a high negative value indicating a flat distribution. These two properties can be used to assess normality: ideally both should be close to zero, but an absolute value greater than unity in either measure suggests significant deviations from normality [53].

The two stress variables σ_y and σ_u were found to exhibit CVs ranging from 5% to 17% with an average of 9.5%, suggesting a fairly narrow distribution with most observations clustered about a reasonably well-defined sample mean (Fig. 7). By contrast, the two variables h and n exhibit CVs ranging from 22% to 43% with an average of 35%, suggesting a much larger scatter of observation (Fig. 8). The accurate evaluation of h and n is highly dependent on precise measurements of strains which the 'stress-only' variables σ_y and σ_u are not, and the higher dispersion is a consequence of the greater difficulty inherent in accurately measuring strains rather than stresses [20,42]. This is because strains are always numerically very small so any disturbances present in the equipment during testing, however minor, are likely have a disproportionate effect on the deduced values.

Each variable was found to exhibit reasonably small SEs and error bars around the sample mean, suggesting that the true population means of these variables are well defined. Nonetheless, the CVs are quite high, especially for the h and n variables within the S235JR grade, suggesting a high experimental scatter even within this relatively large sample. The skew and kurtosis values strongly suggest that each of the four variables σ_y, σ_u, h and n for the well-represented S235JR, S355J2+N and S550MC grades may well follow a normal distribution, though a log-normal distribution has been postulated as more acceptable for the σ_y and σ_u variables [20,23,54].

It is of interest to note that the mean and standard deviation of σ_y for the 120 specimens of S235JR steel are 410.1 MPa and 53.1 MPa, respectively (CV = 12.96%), with minimum and maximum values of 278 MPa and 578 MPa, respectively; all are very high values for such a low grade steel. In their statistical analysis of a much larger data set of 5493 specimens of S235 steel produced in the Czech Republic since 2001, Melcher et al. [54] found much lower mean, standard deviation, minimum and maximum values of 284.5 MPa, 21.5 MPa, 204.0 MPa and 399.0 MPa for σ_y, respectively (CV = 7.56%). A further comparison with the results of Melcher et al. [54] suggests that the estimates and CVs obtained here for the ultimate stress σ_u are also very high, as are the results for the S355 steel grade. Further, the results of Schmidt and Bartlett [23] for the yield stress of 10652 rolled W section flanges made of ASTM A992 grade steel (approximately equivalent to S345) suggests a mean of 393.4 MPa and a similarly small standard deviation of 23.99 MPa (CV = 6.1%). The discrepancy in the CVs appears to reflect the tendency of some steel manufacturers to label higher grade steels as a lower grade if they fail the quality control tests for a higher grade [20] or simply that they can sell large quantities of their stock at a discounted price. This situation leads to inhomogeneity in the sample and makes it very difficult to determine how representative the calculated bounds on the parameters h and n may be for a given steel grade. Melcher et al. [54] also did not explore the yield plateau and strain hardening variables.

The mean values of h and n for the S235JR grade are of ~1% and ~13 respectively while those for the S550MC grade are ~0.9% and ~10, respectively, in both cases of a similar order of magnitude. By contrast, those of the S355J2+N grade exhibit a significantly higher strain hardening ratio h of ~2.3% but a distinctly shorter yield plateau length n of ~6.5. This suggests that the strain hardening ratio h and yield plateau length n are negatively correlated, at least in the present sample. These intercorrelations may potentially be useful for the purposes of predicting values of h and n from the more readily available data for σ_y and σ_u. This is explored in more detail in a regression study in what follows. Lastly, the deduced means and 5th percentile values of h appear to be significantly *lower* than what the general literature would lead one to expect, so perhaps the 20th percentile would be a more forgiving value for design

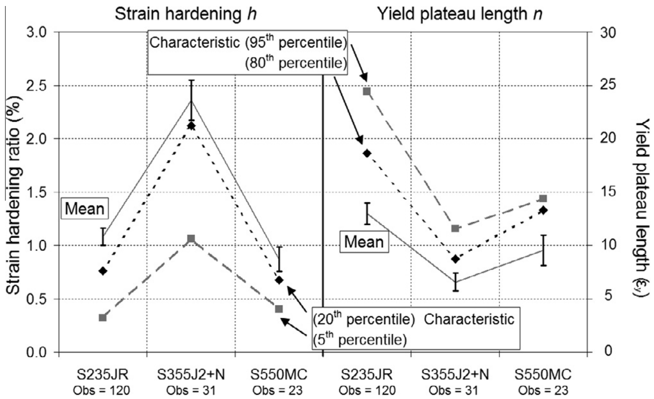

Fig. 8. Line plots of mean and characteristic linear strain hardening moduli h and yield plateau lengths n for each steel grade with error bars denoting the 95% confidence interval (there are no codified values for these properties).

purposes (also shown in Fig. 8). By contrast, a shorter yield plateau is more desirable because it permits strain hardening and full plastic capacity to be attained at lower strains, so the 80th and 95th percentiles are illustrated instead.

It was found that 70 of the 120 S235JR specimens were not straight but slightly curved because they originated from a circular tube. The remaining 50 were flat because they originated either from a rectangular tube, U-section or sheet (Table 1). The circular tubes were made from initially flat sheets by cold forming which introduces additional plastic strains into the material. Further, as these tubes were of similar dimensions with a mean diameter of 47.6 mm (CV = 5.7%) and a mean thickness of 2.8 mm (CV = 12.2%), the degree of cold forming and additional plastic strains would have been similar for all of the curved specimens. Conversely, though the rectangular tubes and U-sections were also made by cold forming, the regions of high plasticity are local and limited to the corners, and the 50 flat specimens were drawn from locations distant from the corners so that they would have been relatively unaffected by cold working. They may therefore be expected to have significantly different post-yield material properties from the 70 curved specimens. Based on this reasoning, the S235JR data set was split into two non-overlapping subsets named 'curved' and 'flat' (Table 3). An assessment of the skew and kurtosis suggests that the two subsets follow quite different sample distributions.

The subset means of the curved specimens were found to be 3.4% higher (σ_y), 2.0% lower (σ_u), 17.7% lower (h) and 16.5% lower (n) than those for the flat specimens. A parametric 2-sample t-test (not assuming equal variances) found that the differences in the subset means of σ_y and σ_u were not statistically significant ($p > 0.05$), while those for h and n were significant with $p = 0.007$ and 0.014, respectively. This conclusion was confirmed by the non-parametric Mann–Whitney U-test for equal medians and the 2-sample Kolmogorov–Smirnov tests for equal distribution shapes [50] which do not require either the assumption of normality or equal variances. This suggests that although additional cold forming may have had little influence on the strength capacity of the S235JR specimens in the present sample, it has markedly reduced both the strain hardening ratio and the yield plateau length. The negative influence of cold forming on ductility is well known in the field of metal forming [27,55] and the two subsets of S235JR are further explored using regression analysis later in this paper.

The majority of the specimens (25 out of 31) for the S355J2+N grade originated from a metal sheet, of which 10 were cut longitudinally while 15 were cut transversely to the rolling direction (Table 1). The data for this grade was thus split into two subsets corresponding to the two perpendicular orientations relative to the direction, with the remaining specimens being left out. With one exception, the variable distributions possibly follow an approximately normal distribution within the two sample subsets. On this basis, a parametric 2-sample t-test (not assuming equal variances) found a significant difference between the sample means of σ_y, h and n for the two orthogonal orientations (confirmed qualitatively by the non-parametric Mann–Whitney U-test for equal medians). The sample means were 6% lower (σ_y), 1.6% lower (σ_u) 8.7% higher (h) and 25% lower (n) for the 'transverse' direction relative to the 'longitudinal' direction (Table 4). This suggests that although the effect of the orientation of the specimen in a metal sheet does not appear to greatly influence the strength or strain hardening modulus, it may have a large influence on the yield plateau length and thus on the onset strain of the beneficial strain hardening effect that is implicitly assumed in plastic design. Casual assumptions of isotropy for the entirety of the stress–strain relationship should therefore be made with great care and further research based on larger datasets is necessary to establish whether this effect is significant.

6. Additional regression analyses on S235JR grade steels

The two stress variables σ_y and σ_u are widely known to be highly positively correlated in steels [20,23,54,56,57] and data on them is widely available. It is desirable to regress h and n on σ_u to generate a useful predictor relationship between these rarely measured variables (h and n) and a widely measured one (σ_u).

Table 3
Summary statistics for two subsets of the S235JR steel grade specimens.

	Curved (obs = 70)				Flat (obs = 50)			
	σ_y	σ_u	h	n	σ_y	σ_u	h	n
Mean	415.7	465.5	1.02	12.9	401.9	474.9	1.24	15.4
Min.	315	383	0.04	3.2	278	331	0.09	7.0
Max.	554	574	2.26	30.9	578	621	2.44	27.9
St. dev.	45.7	40.6	0.39	4.87	61.7	63.8	0.50	6.10
SE	5.5	4.9	0.05	0.58	8.73	9.02	0.07	0.86
CV	10.99	8.71	39.09	37.87	15.36	13.43	40.08	39.60
Skew	0.88	0.76	0.07	0.78	0.10	0.07	0.47	0.29
Kurtosis	2.09	1.24	1.05	1.62	0.30	0.03	0.43	−0.98

Table 4
Summary statistics for two subsets of the S355J2+N steel grade specimens.

	Longitudinal to rolling direction of the sheet (obs = 10)				Transverse to rolling direction of the sheet (obs = 15)			
	σ_y	σ_u	h	n	σ_y	σ_u	h	n
Mean	393.8	564.6	2.41	7.3	369.9	555.8	2.62	5.5
Min.	350	544	2.20	2.3	355	536	1.98	3.6
Max.	431	587	2.64	9.8	387	574	3.09	9.0
St. dev.	22.0	14.4	0.15	1.94	9.5	11.1	0.32	1.75
SE	6.95	4.54	0.05	0.61	2.45	2.87	0.08	0.45
CV	5.58	2.54	6.36	26.57	2.56	2.00	12.15	31.92
Skew	−0.34	0.40	0.08	−2.00	−0.13	−0.16	−0.80	1.23
Kurtosis	1.34	−0.65	−0.81	5.77	−0.62	−0.68	0.11	0.30

The regression on σ_u is preferable because σ_u, corresponding to the peak of the stress-train curve, is a better-defined value than σ_y whose definition may either be a fitted constant value through the yield plateau (Eq. (2)) or the normative 0.2% proof stress. The variables h and n may additionally be regressed on each other as they appear to be strongly negatively correlated for some steel grades.

In what follows, the yield and ultimate stresses σ_y and σ_u are in units of MPa, the strain hardening ratio h is a percentage and the yield plateau length n is dimensionless. Regression coefficients satisfying 0.05 (95%), 0.01 (99%) and 0.001 (99.9%) statistical significance are annotated with *, ** and *** respectively, as per convention. Also shown is the (unadjusted) coefficient of determination r^2 = (explained variance)/(total variance) and the estimated root mean squared error of the regression (Root MSE) in units of the dependent variable. The residuals from each analysis were subjected to the usual set of visual diagnostics to assess approximate normality, randomness and lack of skew, and hence legitimise the significance tests on the regression coefficients [58]. The regressions were again performed using the Minitab 16 v. 16.2 statistical software package [50].

A regression of σ_u on σ_y, h on σ_u, n on σ_u and h on n for the full data set of the best represented S235JR steel grade (Table 5) confirms the very high positive correlation ($r^2 = 0.76$) between the two stress variables σ_u on σ_y, but suggests only very low correlations ($r^2 < 0.25$) between h and n and any other variables. The coefficients, however, are statistically highly significant, suggesting that the sample regressions may be good estimates of the 'true' population relationships (or lack thereof). Despite the low correlations for this sample, the linear strain hardening modulus ratio h is positively correlated with the ultimate stress σ_u but negatively correlated with yield plateau length n which, then, is also negatively correlated with σ_u. This confirms that a longer yield plateau is associated with a lower strain hardening modulus. For this steel grade, for example, an increase in n of 10 leads, on average, to a 0.3% decrease in h. The regression lines (Fig. 9) illustrate the wide scatter and consequent low correlations among h, n and σ_u. Also shown for illustration purposes are 95% confidence interval (CI)

Table 5
Regression summary for S235JR steel grade, full data set (120 observations).

Regression equation (e = error)	Coefficient a	Coefficient b	Coefficient c	Root MSE	r^2
$\sigma_u = a + b\sigma_y + e$	123.23***	0.845***		25.305	0.76
$h = a + b\sigma_u + e$	−0.790*	0.0041***		0.404	0.22
$n = a + b\sigma_u + e$	32.488***	−0.0395***		5.173	0.14
$h = a + bn + e$	1.499***	−0.0279***		0.428	0.12
$\sigma_u = a + b\sigma_y + c\delta + e$	125.19***	0.870***	−21.422***	23.093	0.80
$h = a + b\sigma_u + c\delta + e$	−0.609*	0.0039***	−0.183**	0.396	0.25
$n = a + b\sigma_u + c\delta + e$	35.397***	−0.0421***	−2.935**	4.986	0.20
$h = a + bn + c\delta + e$	1.764***	−0.0341***	−0.306***	0.403	0.22

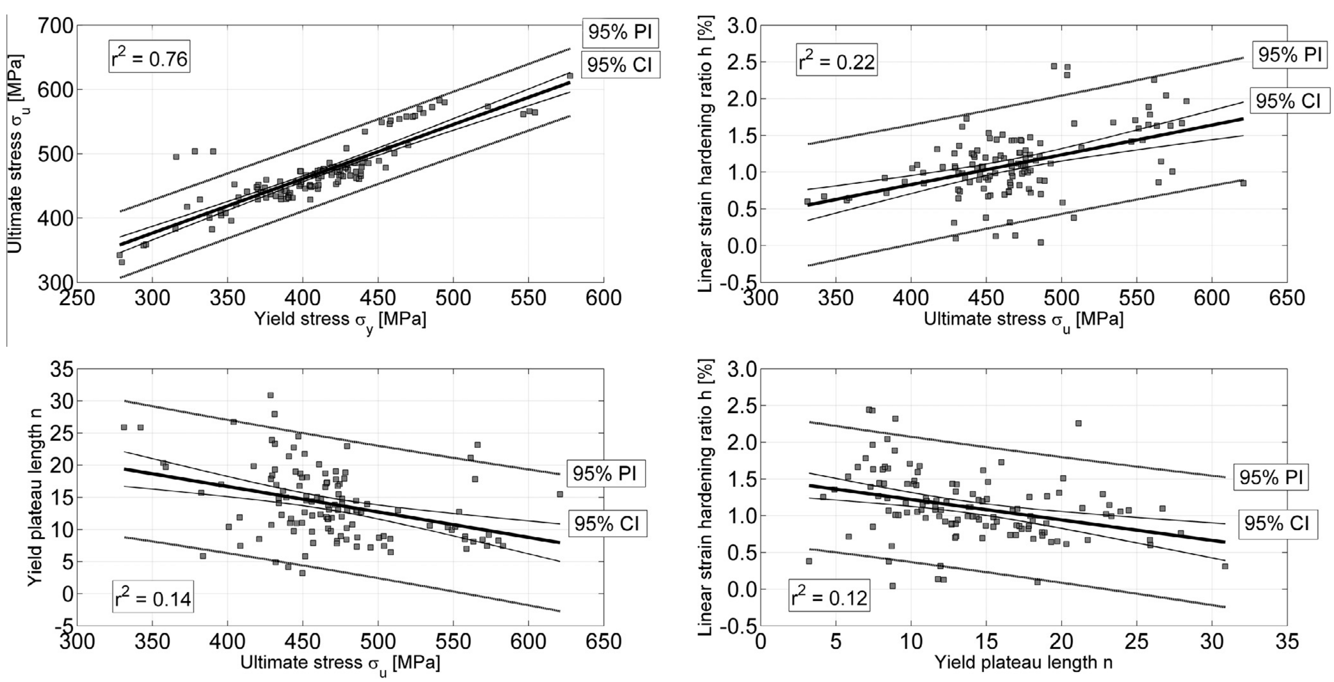

Fig. 9. Selected regression lines (RL), 95% confidence (CI) and prediction (PI) interval bands for the full data set of S235JR steel specimens (120 observations).

Table 6
Regression summary for two sub-clusters of the S235JR steel grade.

Specimen cluster	Regression equation (e = error)	Coefficient a	Coefficient b	Root MSE	r^2
Flat sections (50 observations)	$\sigma_u = a + b\sigma_y + e$	112.58***	0.902***	31.488	0.76
	$h = a + b\sigma_u + e$	−0.835	0.00437***	0.415	0.32
	$n = a + b\sigma_u + e$	49.004***	−0.0708***	4.151	0.55
	$h = a + bn + e$	2.030***	−0.0513***	0.389	0.40
Circular tubes (70 observations)	$\sigma_u = a + b\sigma_y + e$	120.67***	0.830***	14.544	0.87
	$h = a + b\sigma_u + e$	−0.403	0.0031*	0.382	0.10
	$n = a + b\sigma_u + e$	9.032	0.00824	4.897	0.01
	$h = a + bn + e$	1.211***	−0.0149	0.395	0.03

bands (where the mean value of the regressed variable is likely to fall with 95% confidence for any value of the predictor variable) and 95% prediction interval (PI) bands (where a single additional observation of the regressed variable is likely to fall with 95% confidence). The PIs are larger than the CIs because of the additional uncertainty in predicting *any* new value as opposed to the *mean* value.

A second set of regressions on the full set of S235JR specimens assumed a binomial dummy variable δ equal to 1 when the specimen was curved and 0 if it was flat [59]. The inclusion of the dummy variable leads to modest decreases in the Root MSE and increases in r^2, but the coefficients on δ are always highly significant and suggest that the strain hardening ratio and yield plateau length are on average 0.2% and 2.9ε_y *lower* for a curved specimen than for a flat specimen.

The regressions were subsequently repeated on the initially curved and flat specimen subsets individually (Table 6), revealing similar correlations between σ_u and σ_y as for the full data sets but substantial differences in the strength of the correlations between h, n and σ_u. In particular, the subset of initially flat S235JR specimens exhibited reasonable correlations of $r^2 = 0.32$ for h on σ_u and 0.55 for n on σ_u (with mostly highly significant coefficients), distinctly higher than 0.23 and 0.18, respectively for the full data set (Table 5), but very low corresponding correlations of 0.10 and 0.07 for initially curved specimens (with mostly insignificant coefficients). This unfortunately suggests that cold working of the specimen eliminates any meaningful relationship that may exist between the h, n and σ_u. Consequently, any predictor relationships for these variables must be treated with great care unless the exact history of the steel is known, and it may be possible that safe values of h and n for any particular steel can only be obtained reliably by costly testing. It is extremely important to investigate this finding more carefully on a bigger data set as part of future work.

7. Conclusions

This study presents a statistical analysis of the post-yield material properties of several structural grade steels. The properties explored were the yield stress, ultimate stress, initial strain hardening modulus and yield plateau length, all implicitly invoked in modern structural design.

The ultimate stress was always found to be strongly positively correlated with the yield stress, a well-known result. More importantly, the linear strain hardening ratio was found to be positively correlated with the ultimate stress and the yield stress. The length of the yield plateau was found to be negatively correlated with the two stress variables, illustrating the shorter plateaux found in higher strength steels. The strength of the correlation and the statistical significance of the regression coefficients depend closely on the number of observations.

Cold working is known to harden steel, increasing its strength but decreasing its ductility substantially. It was found that curved specimens originating from cold-formed circular tubes exhibit statistically different material properties from flat specimens obtained from rectangular tubes, U-sections or plates, even if the steel grade is nominally the same. In particular, the additional plastic strains to which curved specimens had been subjected reduced both the strain hardening modulus and the length of the yield plateau, whilst also erasing the correlations with the stress variables.

As a result, for cold formed members, it may not always be possible to establish a reliable predictive relationship between the post-yield strain hardening modulus and yield plateau length with the yield and ultimate stresses, although such a relationship would be very helpful. The former properties are rarely quantified whilst the latter are codified and widely available. Unfortunately, the relationship between the post-yield properties and the stresses appears to be strongly affected by the history of manufacture which is rarely known to the designer. This has consequences for the choice of safe values for strain hardening modulus and yield plateau length in design and computational modelling, and more studies are needed to establish safe bounds on these parameters for the most common worldwide steel grades.

The authors hope that the present study will inspire researchers and practitioners worldwide to take a closer look at the data that they may have gathered over many years with a view to performing similar analyses using the approach and methods suggested in this paper. A dedicated, comprehensive and openly disseminated study of the post-yield material properties of the most common grades of structural steels is sorely needed. It should preferably be based on the largest and most varied high quality data sets, thoroughly explored using rigorous statistical analyses.

Acknowledgements

This work was carried out as part of the EU RFCS Combitube research project funded by the European Commission, grant contract RFSR-CT-2011-00034.

The authors are grateful to Professor Marios Chryssanthopoulos of the University of Surrey for his valuable suggestions during the preparation of this paper.

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