



Quantifying and communicating uncertainty in seismic risk assessment

Bruce R. Ellingwood^{a,*}, Kursat Kinali^{b,1}

^a School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, USA

^b Uzun and Case Engineers, LLC, Atlanta, GA, USA

ARTICLE INFO

Article history:

Available online 21 July 2008

Keywords:

Buildings
Earthquakes
Fragility
Reliability
Risk
Structural engineering

ABSTRACT

Modern seismic risk assessment strives to support risk mitigation by providing insight into the performance of civil infrastructure, including buildings, bridges and transportation and utility systems, subjected to severe earthquakes. A fully-coupled seismic risk or safety assessment of a structural system, and its accompanying analysis of uncertainty, provides estimates of the annual probability of exceeding pre-defined performance levels, defined either in terms of structural responses or more qualitatively defined damage states. All sources of uncertainty, both inherent and knowledge-based, should be included in risk assessment; however, the manner in which they are displayed depends on the preferences of the stakeholders and decision-makers. This paper illustrates how such uncertainties are propagated through a seismic risk assessment of steel frame building structures that are typical of regions of low-to-moderate seismicity in the Central and Eastern United States and explores some of the implications for risk-informed evaluation of civil infrastructure.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction

The earthquake hazard is paramount among the natural hazards impacting civil infrastructure in the United States. The impacts of major earthquakes in recent times have provided the impetus for significant advances in engineering practices for earthquake-resistant design of buildings, bridges, lifelines and other civil infrastructure. In the presence of uncertainties, risk to civil infrastructure from earthquakes cannot be eliminated, but must be managed in the public interest by engineers, code-writers and other regulatory authorities. Structural reliability concepts and probabilistic risk analysis tools provide an essential framework to model uncertainties associated with earthquake prediction and infrastructure response and to trade off potential investments in infrastructure risk reduction against limited resources available for this purpose.

Much of the research to date on the performance of civil infrastructure during and after earthquakes has concentrated on areas exposed to high seismic hazard. However, the earthquake hazard in certain regions of the Central and Eastern United States (CEUS) is non-negligible when viewed on a competing risk basis with other extreme natural phenomena hazards. The seismicity in such regions is described, in general, by events that are infrequent in nature but may have high consequences for civil infrastructure and urban populations. The main contributor to seismic risk in

the region is the New Madrid Seismic Zone (NMSZ), located in the central Mississippi Valley extending from northeast Arkansas, through southeast Missouri, western Tennessee, and western Kentucky to southern Illinois. The last major earthquakes occurred in the NMSZ nearly two centuries ago when, in a 3-month period (December 1811–February 1812), a sequence of three powerful quakes with magnitudes ranging from $M_w = 7.8$ to 8.1 shook the area. Today, the largest population center near the NMSZ is Shelby County, Tennessee, including the city of Memphis, with a population of nearly 900,000 people. Building design, regulatory practices, and social attitudes toward earthquake risk differ in such areas, where civil infrastructure generally is not designed to withstand ground motions of the magnitude that modern seismology indicates are possible or probable. As a result, risks to affected communities (measured in terms of economic or social consequences) may be far more severe than commonly has been believed. Communicating these risks effectively to decision-makers with the authority or financial resources to address them effectively is difficult because of their lack of familiarity with earthquake hazards.

The state of the art in uncertainty modeling and risk analysis now has advanced to the point where integrated approaches to earthquake hazard analysis, performance evaluation for civil infrastructure, and seismic risk management are feasible. Consequence-based risk management (CBRM) – a framework to enable the effects of uncertainties and benefits of alternate seismic risk mitigation strategies to be assessed in terms of their impact on the performance of the built environment and on the affected population – is the unifying principle for research being conducted by the National Science Foundation-supported Mid-America Earthquake

* Corresponding author.

E-mail address: ellingwood@gatech.edu (B.R. Ellingwood).

¹ Formerly, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, USA.

Center at the University of Illinois at Urbana-Champaign. This paper reviews recent uncertainty modeling and seismic risk-based decision tools that have been developed as part of CBRM for a spectrum of stakeholders with different backgrounds and decision-making roles – architects, engineers, urban planners, insurance underwriters, and local regulatory agencies – and identifies some of the research issues that must be addressed to make further advances toward risk-informed decision-making for civil infrastructure. The concepts are illustrated with an application to steel frames typical of building practices in regions of the CEUS that are at risk from large but infrequent earthquakes.

2. Framework for risk-informed decision-making

The term “risk” is often used interchangeably with “probability” when confronting a potentially hazardous situation; certainly, the notion of relative likelihood (expressed as probability or annual frequency) is essential to understanding risk in the everyday sense. However, the annual frequencies of earthquakes that pose a substantial threat to civil infrastructure are very small in the CEUS and similar regions of low-to-moderate seismicity, making it difficult to communicate risks to stakeholders and decision-makers because there is little information against which those risks can be benchmarked. Although one often must look beyond annual frequency for a satisfactory risk metric, metrics such as direct economic loss, opportunity loss, or deaths and injuries are context-specific and depend on the preferences of the decision-maker, both of which require information that is unknown and judgments that are outside the scope of the present study. Most risk analyses of civil infrastructure require, at some stage, an estimate of the probability that a structural system enters various safety or serviceability limit states. These structural limit states, in turn, are used to identify damage states (e.g. minor, moderate, and severe) or building performance levels (immediate occupancy, life safety, collapse prevention) which are related to estimated economic losses (e.g. [24,33,14]).² Accordingly, in this paper, we focus on the limit state probability as the risk metric, with the understanding that the consequences often are most important in risk management.

The limit state probability in seismic risk assessment is defined as

$$P_{LS} = \sum_x P[LS|Q = x]P[Q = x] \quad (1)$$

in which Q defines the intensity of the seismic demand on the system (in recent years, the intensity often is expressed as spectral acceleration or displacement) and LS = limit state, defined in terms of random variables that describe the capacity of the system. Eq. (1) often is written in a form involving (annual) mean occurrence rates, under the assumption that the occurrence of significant earthquakes can be described by a Poisson process. Either way, Eq. (1) delineates the two essential ingredients of seismic risk assessment along disciplinary lines: the fragility $P[LS|Q = x]$ (structural engineering) and the seismic hazard, $P[Q = x]$ (seismology). The extension of Eq. (1) from summation to integration in the case of a continuously defined rather than discrete hazard is obvious.

2.1. Modeling, analysis and display of uncertainties

Understanding and quantifying various sources of uncertainty are essential to develop the probabilistic models of behavior needed for seismic risk assessment and interpret the limit state

probability in Eq. (1) in the proper context. The modeling and evaluation of low-probability, high-consequence natural events involve significant uncertainties, some caused by factors that are inherently random at the customary scales of physical modeling and engineering analysis (aleatoric uncertainties) and others that arise from imperfect scientific and engineering modeling, simplifications, and limited databases. Because geophysical processes giving rise to extreme ground motion are not well-understood [23], it follows that epistemic uncertainties play a significant role in any seismic risk assessment, especially when applied to civil infrastructure in regions of low-to-moderate seismicity, such as the CEUS. In contrast to aleatoric uncertainties, which are essentially irreducible in a practical sense, epistemic uncertainties generally can be reduced through additional knowledge provided at the expense of more comprehensive (and costly) data acquisition and analysis. It is essential that all sources of uncertainty – both aleatoric and epistemic – be considered in the risk assessment process. The distinction between them is dependent on the physical modeling process and often is made for convenience. However, as will be shown subsequently, it often is important to maintain this distinction because of the difference in the way that their effects are manifested in seismic risk assessment [1]. We will return to this point subsequently in the assessment of the steel frames.

Useful results for risk-informed decision-making can be obtained by prescribing, *a priori*, probabilistic models of aleatoric uncertainty from reasonable physical arguments, supported to the extent possible by the available databases. Modern seismic risk analysis, beginning with the paper by Cornell [8] and supported by later studies, has indicated that earthquake ground motion intensity (measured by peak ground motion or spectral ordinates) can be represented by the Cauchy–Pareto family of distributions. Similarly, a number of seismic fragility studies of buildings and bridges conducted during the past decade [11,27,28,25,32] have confirmed that the fragility term in Eq. (1) can be modeled by a lognormal distribution. Using these normative models, the risk analyst then can concentrate on the necessary information-gathering to define the descriptive parameters of these models and on methods for communicating the risk so determined.

With the normative selection of the distributions to model the uncertainties in demand and capacity, the fragility, $F_R(x)$ and the hazard, or, probability that ground motion intensity exceeds x , $H(x)$, become

$$F_R(x) = P[LS|Q = x] = \Phi[(\ln x - \ln m_R)/\beta_R] \quad (2)$$

$$H(x) = 1 - \exp[-(x/u)^{-k}] \approx (x/u)^{-k} \\ = k_0 x^{-k} \text{ (at large values of } x) \quad (3)$$

in which $\Phi[\]$ = standard normal probability integral and the fragility parameters, m_R and β_R , define the median capacity and logarithmic standard deviation in capacity; u = scale parameter, k = shape parameter, and constant $k_0 = u^k$. The seismic hazard curves are provided by the US Geological Survey (USGS).³ Substitution of Eqs. (2) and (3) into Eq. (1) and numerical integration leads to a point estimate of P_{LS} . Recognizing that only a small range of x in the integrand of Eq. (1) typically contributes significantly to P_{LS} ,⁴ one may obtain the approximation (e.g. [7])

$$P_{LS} \approx (k_0 m_R^{-k}) \exp[(k\beta_R)^2/2]. \quad (4)$$

³ http://earthquake.usgs.gov/hazmaps/products_data/2002/ceus2002.html.

⁴ Plots of mean seismic hazard provided by the USGS show that the seismic hazard curve is slightly concave downward when plotted on a log-log plot. The linearization of $H(x)$ over the range of seismic intensities contributing to the summation (integral) in Eq. (1) often is an acceptable approximation, particularly in the CEUS where the hazard curves are flat.

² A direct mapping between limit states and damage states often is presumed (e.g. “severe” damage in steel frames may be implied by maximum inter-story drift angles (ISDAs) between, say, 0.02 and 0.05).

With re-arrangement to express the demand and capacity in terms of inter-story drift angle (ISDA), Eq. (4) is the basis of the capacity-demand factor design and assessment method developed in the SAC/FEMA project on steel buildings. Eq. (4) also is the basis for a new approach to seismic design based on uniform risk rather than uniform hazard in DOE 1020-02 [10] and in ASCE Standard 43-05 [2] for nuclear facilities. As will be shown subsequently, in the presence of epistemic uncertainty, the point estimate provided by Eq. (4) approximates the mean limit state probability.

2.2. Point vs. interval estimates of risk (probability)

Eqs. (2)–(4) describe the fragility, hazard and limit state (or damage state) probability of the system when the state of knowledge is essentially perfect, at least within the bounds of normal seismology and structural engineering, and provide a point estimate of P_{LS} . In many decision contexts, this point estimate may be sufficient. Additional sources of uncertainty in capacity and demand arise from assumptions and approximations made in specifying the hazard, modeling the strength and stiffness of structural materials and components, and modeling the structural system by finite element methods, as well as from limitations in the supporting databases [13]. For example, it is generally understood that the seismic hazard curves provided by the USGS represent the *mean* seismic hazard. The shape of this mean seismic hazard curve represents aleatoric uncertainty; epistemic uncertainty is depicted by hazard curves associated with alternate plausible ground motion models, which are averaged to obtain the mean hazard [1]. Similarly, sources of epistemic uncertainty in the estimation of response of steel structures include two-dimensional models of three-dimensional structures, structural models based on beam and column centerline dimensions that neglect beam-column panel zones, assumptions on support conditions, neglecting partition walls and other non-structural elements, connections that are neither fully rigid nor simple, and granularity of finite element modeling.

The presence of epistemic uncertainty in seismic risk assessment implies that the hazard and fragility curves in Eqs. (2) and (3) are themselves uncertain, reflecting incomplete knowledge regarding the distributions and parameters used to model the aleatoric uncertainty. Thus, one might model the hazard and fragility by *families* of distributions, leading to an epistemic uncertainty in the limit state probability determined from Eq. (1) or (4) that is described by a frequency distribution. If the epistemic uncertainty in the hazard and fragility models is small, this frequency distribution is tightly centered on the point estimate P_{LS} ; conversely, if the epistemic uncertainties are large, the frequency distribution of P_{LS} is broad. One can, of course, compute a mean value of P_{LS} by interpreting the unknown parameters as Bayesian variables and using customary analysis. This mean value is the “best” point estimate of P_{LS} in a decision-theoretic sense and now is accepted in the nuclear industry.⁵

While the mean (or other point estimate) of P_{LS} is an unambiguous metric of risk, and is a natural choice in performing expected cost (or loss) analyses, it does not convey a sense of confidence that the analyst has in the risk assessment. Two alternative analyses, one made with limited data and a second made with comprehensive effort, would lead to the same P_{LS} if the estimates of the aleatoric uncertainties were the same. A number of recent studies⁶

have revealed that many decision-makers find this lack of distinction undesirable, and would prefer a statement of confidence in the results of the risk assessment, particularly if the consequences are severe. One way to provide this statement of confidence is through an interval estimate of P_{LS} . Point and interval estimates of limit state probabilities of steel frames typical of building practices in the CEUS will be illustrated subsequently.

In the following sections, seismic risks to three code-compliant steel frames that are typical in building construction in urban areas of the Eastern United States that are exposed to moderate seismic hazards are considered to illustrate the above concepts.

3. Risk assessment of steel moment frames

3.1. Description of steel frames

The steel frames used to illustrate the uncertainty modeling concepts are 2, 4, and 6 stories in height, respectively, and are denoted as Frames 2PR, 4PR, and 6XB. The first two are Partially-Restrained (PR) moment frames, while the third is an X-braced frame. All were designed for loading conditions in the CEUS. The analytical modeling of these frames was performed using OpenSees [22]. All beams and columns were modeled using nonlinear beam-column elements having distributed plasticity and fiber sections divided into 10 layers in the axis of bending. Both material and geometric ($P - \Delta$) nonlinearities were taken into consideration. The force-deformation relation for all beams and columns is bi-linear, with 3% strain hardening. The clear dimensions of members were used in the structural model, i.e., beam-to-column panel zones were modeled explicitly with actual physical dimensions.

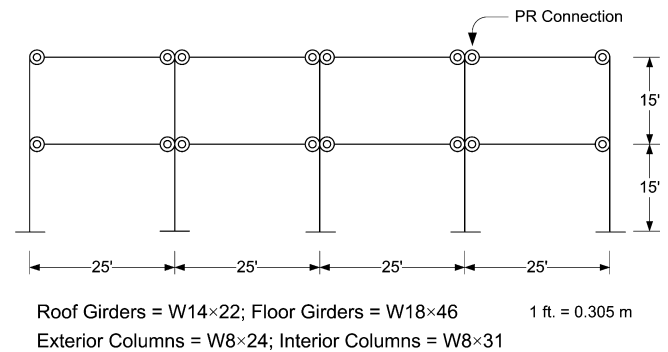


Fig. 1. The analytical model and the member sizes for Frame 2PR.

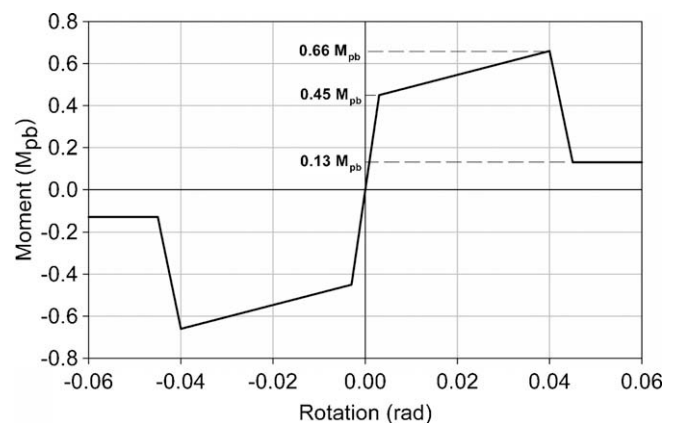


Fig. 2. Typical moment-rotation envelope for PR connections in Frame 2PR.

⁵ In early seismic PRA and margin studies of nuclear plants conducted in the 1980s, the aleatoric and epistemic uncertainties were tracked and propagated through the risk analysis separately. In recent years, however, the US Nuclear Regulatory Commission has adopted the mean value as basis for risk-informed decision-making.

⁶ Stakeholder workshops held in connection with the ATC Project 58, “Guidelines for seismic performance assessment of buildings” have conveyed this message clearly.

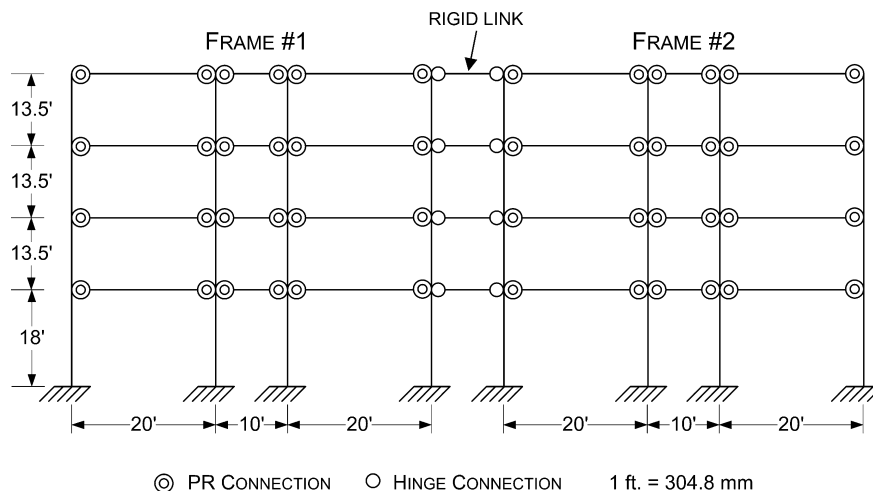


Fig. 3. The analytical model for Frame 4PR.

Frame 2PR is a two-story, four-bay PR moment resisting frame analyzed previously by Barakat and Chen [5]. The analytical model and the member sizes are given in Fig. 1. The frame was designed without any seismic considerations using a 1980-vintage design code. Wind load deflections governed the design of the frame. This frame represents one interior frame of a series of identical frames. The fundamental period of the frame is 1.07 s with 96% first-mode participation ratio. ASTM Grade A36 steel (the nominal yield strength is 3 ksi (248 MPa)) was used for all members in the design. The mean value of the yield strength of Grade A36 steel manufactured in the 1950–1960s is approximately 40 ksi (276 MPa) [18]. The connections in Frame 2PR are top and seat angle connections with double web angles. They were modeled using rotational springs with moment–rotation relationships adjusted to fit the results of a series of experimental tests by Azizinamini et al. [4]. The connection backbone curves used to define the hysteretic behavior of the connections (Fig. 2) were normalized by the plastic moment capacity of the adjoining beams and included a sharp drop in connection strength after relative rotation of 0.04 rad to account for bolt slippage and seat angle failure.

Frame 4PR represents one interior and one exterior frame of a 4-story, 3-bay PR building. In analytical model, these frames were connected to each other via rigid links with hinges at both ends (Fig. 3). These frames were taken from a building that was designed according to practices in the 1950s [21]. Grade A36 steel has been used for all members. The fundamental period of the frame is 1.34 s with 90% first-mode participation ratio. As with Frame 2PR, wind load effects governed the design of the lateral force-resisting system of the building. The connections throughout Frame 4PR are typical T-Stub connections which were modeled using the results of experimental tests. The results of the tests performed by Forcier [17] were used to model the moment–rotation behavior of the connections. Fig. 4 shows the hysteretic behavior in a typical connection in Frame 4PR, along with the mathematical representation in OpenSees. This connection is relatively stiff and develops 65% of the beam capacity.

Frame 6XB (Fig. 5) is an X-braced interior frame taken from a 6-story office building located in Memphis, TN, which was designed using the *Standard Building Code*, 1991 Edition. It was framed with ASTM A572 Grade 50 steel for all beams and columns (nominal yield strength is 50 ksi (345 MPa); mean yield strength is 58 ksi (400 MPa)). The braces are ASTM A500 Grade B steel tubes (nominal and mean yield strengths of 46 ksi (317 MPa) and 48 ksi (331 MPa), respectively). All member sizes are provided elsewhere [20]. All beam-to-column connections in this frame are simple con-

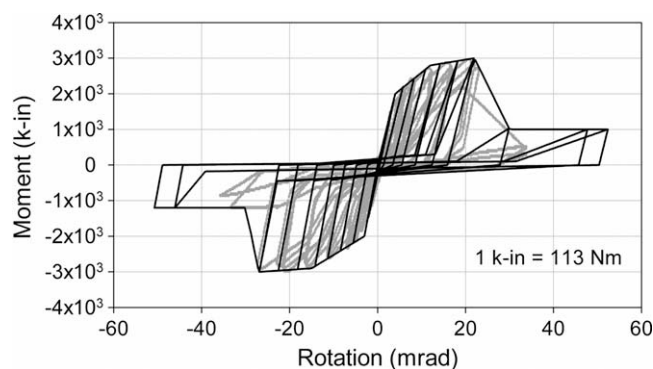


Fig. 4. Experimental hysteretic behavior for PR connection in Frame 4PR [17] and its mathematical representation.

nections; therefore, the lateral stiffness of the frame comes from the truss action in the diagonal bracing system. The fundamental period of the frame is 1.04 s with 73% first-mode participation ratio. Typical cyclic behavior of the braces is illustrated in Fig. 6. A residual compression capacity of 20% of the critical load was assumed to be reached at five times the yielding axial deformation and the post-buckling envelope was modeled as curved rather than straight to better represent the test results.

3.2. Fragility modeling of steel frames

The seismic fragilities are developed through a simulation process involving nonlinear time history analysis (NTHA) of frame response to ground motion records. The ground motion seismic intensity is characterized by the 5% damped spectral acceleration, S_a , at the fundamental period of the frame. The seismic demand on the frames is measured in terms of ISDA⁷ since the performance limit states (structural system capacity) also are measured in terms of ISDA. Unlike the Western United States, there are few ground motion records available for the CEUS that are large enough to threaten the current infrastructure in the region. Accordingly, the ground

⁷ Several single-parameter metrics have been proposed for measuring the response of a structural system in the nonlinear range. Inter-story drift angle is the most common of such metrics, and has been correlated (albeit, subjectively) to performance levels and damage states in FEMA 356 [15], HAZUS Manual [16], and other documents dealing with seismic risk mitigation.

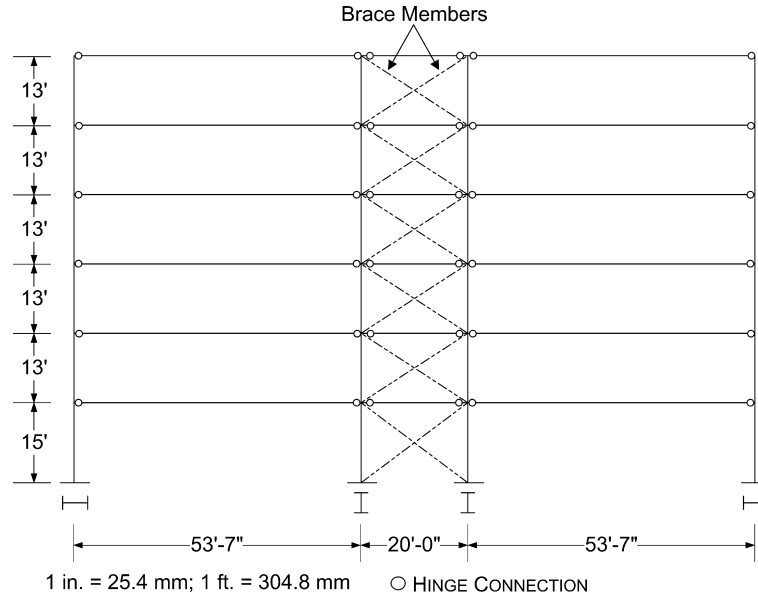


Fig. 5. The analytical model for Frame 6XB (after [20,19]).

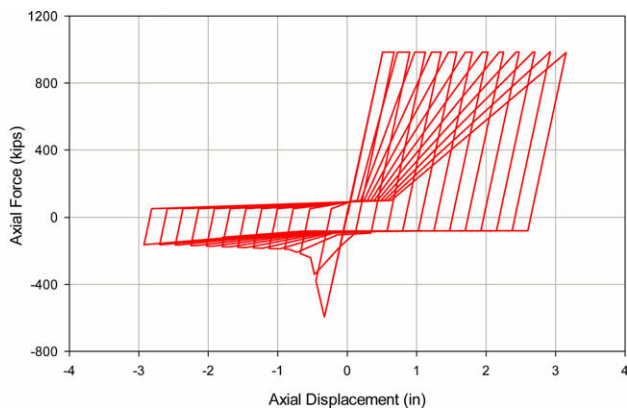


Fig. 6. Typical cyclic behavior of a brace in Frame 6XB.

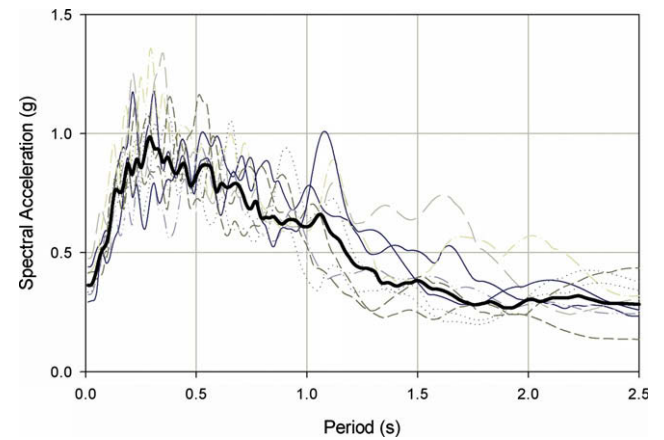


Fig. 7. Wen-Wu Memphis 2%/50 yr ensemble response spectra.

motions used in this study were generated as part of MAE Center research by Wen and Wu [31] for three sites in Mid-America: Memphis, TN, Carbondale, IL, and St. Louis, MO (believed to represent a cross-section of earthquake-prone sites in the central United States). Ensembles of 10 ground motions corresponding to probabilities of 2% and 10% of being exceeded in 50 years (abbreviated 2%/50 yr and 10%/50 yr) equivalently, return periods of 2475 yr and 475 yr, respectively were generated at these sites. The ensemble of response spectra derived from one of these ground motion sets, along with the median spectrum, is shown in Fig. 7; where the variability in the ensemble represents the aleatoric uncertainty in seismic intensity in Memphis, TN for earthquakes of this return period. The yield strength, ultimate strength, and modulus of elasticity of the frame members initially were set equal to their respective mean values, as has been a common practice in seismic fragility analysis, because it has been found (e.g. [26,28]) that the overall response variability is dictated mainly by the seismic demand. The validity of this assumption will be revisited later in the paper.

For frames such as those discussed in this study, the uncertainty in seismic demand on a structural system can be characterized by a relation between the maximum ISDA, θ_{\max} , and the spectral acceleration at the fundamental period of the building, S_a [26]:

$$\theta_{\max} = a S_a^b \varepsilon \quad (5)$$

in which a and b are model constants and ε is a random variable (with median of unity and logarithmic standard deviation, $\sigma_{\ln \varepsilon}$) that describes the uncertainty in the relationship. Estimates of these constants can be determined by performing a set of nonlinear dynamic analyses of the building frame using an ensemble of ground motions and performing a linear regression analysis of $\ln \theta_{\max}$ on $\ln S_a$. The results of these analyses are presented in Fig. 8 for Frame 2PR. To cover a range of spectral accelerations of interest, three ensembles of increasing intensity were utilized. The seismic demand parameters are found to be $a = 0.051$, $b = 0.91$ and $\sigma_{\ln \varepsilon} = 0.19$. Similar values of these parameters were obtained when other ensembles of accelerograms were used [19]. The same approach was used to determine the seismic demands on frames 4PR and 6XB; the results for all three frames are summarized in Table 1. In both the Shome et al. [26] study cited above, which utilized only natural records, and the current study, it was found that the median relation between deformation and S_a was relatively insensitive to the ground motion ensemble selected (at least for steel frames), provided that accelerograms were selected from events of similar magnitude and distance with no directivity (near-field) effects. Thus, one might conclude that the parameters a and b in Eq. (5) are characteristics of a particular building and not strongly

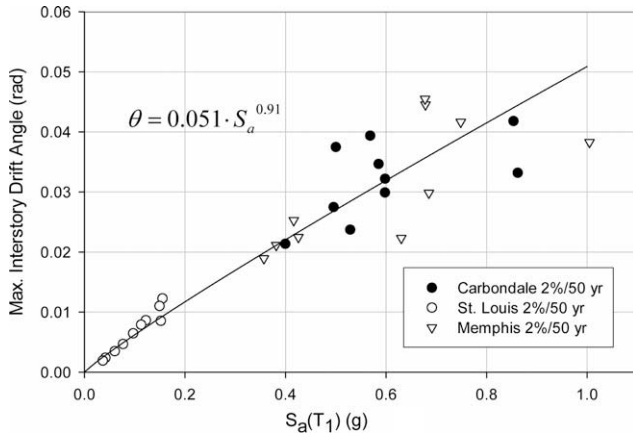


Fig. 8. Seismic demands on Frame 2PR using Wen–Wu 2%/50 yr ensemble.

Table 1
Median seismic demand statistics for model frames per Eq. (6)

Frame	a	b	$\sigma_{\ln \varepsilon}$
2PR	0.051	0.91	0.19
4PR	0.046	0.88	0.19
6XB	0.034	0.98	0.26

Table 2
Performance state limits for model frames in terms of ISDA

Frame	IO (rad)	SD (rad)	CP (rad)
2PR	0.007	0.027	0.100
4PR	0.007	0.030	0.067
6XB	0.004	0.013	0.050

dependent on the ensemble selected for its determination. The fact that parameter b is close to unity is consistent with the so-called *equal displacement rule* first proposed by Veletsos and Newmark [30].

The seismic fragilities reflective of the aleatoric uncertainties can be obtained on the basis of these seismic demand relationships. Substituting Eq. (5) into Eq. (2), we obtain

$$P[LS|S_a = x] = \Phi[\ln(ax^b/m_c)/(\beta_c^2 + \beta_{D|S_a}^2)^{1/2}] \quad (6)$$

in which demand variability $\beta_{D|S_a}$ is assumed equal to $\sigma_{\ln \varepsilon}$ and the “capacity” terms m_c and β_c depend on the performance level of interest. Analogous to FEMA 356 [15], three levels of performance are considered – immediate occupancy (IO), structural damage (SD)⁸ and Collapse Prevention (CP) – each of which is related to maximum ISDA.⁹ Limits IO and CP are associated, respectively, with the onset of inelastic behavior (ISDAs typically between 0.005 and 0.01) and the onset of instability determined from incremental dynamic analyses of ensembles of ground motions [29] (approximately 0.10 for Frame 2PR, 7% for Frame 4PR and 5% for Frame 6XB). The ISDAs defining the intermediate SD limit, a deformation level at which moderate to severe structural damage may be expected to occur, are in the range 0.013–0.03, the lower value being for Frame 6XB and higher value for Frame 4PR. This intermediate level of deforma-

⁸ The intermediate performance level in FEMA 356 [15] is Life Safety, but this level is difficult to define in terms of structural response, because it depends on the performance of non-structural as well as structural components. Accordingly, the intermediate performance level in this study is stipulated as significant damage.

⁹ The mapping between the inter-story drift (or other structural demand parameter) and the performance level (or damage state) is an area where more research is required [13].

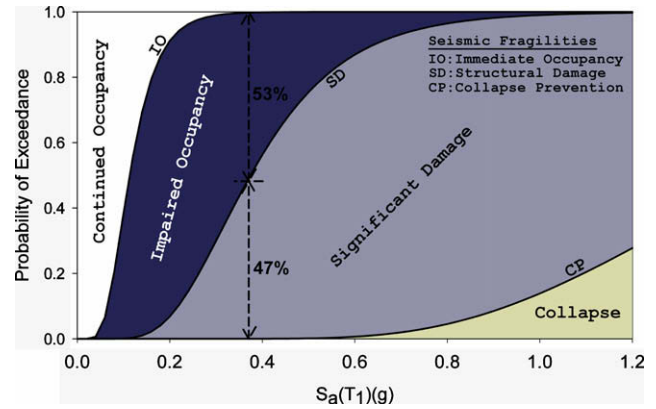


Fig. 9. Seismic fragilities and performance state probabilities for Frame 6XB.

tion was defined [20] as the ISDA at which the global lateral stiffness of the frame determined from a nonlinear static pushover analysis drops to half of its initial value. The median capacities, m_c , associated with the three performance state limits for all model frames are summarized in Table 2. The logarithmic standard deviation, β_c , in Eq. (6) reflects the uncertainty associated with the use of the above-defined drift limits as measures of capacity, and is set equal to 0.25 for IO and SD limits and 0.15 for the CP limit [19].

The seismic fragilities for the IO, SD and CP performance levels are illustrated in Fig. 9 for Frame 6XB. These seismic fragilities can be used to divide the building performance spectrum into four regions – continued occupancy, impaired occupancy, significant damage, and collapse – as illustrated in Fig. 9. This figure provides a convenient way of depicting seismic damage capabilities of earthquakes of different magnitudes. The maximum considered earthquake (MCE) stipulated in the *BSSC/NEHRP Recommended Provisions* [6] and in *ASCE Standard 7-05* [3] is defined by the response spectrum (spectral acceleration) with a probability of 2% of being exceeded in 50 years (2%/50 yr). The 2%/50 yr spectral acceleration for a 1-s fundamental period in Memphis, TN on soft ground is 0.37 g^{10} ; at this level of ground motion, this non-seismically designed Frame 6XB has a relatively high probability of sustaining significant damage (approximately 47%) but a substantial (and surprising) reserve capacity against collapse. A local code official or director of emergency planning likely would find the notion of (conditional) probabilities of moderate and significant damage of 53% and 47%, respectively, under a 2%/50 yr earthquake (or MCE with a spectral acceleration of 0.37 g) easier to grasp than an annual damage probability on the order of 10^{-4} .

3.3. Effects of variability in capacity on the seismic response of steel frames

As noted previously, the yield strength, ultimate strength, and modulus of elasticity of the steel initially were assumed equal to their respective mean values in obtaining the results presented above. This assumption presumes that the total uncertainty can be vested in the record-to-record variability in the ground motion. To test the validity of this assumption, the yield strengths and stiffnesses of the beams and columns as well as the moment–rotation relations of PR connections in Frame 2PR were assumed to be random, and the uncertainties were propagated using Latin Hypercube sampling. Each sample frame was described by the modulus of elasticity of beams, E_b , and columns, E_c ; yield strength of beams,

¹⁰ USGS does not account for local site conditions; hence the difference between this value and that in Fig. 7 at a period of 1.0 s.

F_{yb} , and columns, F_{yc} ; and connection capacity multiplier, M_{conn} , which randomizes the connection strength with respect to the adjoining beams (cf. Fig. 2). Beam and column properties were assumed to be statistically independent. Both beam and column moduli of elasticity were assumed normally distributed with means equal to 29,000 ksi (200 GPa) and coefficient of variation (COV) of 5%. Similarly, the yield strength of beams and columns were assumed log-normally distributed with means equal to 40 ksi (276 MPa) and COVs equal to 10%. The connection behavior was described by a normal distribution, with mean behavior represented by the curve in Fig. 2 and COV of 15% in moment capacity. The Wen–Wu 2%/50 yr records at three sites in the CEUS were used to represent the seismic demand on the frame, each record being treated as equally likely within each ensemble. First-mode spectral acceleration values were updated for each record to reflect the change in the natural period of the frame due to the randomness in the stiffness parameters.

In comparison with Fig. 8, the seismic demand analysis involving frames with random strength and stiffness results in an increase in parameter “a” from 0.051 to 0.052, “b” from 0.91 to 0.93 and $\sigma_{\ln \varepsilon}$ from 19% to 21%. The difference between the median and log-standard deviation in seismic demands is minimal. This statistically insignificant difference supports the practice for steel frames of using mean values for the strength and stiffness and ignoring randomness in structural properties in seismic demand analysis.

4. Point and interval estimates of annual performance level probabilities

Point and interval estimates of annual limit or damage state probabilities require, in addition to the seismic fragilities, descriptions of the seismic hazard, defined previously in Eq. (3). There are striking differences in the seismic hazard characteristics of high-seismic zones (WUS) and regions of moderate seismicity, typified by the CEUS. In the CEUS, the hazard curve tends to be flat (COVs in the annual extreme or 50-yr maximum spectral acceleration often exceeding 100%), and the shape parameter k (Eq. (3)) typically is in the range of 1–2. In contrast, the hazard curve in the WUS is much steeper and k typically is in the range of approximately 3–4. Fig. 10 (prepared from data available on the USGS website noted earlier) compares the mean seismic hazard curves for Memphis, TN; Boston, MA; Charleston, SC; Los Angeles, CA and Seattle, WA for a structure on rock with a fundamental period of 1 s.

The shape and scale parameters, k and k_0 , for these sites determined by least-squares analysis of the tabulated data from the

Table 3

Seismic hazard parameters per Eq. (4) for five US cities ($T_1 = 1$ s)

Site	k	k_0	2%/50 yr S_a (g)	10%/50 yr S_a (g)
Memphis, TN	1.00	1.48×10^{-4}	0.370	0.085
Boston, MA	1.49	7.50×10^{-6}	0.070	0.024
Charleston, SC	0.81	2.22×10^{-4}	0.520	0.068
Los Angeles, CA	2.69	1.66×10^{-4}	0.720	0.410
Seattle, WA	2.14	8.93×10^{-5}	0.500	0.240

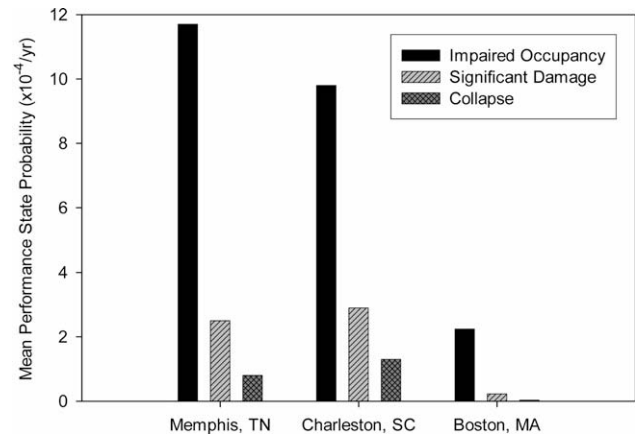


Fig. 11. Mean performance state probabilities for Frame 2PR.

USGS, are given in Table 3 along with 1-s spectral accelerations for both 10%/50 yr and 2%/50 yr earthquakes.¹¹ Note that at very large return periods (on the order of 5000–10,000 years, corresponding to 0.5–1% in 50 yr earthquake), the hazard curves for Charleston, Memphis and Los Angeles approach one another.¹² These seismic hazard curves, as defined in Table 3, are used to calculate the performance level probabilities in the following paragraphs.

Point estimates of risk (measured in terms of annual probability of IO, SD or CP) are obtained by convolving the fragility in Eq. (6) with the (mean) seismic hazard. As an example for Frame 6XB in Memphis, TN, the performance limits in terms of ISDA are 0.004, 0.013 and 0.05 for IO, SD and CP, respectively. The point estimates of P_{IO} , P_{SD} and P_{CP} are $1.5 \times 10^{-3}/\text{yr}$, $4.4 \times 10^{-4}/\text{yr}$ and $1.1 \times 10^{-4}/\text{yr}$, respectively. Similarly, the performance limit probabilities for all frames were calculated for the other sites in the CEUS. Performance state probabilities (as seen in Fig. 9), then, are calculated and the results are depicted in Figs. 11–13. The probabilities of impaired occupancy (damage sustained is higher than the IO limit but lower than the SD limit) are comparable for the three frames at each site. However, the probability of significant damage (damage

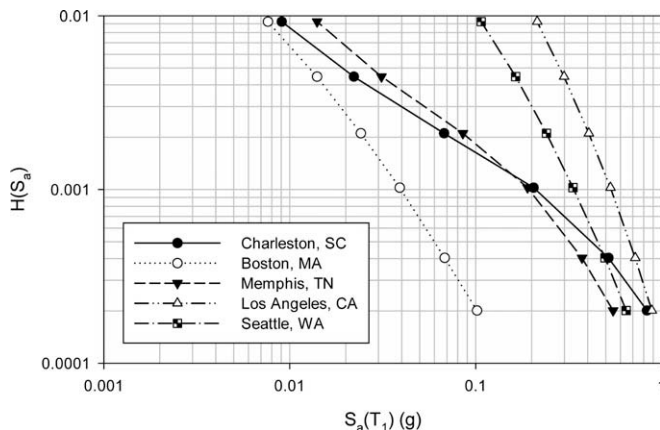


Fig. 10. Comparison of the seismic hazards for five cities in the US.

¹¹ These earthquakes play a central role in earthquake-resistant design. The USGS develops earthquake hazard maps for the 2%/50 yr earthquake, termed the maximum considered earthquake, or MCE. Structural design [3] is based on 2/3 MCE; for sites in the WUS, this corresponds, approximately, to a 10%/50 yr event, which was mapped by the USGS prior to 1997. The implication is that there should be a factor of safety against incipient collapse of 1.5.

¹² This convergence illustrates one of the difficulties in basing probability-based limit states design for regionally dependent extreme natural hazards, such as earthquakes or hurricanes, on characteristic loads with stipulated return periods (50–100 yr) factored with partial load factors greater than 1.0. For example, if one were to design a structure with period of 1.0 s for structural actions due to a 500 year earthquake with a load factor of 1.5, the return period of the ultimate limit state would be approximately 800 years in Memphis and 1900 years in Los Angeles (Fig. 10). Conversely, if incipient collapse is stipulated at the 5000 year return period, and if the factor of safety against collapse is 1.5, then one should design for a 2500 year characteristic event in Memphis and a 1400 year characteristic event in Los Angeles. From the standpoint of uniform risk, it is preferable to design directly at the “checking point” obtained from first-order reliability analysis, with a factor of safety equal to 1.0.

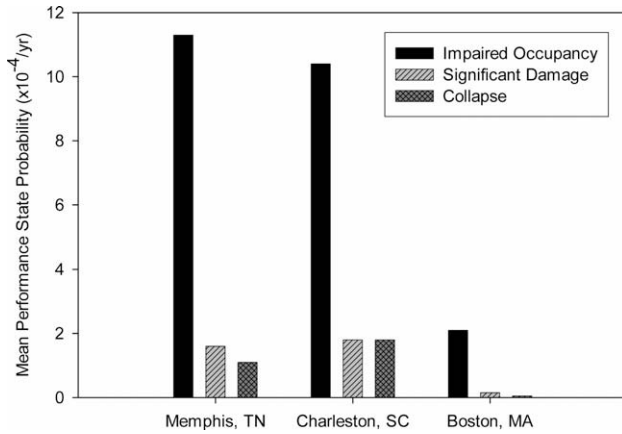


Fig. 12. Mean performance state probabilities for Frame 4PR.

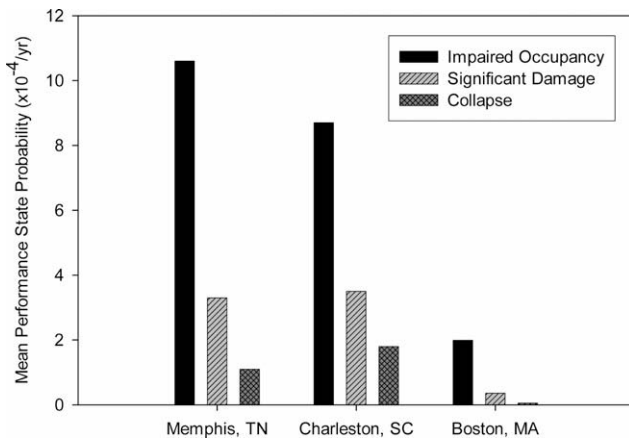


Fig. 13. Mean performance state probabilities for Frame 6XB.

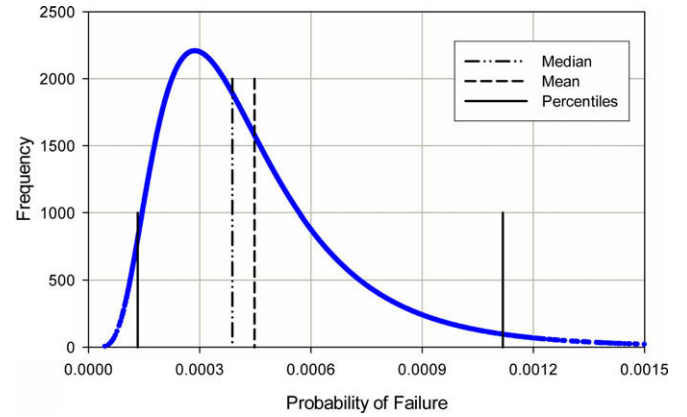


Fig. 14. Frequency distribution of probability of structural damage (SD) for Frame 6XB showing the effect of epistemic uncertainty in fragility and hazard modeling.

abases. This assumption implies that M_R is unbiased and its COV, β_{RU} , is approximately 0.20. This value reflects the refinements (e.g. panel zone modeling, connection fracture, brace buckling and $P-\Delta$ effects) of the finite element modeling approach utilized in this paper. Similarly, the ratio of 85th–15th percentiles of seismic hazard curves postulated by seismic experts at sites in the CEUS is typically around 3,¹⁴ implying that β_{HU} is approximately 0.50.

The resulting frequency distribution of P_{SD} for Frame 6XB determined by numerical integration is shown in Fig. 14. In this illustration, the median and mean damage state probabilities are $3.8 \times 10^{-4}/\text{yr}$ and $4.4 \times 10^{-4}/\text{yr}$, respectively, the latter being equal to the point estimate of P_{SD} determined in the previous section. Of equal interest to the decision-maker is the uncertainty or confidence in these risk estimates. One might state that “the probability of sustaining significant damage level is between $1.3 \times 10^{-4}/\text{yr}$ and $11.0 \times 10^{-4}/\text{yr}$ with 95% confidence (a range of an order of magnitude, reflecting the level of epistemic uncertainty in the analysis). Or, one might say that “the probability of sustaining significant damage is less than $7.7 \times 10^{-4}/\text{yr}$ with 90% confidence.

It is worth emphasizing that the mean value of P_{SD} , which has been averaged over the frequency distribution of epistemic uncertainties, is equivalent to the point estimate (aleatoric) of P_{SD} obtained previously. As a numerical average, the mean is invariant to whether the uncertainties are modeled as aleatoric or epistemic (e.g. [23]). Nonetheless, retaining the distinction between the aleatoric and epistemic uncertainties may be advantageous in some instances. The analysis involving aleatoric uncertainties answers the question, “What is the damage state probability? Propagating the epistemic uncertainties separately answers the question, “How confident can one be that the damage state probability is, in fact, what has been estimated?” As noted previously, many decision-makers would like to see such statements of confidence accompany a risk estimate. Furthermore, there is evidence [9] that society views aleatoric and epistemic uncertainties differently when making judgments on acceptable risk. A risk-averse decision-maker may not be comfortable with the mean as a point estimate, and may prefer a more conservative fractile of the frequency distribution, e.g. 7.7×10^{-4} in the illustration above. Tracking the aleatoric and epistemic uncertainties separately addresses this concern.

Quantitative risk assessment of buildings and other civil infrastructure projects, which traditionally has been regulated by design codes and public regulations, is a relatively new develop-

is higher than SD limit but lower than CP limit) is more likely for Frames 2PR and 6XB than for Frame 4PR. This can be attributed to the fact that the X-braced frame is stiffer and stronger than the PR frames, and Frame 2PR benefits from the fact that it is a low-rise frame, in which $P-\Delta$ effects are negligible.

When epistemic uncertainties are considered in seismic risk analyses, it is customary to vest them in the estimates of the first-order statistics (here, median capacity, m_R , and hazard scale parameter, k_0).¹³ Under this assumption, m_R and k_0 are replaced by (Bayesian) random variables, M_R and K_0 , which are assumed to be modeled by lognormal distributions with medians m_R and k_0 and logarithmic standard deviations, β_{RU} and β_{HU} . Note that β_{RU} and β_{HU} may, in turn, represent a combination of knowledge-based (modeling) uncertainties. This simple representation neglects correlation that may exist among seismic hazard models at different return periods. The limit state probability in Eq. (4), then, becomes a random function of M_R and K_0 , e.g.

$$P_{SD} \approx (K_0 M_R^{-k}) \exp[(k\beta_R)^2/2] \quad (7)$$

in which $\beta_R = (\beta_C^2 + \beta_{D,sa}^2)^{1/2}$, with an associated frequency distribution which can be determined numerically. In the absence of supporting databases, the descriptions of M_R and K_0 must be encoded by judgment. For example, one might assume that M_R can be estimated to within $\pm 30\%$ accuracy with 90% confidence with current nonlinear finite element software and limitations in supporting data-

¹³ Unpublished studies have shown that the contribution of uncertainty in the second-order terms is of lesser importance.

¹⁴ The first author's experience with seismic PRAs of nuclear power plants in the US indicates that this ratio is typical at return periods of relevance in civil infrastructure risk assessment (200–5000 years).

ment. Despite the growth and acceptance of structural reliability as a decision tool [12,13], the question of what constitutes “acceptable risk” in the built environment remains context-specific. Answering this question requires a framework, such as that described in the previous sections of this paper, that guides how uncertainties are modeled and propagated and how limit or damage state probabilities, such as those determined above, are assessed for risk-informed decision-making. Risk communication requires a continuing dialogue among project stakeholders that is aimed at understanding such basic issues and how they can enhance the credibility and acceptance of the decision process. A successful risk-informed decision process is one that can be scrutinized independently by a peer reviewer, provides an easily understood audit trail for key decisions, and can be communicated to decision-makers who are not expert in probabilistic risk analysis but control the resources necessary for risk management and mitigation.

5. Conclusions

Seismic risk analysis tools can be used to test the viability of proposed code provisions, to assess the need for upgrading an existing facility when new information suggests that the original design conditions may not have been sufficient, and to plan for structural maintenance or rehabilitation and repair following an earthquake. Setting target probabilities for managing the consequences of rare events is problematic because experience bases of the sort used for benchmarking probability-based limit states design [12] are limited. Moreover, many decision-makers who control investment resources for risk management are not expert in seismic risk assessment and find small probabilities difficult to grasp; their needs (and the context of the decision process) must be taken into account. The integrated approach summarized in this paper provides project stakeholders with a structured framework for thinking about uncertainty and how a building's safety and serviceability may be threatened by its failure to perform under a spectrum of seismic events. The benefits of such an approach are an improved ability to assess the effectiveness of various risk mitigation strategies in terms of risk reduction per dollar invested, and better allocations of resources for managing risk.

Acknowledgements

The research reported herein was conducted under the sponsorship of the Mid-America Earthquake Center, with additional support from the Georgia Institute of Technology. This support is gratefully acknowledged. The MAE Center is a National Science Foundation Engineering Research Center funded at the University of Illinois at Urbana-Champaign by Award No. EEC-9701785.

References

- [1] Abrahamson NA, Bommer JJ. Probability and uncertainty in seismic hazard. *Earthquake Spectra* 2005;21(2):603–7.
- [2] ASCE – ASCE/SEI Standard 43-05. Seismic design criteria for structures, systems and components in nuclear facilities. Reston (VA): American Society of Civil Engineers; 2005.
- [3] ASCE – ASCE 7-05 Standard. Minimum design loads for buildings and other structures. American Society of Civil Engineers; 2005.
- [4] Azizinamini A, Bradburn JH, Radzinski JB. Static and cyclic behavior of semi-rigid steel beam-column connections. Department of Civil Engineering, Columbia (SC): University of South Carolina; 1985.
- [5] Barakat M, Chen WF. Design analysis of semi-rigid frames: evaluation and implementation. *Eng J AISC* 1991;Second Quarter:55–64.
- [6] BSSC. NEHRP Recommended provisions for seismic regulations for new buildings and other structures. Federal Emergency Management Agency Report 450, Washington (DC): Building Seismic Safety Council; 2003.
- [7] Cornell CA, Jalayer F, Hamburger RO, Foutch DA. Probabilistic basis for 2000 SAC FEMA steel moment frame guidelines. *J Struct Eng ASCE* 2002;128(4):526–33.
- [8] Cornell CA. Engineering seismic risk analysis. *Bull Seism Soc Am* 1968;58:1583–606.
- [9] Corotis RB. Risk and uncertainty. In: Der Kiureghian Madanat, Pestana, editors. Proceedings of the seventh international conference on applications of statistics and probability (ICASP7). Millpress; 2005.
- [10] DOE – DOE STD 1020-2002. Natural phenomena hazards design and evaluation criteria for Department of Energy facilities. Washington (DC): US Department of Energy; 2002.
- [11] Ellingwood B. Validation studies of seismic PRAs. *Nucl Eng Des* 1990;123(2):189–96.
- [12] Ellingwood BR. Probability-based codified design: past accomplishments and future challenges. *Struct Safety* 1994;13(3):159–76.
- [13] Ellingwood BR. Earthquake risk for building structures. *Reliab Eng Syst Safety* 2001;74(3):251–62.
- [14] Ellingwood BR, Wen YK. Risk-benefit based design decisions for low probability/high consequence earthquake events in Mid-America. *Prog Struct Eng Mater* 2005;7(2):56–70.
- [15] FEMA – FEMA 356. NEHRP guidelines for the seismic rehabilitation of buildings. Washington (DC): Federal Emergency Management Agency; 2000.
- [16] FEMA. Multi hazard loss estimation methodology HAZUS technical manual. Washington (DC): Federal Emergency Management Agency; 2003.
- [17] Forcier GP. Seismic performance of older steel frames. PhD thesis, Department of Civil and Environmental Engineering: University of Minnesota; 1994.
- [18] Galambos TV, Ravindra MK. Properties of steel for use in LRFD. *J Struct Div, ASCE* 1978;104(9):1459–68.
- [19] Kinali K, Ellingwood BR. Seismic fragility assessment of steel frames for consequence-based engineering: a case study for Memphis, TN. *Eng Struct* 2007;29(6):1115–27.
- [20] Kinali K. Seismic fragility assessment of steel frames in the central and eastern United States. PhD thesis, Department of Civil and Environmental Engineering, Atlanta (GA): Georgia Institute of Technology; 2007.
- [21] Leon RT, Kim DH. Seismic performance of older PR frames in areas of infrequent seismicity. In: Proceedings of the 13th world conference on earthquake engineering, Paper No. 2696, Vancouver (BC), Canada; 2004.
- [22] Mazzoni S, McKenna F, Fenves GL. OpenSees command language manual. Pacific Earthquake Engineering Center, Berkeley (CA): University of California; 2005.
- [23] McGuire RK, Cornell CA, Toro GR. The case for using mean seismic hazard. *Earthquake Spectra* 2005;21(3):879–86.
- [24] Porter KA, Kiremidjian AS, LeGrue JS. Assembly-based vulnerability of buildings and its use in performance evaluation. *Earthquake Spectra* 2001;17(2):291–312.
- [25] Shinozuka M, Feng NQ, Lee J, Naganuma T. Statistical analysis of fragility curves. *J Struct Eng ASCE* 2000;126(12):1224–31.
- [26] Shome N, Cornell CA, Bazzurro P, Carballo JE. Earthquakes, records, and nonlinear responses. *Earthquake Spectra* 1998;14(3):469–500.
- [27] Singhal A, Kiremidjian AS. Method for probabilistic evaluation of seismic structural damage. *J Struct Eng ASCE* 1996;122(12):1459–67.
- [28] Song J, Ellingwood BR. Seismic reliability of special moment steel frames with welded connections: I and II. *J Struct Eng ASCE* 1999;125(4):357–84.
- [29] Vamvatsikos D, Cornell CA. Incremental dynamic analysis. *Earthquake Eng Struct Dyn* 2002;31(3):491–514.
- [30] Veletsos AS, Newmark NM. Effect of inelastic behavior on the response of simple systems to earthquake motions. Proceedings of the second world conference on earthquake engineering; Japan 1960:895–912.
- [31] Wen YK, Wu CL. Uniform hazard ground motions for mid-America cities. *Earthquake Spectra* 2001;7(2):359–84.
- [32] Wen YK, Ellingwood BR. The role of fragility assessment in consequence-based engineering. *Earthquake Spectra* 2005;21(3):861–77.
- [33] Wen YK, Ellingwood BR, Veneziano D, Bracci J. Uncertainty modeling in earthquake engineering (white paper). Report No. FD-2, Mid-America Earthquake Engineering Center. Urbana (IL): University of Illinois; 2003.