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RELIABILITY BASED OPTIMIZATION OF STRUCTURES UNDER SEISMIC EXCITATION

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Abstract. *A new methodology is proposed for the Reliability-Based Optimal (RBO) design of structures subjected to seismic loading. The proposed methodology combines efficient structural optimization and reliability analysis procedures for the seismic design of steel moment resisting frames. The multi-modal response spectrum analysis procedure is employed for the seismic analysis. The deterministic constraints of the optimization problem are based on the provisions of European design codes, while additional probabilistic constraints are imposed in order to control the uncertainties on load bearing and deformation capacity of the structure. For the solution of the optimization problem the Evolution Strategies algorithm is employed, while the reliability analysis is carried out with the Monte Carlo simulation method incorporating the latin hypercube sampling technique.*

1 INTRODUCTION

Since the early seventies structural optimization has been the subject of intensive research and many different approaches, in terms of the formulation of the problem or the optimization algorithm, have been advocated. Most of the attention of the engineering community has been directed towards the optimum design of structures under static loading conditions with the assumption of linear behavior. For a large number of practical problems this assumption is not adequate and may lead to erroneous results. In particular when sizing optimization, with the objective to minimize the weight of the structure, is carried out special attention should be paid since the cross section dimensions might be reduced substantially during the optimization process. Furthermore, due to the large intensity of the inertia loads imposed during an earthquake, structures usually deform inelastically. This is why modern seismic design codes allow the structures exhibit inelastic behavior under major earthquakes.

Due to the uncertain nature of the earthquake loading, structural design is often based on the design response spectrum of the region of interest and on some simplified assumptions of the structural behavior under earthquakes. In the case of a direct consideration of the earthquake loading the optimization of structural systems requires multiple solution of the dynamic equations of motion which can be orders of magnitude more computational intensive than the case of static loading. Generally, the reliability of deterministically designed structures is in the range of 0.9 to 0.99, since the uncertainties are partially incorporated in the design process, by simply using code recommended safety factors [1]. Reliability-based optimum (RBO) structural design aims to improve the expected performance taking into consideration various probabilistic aspects. In recent years a number of publications have appeared dealing with the reliability-based optimum structural design for various structural performance and seismic hazard levels [2-5].

In the present study the reliability-based sizing optimization of multi-story steel frames under seismic loading is investigated. The objective function is the weight of the structure while the constraints are both deterministic (stress and displacement limitations imposed by the design codes) as well as probabilistic (limitation on the overall probability failure of the structure). Randomness of ground motion excitation and material properties are also taken into consideration using Monte Carlo simulation (MCS). The probability of failure of frame structures, in terms of interstorey drift limits, is determined via a multi-modal response spectrum analysis. The optimization part is solved using Evolution Strategies (ES) method, which in a number of optimization problems as those presented in this work, has been proven more robust and demonstrated a better global behavior than mathematical programming methods [6, 7].

The rest of the paper is organized as follows. In the second section the reliability analysis that has been considered in this study is outlined. In the third section the description of the proposed seismic design methodology is presented. The formulation of the optimization problem is subsequently presented, where the design variables, the objective function and the constraint functions for each analysis type considered are explained. In the fifth section the evolution strategies method considered in the current study for solving the optimization problem is described. Assumptions regarding finite element and inelastic behavior modeling are provided in the sixth section. The two test examples considered are demonstrated in the seventh section and the discussion of the conclusions in given in the final section.

2 STRUCTURAL RELIABILITY ANALYSIS

In the design of structural systems, limiting uncertainties and increasing safety is an important issue to be considered. Structural reliability, which is defined as the probability that the system meets some specified demands for a specified time period under specified environmental conditions, is used as a probabilistic measure to evaluate the reliability of structural systems. The performance function of a structural system must be determined to describe the system's behavior and to identify the relationship between the basic parameters in the system. It should be noted that in the earthquake loading environment the uncertainties related to seismic demand and structure’s capacity are strongly coupled.

In reliability analysis the main goal is to calculate the safety margins, i.e. the maximum probability of failure, within a given lifetime period of the structure. The probability of failure p_f can be determined using a time invariant reliability analysis procedure from the following relationship

$$p_f = p[R < S] = \int_{-\infty}^{\infty} F_R(t)\, f_S(t) dt = 1 - \int_{-\infty}^{\infty} F_S(t)\, f_R(t) dt \tag{1}$$

where R denotes the structure’s bearing capacity and S the external loads, i.e. the demand level. The randomness of R and S can be described by known probability density functions $f_R(t)$ and $f_S(t)$, with $F_R(t)=p[R<t]$, $F_S(t)=p[S<t]$ being the cumulative probability density functions of R and S, respectively.

Most often a limit state function is defined as G(R,S)=S-R and the probability of structural failure is given by

$$p_f = p[G(R,S) \geq 0] = \int_{G \geq 0} f_R(R)\, f_S(S) dR dS \tag{2}$$

It is practically impossible to evaluate p_f analytically for complex and/or large-scale structures, especially in the case of dynamic Reliability-Based Optimization (RBO) problems that are considered in the present study. In such cases the integral of Eq. (2) can be calculated only approximately using either simulation methods, such as the Monte Carlo Simulation (MCS), or approximation methods like the first order reliability method (FORM) and the second order reliability method (SORM), or response surface methods (RSM) [8]. Despite its high computational cost, MCS is considered as an efficient method and is commonly used for the evaluation of the probability of failure in computational mechanics, either for comparison and validation of the results from other approximate methods or as a standalone reliability analysis tool.

2.1 Monte Carlo simulation

In reliability analysis the MCS method is often employed when the analytical solution is not attainable and the failure domain can not be expressed or approximated by an analytical form. This is mainly the case in problems of complex nature with a large number of basic variables where all other reliability analysis methods are not applicable. Expressing the limit state function as $G(x)<0$, where $x=(x_1,x_2,...,x_M)$ is the vector of the random variables, Eq. (2) can be written as

$$p_f = \int_{G(x)\geq 0} f_x(x)dx \tag{3}$$

where $f_x(x)$ denotes the joint probability of failure for all random variables. Since MCS is based on the theory of large numbers (N_∞) an unbiased estimator of the probability of failure is given by

$$p_f = \frac{1}{N_\infty}\sum_{j=1}^{N_\infty} I(x_j) \tag{4}$$

in which $I(x_j)$ is an indicator for "successful" and "unsuccessful" simulations defined as

$$I(x_j) = \begin{cases} 1 & \text{if} \quad G(x_j) \geq 0 \\ 0 & \text{if} \quad G(x_j) < 0 \end{cases} \tag{5}$$

thus in every violation a successful simulation is encountered and the failure counter is increased by 1.

It is important in structural reliability using simulation methods, to efficiently and accurately evaluate the probability of failure for a given performance function. In order to estimate p_f an adequate number of N_{sim} independent random samples is produced using a specific, usually uniform, probability density function of the vector x. The value of the failure function is computed for each random sample x_j and the Monte Carlo estimation of p_f is given in terms of sample mean by

$$p_f \cong \frac{N_H}{N_{sim}} \tag{6}$$

where N_H is the number of successful simulations.

2.2 Latin hypercube sampling

The mathematical formulation of the MCS is relatively simple and the method has the ability of handling practically every possible case, regardless of its complexity. On the other hand, the computational effort involved in crude MCS is excessive, since the sample size needs to be extremely large in order to reduce the statistical error that is inherent in MCS. For this reason various sampling techniques, also called variance reduction techniques, have been developed in order to improve the computational efficiency of the method by minimizing the sample size and improving the accuracy of the prediction. Among them are the: importance sampling, adaptive sampling technique, stratified sampling, latin hypercube sampling, antithetic variate technique, conditional expectation technique. Latin Hypercube Sampling (LHS) is generally recognized as one of the most efficient size reduction techniques, as long as there is not any dependency among the random parameters. The basis of LHS is a full stratification of the sampled distribution with a random selection inside each stratum. In consequence, sample values are randomly shuffled among different variables. Apart from the standard LHS there are also improved LHS schemes, which combine LHS with descriptive sampling [9] or adaptive importance sampling [10] methods, in order to further increase the efficiency of this sampling procedure.

In the LHS method, the range of probable values for each random variable is divided into M non-overlapping segments of equal probability of occurrence. Thus, the whole parameter space, consisting of N parameters, is partitioned into M^N cells. Then the random sample generation is performed, by choosing M cells from the M^N space with respect to the density of each interval, and the cell number of each random sample is calculated. The cell number indicates the segment number that the sample belongs to with respect to each of the parameters. In LHS sampling is realized independently, whereas, matching of random samples is performed either randomly, or in a restricted manner. All necessary random samples are produced and they are accepted only if they do not agree with any previous combination of the segment numbers. The advantage of the LHS approach is that the random samples are generated from all the ranges of possible values, thus giving a more thorough insight into the tails of the probability distributions.

3. DESIGN OF STEEL FRAMES UNDER SEISMIC LOADING

Steel moment resisting frames is a popular structural system, which combines both efficiency and economy. Four alternative procedures for the design or assessment against earthquake loading may be used, namely: simplified and multi-modal response spectrum analysis, inelastic static analysis, also referred as pushover analysis, and the inelastic time history analysis. The first three of the above procedures are based on regional design response spectra. Most seismic codes and standards adopt design procedures that are based on linear analysis methods [11,12]. This is because either the computational cost to carry out a more elaborate and accurate analysis is usually excessive, or because the requirements to carry out this kind of analysis, i.e. engineering judgment and appropriate tools, are not widespread yet.

3.1 Multi-modal Response Spectrum analysis

Although a thorough discussion on methods of seismic design would be beyond the purposes of this paper it is considered useful to shortly describe the design procedure that was adopted. In the case of the multi-Modal Response Spectrum (mMRS) analysis the dynamic equation of equilibrium is written in the form

$$\overline{M}(s_i)\ddot{u}_t + \overline{C}(s_i)\dot{u}_t + \overline{K}(s_i)u_t = \overline{R}_t \quad (7)$$

where

$$\bar{M}_i = \Phi_i^T M_i \Phi_i,\ \bar{C}_i = \Phi_i^T C_i \Phi_i,\ \bar{K}_i = \Phi_i^T K_i \Phi_i \text{ and } \bar{R}_t = \Phi_i^T R_t \tag{8}$$

are the generalized values of the corresponding matrices and the loading vector, while Φ_i is an eigenmode shape matrix and $M(s_i)$, $C(s_i)$, and $K(s_i)$ are the (NxN) mass, damping and stiffness matrices of the i-th design vector s_i; R_t is the external load vector, and $u_t, \dot{u}_t, \ddot{u}_t$ are the displacement, velocity and acceleration vectors of the finite element assemblage, respectively. For simplicity the matrices $M(s_i)$, $C(s_i)$, $K(s_i)$ are denoted as M_i, C_i, K_i.

In the mMRS analysis a number of different formulas have been proposed in order to obtain reasonable estimates of the maximum response. The simplest and most popular formula is the Square Root of Sum of Squares (SRSS) of the modal responses. According to this estimate the maximum displacement is approximated by

$$u_{max} = \sqrt{u_{1,max}^2 + u_{2,max}^2 + \ldots + u_{N,max}^2} \tag{9}$$

where the subscript i that denotes the design vector is omitted for simplification purposes, $u_{j,max}$ corresponds to the maximum displacement calculated from the j-th transformed dynamic equations over the complete time period. The use of the Eq.(9) permits this type of "dynamic" analysis by knowing only the maximum coordinates $u_{j,max}$ of each mode.

The mMRS analysis method is summarized in the following steps, where the subscript i refers to the s_i design vector

1. Obtain first m<N eigenmode shape matrices $\Phi_i = [\varphi_i^1, \varphi_i^2, \ldots, \varphi_i^m]$ and their frequencies ω_i^j; The number of modes m considered is specified by the condition that the sum of modal masses must be equal to or greater than the 90% of the total participating mass of the system.
2. Calculate modal masses, according to the following equation

$$\bar{m}_i^j = \phi_i^{j^T} M_i \phi_i^j \tag{10}$$

calculate coefficients L_i^j

$$L_i^j = \phi_i^{j^T} M_i r \tag{11}$$

where r is the influence vector, which depends on the direction of the seismic excitation.

3. Calculate modal participation factors Γ_i^j using

$$\Gamma_i^j = \frac{L_i^j}{\bar{m}_i^j} \tag{12}$$

4. Calculate the effective modal mass for each design vector and for each eigenmode, by the following equation

$$m_{eff,i}^j = \frac{{L_i^j}^2}{\bar{m}_i^j} \tag{13}$$

5. Calculate spectral accelerations $R_d(T_j^i)$ for each period of the m modes considered. For this step the knowledge of the design response spectrum is necessary.
6. Finally obtain modal displacements according to the relations

$$(SD^i)_j = \frac{R_d(T_j^i)}{\omega_j^{i\,2}} = \frac{R_d(T_j^i) \cdot T_j^{i\,2}}{4\pi^2} \tag{14}$$

$$u_{j,max}^i = \Gamma_i^j \cdot \phi_i^j \cdot (SD^i)_j \tag{15}$$

Total maximum displacement is obtained by superimposing the maximum modal displacements using the SRSS rule of Eq. (9).

3.2 Load Combinations

Eurocode standards consider earthquake loading as a random action, therefore the following loading combination is required to be used for structural design

$$S_d = \sum_j G_{kj} "+" E_d "+" \sum_i \psi_{2i} Q_{ki} \tag{16}$$

where “+” implies “to be combined with”, the sum sign implies “the combined effect of”, G_{kj} denotes the characteristic value of the permanent action j, E_d is the design value of the seismic action, Q_{ki} refers to the characteristic value of the variable action i, and ψ_{2i} is the combination coefficient for quasi permanent value of the variable action i, here taken as 0.30. Design code checks are implemented in the optimization algorithm as constraints. Each structural member should be checked for actions that correspond to the most severe load combination obtained from Eq. (16) and the load combination of the static actions

$$S_d = 1.35 \sum_j G_{kj} "+" 1.50 \sum_i Q_{ki} \tag{17}$$

It should be pointed out that the seismic action is obtained from the design spectrum which is derived from the elastic spectrum reduced by a behaviour factor q. This is done because the structure is expected to absorb energy by deforming inelastically. Maximum values for the q-factor are suggested by design codes and vary according to the material and the type of the structural system.

3.3 Probabilistic definition of seismic response spectra

The inherent probabilistic nature of geometry, material properties and loading conditions involved in structural analysis is an important factor that influences structural safety. Reliability analysis leads to safety measures that a design engineer has to take into account due to the aforementioned uncertainties. Modern conceptual approaches of seismic structural design follow the so-called Performance-Based Earthquake Engineering (PBEE) concepts

[13-15]. The most important ingredient of PBEE is structural reliability: a straightforward consideration of all uncertainties and variabilities that arise in structural design, construction and serviceability in order to calculate the level of confidence about the structure's ability to meet the desired performance targets.

	Earthquake name (Date)	Site \ Soil Conditions	Orientation	M_S	PGA(g)	PGV(cm/sec)	a/v(sec)
1	Victoria Mexico (06.09.80)	Cerro Prieto \ Alluvium	45	6.4	0.62	31.57	19.30
2	Kobe (16.01.95)	Kobe \ Rock	0	6.95	0.82	81.30	9.91
3	Imperial Valley (19.05.40)	El Centro Array \ CWB: D, USGS: C	180	7.2	0.31	29.80	10.32
4	Duzce (12.11.99)	Bolu \ CWB: D, USGS: C	90	7.3	0.82	62.10	12.99
5	San Fernando (09.02.1971)	Pacoima dam \ Rock	164	6.61	1.22	112.49	10.69
6	Gazli (17.05.1976)	Karakyr, CWB: A	90	7.3	0.72	71.56	9.83
7	Friuli (06.05.1976)	Bercis \ CWB: B	0	6.5	0.03	1.33	21.17
8	Aigion (17.05.90)	OTE building \ Stiff soil	90	4.64	0.20	9.76	20.00
9	Central California (25.04.54)	Hollister City Hall \ CWB: D, USGS: C	271	--	0.05	3.90	12.77
10	Alkyonides (24.02.81)	Korinthos OTE building \ Soft soil	90	6.69	0.31	22.70	13.34
11	Northridge (17.01.94)	Jensen filter Plant \ CWB: D, USGS: C	292	6.7	0.59	99.10	5.86
12	Athens (07. 09.99)	Sepolia (Metro Station) \ Unknown	0	5.6	0.24	17.89	13.32
13	Cape Mendocino (25.04.92)	Petrolia \ CWB: D, USGS: C	90	7.1	0.66	89.72	7.24
14	Erzihan, Turkey (13.03.92)	Erzikan East-East Comp \ CWB: D, USGS: S	270	6.9	0.49	64.28	7.56
15	Kalamata (13.09.86)	Kalamata, Prefecture \ Stiff soil	0	5.75	0.21	32.90	6.41
16	Iran (16.09.78)	Tabas \ CWB: S	0	7.4	0.85	121.40	6.89
17	Loma Prieta[1] (18.10.89)	Hollister Diff Array \ CWB: D	255	7.1	0.28	35.60	7.69
18	Loma Prieta[2] (18.10.89)	Coyote Lke dam \ CWB: D	285	7.1	0.48	39.70	11.95
19	Mammoth Lakes (27.05.80)	McGee Creek \ CWB: D	0	5	0.33	8.55	37.29
20	Irpinia, Italy (23.11.80)	Sturno \ Unknown	270	6.5	0.36	52.70	6.66

Ms: Surface moment

Table 1: List of natural accelerograms.

Within this probabilistic framework the seismic hazard is typically expressed in terms of occurrence of earthquakes of a certain (or greater) intensity during the lifetime of the structure, which is usually 50 years. The structural performance in PBEE is measured as the probability that the damages caused by a certain seismic hazard level are kept under a specified level. For example one PBEE goal would be to calculate the probability to have collapse prevention if the earthquake has 2% probability of exceedance in 50 years while another would be to have no loss of human life for the seismic event of a 10% probability in 50 years. Obviously, according to PBEE methodology one can obtain all the "levels of confidence" for the various combinations of structural capacity and seismic demand levels.

The most common approach for the definition of seismic loading is the use of a design code response spectrum. This is a general approach that is not difficult to implement. However, if higher precision is required the use of spectra derived from natural earthquake records is more appropriate. Since significant dispersion on the structural response due to the use of different natural records has been observed, these spectra must be scaled to the same desired earthquake intensity. The most commonly applied scaling procedure is based on the peak ground acceleration (PGA).

In this study a set of twenty natural accelerograms, listed in Table 1, is used. It can be seen that each record corresponds to different earthquake magnitudes and soil properties. The records of this set correspond to a wide range of PGA and peak acceleration over peak displacement ratio (a/v) values. The latter parameter is considered to describe the damage potential of the earthquake more reliably than PGA. The records are scaled, to the same PGA value in order to ensure compatibility between them. The response spectra for each scaled record are shown in Figure 1. It has been observed that the response spectra follow the lognormal distribution [16]. Therefore the median spectrum $\hat{R}_d$, also shown in Figure 1, and the standard deviation δ are calculated from the above set of spectra using the following expressions

$$\hat{R}_d = \exp\left[\frac{\sum_{i=1}^{n} \ln(R_{d,i}(T))}{n}\right] \tag{18}$$

$$\delta = \left[\frac{\sum_{i=1}^{n} \left(\ln(R_{d,i}(T)) - \ln(\hat{x})\right)^2}{n-1}\right]^{1/2} \tag{19}$$

where $R_{d,i}(T)$ is the response spectrum value for period equal to T of the i-th record. For a given period value, the acceleration R_d is obtained as a random variable that follows the log-normal distribution with a mean value equal to $\hat{R}_d$ and a standard deviation is equal to δ.

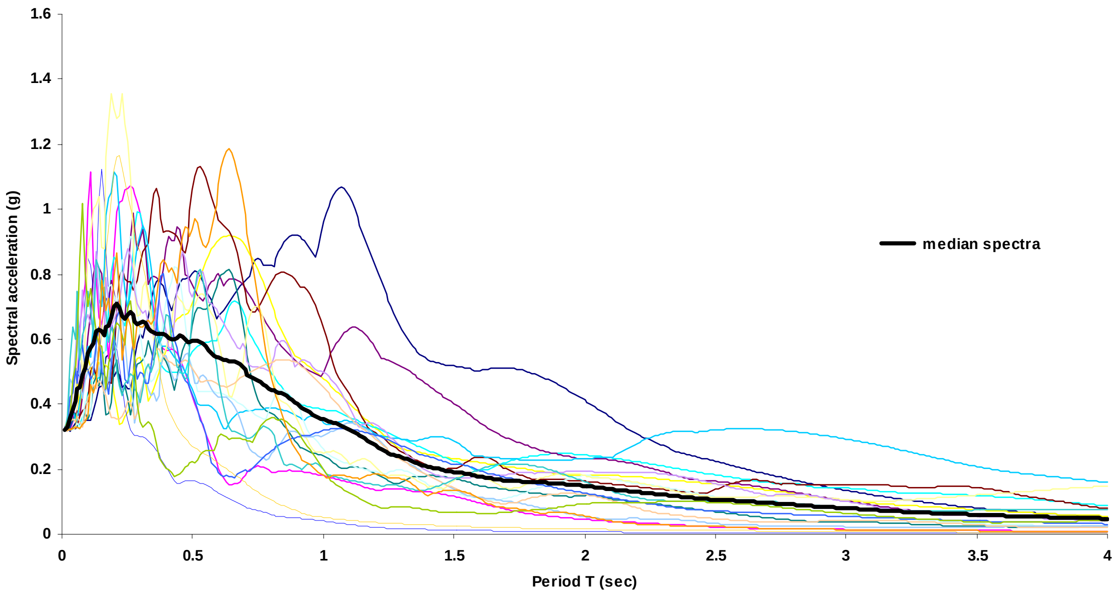

Figure 1: Natural record response spectra and their median.

4 FORMULATION OF THE RBO PROBLEM

In the present study the reliability-based sizing optimization of multi-storey framed structures under earthquake loading is investigated. In deterministic sizing optimization problems the aim is to minimize the weight of the structure under certain deterministic behavioral constraints usually imposed on stresses and displacements. In reliability-based optimal design additional probabilistic constraints are imposed in order to take into account various random parameters and to ensure that the probability of failure of the structure is within acceptable limits. Reliability design criteria are based on characteristic behavioral quantities either on displacements (strain-based limit states), or on forces (stress-based limit states). In most cases the former are considered more appropriate, since they manage to “capture” the inelastic behaviour and to quantify the structural and non-structural damages [17]. In general, maximum interstorey drift is a trustworthy mean for the structural response and is widely used in RBO methodologies. Different drift limits are adopted for the probabilistic and the deterministic constraints. For the deterministic constraints the limits suggested by Eurocode are adopted, while for the probabilistic drift limits that correspond to the actual failure of the structure are used. This is because experience has shown that structures designed in accordance with modern codes, such as EC3, will be at least 50% stronger than initially assumed. Especially for steel frame buildings without connection failures the maximum interstorey drift capacity ranges from 7% to 10% [18].

The probabilistic constraints enforce the condition that the probability of failure of the system is smaller than a certain value. In this work the overall probability of failure of the structure, obtained from successive multi-modal response spectrum analyses, is taken as the global reliability constraint. Failure is detected when the maximum interstorey drift exceeds a threshold value, here considered as 4% of the storey height. Due to engineering practice demands the members are divided into groups of the same design variables. This linking of elements results in a trade-off between the use of more material and the need of symmetry and uniformity of structures due to practical considerations. Furthermore, it has to be to taken into account that due to manufacturing limitations the design variables are not continuous but discrete since cross-sections belong to a certain set.

A discrete RBO problem can be formulated as follows

$$\begin{aligned} &\min && F(s) \\ &\text{subject to} && g_j(s) \le 0 \quad j = 1,...,m \\ &&& s_i \in R^d, \quad i = 1,...,n \\ &&& p_f(d_r > d_{al}) \le p_a \end{aligned} \qquad (20)$$

where F(**s**) is the objective function, s is the vector of design variables, which can take values only from the given discrete set R^d, $g_j(s)$ are the deterministic constraints and p_f is the probability of failure of the structure i.e. the probability that the inter-storey drift d_r exceeds the threshold value d_{al} for the ultimate limit state. Usually the deterministic constraints of the structure are member stresses, nodal displacements or inter-storey drifts. For rigid frames with W-shape cross sections as in this study, the design constraints were taken from the design requirements of Eurocode 3 (EC3) [19] and Eurocode 8 (EC8) [11].

The proposed reliability-based sizing optimization methodology proceeds with the following steps

1. At the outset of the optimization procedure the geometry, the boundaries and the loads of the structure under investigation are defined.
2. The constraints are defined in order to formulate the optimization problem of Eq. (20).
3. The optimization phase is carried out using Evolution Strategies (ES) where feasible designs should be generated at each generation. The feasibility of the designs is checked for each design vector with respect to both the deterministic and the probabilistic constraints of the problem.
4. The deterministic constraints are checked via mMRS analysis.
5. The probabilistic constraints are checked performing reliability analysis using the MCS technique to determine the probability of failure.

If the convergence criteria for the optimization algorithm are satisfied then the optimum solution has been found and the process is terminated, otherwise the whole process is resumed from step 3 with a new generation of design vectors.

4.1 Deterministic analysis procedure and constraints

In this study the widely used in practice multi-modal response spectrum analysis is employed. Before proceeding to any reliability analysis calculations the algorithm calculates the strength ratio of column to beam and also checks whether the sections selected by the optimizer are of class 1 as EC3 suggests. First check is necessary in order to have a design consistent with the strong column-weak beam philosophy, while the latter is necessary in order ensure that the members have the capacity to develop their full plastic moment and rotational ductility.

Subsequently, displacements are obtained for the gravity, live and earthquake loading cases. Earthquake loading is obtained from the design spectrum, which has been reduced by the behavior factor q. The displacements of the earthquake action are obtained according to the previously described mMRS procedure. Design forces are calculated from the envelope of the above mentioned load combinations. Design checks, which are treated by the optimizer as constraints, are carried out as soon as the design actions have been determined. Both ultimate and serviceability limit states are considered in this work. The ultimate limit state is associated with collapse or with other forms of structural failure that may endanger the safety of people. The following constraints should be satisfied for the ultimate limit state. For beams capacity design against shear requires that the following condition is fulfilled

$$\frac{V_{G.Sd} + V_{M.Sd}}{V_{Pl.Rd}} \leq 0.5 \tag{21}$$

where $V_{G.Sd}$ is the shear force due to non seismic actions and $V_{M.Sd}$ is the shear force due to the application of resisting moments with opposite signs at the extremities of the beam. Moreover the applied moment should be less than $M_{pl.Rd}$ and the axial load less than the 15% of $N_{pl.Rd}$.

For columns subjected to bending with the presence of axial load the following formulae should be satisfied

$$\frac{N_{sd}}{\chi_{min} \cdot N_{pl.Rd}} + \frac{\kappa_y \cdot M_{sd}}{M_{pl.Rd}} \leq 1 \tag{22}$$

$$V_{G.Sd} \leq 0.5 V_{Pl.Rd} \tag{23}$$

where χ_{min} is the reduction factor for buckling, taken equal to 0.7, and κ_y is a correction factor that allows for the combined effects of axial load and moment taken equal to 1. Plastic capacities for each member section are determined from the expressions

$$M_{pl,Rd} = \frac{W_{pl} \cdot f_y}{\gamma_{M0}} \tag{24}$$

$$N_{pl,Rd} = \frac{A \cdot f_y}{\gamma_{M1}} \tag{25}$$

$$V_{pl,Rd} = \frac{1.04 \cdot h \cdot t_w \cdot f_y}{\sqrt{3} \cdot \gamma_{M0}} \tag{26}$$

where γ_{M0} and γ_{M1} are safety factors considered equal to 1.10. Second order (P-Δ) effects are not considered since the following condition must be satisfied for all storeys

$$\frac{P_{tot} \cdot d_r}{V_{tot} \cdot h} \leq 0.10 \tag{27}$$

where P_{tot} is the total gravity load at the storey considered, d_r is the inter storey drift, V_{tot} is the total seismic base shear and h is the storey height.

The serviceability limit state is associated with the occurrence of damage beyond which specified service requirements are no longer met. For the serviceability limit state the interstorey drift should be limited to

$$\frac{d_r}{\nu} \leq 0.006 \cdot h \tag{28}$$

where ν is a reduction factor that defines the relationship between ultimate and serviceability limit state, here taken as $\nu = 2.5$.

5 EVOLUTION STRATEGIES

The optimization algorithm employed in this study is the Evolution Strategies (ES) method, which belongs to the class of evolutionary type optimizers. It has been specially tailored to meet the specific characteristics of the problem at hand. The ES optimization methodology is robust and independent on the type of optimization problem, the finite element formulation or the constraints of the problem. Different optimizing algorithms can also be used, however the selection of this optimization algorithm was based on the authors', as well as other researchers', experience regarding the relative computational superiority of ES over conventional optimization methods [6,7]. Evolutionary algorithms, in general, imitate natural processes maintaining a population of potential solutions. They employ selection processes based on the fitness of individuals and recombination operators.

The ES can be divided into the two-membered evolution strategy and the multi-membered evolution strategy. The two-membered scheme is the minimal concept for an imitation of the organic evolution. The multi-membered ES differ from the two-membered strategies in the size of the population and the additional genetic operator of recombination, used. The two step algorithm adopted in this study is defined as follows

Step 1 (recombination and mutation). In every optimization step called generation μ parent vectors produce λ offspring vectors. For every offspring vector a temporary parent vector $\tilde{\mathbf{s}} = [\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n]^T$ is first built by means of recombination. Mutating the temporary parent vector an offspring is being created using the following expression

$$\mathbf{s}_o^{(g)} = \tilde{\mathbf{s}} + \mathbf{z}^{(g)} \tag{29}$$

where $\mathbf{z}^{(g)}$ is a random vector, $\mathbf{s}_o^{(g)}$ is an offspring vector, of the g-th generation. In continuous optimization problems the normal distribution is used for generating the vectors $\mathbf{z}^{(g)}$.

Step 2 (selection). There are two different types of selection schemes employed by the multi-membered ES

(μ+λ)-ES: The best μ individuals are selected from the population of (μ+λ) individuals.

(μ,λ)-ES: The best μ individuals are selected from the population of λ (μ<λ) individuals.

In engineering practice the design variables are not always continuous because the structural parts are usually constructed with a certain variation of their dimensions. Modified operators have to be engaged in order to assure that the generated design variables belong to the discrete design set. For discrete problems the following recombination scheme is used in the current study

$$\tilde{s}_i = s_{a,i} \text{ or } s_{b,i} \text{ randomly} \tag{30}$$

where $\tilde{s}_i$ is the i-th component of the temporary parent vector, $s_{a,i}$ and $s_{b,i}$ are the i-th components of the vectors s_a and s_b which are two parent vectors randomly chosen from the population. In this case the variance of the random vector $\mathbf{z}^{(g)}$ is derived as follows

$$z_i = \begin{cases} (\kappa+1)\delta s_i & \text{for } \ell \text{ randomly chosen components} \\ 0 & \text{for n-}\ell \text{ other components} \end{cases} \tag{31}$$

where δs_i is the difference between two adjacent values in the discrete set and κ is a random integer number which follows the Poisson distribution

$$p(\kappa) = \frac{(\gamma)^{\kappa}}{\kappa!} e^{-\gamma} \tag{32}$$

γ is the standard deviation as well as the mean value of the random number κ. The choice of ℓ depends on the size of the problem and it is usually taken as the 1/5 of the total number of design variables. The offspring vectors are compared and the worst are rejected, while the remaining are considered to be the parent vectors of the new generation. The ES procedure is terminated as soon as the ratio μ_b/μ has reached a given value ε (in our study is selected to be 0.8) where μ_b is the number of the parent vectors in the current generation with the best value of objective function.

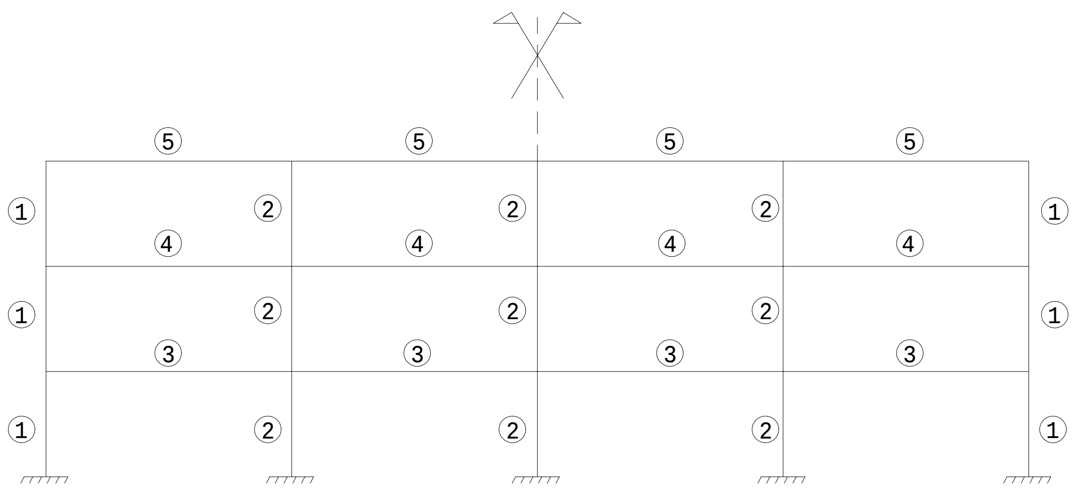

Figure 2: Test example - Geometry and member grouping.

6 TEST EXAMPLE

A typical test example has been investigated in the present study in order to illustrate the efficiency of the proposed methodology. The four-bay, three-storey moment resisting plane frame shown in Figure 2 has been analyzed [20]. The frame consists of rigid moment connections and fixed supports. Each bay has a span of 9.15m (30ft), while each storey is 3.96m (13ft) high. The permanent action G is considered equal to 5kN/m^2 while the variable action Q is equal to 2kN/m^2, both distributed along the beams. The frame is considered to be part of a 3D structure where each frame is 4.5m (15ft) apart. The median spectrum used for the determination of the base shear corresponds to a peak ground acceleration of 0.32g. Structural members are divided into five groups, as shown in Figure 2, corresponding to the five design variables of a discrete structural optimization problem. The cross-sections are W-shape beam and column sections available from manuals of the American Institute of Steel Construction. The objective function is the weight of the structure that is to be minimized.

The deterministic constraints are those discussed in section 4. The probabilistic constraint is imposed on the maximum allowable probability of structural collapse which is set equal to

p_f = 0.001. The probability of failure caused by uncertainties related to seismic loads and material properties of the structure is estimated using MCS and LHS sampling techniques. The earthquake ground motion parameter (Eqs. (18) and (19)), the yield stress and the elastic modulus are considered to be random variables. The type of probability density functions, mean values, and variances of the random parameters are shown in Table 2. The seismic action follows a log-normal probability density function, while the rest of the random variables follow a normal probability density function.

Random variable	Probability density function	Mean value	Standard deviation
E	N	2.1 10^6 MPa	0.10E
σ_y	N	235 MPa	$0.10\sigma_y$
Seismic Load	Log-N	Median Spectrum (Eq. 18)	δ (Eq. 19)

Table 2: Characteristics of the random variables.

For this test case the (μ+λ)-ES approach is used with μ=λ=5, while a sample size of 5000 and 1000 simulations is taken for MCS and LHS techniques, respectively. Table 3 depicts the performance of the optimization procedure for this test case. As it can be seen the probability of failure corresponding to the optimum computed by the deterministic optimization procedure is much larger than the specified target value (i.e. 10^{-3}) of the RBO procedure. For this example the increase on optimum weight achieved, when probabilistic constraints are considered, is approximately 26% compared to the deterministic one, as it can be observed from Table 3.

Optimization Procedure	ES Generations	p_f	Optimum volume (m^3)
Deterministic Optimization	157	0.93 10^{-1}	15.95
RBO-MCS (5000 siml.)	65	0.80 10^{-3}	21.44
RBO-LHS (1000 siml.)	72	1.00 10^{-3}	21.30

Table 3: Performance of the methods.

7 CONCLUSIONS

In most cases optimum design of structures is based on deterministic parameters and is focused on the satisfaction of the associated deterministic constraints. Since there are many random factors that affect the design, the manufacturing and the performance of a structure during its lifetime the deterministic optimum is not indeed a “safe” optimum. In order to find the “real” optimum the designer has to take into account all necessary random parameters and via the reliability analysis of the structure to determine its optimum design taking into account a desired level of probability of structural failure. Only after forming and solving this RBO problem, even with additional cost in weight and computing time, a “global” and realistic optimum structural design can be found.

The aim of the proposed RBO procedure is twofold. To increase the safety margins of the optimized structures under various uncertainties, while at the same time minimizing its weigh. The solution of realistic RBO problems in structural mechanics is an extremely computationally intensive task. As it can be observed from the numerical results the computational cost for the solution of realistic RBO problems can be order(s) of magnitude larger than the corresponding cost for the deterministic optimization. Due to the size and the complexity of RBO problems a stochastic optimization method, such as ES, appears to be the most suitable choice.

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