HW6-nc41, cq4

1. Hidden Markov Models

Q1 H Markov Models

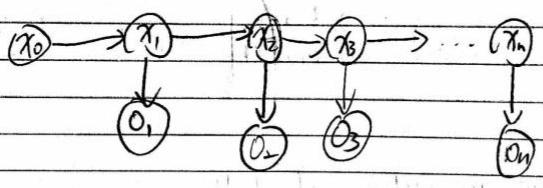
(1) $\pi = [0.5, 0.5]$

$a =$

	healthy	unhealthy
healthy	0.8	0.2
unhealthy	0.2	0.8

$b =$

	low	medium	high
healthy	0.5	0.3	0.2
unhealthy	0.3	0.3	0.4



(2) $P(X_2 = \text{healthy} \mid O_1 = \text{low}, O_2 = \text{low})$

$= \frac{P(X_2 = h, O_1 = l, O_2 = l)}{P(O_1 = l, O_2 = l)} \propto P(X_2 = h, O_1 = l, O_2 = l)$

$\Rightarrow \sum_{X_1} \sum_{X_0} P(X_0, X_1, X_2 = h, O_1 = l, O_2 = l)$

$= \sum_{X_1} \sum_{X_0} P(X_0) P(X_1 \mid X_0) P(X_2 = h \mid X_1) P(O_2 = l \mid X_2 = h) P(O_1 = l \mid X_1)$

$= 0.5 \times 0.8 \times 0.8 \times 0.5 \times 0.5$
$+ 0.5 \times 0.2 \times 0.8 \times 0.5 \times 0.5$
$+ 0.5 \times 0.2 \times 0.2 \times 0.5 \times 0.3$
$+ 0.5 \times 0.8 \times 0.2 \times 0.5 \times 0.3$

$= 0.08 + 0.02 + 0.003 + 0.012$
$= 0.115$

$$\sum_{x_1}\sum_{x_0} P(X_2=\text{unhealthy}, O_1=l, O_2=l, x_1, x_0)$$

$$= \sum_{x_1}\sum_{x_0} P(x_0)P(x_1|x_0)P(x_2=u|x_1)P(O_1=l|x_1), P(O_2=l|x_2=u)$$

$$= 0.5\times0.8\times0.2\times 0.5\times0.3$$
$$+ 0.5\times0.2\times0.2\times0.5\times0.3$$
$$+ 0.5\times0.2\times0.8\times0.3\times0.3$$
$$+ 0.5\times0.8\times0.8\times0.3\times0.3$$

$$= 0.012 + 0.003 + 0.0072 + 0.0288$$
$$= 0.051$$

$$P(X_2=\text{healthy}\,|\,O_1=l, O_2=l) = \frac{0.115}{0.115+0.051} = \frac{0.115}{0.166} = 0.692771$$ #

(3) t=2 the probability of healthy is $0.692771 > 0.5$

So the state X_2 is healthy ##

t=1 $P(X_1|X_2=h, O_1=l, O_2=l)$

$\propto P(X_1, O_1=l, O_2=l, X_2=h)$

$\Rightarrow \sum_{x_0} P(X_1=h, O_1=l, O_2=l, x_2=h, x_0)$

$= \sum_{x_0} P(x_0)P(x_1=h|x_0)P(x_2=h|x_1=h)P(O_1=l|x_1=h)P(O_2=l|x_2=h)$

$= 0.5\times0.8\times0.8\times0.5\times0.5$
$+ 0.5\times0.2\times0.8\times0.5\times0.5$
$= 0.08 + 0.02 = 0.1$

$\sum_{x_0} P(x_1 = \text{unhealthy}, o_1 = l, o_2 = l, x_2 = h, x_0)$

$= \sum_{x_0} P(x_0) P(x_1 = \text{unhealthy} \mid x_0) P(x_2 = h \mid x_1 = u) P(o_1 = l \mid x_1 = u) P(o_2 = l \mid x_2 = h)$

$= 0.5 \times 0.2 \times 0.2 \times 0.3 \times 0.5$
$+ 0.5 \times 0.8 \times 0.2 \times 0.3 \times 0.5$
$= 0.003 + 0.012 = 0.015$

$0.1 > 0.015$ so $x_1 = \text{healthy}$ ✱

∴ the x_1 is **healthy** given $o_1 = l, o_2 = l, x_2 = \text{healthy}$

$t = 0$

$P(x_0 = \text{healthy}, o_1 = l, o_2 = l, x_1, x_2 = h)$

$= \sum_{x_2} \sum_{x_1} P(x_0 = \text{healthy}) P(x_2 \mid x_1) P(x_1 \mid x_0) P(o_1 \mid x_1) P(o_2 \mid x_2)$

$= 0.5 \times 0.8 \times 0.8 \times 0.5 \times 0.5$

$= 0.08$

$P(x_0 = \text{unhealthy}) P(x_2 \mid x_1) P(x_1 \mid x_0) P(o_1 \mid x_1) P(o_2 \mid x_2)$

$= 0.5 \times 0.8 \times 0.2 \times 0.5 \times 0.5$

$= 0.02$

$0.08 > 0.02$

so $x_0 = \text{healthy}$ ✱

2. EM for mixtures of Bernoullis

Q2

(1)

z_k is missing data (unobserved data)

$$\begin{cases} P(z|\pi) = \prod_{k=1}^{K} \pi_k^{z_k} \qquad p(x|z,\mu) = \prod_{k=1}^{K} p(x|\mu_k)^{z_k} \\ P(x|\pi,\mu) = \sum_{k=1}^{K} \pi_k p(x|\mu_k) \\ p(x|\mu_k) = \prod_{i=1}^{D} \mu_{ki}^{x_i} (1-\mu_{ki})^{1-x_i} \end{cases}$$

$$\Rightarrow \ln p(X,Z|\pi,\mu) = \sum_{i=1}^{N}\sum_{k=1}^{K} z_{nk}\left[\ln\pi_k + \sum_{i=1}^{D}\left[x_{ni}\ln\mu_{ki} + (1-x_{ni})\ln(1-\mu_{ki})\right]\right]$$

$$\downarrow \text{ b/c } E_z(x) = k_z(x)$$

$$E_z[\ln p(X,Z|\pi,\mu)] = \sum_{n=1}^{N}\sum_{k=1}^{K} \gamma(z_{nk})\left[\ln\pi_k + \sum_{i=1}^{D}\left[x_{ni}\ln\mu_{ki} + (1-x_{ni})\ln(1-\mu_{ki})\right]\right]$$

$$\gamma = \frac{\pi_k p(x_n|\mu_k)}{\sum_{j=1}^{K}\pi_j p(x_n|\mu_j)}$$

$$\frac{\partial E_z[\ln p(X,Z|\pi,\mu)]}{\partial \mu_{kj}} = \sum_{n=1}^{N}\gamma(z_{nk})\left[\frac{x_n}{\mu} - (1-x_n)\frac{1}{1-\mu}\right]$$

$$= 0 \Rightarrow \frac{\sum_{n=1}^{N}\gamma(z_{nk})x_n}{\mu} = \frac{\sum_{n=1}^{N}\gamma(z_{nk}) - \sum_{n=1}^{N}\gamma(z_{nk})x_n}{1-\mu}$$

$$\sum_{n=1}^{N}\gamma(z_{nk})x_n = \mu\cdot\sum_{n=1}^{N}\gamma(z_{nk})$$

$$\Rightarrow \mu_{kj} = \frac{\sum_{n=1}^{N}\gamma(z_{nk})x_n}{\sum_{n=1}^{N}\gamma(z_{nk})}$$

and $\gamma(z_{nk}) = k_k^n$ #

(2) $p(\mu) = \text{Beta}(\mu|\alpha,\beta) = \frac{1}{B(\alpha,\beta)}\mu^{\alpha-1}(1-\mu)^{\beta-1}$

$\text{Bernoulli}(x_i|\mu) = \mu^{x_i}(1-\mu)^{1-x_i} = P(x|\mu)$

MAP: $P(\mu|x) \propto P(x|\mu)P(\mu)$

$\Rightarrow \prod_i P(x|u) \cdot \text{Beta}(\mu|\alpha,\beta)$

$\Rightarrow$ we need to calculate the $\arg\max_\mu$.

$$B_{max} = \arg\max_\mu \ln P(\mu|X) = \arg\max_\mu \sum_i \ln \text{Bernoulli}(x_i|\mu) + \ln \text{Beta}(\mu|\alpha,\beta)$$

$$= \sum_{n=1}^{N} r(z_{nk}) \left[\ln \pi_k + \sum_{i=1}^{D} \left[x_{ni} \ln \mu_{ki} + (1-x_{ni}) \ln(1-\mu_{ki})\right]\right] + \ln \text{Beta}$$

$$\frac{\partial B_{max}}{\partial \mu_{ki}} = \sum_{n=1}^{N} r(z_{nk}) \left(\frac{x_{ni}}{\mu_{ki}} - (1-x_{ni}) \cdot \frac{1}{1-\mu_{ki}}\right) + \frac{\alpha-1}{\mu_{ki}} - \frac{\beta-1}{1-\mu_{ki}} = 0$$

$$\Rightarrow \frac{\sum_{n=1}^{N} r(z_{nk}) x_{ni} + \alpha - 1}{\mu_{ki}} = \frac{\sum_{n=1}^{N} r(z_{nk}) - \sum_{n=1}^{N} r(z_{nk}) x_{ni} + \beta - 1}{1-\mu_{ki}}$$

$$\sum_{n=1}^{N} r(z_{nk}) x_{ni} + \alpha - 1 - \mu_{ki}\left(\sum_{n=1}^{N} r(z_{nk}) x_{ni} + \alpha - 1\right)$$

$$= \sum_{n=1}^{N} r(z_{nk}) \mu_{ki} - \sum_{n=1}^{N} r(z_{nk}) x_{ni} \cdot \mu_{ki} + \mu_{ki}(\beta - 1)$$

$$\mu_{ki}\left(\sum_{n=1}^{N} r(z_{nk}) + \alpha + \beta - 2\right) = \sum_{n=1}^{N} r(z_{nk}) x_{ni} + \alpha - 1$$

$$\mu_{ki} = \frac{\sum_{n=1}^{N} r(z_{nk}) x_{ni} + \alpha - 1}{\sum_{n=1}^{N} r(z_{nk}) + \alpha + \beta - 2}$$

where $r(z_{nk})_{i=n} = \gamma_k$

#

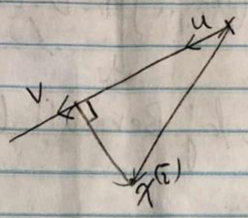
3. Principal Components Analysis

Q3.

$f_u(x) = \text{argmin}_{v \in V} \|x - v\|^2 = u^T x u$

$\text{argmin}_{u: uu^T=1} \sum_{i=1}^{m} \|x^{(i)} - f_u(x^{(i)})\|^2$

$\boxed{f_u(x) = u^T x u}$



$u^T x$ = projected length

so if we want to have v vector we need to $u^T x \cdot u = v$

$\text{argmin}_{u=uu^T} \sum_{i=1}^{m} \|x^{(i)} - f_u(x^{(i)})\|^2 = \text{argmin}_{u:uu^T=1} \|x - u^T x u\|^2$

$= x^T x - 2u^T x x^T u + (u^T x u)^T (u^T x u)$

$= \text{const} - 2u^T x x^T u$

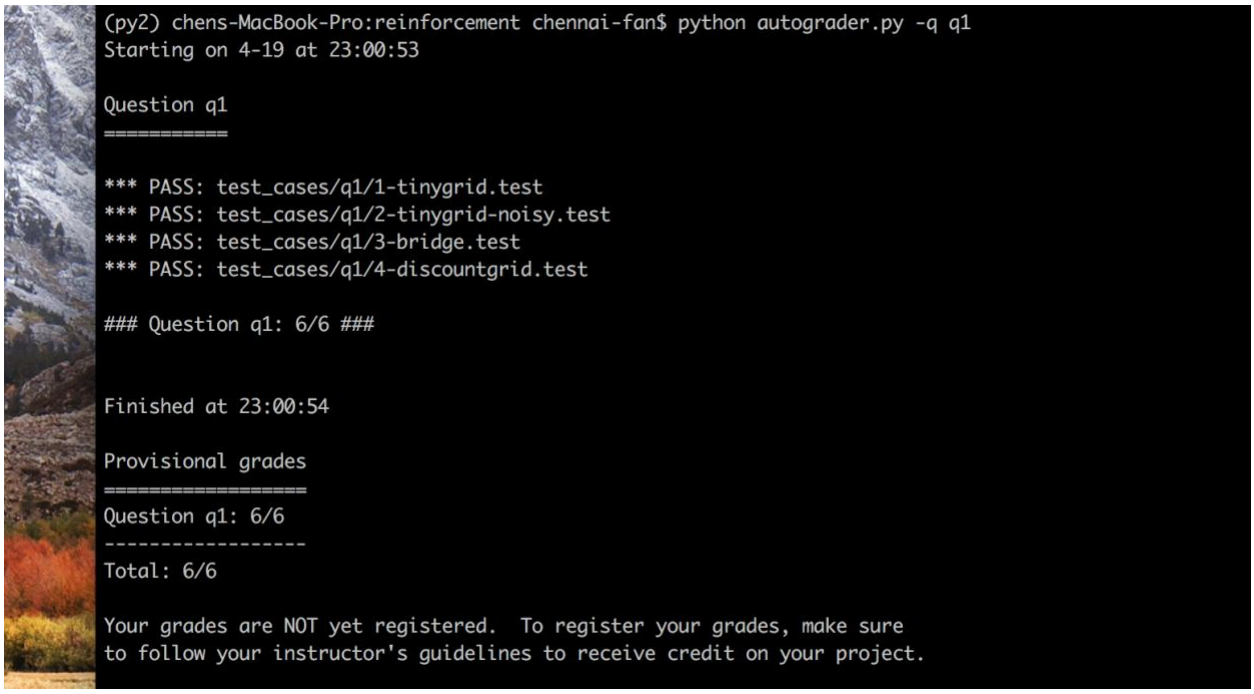
$= \text{const} - 2u^T \Sigma u$

we need to maximize covariance to minimize the distance between $x^{(i)}$ and points which is projected by $x^{(i)}$

We know the pca need to maximize the variance so we proof the reconstruction error need to need be minimize.

Problem 7 Markov Decision Processes and Grid World

Problem 7.1 Value Iteration



Problem 7.2 Bridge Crossing Analysis

Problem 7.3 Policies

```
Question q3
===========

*** PASS: test_cases/q3/1-question-3.1.test
*** PASS: test_cases/q3/2-question-3.2.test
*** PASS: test_cases/q3/3-question-3.3.test
*** PASS: test_cases/q3/4-question-3.4.test
*** PASS: test_cases/q3/5-question-3.5.test

### Question q3: 5/5 ###


Finished at 23:01:35

Provisional grades
==================
Question q3: 5/5
------------------
Total: 5/5

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

Problem 7.4: Q-learning

```
Question q4
===========

*** PASS: test_cases/q4/1-tinygrid.test
*** PASS: test_cases/q4/2-tinygrid-noisy.test
*** PASS: test_cases/q4/3-bridge.test
*** PASS: test_cases/q4/4-discountgrid.test

### Question q4: 5/5 ###


Finished at 23:02:04

Provisional grades
==================
Question q4: 5/5
------------------
Total: 5/5

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

Problem 7.5: Epsilon-greedy policies

```
Starting on 4-19 at 23:02:21

Question q5
===========

*** PASS: test_cases/q5/1-tinygrid.test
*** PASS: test_cases/q5/2-tinygrid-noisy.test
*** PASS: test_cases/q5/3-bridge.test
*** PASS: test_cases/q5/4-discountgrid.test

### Question q5: 3/3 ###


Finished at 23:02:23

Provisional grades
==================
Question q5: 3/3
------------------
Total: 3/3

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

Problem 7.6: Bridge Crossing Revisited

```
Question q6
===========

*** PASS: test_cases/q6/grade-agent.test

### Question q6: 1/1 ###


Finished at 23:02:59

Provisional grades
==================
Question q6: 1/1
------------------
Total: 1/1

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

Problem 7.7: Q-learning and Pacman

```
Win Rate:      100/100 (1.00)
Record:        Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, W
in, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Wi
n, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win
*** PASS: test_cases/q7/grade-agent.test (1 of 1 points)
***     Grading agent using command:  python pacman.py -p PacmanQAgent -x 2000 -n 2100 -l smallGrid -q -f --fixRandomSeed
***     100 wins (1 of 1 points)
***         Grading scheme:
***          < 70:  0 points
***         >= 70:  1 points

### Question q7: 1/1 ###


Finished at 23:04:03

Provisional grades
==================
Question q7: 1/1
------------------
Total: 1/1

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

Problem 7.8: Approximate Q-learning

```
Starting on 4-19 at 23:05:41

Question q8
===========

*** PASS: test_cases/q8/1-tinygrid.test
*** PASS: test_cases/q8/2-tinygrid-noisy.test
*** PASS: test_cases/q8/3-bridge.test
*** PASS: test_cases/q8/4-discountgrid.test
*** PASS: test_cases/q8/5-coord-extractor.test

### Question q8: 3/3 ###


Finished at 23:05:42

Provisional grades
==================
Question q8: 3/3
------------------
Total: 3/3

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```