$$\boxed{P(\theta|y) \propto P(y|\theta)P(\theta)}$$

$$P(y|\theta) = \boxed{\binom{n}{y}} \theta^{y}(1-\theta)^{n-y}$$

$$\downarrow$$

$$C$$

$$P(\theta|a,b) = \boxed{\frac{1}{B(a,b)}} \theta^{a-1}(1-\theta)^{b-1}$$

$$\theta \sim Beta(a,b) \qquad \downarrow$$

$$C$$

$$p(\theta|y) \propto \theta^{y}(1-\theta)^{n-y}\theta^{a-1}(1-\theta)^{b-1}$$

$$\theta^{\boxed{y+a}-1}(1-\theta)^{\boxed{n-y+b}-1}$$

$$\theta|y \sim Beta(y+a, n-y+b)$$

$$p(y|\theta) \propto \theta^{y}(1-\theta)^{n-y} \qquad \text{Binomial}(n,\theta)$$

$$p(\theta) \propto \theta^{a-1}(1-\theta)^{b-1} \qquad \text{Beta}(a,b)$$

$$p(\theta|y) \propto p(y|\theta)\,p(\theta)$$

$$\propto \theta^{\boxed{y+a}-1}(1-\theta)^{\boxed{n-y+b}-1}$$

$$\theta|y \sim \text{Beta}(\underbrace{y+a}_{a\text{-post}}, \underbrace{n-y+b}_{b\text{-post}})$$

$$E(\theta) = \frac{a}{a+b}$$

$$E(\theta|y) = \frac{y+a}{y+a+n-y+b} = \boxed{\frac{y+a}{n+a+b}}$$

$$\frac{y+a}{n+a+b} = \frac{y}{n}\boxed{\frac{n}{n+a+b}} + \frac{a}{a+b}\boxed{\frac{a+b}{n+a+b}}$$