known.

$$y_i \mid \mu, \sigma^2 \sim N(\mu, \boxed{\sigma^2})$$

$$p(y \mid \mu, \sigma^2) = \prod_{i=1}^{n} \boxed{\frac{1}{\sqrt{2\pi\sigma^2}}} \exp\left(-\frac{1}{2\sigma^2}(y_i - \mu)^2\right)$$

$$p(y \mid \mu, \sigma^2) \propto \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \mu)^2\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\left(\cancel{\sum y_i^2} - 2\underbrace{\sum y_i}_{n\bar{y}}\mu + n\mu^2\right)\right)$$

$$\propto \exp\left(\frac{n\bar{y}\mu}{\sigma^2} - \frac{n\mu^2}{2\sigma^2}\right)$$

likelihood.

$$p(\mu \mid \mu_0, \sigma_0^2) \propto \exp\left(-\frac{1}{2\sigma_0^2}(\mu - \mu_0)^2\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma_0^2}(\mu^2 - 2\mu\mu_0 + \mu_0^2)\right)$$

$$\propto \exp\left(-\frac{\mu^2}{2\sigma_0^2} + \frac{\mu\mu_0}{\sigma_0^2}\right)$$

prior.

$$p(\mu \mid y) \propto p(y \mid \mu, \sigma^2)\, p(\mu \mid \mu_0, \sigma_0^2)$$

$$\propto \exp\left(-\frac{n\mu^2}{2\sigma^2} - \frac{\mu^2}{2\sigma_0^2} + \frac{n\bar{y}\mu}{\sigma^2} + \frac{\mu\mu_0}{\sigma_0^2}\right)$$

$$\propto \exp\left(-\frac{1}{2}\mu^2\left(\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}\right) + \mu\left(\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}\right)\right)$$

Aside

$$\mu | y \sim N(\mu_p, \sigma_p^2)$$

$$p(\mu|y) \propto \exp\left(-\frac{1}{2\sigma_p^2}(\mu - \mu_p)^2\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma_p^2}(\mu^2 - 2\mu\mu_p + \mu_p^2)\right)$$

$$\propto \exp\left(-\frac{1}{2}\mu^2\boxed{\frac{1}{\sigma_p^2}} + \mu\boxed{\frac{\mu_p}{\sigma_p^2}}\right)$$

Recall

$$p(\mu|y) \propto \exp\left(-\frac{1}{2}\mu^2\left(\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}\right) + \mu\left(\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}\right)\right)$$

$$\frac{1}{\sigma_p^2} = \frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}$$

$$\boxed{\sigma_p^2 = \frac{1}{\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}}}$$

$$\frac{\mu_p}{\sigma_p^2} = \frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}$$

$$\boxed{\mu_p = \frac{\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}}{\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}}}$$